Logic, Metalogic and Neutrality*

(penultimate version)

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A natural metametaphysical hope is that logic should be able to act as a neutral arbiter of metaphysical disputes, at least as a framework on which all parties can agree for eliciting the consequences of the rival metaphysical theories. An obvious problem for this hope is the proliferation of alternative logics, many of them motivated by metaphysical considerations. For example, rejection of the law of excluded middle has been based on metaphysical conceptions of the future or of infinity. Quantum mechanics has been interpreted as showing the invalidity of one of the distributive laws. Dialetheists believe that paradoxes about sets or change manifest black holes of contradiction in reality itself. There is no core of universally accepted logical principles. Nevertheless, logic seems to have come of age as a mature science. This paper explores some aspects of the situation just sketched. In particular, it assesses the idea that metalogic is the agreed, scientific part of logic. It takes some comments by Georg Henrik von Wright on the past and future of logic as a convenient starting point.

1. In ‘Logic and Philosophy in the Twentieth Century’, first delivered as a lecture in 1991, von Wright claims that “logic has been the distinctive hallmark of philosophy in our era”.[1] But he judges it “unlikely that logic will continue to play the prominent rôle in the overall picture of an epoch’s philosophy which it has held in the [twentieth] century”.[2]

Von Wright offers two reasons for his judgment. One is that he discerns a new mood of pessimism about civilization. The suggestion is that philosophers in the twentieth-first century will at best be too preoccupied with criticizing the dark side of rational thought to have much time for logic, and at worst see it as integral to what they are criticizing. Perhaps: although Gödel and Tarski proved their most philosophically seminal results in 1930s Europe, which had its own urgent grounds for pessimism about civilization.

Von Wright's other reason is this. After an initial period of creative confusion, in which modern logic was mainly concerned with foundational questions of compelling philosophical significance, since the 1930s it has settled down into a period of normal science, in which precise questions of specialised interest are answered by agreed, rigorous methods. Exaggerated hopes are no longer pinned on foundational programmes. According to von Wright, "logic thus transformed ceases to be philosophy and becomes science". [3] One may feel that the comment presupposes an unnecessarily exclusive conception of the relation between philosophy and science, perhaps based on an over-idealized conception of science. Nevertheless, the passage from Bertrand Russell quoted by von Wright is prescient: "Mathematical logic […] is not *directly* of philosophical importance except in its beginnings. After the beginnings, it belongs rather to mathematics than to philosophy."[4] Most work in contemporary logic — as represented, for instance, in *The Journal of Symbolic Logic* — is of no more interest for philosophy than any other work in mathematics. Although mathematical rigour is quite compatible with philosophical significance, the direction of inquiry in logic is now far more likely to be set by mathematical than by philosophical concerns.[5]

Still, it is not obvious that if logic becomes less philosophical, then philosophy becomes less logical. I see no evidence that philosophers on average use logical or formal methods less than they once did. The recent growth of formal epistemology indicates the opposite. More generally, checking arguments by formalization is a standard tool in contemporary philosophy. Of course, such methods should not be applied blindly — they have limits and must be used with discretion and good judgment. But of what scientific method is that not true?

Von Wright concedes that "We can be certain that there will always remain obscure corners in logic too, thus assuring for it a permanent place among the concerns of philosophers".[6] But there is a far more systematic challenge to the philosophical uncontentiousness of logic than von Wright had in mind.

2. One aspect of the process by which logic became, in von Wright's terms, scientific rather than philosophical was that first-order logic — in its classical, non-modal form, of course — came to have the

status of ‘standard logic’. Logic textbooks teach first-order logic; they rarely teach second-order logic, which is marginalized and regarded as exotic. Yet the logical systems of Frege, Russell and Whitehead and others before 1914 were higher-order. Their first-order fragments were isolated as significant only in retrospect. The historical details of the canonization of first-order logic are contested.[7] Surely Gödel’s completeness and incompleteness theorems in 1930-1 played a vital part. They showed that while first-order logic has a sound and complete formal axiomatization, there cannot be a sound and complete formal proof system for second-order logic (as standardly interpreted). In this sense, first-order reasoning can be made purely formal — ‘scientific’, in a narrow sense — while second-order reasoning cannot. Later, Quine offered a well-known philosophical defence of the privileging of first-order logic. He treated second-order ‘logic’ as a misleading façade for set theory, the ontological commitments of the latter being more honestly displayed by its explicit axiomatization in a first-order setting. Quine also dismissed the claims to logical status of other alternatives to standard first-order logic: in particular, of extensions of classical logic such as modal logic and of non-classical logics such as intuitionistic logic.[8]

Quine’s stance has come to seem unduly restrictive. Mathematically, the specific systems to which he denied logical status are well-defined structures that can be studied in the usual way. Philosophically, it seems dogmatic and pointlessly controversial to exclude them. Some extensions to classical logic, most notably modal logic, are routinely used as background logics for philosophical discussion.[9] Many philosophers of mathematics are now convinced that the appropriate background logic for mathematical theories is second-order rather than first-order: most notably, second-order arithmetic fully captures the structure of the natural numbers because all its models are isomorphic to each other, whereas first-order arithmetic or any consistent formal extension thereof has models whose structure is not the intended one — they contain elements that cannot be reached by finitely many applications of the successor operation, starting from zero.[10] Moreover, serious arguments have been given for rejecting classical logic in favour of one or other non-classical logic — many-valued, paraconsistent, intuitionistic, … — in order to give a philosophically satisfying account of the liar paradox, the sorites paradox, metaphysical problems about infinity or the future, and so on. Even if one rejects such arguments (as I do), one cannot simply dismiss them on the grounds that no genuine alternative to classical logic is on offer. Any effective response must engage with the specifics of the proposal in question.

How can this anarchy of different systems be reconciled with the apparently scientific, unphilosophical nature of logic? The answer lies in the role of metalogic. All these systems are normally studied from within a first-order non-modal metalanguage, using classical reasoning and set theory. Scientific order is restored at the meta-level. Not only are the systems susceptible to normal methods of mathematical inquiry with respect to their syntax and proof theory, their model theory is also carried out within classical first-order set theory. Let us consider the example of propositional modal logic.

For modal logic, the decisive technical breakthrough was the development of 'possible worlds' semantics. The central definitions were those of a model and of truth in a model for propositional modal logic. Standardly, a model is any quadruple <W, @, R, V>, where W is a nonempty set, @ is a member of W, R is a binary relation on W (understood as a set of ordered pairs of members of W), and V is a function from atomic formulas to subsets of W.[11] The truth of a formula at a member *w* of W in a given model is defined recursively. An atomic formula **p** is true at *w* if and only $w \in$ V(**p**). The clauses for truth-functional operators such as negation and conjunction are the obvious quasi-homophonic ones: for any formula **A**, **¬A** is true at *w* if and only if **A** is not true at *w*; for any formulas **A** and **B**, **A & B** is true at *w* if and only if **A** is true at *w* and **B** is true at *w*. The clauses for the modal operators of possibility and necessity use existential and universal quantification respectively over W: ◊**A** is true at *w* if and only if **A** is true at some $x \in$ W such that $\langle w, x\rangle \in$ R; □**A** is true at *w* if and only if **A** is true at every $x \in$ W such that $\langle w, x\rangle \in$ R. A formula is true in a model <W, @, R, V> if and only if it is true at @ with respect to that model. A formula is valid in a class C of models if and only if it is true in every member of C.

Those definitions were given in purely mathematical terms. No modal operators were used in the metalanguage, even in the clauses for the modal operators ◊ and □ in the object language. In motivating the semantics informally, one may speak of W as a set of worlds, of @ as the actual world, and of R as a relation of relative possibility between worlds, but those ideas play no role whatsoever in the formal definitions. For instance, one can prove by purely mathematical means that the formula **¬(p & ¬□p)** is not valid in the class of all models <W, @, R, V> such that R is reflexive, symmetric and transitive on W. One can do that simply by specifying a model in which:

W = {0, 1}

@ = 0

$R = \{<0, 0>, <0, 1>, <1, 0>, <1, 1>\}$

V(**p**) = {0}.

Thus **p & ¬□p** is true, and so **¬(p & ¬□p)** not true, in the model. On what may be the intended interpretation of the object language, one is showing that truth does not imply necessity (at least, for models of that kind), but not by producing an example of a contingent truth: this model is a purely abstract mathematical structure, and the fact that the formula **p** is true at 0 in the model is not itself contingent. An eccentric metaphysician who thinks that all truths are necessary but remains mathematically orthodox must still agree that the formula **p & ¬□p** is true in the model, and simply deny that the model fits the intended interpretation of the object language. In effect, the technical study of modal logic has made such dramatic progress over the past fifty years by eliminating all modal considerations from its reasoning.

For so-called modal realists such as David Lewis, the modal is indeed reducible to the non-modal: quantification over worlds in a non-modal language gives a more perspicuous representation of the underlying metaphysical reality than does the use of modal operators. The actual world is merely one world amongst many, just as here is merely one place amongst many, privileged only from its own perspective. But most philosophers who use modal language reject modal realism as hopelessly implausible; they insist that this actual world objectively, albeit contingently, has a privileged metaphysical position. For them, the use of modal operators gives a more perspicuous representation of the underlying metaphysical reality in that respect than does quantification over worlds in a non-modal language. On their view, the formal model theory still plays a worthwhile auxiliary role in facilitating proofs that a particular modal conclusion cannot be derived from particular modal premises. Moreover, it can be argued — using modal considerations! — that for any given assignment of propositions to the atomic formulas, there is a model in which the true formulas exactly coincide with the formulas that are true under that assignment and the intended interpretations of the connectives.[12] It follows that for some class of models, the formulas valid in that class exactly coincide with the formulas that are true under *every* assignment of propositions to the atomic formulas and the intended interpretations of the connectives.[13] Once the right class has been identified, which will again require modal considerations, it can be used as a check on modal reasoning. But these applications are extrinsic to the formal model theory itself, and employ it in a purely instrumental capacity.

A similar phenomenon arises for second-order logic. Its standard model theory is given in a first-order metalanguage with set theory: the second-order variables range over all subsets of the domain over which the first-order variables range. Even such a leading advocate of second-order logic as Stewart Shapiro writes that second-order variables "range over properties, sets or relations on the domain-of-discourse or over functions from the domain to itself".[14] He thereby uses first-order quantification in English, his informal metalanguage, over properties, sets, relations or functions, of exactly the same grammatical category as one uses in saying "first-order variables range over individuals in the domain". But second-order quantification is quantification into predicate position, as opposed to first-order quantification into name position. Shapiro's metalanguage for his favoured second-order object language is first-order.

As for non-classical logics, their metatheory is also normally conducted with classical reasoning. Consider, for example, continuum-valued or fuzzy logic. It is sometimes advocated as a solution to the paradoxes of vagueness, on the grounds that a continuum of degrees of truth is needed to track the way in which a vague sentence such as "She is a child" can gradually shift from true to false through a continuous process. It may also be proposed as part of a solution to the semantic paradoxes, such as the liar. A continuum-valued model for propositional logic is any function from atomic formulas to members of the real interval [0, 1], where 1 represents perfect truth, 0 perfect falsity, and the other numbers intermediate degrees of truth. A distinctive feature of the model theory is that it computes the degrees of truth of complex formulas as a function of the degrees of truth of their constituent formulas, as a generalization of the two-valued truth-tables. Let v(**A**) be the degree of **A**. Then:

$$v(\neg\mathbf{A}) = 1 - v(\mathbf{A})$$

$$v(\mathbf{A}\ \&\ \mathbf{B}) = \text{minimum}\{v(\mathbf{A}), v(\mathbf{B})\}$$

$$v(\mathbf{A} \vee \mathbf{B}) = \text{maximum}\{v(\mathbf{A}), v(\mathbf{B})\}$$

$$v(\mathbf{A} \rightarrow \mathbf{B}) = 1 - (v(\mathbf{A}) - v(\mathbf{B})) \text{ if } v(\mathbf{A}) \geq v(\mathbf{B})$$

$$= 1 \text{ otherwise.}$$ [15]

The idea of the final clause is that the degree of truth of a conditional should be less than perfect just to the extent that there is a loss of degree of truth from its antecedent to its consequent. Count a formula *valid* if and only if it is perfectly true in every model. We can now prove mathematically that the law of excluded middle, $\mathbf{p} \vee \neg\mathbf{p}$, is invalid on this semantics. For in a model in which $v(\mathbf{p}) = 0.5$, the clauses for negation and disjunction make $v(\mathbf{p} \vee \neg\mathbf{p}) = 0.5$ too. This model-theoretic argument was given using classical logic and mathematics. It made no appeal whatsoever to vagueness, semantic paradox or any other phenomenon that was supposed to motivate the shift from two-valued to continuum-valued semantics. Nevertheless, according to proponents of this model theory, the formulas that it validates exactly coincide with those that are perfectly true on every potentially vague or semantically paradoxical interpretation of the atomic formulas.

That example is quite typical of the usual metatheory of non-classical logic. In such cases, classical reasoning in the metalanguage from non-homophonic semantic clauses leads to the conclusion that some classical principle in the object language is invalid.

A sort of tacit Quineanism seems to be operating at the meta-level. Any deviation from classical first-order non-modal logic is permitted, because it can be given a model theory in a classical first-order non-modal metalogic.[16] The maxim is: be as unorthodox as you like in your object language, provided that you are rigidly orthodox in your metalanguage. This attitude may even encourage the impression that differences in logic are merely notational, or at least somehow superficial, because we are all agreed in our metatheory. Since contemporary mathematical logic is largely metalogic, no wonder it uses agreed, scientific methods.

How stable is this combination of diversity of language and logic at the lower level with uniformity of language and logic at the meta-level? We can consider some Procrustean effects of a classical first-order non-modal metalogic as applied to something other than a standard classical first-order non-modal logic for the object language.

3. (i) Intuitionistic logic provides one of the most carefully studied examples of non-classical metalogic. By contrast with classical mathematicians who are interested in intuitionistic logic only for its formal structure, ideological intuitionists in the tradition of Brouwer and Heyting deny the validity of the law of excluded middle when infinite domains are under discussion. In the metatheory of intuitionistic logic, one is discussing an infinite domain of formulas of the language and an infinite domain of proofs. Therefore, ideological intuitionists are committed to denying the validity of the law of excluded middle for

their metatheory. Taking this point seriously, they have tried to develop an intuitionistic metatheory for intuitionistic logic.

The situation is complex, because there are many non-equivalent kinds of semantics for intuitionistic logic. However, for a natural sense of ‘interpretation’ for first-order intuitionistic logic, at least somewhat analogous to Tarski’s model-theoretic concept of an interpretation for first-order classical logic, the position is this. Consider a standard first-order language (with or without identity). Call a formula ‘intuitionistically valid’ if and only if it is true on every intuitionistic interpretation in the sense indicated. Call it ‘intuitionistically provable’ if and only if it is provable in a standard intuitionistic system of natural deduction for the language. Soundness is unproblematic: by metatheoretic reasoning that is both classically and intuitionistically acceptable, one can establish that every intuitionistically provable formula is intuitionistically valid. The problem concerns the converse, completeness. By metatheoretic reasoning that is classically but not intuitionistically acceptable, one can establish that every intuitionistically valid formula is intuitionistically provable. Moreover, by metatheoretic reasoning that is both classically and intuitionistically acceptable, one can establish that *if* every intuitionistically valid formula is intuitionistically provable *then* a certain consequence follows, a consequence that is classically valid but intuitionistically highly implausible. Thus, from the perspective of intuitionistic metatheory, the relevant completeness theorem for first-order intuitionistic logic looks false, even though it is provable in a classical metatheory.

Admittedly, there are other notions of models for first-order intuitionistic logic with respect to which its soundness and completeness can be established by reasoning that is both classically and intuitionistically acceptable. But it is doubtful that they correspond as well as do interpretations in the previous sense to the original intentions of ideological intuitionists for the meaning of the expressions of their object language. Indeed, some may even interpret the completeness theorem for the new semantics as *showing* its inadequacy to the originally intended meanings, given the incompleteness of intuitionistic logic on the old semantics, for if truth in all the new models entails intuitionistic provability while truth on all intuitionistic interpretations does not, it follows that truth on all intuitionistic interpretations does not entail truth in all the new models.[17]

(ii) Let us return to a simpler example: continuum-valued or fuzzy logic as motivated by the problem of vagueness. The obvious objection to the usual procedure of studying it in a classical metatheory is higher-order vagueness. If it is vague whether someone is a child, it is also vague what real number in the interval [0, 1] best measures the degree to which she is a child. Hence vagueness infects the metalanguage too, and if vagueness in the object language makes continuum-valued logic appropriate for the object language, then by parity of reasoning vagueness in the metalanguage should make continuum-valued logic appropriate for the metalanguage too. Hence continuum-valued logicians should not rely on the law of excluded middle and similar principles in their metatheory. To this they may reply as follows:

> One must distinguish between truth theory and model theory. A truth theory for an interpreted language should be faithful to the actual meanings of the non-logical atomic expressions, and the problem of higher-order vagueness does indeed arise. But model theory abstracts from the actual meanings of the non-logical atomic expressions. It generalizes over all assignments to them of semantic values of the appropriate type. More specifically, a model for continuum-valued propositional logic is simply any function from the atomic formulas to real numbers in the interval [0, 1]. To generalize over such functions one requires only precise mathematical and syntactic vocabulary; thus the problem of higher-order vagueness does not arise. One can legitimately use a classical metalogic in the model theory of continuum-valued logic for a vague language.[18]

The danger for this response is of making the gap between model theory and truth theory too wide. On a model-theoretic conception, logical truth is truth in all models, and logical consequence is truth-preservation in all models. But logical truths should be true, and a logical consequence of true premises should also be true. The most straightforward way of satisfying these constraints is by having one or more *intended models*, corresponding to the actual meanings of the object language expressions: a sentence is true in a given intended model if and only if it is true *simpliciter*. Since a logical truth is true in all models, in particular it is true in the intended model, and therefore true *simpliciter*; likewise for logical consequence. On a degree-theoretic conception, the degree of truth in an intended model will be equal to its actual degree of truth. But if models for continuum-valued logic are the purely mathematical structures that the response requires them to be, then a language with higher-order vagueness has no intended models. The responder might still hope that the model theory fulfils its instrumental purpose through some less direct way in which truth in all models implies truth *simpliciter* and truth-preservation in all models implies truth-preservation *simpliciter*. But even that hope is vain.

Here is an example. If one studies continuum-valued logic from within a classical metatheory, one will judge this formula valid:

(#) $$(\mathbf{p} \rightarrow \mathbf{q}) \vee (\mathbf{q} \rightarrow \mathbf{p})$$

For in any given model, either v(**p**) ≤ v(**q**), in which case v(**p** → **q**) = 1, or v(**q**) ≤ v(**p**), in which case v(**q** → **p**) = 1; either way, v(#) = 1. Now the degree theorist's original motivation for rejecting the law of excluded middle **p** ∨ ¬**p** for a vague language was that, in a borderline case, both disjuncts seem not perfectly true but only true to some intermediate degree, which on the degree theorist's conception means that the disjunction is not perfectly true, for since the degree of truth of the disjunction is the maximum of the degrees of truth of its disjuncts, the perfect truth of a disjunction requires the perfect truth of at least one disjunct. Now suppose that **p** and **q** are unrelated borderline cases, and that both display higher-order vagueness. For instance, **p** might be interpreted as "She is a child" and **q** as "This is a heap". Just as it may be utterly unclear whether she is a child and whether this is a heap, so it may also be utterly unclear how to rank the degree to which she is a child against the degree to which this is a heap, and so too for the corresponding degrees of truth. In degree-theoretic terms, both disjuncts of # seem not perfectly true but only true to some intermediate degree, which means that # is not perfectly true. Thus the original objection to the law of excluded middle generalizes to #, even though the continuum-valued semantics implies by classical reasoning that # is valid, a logical truth.

The problem with # can also be brought out by considering its special case in which **q** is ¬**p**:

(##) $$(\mathbf{p} \rightarrow \neg\mathbf{p}) \vee (\neg\mathbf{p} \rightarrow \mathbf{p})$$

The first disjunct is perfectly true if and only if the degree of truth of **p** is at most 0.5; the second disjunct is perfectly true if and only if the degree of truth of **p** is at least 0.5. If **p** is a borderline case with some considerations pulling in favour of **p** and some pulling against, it may seem neither perfectly true that the degree of truth of **p** is at most 0.5 nor perfectly true that the degree of truth of **p** is at least 0.5. Some considerations pull towards a degree of truth for ## less than 0.5, others pull towards a degree of truth greater than 0.5, and how to balance them against each other remains utterly unclear. Thus a classical metatheory for continuum-valued logic validates formulas that degree theorists must reject if they are to give a principled treatment of vagueness.

Philosophically, the obvious move for the degree theorist is to use a continuum-valued metalogic. Technically, however, this move causes grave problems. It is not only that continuum-valued logic is very weak, that proving serious metalogical results in it is likely to be very difficult, and that degree theorists have made hardly any attempt to do so. It is unclear even in principle how to work out for the first time which principles are valid in this logic, if we have to use it in the metalanguage too. For if one starts out not yet knowing the validity of any principle in this logic, by the same token one has no metalogical principles on which one can rely to deduce the validity of principles in the logic. Thus one never gets started.

One might try cutting down rather than building up: starting with classical logic as our system S_0, then defining S_1 as the system of all principles that can be validated by the continuum-valued semantics using S_0 as the metalogic, S_2 as the system of all principles that can be validated by the continuum-valued semantics using S_1 as the metalogic, and so on. The law of excluded middle is in S_0 but not in S_1; # and ## will be in S_0 and S_1 but presumably not in S_2, since the reasoning needed to validate them involves something similar to the metalogical law of excluded middle (it assumes that either $v(\mathbf{p}) \leq v(\mathbf{q})$ or $v(\mathbf{q}) \leq v(\mathbf{p})$). In general, S_{n+1} will be the logic consisting of all principles that can be validated by the continuum-valued semantics using S_n as the metalogic. This process can be iterated through the ordinals. Principles may be lost but are never gained as the subscript increases.[19] Eventually the process will reach a fixed point S_α such that $S_{\alpha+1} = S_\alpha$. This logic in the metalanguage will validate itself by the continuum-valued semantics for the object language, and so is a natural candidate to be the principled continuum-valued logic. But it is still very unclear which principles will belong to this fixed point logic. Indeed, although we know which principles are in S_1 (whose metalogic is classical), it is not even remotely clear which principles are in S_2 (whose metalogic is the non-classical S_1). The fixed point logic may well turn out to be hopelessly weak. Even the argument for the existence of the fixed point may use classical principles which do not survive the cutting-down process. Nevertheless, principled continuum-valued logic as a treatment of vagueness can only be given a fair trial if it is used as its own metalogic, however unexplored the territory into which that takes us.[20]

(iii) Similar phenomena arise for classical extensions of first-order non-modal logic. Consider, for instance, the Barcan schema in first-order modal logic:[21]

(BF) $$\Diamond \exists \mathbf{x}\, \mathbf{A(x)} \rightarrow \exists \mathbf{x} \Diamond \mathbf{A(x)}$$

Informally, it says that if there could have been something that met a certain condition, then there is something that could have met that condition. Many philosophers hold that there are actual counterexamples to BF. For instance, Queen Elizabeth I never had a child, but she could have done. By BF, it follows that there is something that could have been a child of Elizabeth I. But what is it? Given the essentiality of one's actual origins, as defended by Kripke, no actual person could have had Elizabeth I as a parent. Although some actual collection of atoms could have *constituted* a child of Elizabeth I, the collection would not have been *identical* with the child. According to those philosophers, there is actually nothing that could have been a child of Elizabeth I. Thus BF is false. Again, given the necessity of identity, BF implies that there could not have been more things than there actually are; but many philosophers regard the size of the universe as contingent.

Kripke showed how to model counterexamples to BF in possible worlds semantics. Each member w of the set W (each 'world') is associated with a set D(w), the domain of w; first-order quantification is evaluated at w as restricted to D(w). Thus $\exists \mathbf{x}\, \mathbf{A(x)}$ is true at w if and only if for some member o of D(w), $\mathbf{A(x)}$ is true at w under the assignment of o as value to the variable $\mathbf{x}$ (the values of all other variables being held fixed). Different members of w can have different domains. Informally, the domain of w is conceived as the set of individuals that exist in the world w, but this plays no role in the semantics. To construct a counter-model to BF when $\mathbf{A(x)}$ is the atomic formula $\mathbf{Fx}$, we need a model in which its antecedent is true and its consequent false at the designated member @ of W ('the actual world'). For simplicity, let us consider a model in which all pairs of members of W belong to the relation R, which validates the modal system S5; informally, every world is possible relative to every world, and necessity and possibility are not themselves contingent matters. To verify the antecedent of BF, suppose that the extension of the atomic predicate **F** at a world *w* contains the individual $o \in D(w)$. To falsify the consequent of BF, suppose that no individual $o^* \in D(@)$ is in the extension of **F** at any world, from which it follows that $o \notin D(@)$. Formally, these constraints are easily combined. For instance, let W = {0, 1}, @ = 0, *w* = 1, D(0) = {2}, D(1) = {2, 3}; let the extension of **F** at 0 be {} and the extension of **F** at 1 be {3}. Then $\Diamond\exists$ **x Fx** is true at @, because $\exists$ **x Fx** is true at 1, and $\exists$ **x**$\Diamond$ **Fx** is false at @, because $\Diamond$ **Fx** is false at @ when **x** is assigned the value 2, the only member of D(@). So far, the model theory seems to be perfectly consonant with the intuitions of those philosophers.[23]

But now try to make such a counterexample work on the *intended* interpretation of the object language. Thus W is not a pair of natural numbers but a set of possible worlds, and @ is the actual world. D(@), the domain of the actual world, is the set of everything that actually exists. A counter-model to BF requires there to be an individual o that is a member of some D(*w*) but not of D(@), and therefore an object that does not actually exist. Thus, in describing the counter-model in the non-model metalanguage in which possible worlds semantics is pursued, one is committed to saying that there is something, o, that does not actually exist. For a modal realist who thinks of actually existing as a matter of being within this particular space-time system, that result may be acceptable. But most philosophers who reject BF are not modal realists. Rather, they hold that *whatever there is* actually exists in the relevant sense. Thus, in describing the counter-model to BF, they are committed to saying this: there is something that there is not. That is a contradiction. All counter-models to BF interpret the object-language quantifier at the actual world as more restricted than the quantifiers in the metalanguage. But the metaphysically most interesting readings of BF involve no such unnecessary restriction. If one of the possible worlds models provides such an intended interpretation of the object language, then BF holds. Contrary to first appearances, the model theory provides no explanation of how BF could fail on an unrestricted reading of its quantifiers.

It does not follow that opponents of BF on its unrestricted reading should give up. Rather, the natural line for them to take is to ascribe a purely instrumental role to the possible worlds model theory. On their view, the formulas true in all such models in some class may exactly coincide with the formulas that are valid in some other sense on the intended interpretations of the modal operators, quantifiers and other logical constants, but not because the models capture the intended meanings of those expressions. Some less direct argument would need to be given for the coincidence. The intended meanings of the expressions of the modal object language would have to be captured in a modal meta-language. The primary standard for assessing principles of the modal metalogic would itself be modal. There would be no explanation in non-modal terms of the failure of BF.

A little work has indeed been done in developing semantic theories for modal object languages in modal metalanguages.[24] It is a laborious business, by comparison with possible worlds semantics in a non-modal metalanguage; even very simple results are very hard to prove. Nevertheless, such work may have to be done if we are to give a fair assessment of modal principles such as BF.

(iv) For an example involving classical first-order non-modal logic itself, consider the logic of absolutely unrestricted generality, in which the first-order quantifiers are mandatorily interpreted as ranging over everything whatsoever. In view of Russell's and Burali-Forti's paradoxes in set theory, the intelligibility and coherence of such quantification is highly controversial, but I have defended it elsewhere.[25] It can be proved that for a standard first-order language an argument is truth-preserving on all such unrestricted interpretations if and only if it is truth-preserving in every standard set-theoretic model with an infinite but set-sized domain. One can therefore give it a sound and complete axiomatization simply by adding as new axioms the usual formalizations of 'There are at least *n* things' for each natural number *n*.[26] Given the theorem of standard set theories such as Zermelo-Fraenkel set theory that there is no universal set, no set-sized model gives the quantifiers the intended unrestricted interpretation; nevertheless, larger models are not needed to give an extensionally correct characterization of validity on the unrestricted interpretation of the quantifiers.

Those results may seem to show that we can avoid this contentious sort of quantification in the metalanguage. But that is too quick. The reason is not just that one must make initial use of unrestricted quantification in the metalanguage to prove the soundness and completeness theorems for the logic of unrestricted quantification in the object language. As Harvey Friedman has shown, the proof of the completeness theorem for unrestricted quantification makes essential use of the assumption that there is a linear ordering of absolutely everything. This is a comparatively weak but still controversial consequence of the Axiom of Global Choice. If there is no linear ordering of everything, then the first-order formula that in effect says that **R** does not express a linear ordering of everything will be true on all unrestricted interpretations:

(NLO) $\neg \forall x \forall y \forall z\, (\mathbf{Rxx} \;\&\; (x \neq y \rightarrow (\mathbf{Rxy} \leftrightarrow \neg \mathbf{Ryx})) \;\&\; ((\mathbf{Rxy} \;\&\; \mathbf{Ryz}) \rightarrow \mathbf{Rxz}))$

Of course NLO is false in some infinite set-sized models, for instance where the domain is the set of natural numbers and **R** is the usual ordering of them. Thus which formulas are valid on the unrestricted interpretation of the quantifiers is sensitive to delicate issues about the structure of everything that there is.[27] Ascent to the metalanguage does not avoid such controversy.

A further feature of the example is that in order to generalize properly in the metalanguage over all unrestricted interpretations of the object language, one needs a second-order metalanguage. For if one tries to use a first-order metalanguage, all the semantic values available for the non-logical atomic predicates will also be available as values of the first-order variables, which generates a version of Russell's paradox unless the interpretation of the non-logical atomic predicates is constrained in some unintended way. This problem can be avoided in a second-order metalanguage, in which the relevant generality is achieved by second-order quantification, not by first-order quantification over the semantic values of atomic predicates but by second-order quantification. Atomic predicates are not assigned semantic values. Even the noun 'interpretation' must be replaced by an appropriate higher-order term. Any attempt to give the semantics of the second-order metalanguage in a first-order metametalanguage would reintroduce Russell's paradox or be unfaithful in some other way.[28] In a case like this, semantic ascent from object-language to metalanguage is a move in the direction of more controversy, not less.

4. To deny the reality of an apparent disagreement in logic is to be a *logical pluralist*, at least about that case. Logical pluralism takes many forms. Sometimes the apparently opposed logics are treated as mere uninterpreted formal systems, which raise no question of truth or falsity. That is one of the commonest ways of shifting all questions of truth or falsity to the metalanguage in the attempt to restore

scientific law and order to logic. However, it is also one of the least interesting forms of logical pluralism, because it ignores the fundamental role of logic in the task of getting from true premises to a true conclusion.

More interesting forms of logical pluralism treat the object language as interpreted, but postulate a difference in its interpretation between the apparently opposed sides. For example, it has often been suggested that intuitionists and classical logicians mean different things by ‘not’ and other logical connectives. This view receives at least superficial support from the radically non-classical structure of models for intuitionistic logic. Similarly, the non-classical structure of models for fuzzy logic gives at least superficial support to the view that fuzzy and classical logicians mean different things by the logical connectives. Postulating such semantic differences might be justified by appeal to a principle of charity in interpretation. A more direct line of argument seems open to pluralists who identify the meanings of the connectives with their inferential roles: if a connective has different inferential roles in two logics, it has different meanings in those logics.

Such arguments for logical pluralism do not withstand much scrutiny. Schematically, we must distinguish three candidates for the inferential role of a connective C as used by a subject S: (i) how S ought to reason with C; (ii) how S thinks S ought to reason with C; (iii) how S actually does reason with C. If S advocates a non-classical logic, that is primarily a matter of unorthodoxy with respect to (ii). But (ii) is by far the least plausible of (i)-(iii) as a candidate for the meaning of C as used by S. One’s theories even about one’s own idiolect are by no means guaranteed to be true. S may misinterpret some examples and think that nobody ought to reason with C according to a principle P, although P is in fact a perfectly valid principle to which S’s reasoning does and ought to conform. S may mean exactly the same as everyone else by C and simply have a false theory about its logical powers, just as a linguist may have a false semantic theory about the word ‘only’ in English while continuing to mean exactly the same by ‘only’ as other native speakers of English. Even if S matches practice to theory and stops reasoning with C according to P, it does not follow that S ought not to reason with C according to P. It is possible to fail quite systematically to make deductions of a valid form. Many deviant logicians make it clear that they are not stipulating idiosyncratic new meanings for the connectives but rather are proposing new theories about the old meanings of those connectives, according to which their logical powers have been mischaracterized by classical logic. Normatively, they are holding themselves responsible to the meanings of the connectives in a public language, employing the connectives with those publicly available meanings in order to engage in unequivocal public debate with defenders of orthodoxy. Since S's deviance with respect to (iii) does not imply that S uses C with a different meaning, (iii) is also a bad candidate for identification with the meaning of S as used by C. Of (i)-(iii), the best candidate for that identification is (i). But since all parties to the debate are to be held responsible to the public meaning of C, (i) is the same for all of them. S ought to reason with C according to P if and only if the defender of orthodoxy ought to reason with C according to P. Thus, on pain of implausibly individualistic consequences, inferential role semantics does not support the idea that deviant logicians use the logical connectives of their native language with deviant meanings.

A similar conclusion holds on a referentialist view of the meanings of the logical connectives, which is in any case better integrated with the most developed and adequate forms of systematic semantic theory for the language as a whole. The reference of a connective — for instance, to a truth-function — is determined at the level of the public language. All parties are employing the connectives with those publicly available meanings in order to engage in unequivocal public debate. No sound principle of charity in interpretation requires us to postulate equivocation rather than disagreement in such cases.[29]

For all that has been said, different logicians may use a given artificial symbol in a formal language with different stipulated meanings. Indeed, they sometimes actually do so. For example, the symbol '→' is sometimes used to mean material implication, sometimes to mean a stricter sort of implication. But since most logical disputes of philosophical significance can be articulated in a natural language, it is legitimate to focus on that case.

Logical disputes in natural language do not depend on equivocation with the logical connectives. That is so just as much when the natural language is functioning as a metalanguage as when it is functioning as the object-language. However, equivocation in the metalanguage is possible at another point: the interpretation of technical vocabulary such as 'valid', 'model', 'consequence', and 'proof'. Although such words are meaningful terms of ordinary language, they are frequently used in metalogical discourse with stipulated technical meanings. The argument against equivocation in words such as 'not', 'if', 'and', 'or', 'all', and 'some' does not generalize to such cases, because it depends on the absence of special stipulations. A recent form of logical pluralism targets exactly such technical metalogical vocabulary.[30]

Perhaps some confused disputes in metalogic have turned on equivocation over technical terms. However, that is not the key to any of the disputes discussed in this paper.

In many cases, the disputes already arise as disagreements about whether to accept or reject a given claim in the object-language. For example, one philosopher accepts the claim 'Either there will be a sea battle tomorrow or there won't be'; another rejects it. Although this could lead to a dispute concerning the validity of the law of excluded middle, the two issues are not equivalent. Those who reject the law of excluded middle usually still accept many particular instances of it. Even if the two philosophers differ in the range of cases they stipulate relevant to the application of the technical term 'valid', that is not why one of them accepts this particular instance of the law of excluded middle while the other rejects it. Similarly, no equivocation over technical metalogical vocabulary explains why one philosopher accepts and another rejects the dialetheist claim 'The Russell set is a member of itself and it isn't'.

In other cases, the dispute is formulated in explicitly metalogical terms with explicitly agreed meanings. For example, we saw in section 3(i) that classical logicians and intuitionists disagree as to whether every intuitionistically valid formula of first-order logic is intuitionistically provable, for some stipulated definitions of the technical terms 'intuitionistically valid' and 'intuitionistically provable', even when the propriety of those definitions is not itself at issue.

Although the structure of some of the earlier disputes is more complex than in these examples, none of them can be resolved by disambiguation of metalogical vocabulary. In every case, deeper issues are at stake.

Here is a simple schematic case. Suppose that two logics, L and L*, have been proposed for a given object-language. L and L* have extensionally distinct consequence relations. The metalanguage is an extension of the object-language. Using L in the metalanguage, and agreed assumptions, one can derive the conclusion that L is the correct logic for the object-language. Using L* in the metalanguage, and the same agreed assumptions, one can derive the conclusion that L* is the correct logic for the object-language. Since L and L* are distinct, they cannot both be *the* correct logic for the object-language. If we could find some equivocation in the phrase 'the correct logic', perhaps it would dissolve the apparent conflict. However, merely switching between reasoning by L and reasoning by L* in the metalanguage does not by itself switch the reference of metalinguistic expressions. When we look to see how the phrase 'the correct logic' was defined, we may find exactly the same definition in both derivations. As linguists know, ambiguity and reference-shifting are not phenomena to be postulated whenever one feels like it. Postulating them requires specific evidence. Nothing in the envisaged scenario provides such evidence. We may simply have to accept the obvious, and classify L and L* as genuine rivals.

None of this is to deny a variety of more innocuous claims that may be associated with the vague phrase 'logical pluralism'. Sometimes two people genuinely do mean different things by a given term of metalogic. For example, some modal logicians use 'valid' with respect to a class of models to mean true at the actual world of each model ('real world valid'), while others use it to mean true at each world in each model ('generally valid'). The rule of necessitation preserves general validity with respect to every class of models; it does not preserve real world validity with respect to every class of models. Often, when a metalogical term is used unequivocally, it does not single out a unique logic and is not intended to do so. For example, there is no sound and complete formal system of second-order logic (with respect to its standard semantics), so we may apply the word 'correct' to any of the many sound (but incomplete) formal systems of second-order logic. In that sense there is no such thing as *the* correct formal system of second-order logic.[31] But such examples are quite superficial. They provide no serious support for the idea that the appearance of genuine theoretically motivated disagreements in metalogic of the kinds illustrated in the previous section is illusory, just as similar examples in other disciplines provide no serious support for the idea that the appearance of genuine theoretically motivated disagreements in those disciplines is illusory. Instead, attempts to dissolve such appearances by claims of equivocation have turned out to depend on confused and implausible assumptions about meaning. As with so many other terms in philosophy, we may have to choose between using 'logical pluralism' to denote a boring truth and using it to denote an exciting falsehood.

At this point, some may be tempted by more evasive forms of logical pluralism, such as relativism, in some sense. But even if they could explain that sense properly, what would they hope to gain? The original attraction of logical pluralism was that it promised to preserve the scientific status of logic, by showing its results not to be threatened by what look like intractable philosophical disputes in logic itself. A more sophisticated version of the view shifted the promise from logic to metalogic. When even that promise cannot be made good, the suggestion is to defend a version of the view by interpreting metalogical discourse as some form of relativistic doubletalk. That is a reliable way not to preserve the scientific status of metalogic but to undermine it.

5. In 'Logic and Philosophy in the Twentieth Century', von Wright wrote as though most of the philosophically interesting technical work in logic had already been done. The examples above hint that much of it has only just been started. To an extent much greater than is widely realized, unorthodoxy in the object language can be fully explored and fairly assessed only through unorthodoxy in the metalanguage. Sometimes the unorthodoxy is in the deductive power of the logic, sometimes in the expressive power of the language. Both types of unorthodoxy result in contentious ways of doing metalogic, for instance through unfamiliar restrictions on deduction or unfamiliar freedoms of expression. Since the motivation for the work is primarily philosophical, and the necessary techniques often have a philosophical flavour, we cannot expect the mathematicians to do it for us. We will have to do it ourselves. One of the greatest pleasures in philosophy is to imagine one's way into a radically different pattern of thinking. To watch logical differences reassert themselves in metalogic is to experience just how radical such differences can be.

The conception of metalogic as a neutral arbiter between different logics is the last refuge of the conception of logic as a neutral arbiter between different substantive theories. If the ubiquity of alternative logics undermines the conception of logic as a neutral arbiter in the object language, their reappearance in the guise of alternative metalogics undermines the conception of logic as a neutral arbiter in the metalanguage.

The conception of logic as a neutral arbiter is influential in contemporary philosophy of logic. For example, we find David Kaplan writing of possible worlds semantics (PWS): "if PWS is to serve for intensional *logic*, we should not build […] metaphysical prejudices into it. We logicians strive to *serve* ideologies not to constrain them."[32] A similar view is presupposed in John Etchemendy's book *The Concept of Logical Consequence*. He considers the principle:

> If a universal generalization is true, *but does not make a substantive claim*, then all of its instances are logically true.

Of it, he writes "This new principle seems basically right. Indeed, it seems right because it is nothing more than a vague restatement of principle *(ii)*".[33] Principle *(ii)* says:

> If a universal generalization is logically true, then all of its instances are logically true as well.

Presumably, Etchemendy is taking "is true, but does not make a substantive claim" as a vague paraphrase of "is logically true".

Although both Kaplan and Etchemendy commit themselves to the conception of logic as not making "ideological" or "substantive" claims, neither of them provides a non-circular criterion for identifying such claims. In a footnote to his statement that logicians strive not to constrain ideologies, Kaplan adds the qualification "Except to valid argument, of course". He is surely aware that this reduces what he is saying to the proposition that logicians strive not to constrain ideologies except by logic, which does not tell us much about the bounds of logic. Similarly, Etchemendy's assumption that "non-substantive truth" is just a vague paraphrase of "logical truth" implies that "Logical truths are non-substantive truths" is just a vague paraphrase of "Logical truths are logical truths", and so tells us nothing useful. Nevertheless, Kaplan and Etchemendy make the quoted comments in the course of arguing for highly non-trivial conclusions about the bounds of logic. Kaplan is arguing that a specific formula in an intensional language should not count as a logical truth, despite its validity in possible worlds semantics. Etchemendy is arguing against Tarski's model-theoretic conception of logical consequence, and like Kaplan takes himself to be able to recognize a substantive claim when he sees one: for instance, he is confident that an existential sentence such as $\exists \mathbf{x} \exists \mathbf{y}\ \mathbf{x} \neq \mathbf{y}$ is too substantive to be a logical truth.[34] Again, in second-order logic there are two formulas, CH and NCH, such that CH is a model-theoretic logical truth if and only if Cantor's continuum hypothesis holds, while NCH is a model-theoretic logical truth if and only if the continuum hypothesis does not hold. Thus, classically, either CH is a model-theoretic logical truth or NCH is, but we do not know which, for we do not know whether the continuum hypothesis holds. Etchemendy assumes that both CH and NCH are therefore too substantive to deserve to count as logical truths, and on that basis objects to the model-theoretic conception of logical truth.[35]

Once we have seen that the contentiousness of logic is radical enough to reach metalogic, we should be suspicious of any attempt to bound logic or metalogic to the insubstantive, the non-ideological. Much though we may long for such a neutral arbiter to discipline philosophical debate, we cannot always have one.[36] Logical positivism required a clean break between logic and metaphysics, but logical positivism was wrong. Logic *is* a science, and the parts of it that overlap metaphysics are science too. Since when was science uncontentious?

Notes

* Thanks to participants in the 2008 Barcelona workshop on metametaphysics, to lecture audiences at South-western University Chongqing and St Petersburg State University, and to Matti Eklund and Peter Pagin for helpful comments on earlier versions of various parts of this material.

1 Published in Dag Prawitz, Brian Skyrms and Dag Westerståhl (editors), *Logic, Methodology and Philosophy of Science, IX (Uppsala 1991)* (Amsterdam: North-Holland, 1994), pp. 9-25, and in von Wright, *The Tree of Knowledge and Other Essays* (Leiden: E.J. Brill, 1993), pp. 7-24. The quotation in the text is from p. 16 of the latter.

2 Ibid., p. 24.

3 Ibid., p. 23.

4 *Our Knowledge of the External World, As a Field for Scientific Method in Philosophy* (London: Allen & Unwin, 1914), p. 50. The passage occurs in a chapter called ‘Logic as the Essence of Philosophy’. The title of the book indicates that Russell’s conception of the relation between science and philosophy was less exclusive than von Wright’s.

5 Of course, computer science has become almost as important as mathematics *qua* home for research in logic; sociologically, philosophy comes third.

6 Von Wright, op. cit., p. 24.

7 See Matti Eklund, ‘On How Logic Became First-Order’, *Nordic Journal of Philosophical Logic* 1 (1996), pp. 147-67, and references therein.

8 See, for example, W.V. Quine, *Philosophy of Logic* (Englewood Cliffs, N.J.: Prentice-Hall, 1970), pp. 61-94. See also Ignacio Jané, 'A Critical Appraisal of Second-Order Logic', *History and Philosophy of Logic* 14 (1993), pp. 67-86.

9 According to von Wright, "When viewing the history of modern logic as a process of 'rational disenchantment' in areas of conceptual crisis or confusion, one is entitled to the judgement that the most exciting development in logical theory after the second world war has been the rebirth of modal logic" (op. cit., p. 19).

10 See Stewart Shapiro, *Foundations without Foundationalism: A Case for Second-Order Logic* (Oxford: Clarendon Press, 1991).

11 For many purposes in modal logic it is unnecessary to designate a particular member @ of W in a model; it is designated here for expository purposes.

12 Using modal considerations, one argues that the formulas true under that assignment and the intended interpretations of the connectives constitute a maximal consistent set in K, the weakest normal modal logic. Thus there is a point in the canonical model (in a sense of model on which no point is designated the 'actual world') for K at which all and only those formulas are true (see G.E. Hughes and M.J. Cresswell, *A New Introduction to Modal Logic* (London: Routledge, 1996), pp. 112-20). That point can be taken as the actual world for the canonical model.

13 Just take the union of the classes of models associated as before with particular assignments.

14 S. Shapiro (ed.), *The Limits of Logic: Higher-Order Logic and the Löwenheim-Skolem Theorem* (Aldershot: Dartmouth, 1996), p. xi.

15 See Kenton F. Machina, 'Truth, Belief and Vagueness', *Journal of Philosophical Logic* 5 (1976), pp. 47-78.

16 Adolf Lindenbaum proved what was in effect a very general result along these lines, by showing that any given logic S for a language L is sound and complete with respect to a semantics in which the values assigned to formulas of L are equivalence classes of formulas of L under the relation of logical equivalence in S and the equivalence class of theorems is the designated value, provided that logical equivalence in S is a congruence relation with respect to the operators of L (he constructed what is now known as the Lindenbaum algebra for S). See for example, Michael Dummett, *Elements of Intuitionism* (Oxford: Clarendon Press, 2nd edition 2000), p. 122.

17 See Dummett, op. cit., pp. 154-204, for a detailed discussion and further references. The first notion of completeness discussed here is what he calls 'internal completeness'; see particularly Theorems 5.36 and 5.37. The new models are Wim Veldman and Harry de Swart's generalized Beth trees, in which the falsity constant $\perp$ can be verified at a node provided that all atomic formulas are too. D.C. McCarty gives an argument that even intuitionistic propositional logic is incomplete for infinite sets of premises in "Intuitionism in Mathematics", in Stewart Shapiro, ed., *The Oxford Handbook of Philosophy of Mathematics and Logic* (Oxford: Oxford University Press, 2007), at pp. 372-3.

18 See Hartry Field, *Saving Truth from Paradox* (Oxford: Oxford University Press, 2008), pp. 108-14, for related considerations, although Field's preferred logic is not the continuum-valued one.

19 Let f(S) be the system of all principles validated using S as the metalogic. Thus $S_{\alpha+1} = f(S_\alpha)$. If α is a limit ordinal, S_α is the system of all principles in every S_β for $\beta < \alpha$; consequently $S_{\alpha+1} \subseteq S_\alpha$. Note that if $S \subseteq T$ then $f(S) \subseteq f(T)$. Of course, $S_1 \subseteq S_0$ because any bivalent model is a special case of the continuum-valued semantics in which all formulas are assigned either 0 or 1. One then proves by induction on α that if $\beta \leq \alpha$ then $S_\alpha \subseteq S_\beta$, so the sequence is monotonically decreasing. Since the total number of principles (in any reasonable precise sense) will be bounded by the size of the language, a fixed point will eventually be

reached. The construction is reminiscent of that in Saul Kripke, ‘Outline of a Theory of Truth’, *Journal of Philosophy* 72 (1975), pp. 690-716.

20 For more discussion see T. Williamson, *Vagueness* (London: Routledge, 1994), pp. 127-31, which defends a classical alternative to continuum-valued logic.

21 The principle is named after Ruth Barcan Marcus, who discovered and was the first to formalize it, but it was already known to Ibn Sina (Avicenna, 980-1037); see Zia Movahed, ‘Ibn-Sina’s anticipation of Buridan and Barcan formulas’, in A. Enayat, I. Kalantari and M. Moniri (eds.), *Logic in Tehran: Proceedings of the Workshop and Conference on Logic, Algebra and Arithmetic, held October 18-22, 2003*, ASL Lecture Notes in Logic 26 (Natick, Mass.: A.K. Peters, 2006).

22 See S.A. Kripke, ‘Semantical Considerations on Modal Logic’, *Acta Philosophica Fennica* 16 (1963), pp. 83-94.

23 For more discussion see T. Williamson, ‘Bare Possibilia’, *Erkenntnis* 48 (1998), pp. 257-73, which defends BF and its converse.

24 See Kit Fine, ‘Prior on the Construction of Possible Worlds and Instants’, in A.N. Prior, *Worlds, Times and Selves* (London: Duckworth, 1977), reprinted in Fine, *Modality and Tense: Philosophical Papers* (Oxford: Clarendon Press, 2005); Martin Davies, ‘Weak Necessity and Truth Theories’, *Journal of Philosophical Logic* 7 (1978), pp. 415-39; Christopher Peacocke, ‘Necessity and Truth Theories’, *Journal of Philosophical Logic* 7 (1978), pp. 473-500; Anil Gupta, *The Logic of Common Nouns: An Investigation in Quantified Modal Logic* (New Haven: Yale University Press, 1980); for model theory, Lloyd Humberstone, ‘Homophony, Validity, Modality’, in J. Copeland, ed., *Logic and Reality: Essays on the Legacy of Arthur Prior* (Oxford: Clarendon Press, 1996).

25 See T. Williamson, 'Everything', in J. Hawthorne and D. Zimmerman, eds., *Philosophical Perspectives 17: Language and Philosophical Linguistics* (Oxford: Blackwell, 2003), and for more discussion, A. Rayo and G. Uzquiano, eds., *Absolute Generality* (Oxford: Clarendon Press, 2006).

26 H. Friedman, 'A Complete Theory of Everything: Validity in the Universal Domain', www.math.ohio-state.edu/~friedman/ (1999); Agustín Rayo and T. Williamson, 'A Completeness Theorem for Unrestricted First-Order Languages' and Vann McGee, 'Universal Universal Quantification: Comments on Rayo and Williamson', in J.C. Beall, ed., *Liars and Heaps: New Essays on Paradox* (Oxford: Clarendon Press, 2003).

27 Friedman, op. cit.

28 See Williamson, 'Everything', op. cit., and Rayo and Williamson, op. cit.; for earlier work on semantics for a second-order language in a second-order metalanguage see George Boolos, 'Nominalist Platonism', *Philosophical Review* 94 (1985), pp. 327-44, and A. Rayo and G. Uzquiano, 'Towards a Theory of Second-Order Consequence', *Notre Dame Journal of Formal Logic* 40 (1999), pp. 315-25.

29 For a more detailed argument along the lines of this paragraph, see T. Williamson, *The Philosophy of Philosophy* (Oxford: Blackwell, 2007), pp. 85-116. Similar considerations tell against the (independently implausible) idea that the disputing parties are using the logical connectives with the same context-sensitive meaning but in different contexts, so that their reference shifts. As in other debates, the cooperative norms of communication tend to create a unified context in relevant respects, precisely in order to avoid equivocation. Claims of non-equivocation or sameness of meaning in the text should be read as tacitly including claims of sameness of reference. A further objection to the charge of equivocation is that in many cases, including the dispute between classical and intuitionistic logic, the attempt to combine two sets of connectives, one set conforming to one logic and the other to the other, leads to the collapse of one logic into the other; see T. Williamson, 'Equivocation and Existence', *Proceedings of the Aristotelian Society* 88 (1987/88), pp. 109-27. As emphasized in J. Schechter, 'Juxtaposition: A New Way to Combine Old

Logics', *The Review of Symbolic Logic*, forthcoming, the collapse results are sensitive to differences in the way the logics are axiomatized. The collapse result still goes through for a classical logician committed to the classical natural deduction rules and an intuitionist committed to the intuitionistic natural deduction rules, provided that neither party's commitment is qualified by a tacit restriction on the vocabulary of the language.

30 See J.C. Beall and G. Restall, *Logical Pluralism* (Oxford: Oxford University Press, 2006), and, for more discussion, H. Field, 'Pluralism in Logic', *The Review of Symbolic Logic*, 2 (2009), pp. 342-59.

31 Given an interpreted higher-order language and a decision as to which of its atomic expressions are to count as non-logical on which all of the latter may be replaced by distinct quantifiable variables of the corresponding type, there is a good Tarskian notion of logical truth according to which a formula is logically true if and only if its universal closure is simply true. Nothing in the text undermines the claim that in such a setting there is a unique set of the logical truths of the language.

32 D. Kaplan, 'A Problem in Possible-World Semantic', in W. Sinnott-Armstrong, D. Raffman and N. Asher, eds., *Modality, Morality, and Belief: Essays in Honor of Ruth Barcan Marcus* (Cambridge: Cambridge University Press), at p. 42.

33 J. Etchemendy, *The Concept of Logical Consequence* (Cambridge, Mass.: Harvard University Press), at p. 143.

34 Etchemendy, op. cit., p. 111.

35 Etchemendy, op. cit., pp. 123-4, and Shapiro, *Foundations without Foundationalism*, op. cit., pp. 105-6.

36 Appeals to the concept of analyticity will not help here; see *The Philosophy of Philosophy* (op. cit.), pp. 48-133.