Chapter 3

The Principles of Contradiction, Sufficient Reason, and Identity of Indiscernibles¹

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Leibniz was a philosopher of principles: the principles of Contradiction, of Sufficient Reason, of Identity of Indiscernibles, of Plenitude, of the Best, and of Continuity are among the most famous Leibnizian principles.² In this article I shall focus on the first three principles; I shall discuss various formulations of the principles (sect. 1), what it means for these theses to have the status of principles or axioms in Leibniz's philosophy (sect. 2), the fundamental character of the Principles of Contradiction and Sufficient Reason (sect. 3), some attempts to demonstrate the Principles of Contradiction and Sufficient Reason (sect. 4), and one attempt to demonstrate the Principle of Identity of Indiscernibles (sect. 5). The main results of the chapter are summarized in a short conclusion (sect. 6).

1. Formulations of the Principles of Contradiction, Sufficient Reason, and Identity of Indiscernibles.

Leibniz gives several different formulations of the Principle of Contradiction, among them the following ones:

¹ I am grateful to Maria Rosa Antognazza, Paul Lodge, and Brandon Look for comments on a previous version of this paper.

² According to Ortega y Gasset Leibniz is both the philosopher who used the greatest number of maximally general principles and the philosopher who introduced the greatest number of new principles (1979: 12).

PC1: For any two contradictory propositions p and q, one is true and the other is false (GP VI 413).³

PC2: For any proposition *p*, *p* is either true or false (A 6 4 670/MP 93; GP VII 299/L225; GP VII 420/LC 96; A 6 6 362/NE 362).⁴

PC3: For any proposition p, p is not both true and false (GP VII 299/L 225; GP VII 355/LC 15).

PC4: For any proposition p, if p implies a contradiction, then p is false (GP VI 612/L 646; A 6 4 1445/AG 19; GP VII 199).

PC5: For any proposition p, if p is false, then not-p is true (G VI 612/L 646).

PC6: For any proposition p, if p is an identical proposition, then p is true (A 6 4 1616/MP 75).⁵

³ At A 2 1 387 Leibniz formulates the Principle of Contradiction as 'two contradictories cannot be', which I take to mean something like 'it is impossible that two contradictory propositions be true'. It is reasonable to think, given his commitment to PC2, that Leibniz took this to mean that, necessarily, of two contradictory propositions, one is true and the other false.

⁴ 'MP' abbreviates *Philosophical Writings*, edited by G. H. R. Parkinson, translated by M. Morris and G. H. R. Parkinson, London: Dent & Sons Ltd., 1973. 'LC' abbreviates *The Leibniz–Clarke Correspondence*, edited by H. G. Alexander, Manchester and New York: Manchester University Press, 1956.

An identical proposition, or identity, is an affirmative proposition in which the predicate is explicitly included in the subject, or a negative proposition in which the non-inclusion of the predicate in the subject is explicit – for instance truths of the form 'A is A', 'A is not non-A' (A 6 4 1644/AG 30–1), 'AB is B' (C 11) and, surely, 'AB is not non-B'.⁶

The six principles above are different. Some of them can be derived from others. For instance, as noted by Sleigh (1983: 196), PC6 can be derived from PC4 and PC5 plus the idea that the opposite of an identical proposition is a contradiction. I do not know, however, of any text in which Leibniz proposes such a derivation.

One might think that the fact that Leibniz referred to such a variety of principles as the Principle of Contradiction does not necessarily suggest confusion on the part of Leibniz, since in at least one text Leibniz formulates several principles having to do with truth and falsity, including PC2 and PC3, and says that all of them are usually included in one designation, 'Principle of Contradiction' (GP VII 299/L225). Thus one might think that Leibniz used the phrase 'Principle of Contradiction' as a collective noun. But note that this text would strongly support only the claim that Leibniz thought of 2 and 3 as being included in the referent of a collective noun 'Principle of Contradiction'; it would support that Leibniz saw the other versions of the principle as possible referents of such a collective noun much

⁵ PC2–PC6 appear in Sleigh 1983: 196; I have supplied references to PC2 and PC4, which are missing from Sleigh's text, and I have omitted some of the textual references given by Sleigh. Note that sometimes Leibniz conjoins PC4 and PC6 into a single principle (A 6 4 124). Mates gives a slightly different list of Principles of Contradiction for Leibniz (Mates 1986: 153).

⁶ Sometimes Leibniz gives a different characterization of identities, as propositions whose opposites contain an explicit contradiction (GP VI 612/AG 217).

less strongly. But, overall, there is little evidence for this collective noun hypothesis. The texts suggest that on different occasions he used 'Principle of Contradiction' to refer to different principles. This is puzzling, given the subtlety and power of Leibniz's mind, for it suggests that he did not really distinguish between the different versions of the principle.

A hypothesis that would save Leibniz's clarity of mind with respect to the distinction between these principles is that he thought of 'Principle of Contradiction' as a name of whatever principle played a certain function in his theory – roughly, a principle that, in his view, excluded true contradictions and served to ground mathematical and necessary truths in general. According to this hypothesis the meaning of 'Principle of Contradiction' is relatively constant throughout Leibniz's work, but in different texts he proposes different principles as playing that role. Thus calling these different principles 'Principle of Contradiction' is consistent with him having noticed that they are different. I think this hypothesis is more likely than the collective noun one, but it needs to be properly established and developed. The fact that Leibniz included so many different principles under the label 'Principle of Contradiction' will, until properly explained, remain a puzzling one.

In any case, Leibniz did see some of these principles as more fundamental than others. For instance, in the *New Essays* Leibniz says that the Principle of Contradiction, stated as PC2 above, contains two assertions, which correspond to PC1 and PC3 above (A 6 6 362/NE 362). And in his second letter to Clarke Leibniz seems to equate PC3 and PC6:

The great foundation of mathematics is the principle of contradiction, or identity, that is, that a proposition cannot be true and false at the same time; and in this way A is A, and cannot be not A (GP VII 355/LC 15).

Here Leibniz seems to equate PC3 with the proposition that *A is A*. Now, Leibniz thinks that *L* and *L is true* are coincident propositions, i.e. that they can be substituted for one another without loss of truth (A 6 4 748/LP 54).⁷ Therefore *A is A* is coincident with *A is A is true* and, in general, every identity is coincident with a proposition saying that it is true. But then, by universal generalization, one obtains PC6. However, I am not aware that Leibniz ever proposed such a derivation of PC6. Furthermore, it is not clear that PC3 and PC6 are equivalent and Leibniz does not explain why this is supposed to be so.⁸

Let us discuss the formulation of the Principle of Sufficient Reason. The essence of the Principle of Sufficient Reason is that there are no brute facts or truths, that is, there are no facts or truths for which no explanation can be given. Leibniz also gives various formulations of this principle, formulations that are not equivalent – at least not without presupposing further Leibnizian ideas. Here are three such formulations:

⁷ 'LP' translates Leibniz, *Logical Papers*, edited and translated by G. H. R. Parkinson, Oxford: Oxford University Press, 1966.

⁸ Note that I have altered Clarke's translation of Leibniz's. Where I have 'in this way', Clarke had 'therefore'. In this passage 'ainsi' does not seem to have the meaning of 'therefore'. It should also be noted that translating 'ainsi' as 'therefore' would not have been more charitable to Leibniz, since it is not clear, and Leibniz does not explain how it is supposed to be, that the truth of *A is A* follows from PC3 alone.

PSR1: Nothing occurs without a sufficient reason why it is so and not otherwise (A 2 2 65; A 6 4 1616/MP 75; A 6 4 1645/AG 31; A 6 6 179/NE 179; GP VI 127/H 147; GP VI 602/ AG 210; GP VI 612/AG 217; GP VII 356/LC 16;).

PSR2: Nothing occurs without a cause (A 6 4 1616/MP 75; A 6 4 1645/AG 31; GP V 127/H 147).

PSR3: Every truth has an *a priori* proof (A 2 2 65).⁹

Note that the scope of PSR1 is as wide as possible. 'Nothing' there ranges over facts (GP VI 612/AG 217), events (GP VII 393/LC 60) and true propositions (GP VI 612/AG 217). I am not suggesting that Leibniz made a precise distinction between facts and events – he did not, as far as I know. My point is that Leibniz wanted to apply the Principle of Sufficient Reason both to true propositions and whatever in the world corresponds to them. Leibniz also assimilates causes and reasons (A 6 6 475/NE 475), which makes PSR1 and PSR2 closer to each other than one would otherwise think.¹⁰

By an *a priori* proof Leibniz understands a proof that reduces what is to be demonstrated to an identity (A 6 4 1645/AG 31). Thus, an *a priori* proof of a proposition p shows that its predicate is implicitly contained in its subject. Since

⁹ Note that at A 6 4 1616/MP 75 Leibniz restricts PSR3 to truths that are not self-evident ('per se notae'), and he says that identities are the only self-evident truths. Thus, according to A 6 4 1616/MP 75, all truths except identities have an *a priori* proof.

¹⁰ For a discussion of this assimilation in the context of a discussion of the Principle of Sufficient Reason see Frankel 1986: 330–32. But texts like GP VI 127/H 147, quoted below, seem to distinguish between causes and reasons.

identities have their predicate explicitly included in their subjects, there are no *a priori* proofs of them. There is a further exception to PSR3, since Leibniz developed a theory of contingency according to which contingent truths are those that cannot be analysed into an identity in a finite number of steps, and for Leibniz the notion of a proof or demonstration is the notion of a finite analysis (A 6 4 1650). Taking note of this, Sleigh argues that Leibniz rejected PSR3 and replaced it by something like *Every truth has a proof sequence* (Sleigh 1983: 200–01). A proof sequence is like an *a priori* proof except that either it terminates in an identical proposition (like an *a priori* proof) or it merely converges upon one (Sleigh 1983: 200).

Even if not every proposition has a proof, according to Leibniz the predicate of every true proposition is either implicitly or explicitly contained in its subject. In this sense every truth is analytic in Leibniz's system. This has led some commentators to identify the Principle of Sufficient Reason with the claim that *every true proposition is analytic* (Couturat 1901: 214–15, Rescher 1967: 25).

One might also attempt here the hypothesis that 'Principle of Sufficient Reason' designates whatever principle plays a certain role in Leibniz's philosophy. But since there is at least one passage in which PSR1, PSR2, and PSR3 are run together as equivalent versions of the same principle, there is here more of case for thinking that Leibniz identified these principles:

... a principle of the need for giving a reason, to the effect that every true proposition which is not known *per se* has an *a priori* proof, or, that a reason

can be given for every truth, or as is commonly said, that nothing happens without a cause (A 6 4 1616/MP 75).¹¹

The Principle of Identity of Indiscernibles is also formulated in different ways. Typically, Leibniz formulates it in these ways:

PII1: No two beings are perfectly similar (GP IV 514/AG 164; GP VII 372/LC 36).

PII2: There cannot be two perfectly similar beings (A 6 4 554; A 2 2 48/AG 73; A 6 4 1645/AG 32).

These formulations differ with respect to their modal force: PII2 states that the absence of perfect similarity is necessary, while PII1 simply states that there is no perfect similarity. Sometimes Leibniz declares that there are, or can be, no perfectly similar substances, or bodies, minds, or states (A 6 4 1541/AG 41–2; GP II 264/L 534–35; A 6 6 110/NE 110; A 6 6 305/NE 305; GP II 225/L 524; GP II 249/AG 174; GP VI 608/AG 214; A 6 4 554; A 6 4 16390; A 6 6 308/NE 308; GP VII 364/LC 26), but I take these to be specific instances of the Principle of Identity of Indiscernibles rather than different formulations of it. Most commentators take the Principle of Identity of Indiscernibles to be necessary (Russell 1992: 65, Parkinson 1965: 134, Rescher 1967: 48, Adams 1979: 11–12). Although formulating it without modal

¹¹ Lois Frankel (1986) argues extensively for the claim that Leibniz thought of the Principle of Sufficient Reason as a single principle, and she cites this passage in support of her interpretation (1986: 323).

force, like PII1, does not necessarily mean Leibniz took the principle to be contingent, in some passages of the Correspondence with Clarke Leibniz appears to have taken it as contingent (GP VII 394–95/LC 61–2).¹²

It is important to note that the principle means, according to Leibniz, that things must differ intrinsically. A mere relational difference is not sufficient (C 8/MP 133; A 6 6 110/NE 110). Furthermore, for Leibniz things that differ must differ qualitatively. That is, numerically different things must differ with respect to *how* they are intrinsically, and not with respect to *which* ones they are. Thus if *a* and *b* are different but their difference is simply due to the fact that one of them is *a* and the other is *b*, *a* and *b* will differ *solo numero*, and this would be a violation of the Identity of Indiscernibles. For *a* and *b* to satisfy the Identity of Indiscernibles they must difference must be more than numerically in the sense that their difference must be grounded in a difference in their intrinsic qualities (A 6 6 57/NE 57).

Another way to see that this is the case is that for Leibniz there are no purely extrinsic denominations (A 6 4 1645/AG 32). Thus all denominations of a thing must be founded on intrinsic denominations. But the intrinsic denominations of a thing are those that can be known by inspecting the thing in itself and on its own. But, according to Leibniz, what can be known of a thing by inspecting it on its own are its qualities (GM V 180/L 255, GM VII 19/L 667). Thus all the denominations of a thing

¹² See Chernoff (1981), Cover and O'Leary-Hawthorne (1999: 204–09), and Jauernig (2008) for discussion of the modal force of the Identity of Indiscernibles. I do not mean to suggest that there are no interesting issues about the modal force of the Principles of Contradiction and Sufficient Reason. However, these issues do not come to the fore, when considering the formulations of those principles, as immediately as they do in the case of the Principle of Identity of Indiscernibles.

must be founded upon its qualities and therefore the difference between two things must be a qualitative difference.

2. The axiomatic status of the Principles of Contradiction, Sufficient Reason, and Identity of Indiscernibles.

It is common for Leibniz's scholars to speak of the *Principle* of Contradiction, the *Principle* of Sufficient Reason, and the *Principle* of Identity of Indiscernibles. Leibniz himself typically calls each of them *principles* (GP VI 127/H 147; GP VI 612/AG 217; GP VII 372/L IV, 5). He also calls them *axioms* (GP I 382; A 6 4 671/MP 94; A 6 4 1645/AG 31; GP VII 301/L 227; GP IV 514/AG 164; GP II 249/AG 175).¹³ In one passage Leibniz equates first principles and identical axioms (A 6 6 101).

A principle is normally understood in the sense of a *first* principle, that is, a proposition that is not derived or demonstrated from any other in a system of propositions. This permits to speak of *derived* principles, that is, propositions that are relatively high in the system of propositions in question, in the sense that, although they derive from some other propositions, they figure prominently in the derivation or demonstration of other propositions. But the word *axiom* is normally used with a meaning of ultimate priority that makes it contradictory to speak of derived or demonstrated axioms.

However, Leibniz had a conception of axioms on which it was possible to demonstrate and prove an axiom; indeed, in the *Critical Thoughts on the General Part of the Principles of Descartes*, he insists that demonstrating axioms and

¹³ Note that at GP I 382 Leibniz refers to the Principle of Contradiction, directly, as a *principle* but, by implication, he implicitly calls it an *axiom*.

principles is good for the perfection of science and even as a defense from scepticism about the sciences (GP IV 355/L 383–84).¹⁴

Now, this is a non-technical use of the word 'axiom', even with regard to Leibniz's use of the word. For Leibniz himself had a technical use of the word according to which an axiom is a necessary and indemonstrable proposition. And he adds that the true and indemonstrable axioms are identical propositions (C 186).¹⁵ That this is a technical use of the word 'axiom' is clear from the fact that in that very same text Leibniz speaks of a method that will not leave any axiom without proof, except definitions and identical propositions (C 187). So, as Couturat explains, Leibniz distinguished between identical and non-identical axioms, and the latter are the ones to be demonstrated (Couturat 1901: 201).

¹⁵ Compare C 186 with the following: a letter to Conring, where Leibniz says that only identical propositions are indemonstrable, and that axioms are demonstrable (A 2 1 602), *On Universal Synthesis and Analysis*, where Leibniz says that identical propositions are the true axioms and what are popularly called axioms can be reduced to identities by means of analysis (A 6 4 543/L 231–32), a letter to Bernoulli, where Leibniz says that identical propositions are indemonstrable axioms and the rest are demonstrable ones (GM II 321), the *New Essays*, where Leibniz calls *secondary* axioms those that are demonstrable and *primitive* those that are not (A 6 6 407–08/ NE 408), and the *Monadology*, where Leibniz says that there are axioms or primitive principles, which cannot and need not be proved, and these are identical propositions (GP VI 612/AG 217).

¹⁴ The idea that axioms are demonstrable is not original to Leibniz: he refers to Apollonius, Proclus, and Roberval as geometers who attempted to prove axioms (GP IV 355/L 383). Undoubtedly, the idea of indemonstrability associated with axioms was also present at Leibniz's time. Two of Leibniz's correspondents, Hermann Conring and Johann Bernoulli, expressed amazement at Leibniz's claim that axioms needed to be demonstrated (A 2 1 596; GM II 316). And Leibniz refers that Roberval was laughed at in Paris because he wanted to demonstrate Euclid's axioms (A 6 6 107). Leibniz does not identify those who mocked Roberval, but Olaso thinks Pascal might have been one of them (Olaso 1974: 164).

Thus, to the extent that Leibniz considered the Identity of Indiscernibles to be an axiom, he considered it to be a non-identical axiom, since he attempted to prove it (A 6 4 1541; A 6 4 1645; GP VII 393/LC 61). But it is interesting to note that in section 9 of the *Discourse on Metaphysics* Leibniz calls the Identity of Indiscernibles a *paradox* (A 6 4 1541). So the Identity of Indiscernibles is both an axiom and a paradox. This is paradoxical in itself! But this paradox has an easy resolution. For, as pointed out by Martinello (2006: 47), Leibniz has a conception of paradoxes as those propositions that must be proved in order to be believed (A 6 4 90).¹⁶ Thus the Identity of Indiscernibles can be both an axiom and a paradox.

3. The fundamental character of the Principles of Contradiction and Sufficient Reason.

Leibniz's fundamental principles are the Principle of Contradiction and Principle of Sufficient Reason. Consider the following passages, from *Theodicy* §44 and *Monadology* §31–32 respectively:

There are two great principles of our arguments. The one is the principle of contradiction, stating that of two contradictory propositions the one is true, the other false; the other principle is that of the determinant reason: it states that nothing ever comes to pass without there being a cause or at least a reason determining it, that is, something to give an *a priori* reason why it is existent

¹⁶ See also GP I 385, GP VII 294/L230, where Leibniz calls *paradoxical properties* those whose possibility can be doubted.

rather than non-existent, and in this wise rather than in any other (GP VI 127/H 147).

Our reasonings are based on two great principles, that of contradiction, in virtue of which we judge that which involves a contradiction to be false, and that which is opposed or contradictory to the false to be true. And that of sufficient reason, by virtue of which we consider that we can find no true or existent fact, no true assertion, without there being a sufficient reason why it is thus and not otherwise, although most of the time these reasons cannot be known to us (GP $VI \, 612/AG \, 217$).¹⁷

Although these passages do not explicitly assert it, and do not entail it either, the natural reading of them is that these two principles are equally basic and fundamental. Indeed in the Correspondence with Arnauld Leibniz says that both principles are primitive truths (A 2 2 65).¹⁸

There are passages where Leibniz says that some truths must be presupposed, otherwise there could be no demonstration, since proofs cannot proceed to infinity. And he says he will assume or presuppose the Principle of Contradiction (GP I 382). Why assume the Principle of Contradiction rather than other truths? Because, Leibniz

¹⁷ For a similar statement see A 6 4 1616/MP 75.

¹⁸ But there are texts that suggest that only one of the principles is fundamental in one way or another. Thus C 11 and A 6 4B 1379 call the Principle of Sufficient Reason the fundamental principle of reasoning, in A 6 4 543/L 232 Leibniz says that the scholastics correctly noted that all axioms (except identities) reduce to the Principle of Contradiction, in A 6 6 364/NE 364 he calls the Principle of Contradiction the only primitive principle, and in C 514/MP8 Leibniz says that the Principle of Sufficient Reason has, *after* the Principle of Contradiction, the greatest use in all sciences.

says, one presupposes the Principle of Contradiction by writing and reasoning; otherwise one would be able to defend the opposite of what one is saying (GP I 382). Leibniz thinks that in the absence of the Principle of Contradiction all belief, affirmation, and negation would be pointless (A 66 498/NE 498). In other words, every meaningful assertion presupposes the Principle of Contradiction.¹⁹ Thus this is a sort of Aristotelian transcendental justification of the Principle of Contradiction: it is justified because it is a necessary presupposition of rational discourse and thought in general.

It is interesting to note that Leibniz sometimes makes what might be seen as a similar move with respect to the Principle of Sufficient Reason. At the end of his fifth letter to Clarke Leibniz considers whether the Principle of Sufficient Reason needs to be proved, and he says that without it one cannot prove the existence of God, nor give a reason for many other important truths (GP VII 419/LC 95). Here one might see Leibniz as making the point that the Principle of Sufficient Reason is a necessary presupposition of a certain type of inquiry. But Leibniz's short remark is not sufficient to support this interesting interpretation. Furthermore, Leibniz only uses the Principle of Sufficient Reason in his cosmological argument for God's existence, not in his ontological argument.

Leibniz tends to assign different domains to these two principles. The domain of necessary truths and mathematical truths is assigned to the Principle of Contradiction, while the domain of contingent truths and truths from natural philosophy is assigned to the Principle of Sufficient Reason (A 6 4 1616/MP 75; GP

¹⁹ Cf. A 6 6 364/NE 364, where Leibniz says that the Principle of Contradiction assumes nothing.

VII 355–56/LC 15–16).²⁰ This suggests that each principle reigns over a definite and distinct area, and neither principle is prior to the other: they are like two kings each with his own territory. But the situation is more complicated. For the Principle of Sufficient Reason also applies in the domain of necessary truths (A 6 4 1650; GP VI 414; see also Broad 1975: 12, 34, and Martin 1964: 15–16),²¹ and therefore the two principles share some territory. And since not even contingent truths violate the Principle of Contradiction, the Principle of Contradiction also applies in the domain of all truths.

But not only do they share their territory: they are in each other's territory. For the Principle of Contradiction is a necessary truth, and so the Principle of Sufficient Reason has the Principle of Contradiction in its domain. Furthermore, since the Principle of Sufficient Reason too is a necessary truth (GP VI 414; see also Russell 1992: 36), the Principle of Contradiction has the Principle of Sufficient Reason in its domain. This would be a problem if the propositions in the domain in which the principles applied depended for their grounding on those principles, for in that case the Principle of Contradiction and the Principle of Sufficient Reason would circularly ground each other. But that a principle applies in a certain domain simply means that

²⁰ Sometimes the Principle of Sufficient Reason is referred to the domain of metaphysics (GP VI 602/AG 209) and, indeed, on more than one occasion Leibniz says that without the Principle of Sufficient Reason one would not be able to prove the existence of God (A 6 6 179/NE 179; GP VII 419/LC 95), which is clearly a metaphysical truth.

²¹ Note that sometimes Leibniz suggests that identities do not satisfy the Principle of Sufficient reason (GP III 506).

all the propositions in that domain satisfy it, not that they are grounded in it.²² And when Leibniz refers to what these principles ground, he typically claims that the Principle of Contradiction grounds mathematical and necessary truths and the Principle of Sufficient Reason grounds contingent truths from natural philosophy and science (A 6 4 1616/MP 75; A 6 4 1649/AG 28; GP VII 355–56/LC 15–16). Thus, since the Principle of Sufficient Reason is a necessary truth, it should be grounded, if grounded at all, on the Principle of Contradiction.

4. Some attempts to demonstrate the Principles of Contradiction and Sufficient Reason.

As I have said, Leibniz often says that these two principles are fundamental, ungrounded grounders. But sometimes he attempts to prove them – and given his conception of axioms this should not be odd. Indeed, one should expect him to attempt to prove the Principle of Sufficient Reason, since it does not seem to be an identity: its opposite, on any of its versions PSR1–3, is not an explicit contradiction.

And neither is the Principle of Contradiction, on any of its versions PC1–6, an identical proposition. This creates a problem for Leibniz, for sometimes he says that the Principle of Contradiction is indemonstrable (A 6 4 124, A 6 4 670/MP 93), and it is clear that it should be so for Leibniz. For the Principle of Contradiction, at least when understood as PC6, is presupposed in every demonstration. For a

²² This distinction seems to be what Sleigh is getting at when he points to a distinction between the domain in which a principle holds and the realm of propositions that depend on that principle (1983: 195), although Sleigh does not explain the nature of the distinction.

demonstration, according to Leibniz, is a reduction to identities, and this would not be what a demonstration is if identities were not true.

But consider, for instance, PC3, which might seem to be an identity. That principle is not an identity, however. It does not say that for any true proposition p, p is not not-true. It says that for any proposition p, p is not both true and false. But this is not an identity. It is, however, reducible to an identity via the definition of *false* as *not-true* (A 6 4 749/LP 54). But this shows that the Principle of Contradiction, if formulated as PC3, is demonstrable, since it is reducible to an identity by means of definitions. At A 2 1 387 Leibniz says that the Principle of Contradiction is an identical axiom, and he formulates it as 'two contradictories cannot be', which I interpret as a version of PC1 (see footnote 3 above). But this principle is not an identity either.

Thus it seems that the Principle of Contradiction should be demonstrable. And indeed, Leibniz attempted to give a demonstration of it, understood as PC2, from the definition of *false* as *not-true* and of *true* as *not-false* (A 6 4 749–50/LP 54–5), and PC6 can be reduced to an identity by means of the thesis that every true proposition is or is resolvable into an identity. In any case, if Leibniz had formulated PC3 as *For any proposition p, p is not both true and not-true*, then the Principle of Contradiction, understood in this way, would have been an identity, and therefore indemonstrable, according to Leibniz. But the point remains that the principles Leibniz typically refers to as the Principle of Contradiction, although necessary, are not identities, and so he should not have thought of any of them as indemonstrable.

Let us go back to the Principle of Sufficient Reason. In several texts, most notably the *Demonstratio Propositionum Primarum* (A 6 2 483), he tries to prove the Principle of Sufficient Reason on the basis of definitions of *sufficient reason* and *requisite*. A requisite is, roughly, a necessary condition, while a sufficient reason is a sufficient condition. Leibniz says that no thing can exist without all its requisites. Leibniz then asserts that given all the requisites of a thing, it must exist, and that therefore all its requisites are its sufficient reason, and so there must be a sufficient reason for everything that exists. This argument is clearly invalid, and has been thought to be question-begging (Sleigh 1983: 204, Look 2011: 204). Since I have nothing more to add about it, I shall not discuss it further.

In the *Theodicy* Leibniz claims that the Principles of Contradiction and Sufficient Reason are contained in the definition of the true and the false. What might this mean? According to Leibniz an affirmative proposition is true if and only if the predicate or consequent is in the subject or antecedent, or equivalently, if and only if the concept of the predicate or consequent is in the concept of the subject or antecedent (A 6 4 1644/AG 31, A 2 2 49; A 2 2 80; A 6 4 671/MP 93–4; GP VII 300/L 226). This theory is known as the Predicate-in-Notion, or Concept-Containment, theory of truth. As Sleigh notes, the texts suggest that the idea that the concept of the predicate is included in the concept of the subject is an analysis of the idea that the predicate is included in the subject (Sleigh 1983: 198). In any case, for present purposes, I shall formulate the theory in this way:

PIN (Predicate-in-Notion): For any proposition p, p is true if and only if the predicate is in the subject, that is, if and only if p is or is resolvable into an identity (A 6 4 1644/AG 31).²³

²³ But note that, as formulated, PIN makes no reference to any notions or concepts and so 'PIN' is a slightly misleading label. Some, like Couturat (2001: 208–18), seem to maintain that Leibniz accepted PIN because he accepted the Principle of Sufficient Reason. Sleigh disagrees, and suggests that PIN

It is easy to see how the Principle of Contradiction, understood as PC6 above, follows from PIN, for in identical propositions the predicate is explicitly included in the subject (A 6 4 1644/AG 31). Furthermore, in *Primary Truths* Leibniz argues that the Principle of Sufficient Reason follows from PIN, because, for any true proposition p, that the predicate is included in the subject is a reason why p is true. 'Otherwise', Leibniz says, 'there would be a truth (...) which could not be resolved into identities' (A 6 4 1645/AG 31). So this is the sense in which the Principle of Contradiction and the Principle of Sufficient Reason are contained in the definition of the true.²⁴

Primary Truths is a very interesting text in this connection because in it are present the Principle of Contradiction, understood as PC6 above, PIN, and the Principle of Sufficient Reason. Leibniz explicitly states PIN and the Principle of Sufficient Reason in *Primary Truths*. But it should be clear that he is implicitly committed to PC6 in this text, since he says that 'primary truths are those which assert the same thing of itself, or deny the opposite of its opposite. For example, "A is A", "A is not not-A"' (A 6 4 1644/AG 30). What Leibniz is saying here is that

provided Leibniz with some tools to formulate what he regarded as a deep analysis of the Principle of Sufficient Reason (Sleigh 1982: 234).

²⁴ And the definition of the false? Perhaps Leibniz thought that PIN was an implicit definition of falsehood, since he defined false as not-true (A 6 4 749/LP 54). In any case, it should be noted that there are commentators who oppose the primacy of PIN. Baruch Brody, for instance, argues that Leibniz's fundamental principle is the Principle of Sufficient Reason, understood as PSR1 as applied to truths, and that from this principle Leibniz adopts PIN, for if PIN is true, the demand for a sufficient reason is thereby satisfied. Thus, according to Brody, when Leibniz says that the Principle of Sufficient Reason is a corollary of PIN all he means is that the predicate's being contained in the subject is the way in which the Principle of Sufficient Reason is satisfied (Brody 2004: 94).

identities are primary truths, and therefore he is implicitly committed to the idea that identities are truths, which is what PC6 asserts.

But although in *Primary Truths* Leibniz derives the Principle of Sufficient Reason from PIN, he does not derive the Principle of Contradiction from PIN. He states something that commits him to the Principle of Contradiction, then he states PIN, and then he states the Principle of Sufficient Reason and derives it from PIN (AG 30–31). It must also be stressed that in *Primary Truths* Leibniz does not use the Principle of Contradiction to derive the Principle of Sufficient Reason. This is a point that needs stressing for, given that the Principle of Sufficient Reason is a necessary truth, one would expect it to be founded on the Principle of Contradiction, which is the principle of necessary truths. Leibniz says:

... the received axiom that *nothing is without reason* ... directly follows from these considerations; otherwise there would be a truth which could not be proved *a priori*, that is, a truth which could not be resolved into identities (A 6 4 1645/AG 31).

Thus Leibniz argues that that if the Principle of Sufficient Reason did not hold, there would be a truth that neither is an identity nor resolves into one. The implication is that the existence of a truth that neither is nor resolves into an identity would be absurd – and it would be absurd because it would violate the idea that for every p, if p is true, p is an identity or is resolvable into one. But this is only half of PIN, and it is

the half that is closer to the converse of PC6 than to PC6 itself. Thus the Principle of Contradiction is not used in the derivation of the Principle of Sufficient Reason.²⁵

What does 'these considerations' ('ex his', A 6 4 1645) refer to, then? It might be thought that we should take the use of the plural expression seriously, and that therefore there must be more than one consideration at play, and that the Principle of Contradiction, implicit at the beginning of the text, is an obvious candidate as one of the referents. But even taking the use of the plural seriously, given that he does not use the Principle of Contradiction in its derivation of the Principle of Sufficient Reason, there is a more plausible interpretation of the referents of 'these considerations'. This is how Leibniz introduces PIN in *Primary Truths*:

Therefore, the predicate or consequent is always in the subject or antecedent, and the nature of truth in general or the connection between the terms of a statement, consists in this very thing, as Aristotle also observed. The connection and inclusion of the predicate is explicit in identities, but in all other propositions is implicit and must be shown through the analysis of notions; *a priori* demonstration rests on this (A 6 4 1644/AG 31).

There are two ideas here that are used in Leibniz's derivation of the Principle of Sufficient Reason. One is that for every p, if p is true, p is an identity or is resolvable into one. The other is that such resolution into identities is what *a priori* demonstration consists in. These are the referents of 'these considerations'.

²⁵ My reading of *Primary Truths* thus differs from Look's, who suggests that Leibniz derives the Principle of Sufficient Reason from the Principle of Contradiction together with PIN (Look 2011: 205, 207–08, 208–09).

We saw first that, given the way Leibniz derives the Principle of Sufficient Reason, there is no need to make the Principle of Contradiction one of the referents of 'these considerations'. We have now seen that not doing so is consistent with the plural phrase having a plurality of referents. This is evidence that Leibniz did not intend to use the Principle of Contradiction in the derivation of the Principle of Sufficient Reason.²⁶

The other text I would like to comment on is a paragraph at the end of Leibniz's fifth letter to Clarke. Clarke had asked for a justification of (Leibniz's understanding of) the Principle of Sufficient Reason. Leibniz responds that no one has ever provided a counterexample to the Principle of Sufficient Reason, and there are infinitely many positive instances of the principle that make one judge reasonably that the principle is satisfied in other cases as well, 'according to the method of experimental philosophy, which proceeds *a posteriori*' (GP VII 420/LC 96). This is an inductive justification of the principle. This is not the kind of justification of the Principle of Sufficient Reason one would expect from a 'rationalist' philosopher like Leibniz and, furthermore, this method of establishing the Principle of Sufficient Reason would not establish its necessity. But Leibniz is not rejecting the possibility of an *a priori* justification of the Principle of Sufficient Reason in this passage. All he is saying is that there is *a posteriori* justification for the Principle of Sufficient Reason, justification that is independent from any *a priori* justification. Indeed Leibniz says,

²⁶ It is interesting to compare *Primary Truths* with the text at GP VII 209–301. In this text Leibniz states both PIN and the Principle of Contradiction. Unlike *Primary Truths*, Leibniz there explicitly states the Principle of Contradiction, though not as PC6, but as PC2 and PC3. But there the Principle of Sufficient Reason is not derived from the Principle of Contradiction but from the idea, which derives from PIN, that every true proposition which is not identical can be proved *a priori* (GP VII 300/L 226), which is, again, not the Principle of Contradiction in any of its guises.

after stating the inductive justification of the principle, that this would be enough to justify it even if the principle were not otherwise justified on the basis of pure reason or *a priori* (GP VII 420/LC 96), which indicates that Leibniz thought, in that letter, that the principle is also justified *a priori*.²⁷

Note that this inductive justification of the Principle of Sufficient Reason contradicts Leibniz's skepticism about knowledge based purely on induction. In the *New Essays* Leibniz says that 'however often one experienced instances of a universal truth, one could never know ('asseurer') inductively that it would always hold unless one knew through reason that it was *necessary*' (A 6 6 80/NE 80, my italics). It is not that Leibniz thinks that induction never gives reasonable grounds for a universal judgement. In some cases, when we have had sufficiently many experiences, it does. But without knowing the reason why our experiences are what they are, Leibniz says in *On what is independent of sense and of matter*, one would never have absolute certainty of the corresponding universal generalization (GP VI 505/L 550).

In a marginal note to *An introduction to a secret encyclopedia* Leibniz said that what is confirmed by many indications, and which can hardly concur except in the truth, is morally certain, as opposed to metaphysically certain. And he added that morally certain is that which is incomparably more probable than its opposite (C 515/MP 9). Perhaps Leibniz, at the end of his fifth letter to Clarke, was simply trying

²⁷ It is interesting to note that many years before, in *On Contingency*, Leibniz had anticipated the idea of an empirical justification of the Principle of Sufficient Reason, and he also claimed that it has a justification based on reason. Indeed he said that the Principle of Sufficient Reason is 'confirmed ('firmatur') both by reason and experience' (A 6 4 1651/AG 29). Note that the sense in which experience confirms the Principle of Sufficient Reason cannot be that of a mere corroboration, since it must be the same sense as that in which it is confirmed by reason. My interpretation is that 'confirmed', in this text, must be understood in the sense of 'established'.

to establish the moral certainty, as opposed to the metaphysical certainty, of the Principle of Sufficient Reason?

This seems to contradict other texts. For instance, in the same page where he explains what moral certainty is, Leibniz classifies the Principle of Sufficient Reason, stated simply as 'Nothing is without reason', as a metaphysically certain principle (C 515/MP 9). And in the *Preface to an edition of Nizolius* Leibniz says that without reasons not based on induction, induction does not even produce moral certainty (A 6 2 432/L 129–30).

Note that in these texts Leibniz is saying that without knowledge of the reason why our experiences are as they are, we cannot have knowledge of the corresponding universal generalization. Now, according to the passage from the fifth letter to Clarke, for every observed fact or event, there is a reason why it occurs. The corresponding universal generalization is that every fact or event has a reason why it occurs. But the only reason why, for every observed fact or event, there is a reason why it occurs, that would support inferring that every fact or event has a reason why it occurs, is that every fact or event *must* have a reason why it occurs, and this is how Leibniz understood the Principle of Sufficient Reason in that letter to Clarke (he formulated it as 'le principe du besoin d'une raison suffisante', which Clarke translated as 'the principle of the want of a sufficient reason': GP VII 393, 419/LC 60, 95). So trying to combine the justification of the Principle of Sufficient Reason Leibniz suggests in the fifth letter to Clarke with what he says in the passages quoted from *On what is independent of sense and of matter* and the *Preface to an edition of Nizolius*, would lead to a circular justification of the Principle of Sufficient Reason.

Thus it would seem as if what Leibniz says in his fifth letter to Clarke represents a change of mind with respect to induction. According to Paul Lodge, in his later years Leibniz became preoccupied with the metaphysical structure of the actual world and, as a result, Leibniz had to give experience a justificatory role in relation to some of his metaphysical principles (Lodge 2010: 24–5). The passage of Leibniz's fifth letter to Clarke might be seen as an instance of the argumentative strategy Lodge sees in Leibniz's later philosophy. But such an argumentative strategy might fail in this case. For, in the correspondence with Clarke and many other texts, Leibniz applies the Principle of Sufficient Reason to the will of God, and he says that this principle dictates that the bare will of God must be determined by some prior motive. Indeed for Leibniz the notion of will requires a determining reason (GP VII 371–72, 392/LC 36, 59). But there is no *empirical* evidence whether the world was created by a will. Furthermore, in *Monadology* §32 Leibniz says that although there is always a sufficient reason, most of the time these reasons cannot be known by us.

5. An attempt to demonstrate the Principle of Identity of Indiscernibles.

Although on one occasion Leibniz calls the Principle of Identity of Indiscernibles a most obvious ('manifestissimum') axiom and expresses amazement at the fact that people have not used it (GP II 249/AG 175), his demonstrations of the principle use controversial Leibnizian theses. He had two main argumentative strategies to demonstrate it. The first main strategy is exemplified in three related texts from the 1680s: *Notationes Generales* (1683–85), *Discourse on Metaphysics* (1686) and *Primary Truths* (1689). What is common to these texts is that in all three of them

²⁸ I owe the point about the *Monadology* to Maria Rosa Antognazza.

Leibniz argues, in one way or another, for the Principle of Identity of Indiscernibles from his theory of truth, according to which in every true affirmative proposition the predicate is included in the subject, or the concept of the predicate is included in the concept of the subject. It is also common to them, therefore, that they seem to support the necessary version of the Identity of Indiscernibles.

Although sometimes these texts are seen as presenting the same argument (Jauernig 2008: 200, 201 fn. 32), they do not present exactly the same argument. Here are three interesting differences: (a) in *Primary Truths* the Principle of Identity of Indiscernibles derives from the Principle of Sufficient Reason, but this is not so in *Notationes Generales* or the *Discourse*;²⁹ (b) though in both *Primary Truths* and *Notationes Generales* Leibniz states the Principle of Identity of Indiscernibles as applying to all individuals, in the *Discourse on Metaphysics* his conclusion is that no substances are perfectly similar; (c) in *Primary Truths* and *Notationes Generales* the Principle of Identity of Indiscernibles is presented as necessary, but in the *Discourse on Metaphysics* he corresponding claim about substances is not presented as necessary.³⁰

The second main strategy appears in the correspondence with Clarke. There Leibniz derives the Principle of Identity of Indiscernibles from the Principle of

²⁹ But note that Leibniz comes close, in section 8 of the *Discourse*, where Leibniz formulates the elements from which he will deduce the Principle of Identity of Indiscernibles in section 9, to stating PSR3, when he says that God, seeing Alexander's individual notion, sees in it the reason for all of his predicates, and he says that God knows *a priori* those predicates (A 6 4 1540–41/AG 41). Nevertheless, this is short of formulating the Principle of Sufficient Reason.

³⁰ Note that originally Leibniz had written that it was not possible that two substances resemble each other entirely and differ *solo numero*. He then deleted it, and wrote that it is not true that two substances resemble each other entirely and differ *solo numero* (A 6 4 1541).

Sufficient Reason as applied to God. The argument is, in a nutshell, that if there were indiscernibles, God would have preferred the actual world to an indiscernible world, but having a preference for one of two indiscernible worlds would be to violate the Principle of Sufficient Reason, and therefore there are no indiscernibles (GP VII 393/LC 61).

For reasons of space I shall here discuss only what Leibniz says in the *Discourse*, the most important of these four texts.³¹ In section 8 of the *Discourse* Leibniz states that in a true proposition the subject term must contain the predicate term, so that one who understands perfectly the notion of the subject would also know that the predicate belongs to it. Since this is so, Leibniz claims, the nature of an individual substance or complete being is to have a notion so complete that it is sufficient to contain and to allow us to deduce from it all the predicates of the subject to which this notion is attributed. But from this idea that an individual substance has a complete concept, Leibniz says in section 9, several paradoxes follow; in particular it follows that it is not true that two substances resemble each other completely and differ only in number (A 6 4 1541/AG 41-2).³²

³¹ I discuss Leibniz's arguments in the correspondence with Clarke and *Primary Truths* in my 1999 and 2004 respectively. But I have changed my mind about some of the issues; I develop my new views in my 2011.

³² Two interesting things to note: (a) in another text from roughly the same time, *De mundo praesenti*, Leibniz claims that individual substances are unique (which I take to be an expression of the Identity of Indiscernibles applied to substances) because their concepts are complete (A 6 4 1507), but in this text there is no mention of the theory of truth; (b) although the Identity of Indiscernibles features in the *Remarks on Arnauld's letter* (A 2 2 48), at the end of this text Leibniz states the consequences of the proposition that the notion of a substance involves everything that happens to it, but he does not include the Principle of Identity of Indiscernibles in that list, while in the *Discourse*, in section 9, it is the first consequence to be mentioned.

Leibniz does not explain how the Principle of Identity of Indiscernibles is supposed to follow from the idea that substances have complete individual concepts. The impression the *Discourse* gives is that he seems to have thought that the Identity of Indiscernibles follows more or less immediately from the idea that substances have complete individual concepts. Does it?

That substances have complete individual concepts means that a substance's individual concept includes everything that is true of it. The Identity of Indiscernibles means that no two substances resemble each other perfectly. *Prima facie*, the former thesis does not entail the latter. For why couldn't *A* and *B*, two different substances, have the same complete concept? Nothing in the doctrine of complete individual concepts seems to exclude such a possibility. But if *A* and *B* have the same complete concept, they are perfectly similar.

But, it might be claimed, there is something that excludes such a possibility. For if a substance's concept contains everything that is true of it, then substance *A*'s concept includes the concept of *being identical to A*. If so, no other substance can share its concept with *A*. So, it would seem, Leibniz is in a position to guarantee that every concept picks one and only one substance. And it seems to follow from this that the Principle of Identity of Indiscernibles is true and therefore that two substances do not resemble each other perfectly.

But there is a problem with this line of thought. For either being identical with a certain substance consists in having certain qualitative intrinsic features, or being identical with a certain substance is something additional to having any qualitative intrinsic features.

Suppose being identical with a certain substance consists in something additional to having certain qualitative intrinsic features. Then even if concepts like being identical to A are included in the individual concepts of substances, that such concepts include everything that is true of substances does not entail the Identity of Indiscernibles. For that A and B differ with respect to being identical to A (and other identity related conditions like being numerically different from B, being intelligent and identical to A, etc.) does not mean that A and B do not perfectly resemble each other. Two things that differed only with respect to identity conditions like those just mentioned would be only numerically different and therefore perfectly similar. For, according to Leibniz, similarity is sharing of qualities and qualities are what can be known about a thing by inspecting it by itself, without comparing it to other things (GM VII 19/L 667; GM V 180/L 255). But, unless being identical to A reduces to having certain qualitative intrinsic features, one cannot know that a substance is identical to A (and other identity related conditions), A and B do not differ with respect to any qualities and are therefore perfectly similar.

Thus including such conditions like *being identical to A* in individual concepts might deliver a version of the Identity of Indiscernibles according to which there are no two things that satisfy exactly the same predicates or concepts, but this is not Leibniz's Identity of Indiscernibles. Indeed, why couldn't the individual concept of *A* include the concept of *being perfectly similar to B*? Thus, if being identical to a certain substance is something additional to having qualitative intrinsic features, then if substances' complete concepts allow one to deduce everything that is true of a substance, then every individual concept picks out only one substance. But this is not sufficient to deliver the thesis that no two substances resemble each other completely.

Now, if being identical to a certain substance consists in having some qualitative intrinsic features, then of course no two things are perfectly qualitatively intrinsically similar. For in that case *being identical to A* would be to be qualitatively intrinsically thus-and-so. And, if there cannot be two things that are perfectly qualitatively intrinsically alike, Leibniz's Identity of Indiscernibles is true. I believe that Leibniz thought that identity was reducible to qualitative intrinsic character, i.e. that he thought that being identical to a certain substance consists in having some intrinsic features. For reasons of space I cannot argue for such a claim in this article.³³

Thus Leibniz claims to derive the Identity of Indiscernibles from his thesis that substances have complete individual concepts. But he can only establish the Identity of Indiscernibles if he assumes a thesis (that identity reduces to intrinsic qualitative character) that establishes by itself the Identity of Indiscernibles.

But note that if Leibniz assumes that identity reduces to intrinsic qualitative character, then not only can he derive the Identity of Indiscernibles, he can derive it from the thesis that substances have complete concepts. For if identity reduces to intrinsic qualitative character, individual concepts are purely qualitative. On this understanding of complete concepts the Identity of Indiscernibles follows, pretty much directly, from the thesis that substances have complete concepts.

Thus Leibniz's argument for the Identity of Indiscernibles is a simple one. The argument is that individual substances have complete concepts that permit to deduce everything that is true of them. Since they permit to deduce everything about them, they permit to deduce facts about the identity of substances. But those complete concepts are purely qualitative. Therefore, there cannot be two substances that

 $^{^{33}}$ Adams (1979: 9 – 11) also makes the claim and offers a longer discussion.

resemble each other perfectly. This is, I think, a valid argument, but with very controversial premises.³⁴

6. Conclusion.

We have seen that Leibniz gave different formulations of these three important principles, and that sometimes the relation between those different formulations is not clear. Although the Principle of Contradiction and the Principle of Sufficient Reason

³⁴ Broad (1975: 40) thought that the Identity of Indiscernibles does not follow from the thesis that substance have complete individual concepts (a thesis he called, rather inaccurately, the Predicate-in-Notion Principle, see Broad 1975: 6). But he did not argue for this; he just asserted it. Mates says, in reference to the Discourse, that the Principle of Identity of Indiscernibles follows from the fact that every individual concept is complete '[m]aybe via the Principle of Sufficient Reason, which itself is said to follow from the fact that every individual concept is complete', and that perhaps 'the point is that if individuals A and B express the universe in different ways, they can't fall under all the same concepts' (Mates 1986: 134-5, fn. 56). It is not clear to me how Mates understands the derivation, for if A and B are indiscernible they should express the universe in the same way. And it is not clear what role is the Principle of Sufficient Reason playing here. But, in any case, nowhere in the Discourse does Leibniz mention or suggest the Principle of Sufficient Reason as the bridge between the idea that individuals have complete concepts and the Identity of Indiscernibles. Rutherford denies that the Identity of Indiscernibles as applied to substances follows from the thesis that substances have complete individual concepts; for him it depends on an independent commitment to the Principle of Identity of Indiscernibles understood as 'the principle that for any two numerically non-identical things, there must be some discernible difference between them' (Rutherford 1995: 142). Perhaps so, but this does not help us to make sense of the argument in the Discourse since the only version of the Identity of Indiscernibles that Leibniz mentions in the *Discourse* is the one about substances, and so there is no indication that Leibniz is making his claim about substances differing more than numerically depend on a claim about things in general differing more than numerically.

are fundamental principles for Leibniz, to the extent that these principles are not identities, he should not have thought of them as indemonstrable. He attempted some demonstrations of them. I briefly described some of these attempts and examined in more detail the derivation of the Principle of Sufficient Reason in *Primary Truths*, about which I argued that Leibniz does not use the Principle of Contradiction in that derivation. I also discussed an interesting attempt, in the correspondence with Clarke, to prove the Principle of Sufficient Reason empirically. This attempted demonstration is, given what Leibniz says in other texts, problematic. Finally I examined the argument for the Principle of Identity of Indiscernibles in the *Discourse*, and concluded that it is valid but with very controversial premises.

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