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Space and Geometry in the B Deduction*

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1. INTRODUCTION

Kant's reformulation of the Transcendental Deduction of the Categories in the second (1787) edition of the *Critique of Pure Reason* culminates in the notoriously difficult § 26, entitled "Transcendental Deduction of the Universally Possible use in Experience of the Pure Concepts of the Understanding." Kant states the problem that he is now addressing as one of explaining "the possibility of cognizing a priori, *by means of categories*, whatever objects *may present themselves to our senses*—not, indeed, with respect to the form of their intuition, but with respect to the laws of their combination . . . [, f]or if [the categories] were not serviceable in this way, it would not become clear how everything that may merely be presented to our senses must stand under laws that arise a priori from the understanding alone" (B159-160). Kant begins, therefore, by emphasizing his fundamental distinction between sensibility and understanding: what has now to be explained is the possibility of cognizing a priori, by means of the pure concepts of the understanding, whatever may be presented to our sensibility.

The faculty of sensibility is our passive or receptive faculty for receiving sensory impressions. In sharp contrast with all forms of traditional rationalism from Plato through Leibniz, however, Kant takes this receptive faculty to be a source of a priori cognition: notably, the science of geometry as grounded in our outer (spatial) sensible intuition. What makes such a priori cognition possible, for Kant, is a second fundamental distinction, articulated at the beginning of the Transcendental Aesthetic, between the matter and form of sensibility:

> I call that in the appearance which corresponds to sensation its *matter*, but that which brings it about that the manifold of appearances can be ordered in certain relations I call the *form* of appearance. Since that within which sensations can alone be ordered and arranged in a

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> certain form cannot itself be sensation in turn, the matter of all appearance, to be sure, is only given to us a posteriori, but its form must already lie ready for it in the mind a priori and can therefore be considered separately from all sensation. (A20/B34)[1]

In particular, it is because the form of sensibility (e.g., the spatial form of all outer appearances) is invariant under all changes in the content or matter taken up or received therein, that a priori cognition of this content (provided, e.g., by the geometrical structure governing all outer appearances) is possible.

The faculty of understanding, by contrast, is our active or spontaneous faculty of thought, which, considered by itself, has no intrinsic relation to our spatio-temporal faculty of sensibility. Indeed, it is for precisely this reason, for Kant, that we can *think*, but not theoretically *cognize*, supersensible objects—such as God and the soul, for example—by means of the pure concepts of the understanding.[2] So how can we be sure that these same purely intellectual categories—such as substance and causality, for example—also necessarily apply to all objects of sensibility? If the a priori concepts of the understanding originate in the understanding, entirely independently of sensibility, how can we show that they also relate a priori to all possible objects of our (human) sense experience? The pure forms of sensibility are not subject to this difficulty, since, assuming that there are such forms, they are precisely the forms of what is sensibly received or given. They therefore relate, necessarily, to all possible objects of our senses, that is, to all possible objects in space and time. But the categories are pure forms of thought, not forms of sensory perception, and so, in this case, an additional step is needed: a transition from the pure forms of sensibility to which the matter of appearance is already necessarily subject to the conclusion that the resulting appearances, precisely as such, are also necessarily subject to the pure forms of thought

One way to see the force of this problem is to observe that in the rationalist tradition that preceded Kant supersensible objects such as God and the soul are paradigmatic instantiations of the intellectual concept of substance, and their actions—especially the creative activity of God—are paradigmatic instantiations of the intellectual concept of

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[1] In B the words "can be" in the first sentence replace "are" in A.

[2] See the important footnote to the second edition Preface (Bxxvi,n): "In order to *cognize* an object it is required that I can prove its possibility (whether in accordance with the testimony of experience from its actuality or a priori through reason). But I can *think* whatever I wish, as long as I do not contradict myself—i.e., if my concept is only a possible thought, even if I cannot guarantee whether or not an object corresponds to it in the sum total of all possibilities." Kant indicates in the remainder of the note that one may be able to cognize such supersensible objects through reason from a *practical* as opposed to purely *theoretical* point of view.

causality. So it is by no means clear, in this tradition, that such concepts can apply to sensible appearances at all. In the preceding empiricist tradition of Locke and Hume, by contrast, the very existence of purely intellectual concepts is in doubt, and we must instead resort, in all cases, to experience. Kant's complaint against this tradition, therefore, is that it cannot capture the rational necessity that such concepts demand.[3] His radical solution to the resulting dilemma involves deriving the rational necessity in question from the *schematization* of the pure concepts of the understanding in space and time, with the result that only spatio-temporal appearances can thereby be cognized theoretically. I intend my discussion to clarify Kant's solution.

Kant proceeds to articulate the relationship between pure forms of thought and sensible intuition in § 26 by noting that, "under *the synthesis of apprehension*," he "understand[s] the composition [*Zusammensetzung*] of the manifold in an empirical intuition, whereby perception, i.e., empirical consciousness of [the empirical intuition] (as appearance), becomes possible" (B160). The synthesis of apprehension, he continues, "must always be in accordance with" our "a priori *forms* of outer and inner sensible intuition in the representations of space and time" (ibid.). This is relatively straightforward, because it merely reiterates that space and time *are* our two forms of outer and inner intuition. The reminder of the argument, however, is by no means straightforward:

> But space and time are represented a priori, not merely as *forms* of sensible intuition, but as *intuitions* themselves (which contain a manifold) and thus [represented a priori] with the determination of the *unity* of this manifold (see the Transcendental Aesthetic*). Therefore, *unity of the synthesis* of the manifold, outside us or in us, and thus a *combination* with which everything that is to be represented in space or time as determined must accord, is itself already given simultaneously, with (not in) these intuitions. But this synthetic unity can be no other than that of the combination of the manifold of a given *intuition in general* in an original consciousness, in accordance with the categories, only applied to our *sensible intuition*. Consequently all synthesis, even that whereby perception becomes possible, stands under the categories, and, since experience is knowledge through connected perceptions, the categories are conditions of the possibility of experience, and thus are a priori valid for all objects of experience. (B160-161)

[3] For this complaint see the discussion of causal necessity in a preliminary discussion of the Deduction (A91-92/B123-124), and compare the criticism of Locke's and Hume's attempt at an "*empirical* derivation" of the categories that follows several pages later in the second edition (B127-128).

Thus, although Kant begins by reminding us that space and time are unified or unitary representations in a sense already articulated in the Aesthetic, he then appears to claim that this same synthetic unity is actually due to the understanding rather than sensibility. It is for precisely this reason, it appears, that we can now conclude that the pure categories of the understanding are in fact "a priori valid for all objects of [sensible] experience" (B161).

It is for this reason, it also appears, that Kant insists in the second sentence that the synthetic unity in question is given "with" rather than "in" the intuitions of space and time themselves. Indeed, Kant has already insinuated his doctrine of the transcendental unity of apperception—the highest and most general form of unity of which the understanding is capable—by using the notion of "combination [*Verbindung*]" here (which is then repeated in the third sentence). This notion is introduced as a technical term at the very beginning of the Deduction (§ 15) to designate the activity most characteristic of the understanding:

> [T]he *combination* (*conjunctio*) of a manifold in general can never come into us through the senses, and can thus not be simultaneously contained in the pure form of sensible intuition; for it is an act of the spontaneity of the power of representation, and since one must call this, in distinction from sensibility, understanding, all combination—whether we are conscious of it or not, whether it is a combination of the manifold of intuition or several concepts, and, in the first case, of sensible or non-sensible intuition—is an action of the understanding, which we would designate with the title *synthesis* in order thereby to call attention, at the same time, to the fact that we can represent nothing as combined in the object without ourselves having previously combined it. (B129-130)

So the "combination" introduced in the second sentence of the main argument of § 26 (B160-161) is precisely an activity of the understanding—here figuring, in particular, as "a combination of *the manifold of intuition*" (B130; emphasis added).

The discussion of combination in § 15 continues in the following paragraph:

> Combination is the representation of the *synthetic* unity of the manifold. The representation of this unity can therefore not arise from combination; rather, it makes the concept of combination possible in the first place, in so far as it is added to the representation of the manifold. This unity, which precedes all concepts of combination, is not, for example, the category of unity (§ 10); for all categories are based on logical functions in judging, but in these combination, and

> thus unity of given concepts, is already thought. The category thus already presupposes combination. Therefore, we must seek this unity (as qualitative § 12) still higher, namely, in that which contains the ground of the unity of different concepts in judging, and thus of the possibility of the understanding, even in its logical use. (B130-131)

The synthetic unity in question, therefore, "precedes all *concepts* of combination" and thus "all *categories*" (B131; emphasis added). Moreover, the sought after "higher" ground of this unity, according to the next section (§ 16), is just "the original-synthetic unity of apperception"—namely, the representation "*I think*, which must be *able* to accompany all my representations" (ibid). Indeed, according to the following section (§ 17), "[t]he principle of the synthetic unity of apperception is the *highest* principle of all use of the understanding" (B136; emphasis added).

The conclusion of the main argument of § 26 thus appears to be that the same unity that was first introduced in the Aesthetic as characteristic of space and time themselves can now be seen to be due to the understanding after all. Indeed, if he were not asserting this it would be hard to understand how Kant could arrive at the claim that the synthesis of apprehension "stands under the *categories*" (B161; emphasis added). The unity of apperception—"the highest principle of all use of the understanding" (B136)—must be the ultimate ground of both the categories and the characteristic unity of space and time.

This conclusion, however, is puzzling in the extreme. The difficulty arises in the first sentence of the main argument (B160), which contains the justificatory reference back to the Aesthetic. For the primary claim of the Aesthetic, in this connection, is that the characteristic unity of space and time is intuitive rather than conceptual. So how can we possibly begin with a unity that was earlier explicitly introduced as *non-conceptual* and conclude that this same unity is due to the understanding after all? It is for this reason, among others, that some of the most important philosophers in the following German tradition, which rejected Kant's dualistic conception of the faculties of sensibility and understanding in favor of a deeper and more fundamental original unity, appealed to what Kant himself says in § 26 to motivate this rejection. In particular, there has been a sustained attempt, arising within the first stirrings of post-Kantian German idealism, to find in what Kant calls "figurative synthesis" or "transcendental synthesis of the imagination" (§ 24) the mysterious "common root" of the two faculties that is speculatively mentioned at the end of the Introduction to the second edition (B29).[4]

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[4] I am here indebted to the rich discussion of the historical background to Heidegger's notorious "common root" interpretation in Henrich (1955), and I am particularly indebted to Desmond Hogan for calling my attention to this important paper. It would be illuminating to discuss Henrich's later classic discussion of

Thus Hegel, for example, began his philosophical journey by appealing to the transcendental imagination in the context of § 26 in support of the project of overcoming Kantian dualism.[5] The Marburg School of neo-Kantianism, led by Hermann Cohen, then read § 26 as demonstrating that Kant had now become clear that the transcendental synthesis of the imagination is nothing more nor less than an activity of the understanding—which can now ground experience all by itself without an appeal to an independently structured faculty of sensibility.[6] And Martin Heidegger, in explicit reaction against the Marburg School, undertook to overthrow the hegemony of the intellect in the Western tradition once and for all by returning to the first edition version of the Transcendental Deduction in his reading of Kant—where he found the sought after "common root" of the two faculties in the (finite) human temporality already articulated in *Being and Time*.[7]

My own strategy, by contrast, is to focus on the footnote to § 26 in the second edition version, which is attached to the reference back to the Aesthetic in the main text. In the first sentence of this footnote Kant illustrates his point by the example of "[s]pace, represented as *object* (as is actually required in geometry)" (B160n). I shall focus, accordingly, on the role of the transcendental synthesis of the imagination in the science of geometry, and I shall argue that we can thereby illuminate both the way in which the

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the B Deduction (1968-69) against the background of his earlier (1955) paper, but this will have to wait for another occasion.

[5] See, in particular, Hegel's early (1802) discussion of the Kantian, Jacobian, and Fichtean philosophies—where Hegel invokes § 26 near the beginning of his discussion of the Kantian philosophy (1962 ed., p. 16): "One glimpses this idea [of 'the identity of such inhomogeneities'] through the surface of the Deduction of the Categories, and, in relation to space and time, not there, where it should be, in the Transcendental Exposition of these forms, but in what follows, where the original synthetic unity of apperception first comes to the fore and also becomes known as principle of figurative synthesis or the forms of intuition, and space and time themselves [become known] as synthetic unities, and the productive imagination, spontaneity, and absolute synthetic activity are conceived as principles of sensibility, which had been previously characterized only as receptivity."

[6] This process begins with Cohen (1871), and the tie between forms of intuition and the unity of apperception becomes successively stronger in later editions. In all editions Cohen begins with the passage from § 26 according to which space and time are not merely forms of intuition but "*intuitions* themselves" (B160). Cohen (1871, p. 46) continues: "But through this equation we guard against the suspicion that a form that 'lies *ready*' could be a '*completed*' form. Intuition, even pure intuition, is generated. It lies 'ready' but is not 'complete'. Such errors are only possible if one treats transcendental aesthetic without transcendental logic, if one severs the unity of the Kantian critique." The second edition (1885, p. 156) adds: ". . . , if one has not made clear to oneself the form of space as contribution and instrument of the highest principle of the transcendental unity of apperception." The third edition (1918, p. 207) inserts "singular [*einzeln*]" before both "contribution" and "instrument" in this last clause. I am indebted to Frederick Beiser for emphasizing to me Cohen's development after the first edition.

[7] Heidegger's contention that Kant "shrank back" in the second edition from the radicalism of his discoveries in the first is developed most fully in (1929, § 31). His most detailed discussion of § 26, however, is found in his lecture course on the *Critique of Pure Reason* from 1927-28 (1977, § 9). I shall return to this in note 18 below.

understanding functions in that science and the corresponding independent contribution of the faculty of sensibility. I shall here have occasion, as well, to discuss the relationship between §§ 26 and 24 in some detail.

2. READING THE FOOTNOTE TO § 26 (B160-161n)

Before focussing on the footnote and the example of geometry, let us first consider the earlier passage from the Aesthetic to which Kant apparently refers in the main text. The passage in question is the third paragraph of the Metaphysical Exposition of Space, where Kant appeals to the characteristic unity and singularity of our representation of space to argue that it must be a pure intuition rather than a concept:

> Space is not a discursive, or, as one says, general concept of relations of things in general, but a pure intuition. For, first, one can only represent to oneself a single [*einigen*] space, and if one speaks of many spaces, one understands by this only parts of one and the same unique [*alleinigen*] space. These parts cannot precede the single all-encompassing [*einigen allbefassenden*] space, as it were as its constituents (out of which a composition [*Zusammensetzung*] would be possible); rather, they can only be thought *within it*. It is essentially single [*einig*]; the manifold in it, and the general concept of spaces as such, rests solely on limitations. From this it follows that an a priori intuition (that is not empirical) underlies all concepts of space. Thus all geometrical principles, e.g., that in a triangle two sides together are greater than the third, are never derived from general concepts of line and triangle, but rather from intuition, and, in fact, with apodictic certainty. (A24-25/B39)

The characteristic properties of space to which Kant appeals are, first, that it is singular, so that *spaces* (in the plural) are only parts of the one singular space, and, second, that such parts cannot precede this singular (whole) space but can only be thought within it. It is for this reason, in fact, that "the general concept of *spaces*" (emphasis added)—that is, the finite spatial regions that are *parts* of the "single all-encompassing space"—"rests solely on limitations," which carve out such regions from the single infinite space that contains them all. The final sentence of the passage makes it clear that the science of geometry is implicated in the distinctive whole-part structure that Kant is attempting to delineate, a point to which I shall return below.

The crux of this passage is that space is a singular individual representation, whose whole-part structure is completely different from that of any general concept. In particular, the whole "all-encompassing" space precedes and makes possible all of its limited parts (finite spatial regions), whereas the "parts" of any concept—that is, the marks or "partial concepts [*Teilbegriffe*]" that are constitutive of its definition (intension)—precede and make possible the (conceptual) whole. The unity of a general concept, in this sense, is essentially different from that of our representation of space (and similarly for our representation of time), and this is the primary reason, in the Aesthetic, that space and time count as intuitive rather than conceptual representations for Kant.[8]

Yet, as observed, the main argument of § 26 appears to appeal to precisely the characteristically *non-conceptual* unity of space and time to argue that this same unity is actually a product of the "unity of synthesis" that is most characteristic of the *understanding*—"even in its logical use" (B131). The appended footnote, moreover, only compounds the appearance of paradox:

> Space represented as *object* (as is actually required in geometry) contains more than the mere form of intuition—namely, [it contains] the *grasping together* [*Zusammenfassung*] of the manifold, given in accordance with the form of sensibility, in an *intuitive* representation, so that the *form of intuition* gives merely a manifold, but the *formal intuition* [also] gives unity of representation. In the Aesthetic I counted this unity [as belonging] to sensibility, only in order to remark that it precedes all concepts, although it in fact presupposes a synthesis that does not belong to the senses but through which all concepts of space and time first become possible. For, since through it (in that the understanding determines sensibility) space or time are first *given* as intuitions, the unity of this a priori intuition belongs to space and time, and not to the concept of the understanding (§24). (§ 26, B160-161n)

Thus, the "unity of representation" mentioned in the first sentence appears to be the "all-encompassing [*allbefassenden*]" unity of space discussed in the third paragraph of the Metaphysical Exposition (A24-25/B39). The second sentence confirms this idea, but also emphasizes that the unity in question presupposes a distinctively *non-sensible* synthesis. The third sentence, however, appears to take this back, and even to contradict itself; for,

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[8] The crucial difference in whole-part structure is emphasized even more clearly in the immediately following fourth paragraph of the Metaphysical Exposition in the second edition (B39-40): "Space is represented as an infinite *given* magnitude. Now one must certainly think every concept as a representation that is contained in an infinite aggregate of different possible representations (as their common mark), and it therefore contains these *under itself*. But no concept, as such, can be so thought as if it were to contain an infinite aggregate of representations *within itself*. However space is thought in precisely this way (for all parts of space *in infinitum* exist simultaneously). Therefore, the original representation of space is an a priori *intuition*, and not a *concept*."

after reiterating that the synthetic unity in question is a product of the understanding, Kant appears to *deny* that it is due to the understanding after all.

I have recently proposed a solution to these apparent paradoxes that emerged out of my evolving work on Kant's theory of geometry. My original interpretation of this theory in Friedman (1985) emphasized the importance of Euclidean constructive reasoning for Kant and, in particular, appealed to Kant's understanding of such reasoning to explain the sense in which geometry, for him, is synthetic rather than analytic—an essentially intuitive rather than purely logical science. However, I did not there explain the necessary relation between the science of geometry and what Kant calls our pure form of outer intuition: the (three-dimensional) space of perception within which all objects of outer sense necessarily appear to us. I first proposed such an explanation, which establishes a link between geometry and our passage from the Aesthetic (A24-25/B39), in Friedman (2000).[9] And I found the missing link, in turn, in Kant's discussion of the relationship between what he calls "metaphysical" and "geometrical" space in his comments on essays by the mathematician Abraham Kästner in 1790.[10]

Kant's comments first describes the relationship between the two types of space as follows:

> Metaphysics must show how one *has* the representation of space, but geometry teaches how one can *describe* a space, i.e., can present it in intuition a priori (not by drawing). In the former space is considered as it is *given*, prior to all determination of it in accordance with a certain concept of the object, in the latter a [space] is *made*. In the former it is *original* and only a (single [*einiger*]) *space*, in the latter it is *derivative* and here there are (many) *spaces*—concerning which, however, the geometer, in agreement with the metaphysician, must admit, as a consequence of the fundamental representation of space, that they can all be thought only as parts of the single [*einigen*] original space. (20, 419)

[9] I thereby attempted to build a bridge between the "logical" interpretation of Kant's theory of geometry developed in my earlier paper (an approach that was first articulated by Jaakko Hintikka) and the "phenomenological" interpretation" articulated and defended by Charles Parsons and Emily Carson. For further discussion of this issue see also Parsons (1992).

[10] Kästner's three essays on space and geometry were first published in J. A. Eberhard's *Philosophisches Magazin* in 1790. Eberhard's intention was to attack the *Critique of Pure Reason* on behalf of the Leibnizean philosophy, and Kästner's essays were included as part of this attack. Kant's comments on Kästner, sent to J. G. Schulze on behalf of the latter's defense of the Kantian philosophy in his reviews of Eberhard's *Magazin*, were first published by Wilhelm Dilthey in the *Archiv für Geschichte der Philosophie* in 1890. They are partially translated in Appendix B to Allison (1973), which also discusses the historical background in Chapter I of Part One. Kant's comments have played a not inconsiderable role in the subsequent discussion of space and geometry in § 26, and, after presenting my own interpretation, I shall touch on some of this discussion below.

So it appears, in particular, that the (plural) *spaces* of the geometer—i.e., the figures or finite spatial regions that are iteratively constructed in Euclidean proofs—are prominent examples of the *parts* of the "single all-encompassing space" according to our passage from the Aesthetic (A24-25/B39).[11]

Kant's comments go on to discuss the different types of infinity belonging to geometrical and metaphysical space:

> [A]nd so the geometer grounds the possibility of his problem—to increase a given space (of which there are many) to infinity—on the original representation of a single [*einigen*], infinite, *subjectively given* space. This accords very well with [the fact] that geometrical and objectively given space is always finite, for it is only given in so far as it is *made*. That, however, metaphysical, i.e., original, but merely subjectively given space—which (because there are not many of them) can be brought under no concept that would be capable of a construction, but still contains the ground of construction of all possible geometrical concepts—is *infinite*, is only to say that it consists in the pure form of the mode of sensible representation of the subject as a priori intuition; and thus in this form of sensible intuition, as singular [*einzelnen*] representation, the possibility of all spaces, which proceeds to infinity, is *given*. (20, 420-421)

Thus, whereas geometrical space is only *potentially* infinite (as it emerges step-by-step in an iterative procedure), metaphysical space, in a sense, is *actually* infinite—in so far as the former presupposes the latter as an already given infinite whole. Geometrical construction presupposes a single "*subjectively given*" metaphysical space within which all such construction takes place.[12]

In Friedman (2000) I interpreted the relationship between these two kinds of space as follows. Metaphysical space is the manifold of all oriented perspectives that an

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[11] This point becomes clearer in light of the final sentence of our passage from the Aesthetic—which brings Euclid's geometry explicitly into the picture (the example there is Proposition I.20 of the *Elements*).

[12] Immediately preceding this passage Kant illustrates the distinction by contrasting geometry with arithmetic (20, 419-420): "Now when the geometer says that a line, no matter how far it has been continually drawn, can always be extended still further, this does not signify what is said of number in arithmetic, that one can always increase it by addition of other units or numbers without end (for the added numbers and magnitudes, which are thereby expressed, are possible for themselves, without needing to belong with the preceding as parts to a [whole] magnitude). Rather [to say] that a line can be continually drawn to infinity is to say as much as that the space in which I describe the line is greater than any line that I may describe within it." Thus, while the figures iteratively constructed in geometry are only potentially infinite, like the numbers, the former, but not the latter, presuppose a single "all-encompassing" magnitude within which all are contained as parts: i.e., the space "represented as an infinite *given* magnitude" of note 8 above (B39).

idealized perceiving subject can possibly take up. The subject can take up these perspectives successively by operations of translation and rotation—by translating its perspective from any point to any other point and changing its orientation by a rotation around any such point. In this way, in particular, any spatial object located anywhere in space is perceivable, in principle, by the same perceiving subject. The crucial idea is then that the transcendental unity of apperception—the highest principle of the pure understanding—thereby unifies the manifold of possible perspectives into a single "all-encompassing" unitary *space* by requiring that the perceiving subject, now considered as also a thinking subject, is able, in principle, to move everywhere throughout the manifold by such translations and rotations. But this then implies that Euclidean geometry is applicable to all such objects of perception as well, since Euclidean constructions, in turn, are precisely those generated by the two operations of translation (in drawing a straight line from point to point) and rotation (of such a line around a point in a given plane yielding a circle).[13]

I appealed to these ideas in proposing an interpretation of the problematic footnote to § 26 in Friedman (2012a). The "unity of representation" mentioned in the second sentence of this footnote is indeed that considered in our passage from the Aesthetic (A24-25/B39), and Kant is indeed saying that this unity is a product of the understanding. It does not follow, however, that it is a *conceptual* unity—that it depends on the unity of any particular concept. It does not depend on the unity of any geometrical concept, for example, for the schemata of all geometrical concepts are generated by Euclidean (straight-edge and compass) constructions, and these presuppose, according to Kant, the prior unity of (metaphysical) space as a single whole. Nor does it depend on the unity of any category or pure concept of the understanding. For, by enumeration, we can see that none of their schemata result in any such object, i.e., space as a singular given object of intuition.

Rather, the unity of space as a singular given whole results directly from the transcendental unity of apperception, prior to any particular category, in virtue of the

[13] This connection between Euclidean constructions and the operations in question is suggested by Kant himself (20, 410-411): "[I]t is very correctly said [by Kästner] that 'Euclid assumes the possibility of drawing a straight line and describing a circle without proving it'—which means without proving this possibility *through inferences*. For *description*, which takes place a priori through the imagination in accordance with a rule and is called construction, is itself the proof of the possibility of the object. . . However, that the possibility of a straight line and a circle can be proved, not *mediately* through inferences, but only immediately through the construction of these concepts (which is in no way empirical), is due to the circumstance that among all constructions (presentations determined in accordance with a rule in a priori intuition) some must still be *the first*—namely, the *drawing* or describing (in thought) of a straight line and the *rotating* of such a line around a fixed point—where the latter cannot be derived from the former, nor can it be derived from any other construction of the concept of a magnitude."

circumstance that the former unity, as suggested, results from requiring that the perceiving subject (which has available to it the manifold of all possible perspectives) is also a *thinking* subject. For the latter, as Kant says in § 16, must be "one and the same" in all of its conscious representations (B132).[14] The unity of apperception, as Kant says in § 15, is not that of any particular category but something "still higher"—namely, "that which itself contains the ground of the unity of different concepts in judging, and hence of the possibility of the understanding, even in its logical use" (B131). This is why Kant can correctly say, in the last sentence of the footnote to § 26 (B161n; emphasis added), that "the unity of this a priori intuition belongs to space and time, and not to the *concept* [i.e., *category*—MF] of the understanding (§24)."[15]

If we follow the reference of this sentence back to § 24, moreover, we find that Kant there describes the figurative synthesis or transcendental synthesis of the imagination as "an action of the understanding on sensibility and its *first* application (at the same time the ground of all the rest) to objects of the intuition possible for us" (B152; emphasis added). He then proceeds to illustrates this synthesis by Euclidean constructions and explains that it also involves motion "as action of the subject":

> We also always observe this [the transcendental synthesis of the imagination] in ourselves. We can think no line without *drawing* it in thought, no circle without *describing* it. We can in no way represent the three dimensions of space without *setting* three lines at right angles to one another from the same point. And we cannot represent time itself without attending, in the *drawing* of a straight line (which is to be the outer figurative representation of time), merely to the action of synthesis of the manifold, through which we successively determine inner sense, and thereby attend to the succession of this determination in it. Motion, as action of the subject (not as determination of an object*), and thus the synthesis of the manifold in space—if we abstract from the latter and attend merely to the action by which we determine *inner* sense in accordance with its form—[such motion] even first produces the concept of succession. (B154-155)

[14] More fully (B132): "[A]ll the manifold of intuition has a necessary relation to the *I think* in the same subject in which this manifold is encountered. But this representation is an act of *spontaneity*, i.e., it cannot be viewed as belonging to sensibility. I call it *pure apperception*, in order to distinguish it from the *empirical*, or also *original apperception*, because it is that self-consciousness, which—in so far as it brings forth the representation *I think* that must be able to accompany all others, and in all consciousness is one and the same—can be accompanied in turn by no other." Thus, the *I think* is the subject of which all other representations are predicated, whereas it can be predicated of no other representation in turn, and it is in precisely this sense that the *I think* cannot itself be a concept.

[15] It is at this point that two earlier themes from § 15 converge: that the original combination exercised by the understanding can act on either *the manifold of intuition* or several concepts (B130), and that the unity effected by this act is not that of any particular *category* (B131).

Thus, Kant begins with the two fundamental geometrical constructions (of lines and circles), and, after referring to a further construction (of three perpendicular lines), he emphasizes the motion ("as action of the subject") involved in drawing a straight line (and therefore involved in any further geometrical construction as well).

In the appended footnote, finally, Kant says that the relevant kind of motion (as an action of the subject rather than a determination of an object), "is a pure act of successive synthesis of the manifold in outer intuition in general through the productive imagination, and it belongs not only to geometry [viz., in the construction of geometrical concepts—MF], but *even to transcendental philosophy* [presumably, in the unification of the whole of space, and time, as *formal intuitions*—MF]" (B155n; emphasis added).[16] And one should especially observe how the representation of time necessarily enters here along with that of space. In particular, the motion involved "in the *drawing* of a straight line" is what Kant calls "the outer figurative representation of **time**" (B154; bold emphasis added).[17]

3. SPACE, THE CATEGORIES OF QUANTITY, AND THE UNITY OF APPERCEPTION

I shall return to the "figurative" representation of time in the last section of this essay. For now, however, I shall add some further reflections on what we have already learned about space. This will help to clarify the special role of space and geometry, for Kant, among the mathematical sciences. It will thereby clarify, as well, the distinctive contribution of space and geometry within his conception of the necessary a priori conditions underlying all human experience.

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[16] The footnote reads in full (B155n): "*Motion of an *object* in space does not belong in a pure science and thus not in geometry. For, that something is movable cannot be cognized a priori but only through experience. But motion, as the *describing* of a space, is a pure act of successive synthesis of the manifold in outer intuition in general through the productive imagination, and it belongs not only to geometry, but even to transcendental philosophy."

[17] Although there is no doubt that this "figurative" representation of time involves motion—and thus, in the words of the footnote, an "act of successive synthesis of the manifold in *outer* intuition in general" (B155n; emphasis added)—Kant is still clear in the main text that in order thereby to represent "time itself" we must attend solely "to the action of synthesis of the manifold, through which we successively determine *inner* sense, and thereby attend to this determination in *it*" (B154; emphasis added). Moreover, the next sentence insists that, in the motion question, "we abstract from ['the synthesis of the manifold in space'] and attend merely to the action by which we determine *inner* sense in accordance with its form" (B155; emphasis in the original). In the required representation of "time itself," therefore, we abstract from the circumstance that the representation of space (in the drawing of a straight line) is also involved and attend only to the act of successive synthesis (in time) by which different *times* are thereby determined as successive: for example, the time at which I have drawn a (completed) line segment is thereby determined as later than any time at which I have drawn only a (proper) *part* of this segment.

I have argued that an adequate understanding of the problematic footnote to § 26 involves the distinction Kant makes explicit in his comments on Kästner between metaphysical and geometrical space—where the latter is generated step by step via Euclidean constructions of particular figures (lines, circles, triangles, and so on), and the former is given all at once, as it were, as *actually* rather than merely *potentially* infinite. Metaphysical space is thus the single "all-encompassing" whole within which all Euclidean constructions—along with the schemata of all geometrical concepts—are thereby made possible. It is in precisely this way that its characteristic unity precedes and makes possible "all *concepts* of space" (B161n; emphasis added), that is, all concepts of determinate regions of space ("spaces" in the plural) constituting particular geometrical figures [*Gestalten*].

Kant first discusses the characteristic unity of concepts in relation to our cognition of their corresponding objects in § 17. The *understanding*, he says, is "the faculty of *cognitions*," where these "consist is the determinate relation of given representations to an object" (B137). But an *object*, Kant continues, "is that in whose concept a given intuition is *united*," and "all unification of representations requires the unity of consciousness in their synthesis" (ibid.). He illustrates these claims by the unification of a given spatial manifold under the concept of a line (segment), whose object is just the determinate spatial figure (the determinate line segment) thus generated (B137-138): "[I]n order to cognize anything in space, e.g., a line, I must *draw* it, and therefore bring into being synthetically a determinate combination of the manifold, in such a way that the unity of this action is at the same time the unity of consciousness (in the concept of a line), and only thereby is an object (a determinate space) first cognized."

Yet when Kant discusses "[s]pace, represented as *object* (as is actually required in geometry)" in the problematic footnote to § 26 (B160n), he does not mean an object in *this* sense: he does not mean the object of any particular geometrical concept (or, indeed, of any other concept). The single unitary space discussed in the first sentence of the footnote is not geometrical space but rather the metaphysical space that precedes and makes possible all (geometrical) "*concepts* of space" (B161n; emphasis added). Kant's philosophical (or "metaphysical") claim is then that these (geometrical) concepts, together with their finite bounded objects (particular spatial figures), are themselves only possible in virtue of the prior "all-encompassing" metaphysical space in which all such bounded objects appear as parts. This prior metaphysical space—the whole of space as a formal intuition—is not an object *of* the science of geometry but rather an object considered at an entirely different level of abstraction (peculiar to what Kant calls "transcendental philosophy"), which, from a philosophical as opposed to a purely geometrical point of view, can nevertheless be seen as *presupposed* by the science of geometry.[18]

Kant's more general philosophical claim concerns the role of space as a condition of the possibility of experience (empirical cognition)—and therefore its relationship, more specifically, to the pure concepts or categories of the understanding. The relevant concepts here are the categories of *quantity* or *magnitude* [*Größe*], and Kant emphasizes their role in his first illustration following the main argument of § 26:

> Thus, e.g., if I make the empirical intuition of a house into perception through apprehension of the manifold [of this intuition], the *necessary unity* of space and of outer sensible intuition in general lies at the basis, and I draw, as it were, its figure [*Gestalt*] in accordance with this synthetic unity of the manifold in space. Precisely the same synthetic unity, however, if I abstract from the form of space, has its seat in the understanding, and is the category of the synthetic unity of the homogeneous in an intuition in general, i.e., the category of *magnitude* [*Größe*], with which this synthesis of apprehension, i.e., the perception, must therefore completely conform. (B162)

All objects of outer sense, in other words, occupy determinate regions of space, and are therefore conceptualizable as measurable geometrical magnitudes (in determining, for example, how many square meters of floor space there are in a particular house).

Yet the pure intellectual concept of magnitudes as such, in contrast to the sub-species of specifically spatial (geometrical) magnitudes, "abstracts" from the form of space and considers only "the synthetic unity of the homogenous in an intuition in general"—or, as Kant puts it more fully in the Axioms of Intuition, it involves "the composition [*Zusammensetzung*] of the homogeneous and the consciousness of the

[18] I observed that interpreters have appealed to Kant's comments on Kästner while discussing space and geometry in § 26 (see note 10 above): notably, Martin Heidegger, in his lecture course on *Phenomenological Interpretation of Kant's* ***Critique of Pure Reason*** in the winter semester of 1927-28 (1977, § 9), and Michel Fichant (1997), published along with his French translation of Kant's comments. Both Heidegger and Fichant, however, interpret space as a "formal intuition" in the footnote to § 26 as *geometrical* space in the terminology of the comments on Kästner—so that, according to them, the formal intuition of space is derivative from the more original "form of intuition" within which geometrical construction takes place. But this reading is incompatible with Kant's claim in the footnote that space as a formal intuition is both unified and singular in the sense of the Aesthetic—and, most importantly, that it precedes and makes possible all *concepts* of space. Here I am in agreement with Béatrice Longuenesse: for her comments on Heidegger in this connection see Longuenesse (1998a, pp. 224-225); for her parallel comments on Fichant see Longuenesse (1998b/2005, pp. 67-69). I shall briefly return to the relationship between my reading and Longuenesse's below.

synthetic unity of this (homogeneous) manifold" (B202-203).[19] By "the composition of the homogeneous" Kant has primarily in mind the addition operation definitive of a certain magnitude kind (such as lengths, areas, and volumes), in virtue of which magnitudes within a single kind (but not, in general, magnitudes from different kinds) can be composed or added together so as to yield a magnitude equal to the sum of the two. Kant has primarily in mind, in other words, the Ancient Greek theory of ratios and proportion (rigorously formulated in Book V of the *Elements*), but now extended well beyond the realm of geometry proper to encompass a wide variety of physical magnitudes (including masses, velocities, accelerations, and forces) in the new science of the modern era.[20]

Nevertheless, despite this envisioned extension, Kant takes specifically geometrical magnitudes to be primary. In the Axioms of Intuition he again appeals, in the first place, to the successive synthesis involved in drawing a line (A162-163/B203): "I can generate no line, no matter how small, without drawing it in thought, i.e., by generating all its parts successively from a point, and thereby first delineating this intuition." He then refers to the axioms of geometry (B163/B204): "On this successive synthesis of the productive imagination in the generation of figures is grounded the mathematics of extension (geometry), together with its axioms, which express the conditions of a priori sensible intuition under which alone the schema of a pure concept of outer intuition can arise." And he finally asserts that the axioms of geometry, in this respect, are uniquely privileged (ibid.): "These are the axioms which properly concern only magnitudes (*quanta*) as such."

The sense in which geometry is thereby privileged becomes clearer in the immediately following contrast with *quantity* (*quantitas*) and the science of arithmetic (A163-164/B204): "But in what concerns quantity (*quantitas*), i.e., the answer to the question how large something is, there are in the proper sense no axioms, although various of these propositions are synthetic and immediately certain (*indemonstrabilia*)." Kant illustrates the latter with "evident propositions of numerical relations," such as "7 +

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[19] More fully (B202-203): "All appearances contain, in accordance with their form, an intuition in space and time, which lies at the basis of all of them a priori. They can therefore be apprehended in no other way—i.e., be taken up in empirical consciousness—except through the synthesis of the manifold whereby a determinate space or time is generated, i.e., through the composition [*Zusammensetzung*] of the homogeneous and the consciousness of the synthetic unity of this (homogeneous) manifold. But the consciousness of the homogeneous manifold in intuition in general, in so far as the representation of an object first becomes possible, is the concept of a magnitude (*quanti*)."

[20] For discussion of the Ancient Greek theory of ratios and proportion see Stein (1990). For further discussion of this theory in relation to Kant see Friedman (1990), Sutherland (2004a) (2004b) (2006).

5 = 12," which are "singular" and "not general, like those of geometry" (A164/B205).[21] The import of this last distinction, in turn, becomes clearer in Kant's important letter of November 25, 1788 (to his student Johann Schultz) concerning the science of arithmetic (10, 555): "Arithmetic certainly has no *axioms*, because it properly has no *quantum*, i.e., no object [*Gegenstand*] of intuition as magnitude as object [*Obiecte*], but merely *quantity* [*Quantität*], i.e., the concept of a thing in general through determination of magnitude." Instead, Kant continues, arithmetic has only "*postulates*, i.e., immediately certain practical judgements," and he illustrates the latter by the singular judgement "3 + 4 = 7" (10, 555-556).

Kant's claim, therefore, is that arithmetic, unlike geometry, has no proper domain of objects of its own—no *quanta* or objects of intuition as magnitudes. Arithmetic is rather employed in calculating the magnitudes of any such *quanta* there happen to be, but the latter, for Kant, must be given from outside of arithmetic itself. Kant thus does not understand arithmetic as we do: as an axiomatic science formulating universal truths about the (potentially) infinite domain of natural numbers. Nor, in Kant's own terms, is arithmetic an axiomatic science like geometry, which formulates universal truths about the (potentially) infinite domain of geometrical figures generated by Euclidean constructions—which, as we have seen, can be given or constructed in *pure* (rather than *empirical*) intuition. In particular, the potential infinity of this domain is guaranteed by the single, all-encompassing, and actually infinite formal intuition of space, which, in the end, constitutes the pure form of all outer (spatial) perception.

My reading of how the transcendental unity of apperception originally unifies our pure form of spatial intuition into a corresponding "all-encompassing" formal intuition is thus essentially connected with the science of geometry—the most fundamental science of mathematical magnitude. For I understand the pure form of intuition of space as a mere (not yet synthesized) manifold of possible spatial perspectives on possible objects of outer sense, where each such perspective comprises a point of view and an orientation with respect to a local spatial region in the vicinity of a perceiving subject. The unity of apperception then transforms such a not yet unified manifold into a single unitary *space* by the requirement that any such local perspective must be accessible to the same perceiving subject via (continuous) motion—via a (continuous) sequence of translations

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[21] Kant here illustrates the generality of geometry by the Euclidean construction of a triangle in general (A164-165/B205): "If I say that through three lines, of which two taken together are greater than the third, a triangle can be drawn, I have here the mere function of the productive imagination, which can draw the lines greater or smaller, and thereby allow them to meet at any and all arbitrary angles." (This is Proposition I.22; compare note 11 above.)

and rotations. And this implies, as we have seen, that the science of geometry must be applicable to all outer objects of perception. Space is thereby necessarily represented as comprising all specifically *geometrical* mathematical magnitudes.

It does not follow, however, that the transcendental unity of apperception and the pure concepts of the understanding take over the role of our pure forms of intuition, that there is no independent contribution of sensibility as in the conception of the Marburg School. Rather, the representation of space as a formal intuition—as a single unitary (metaphysical) space within which all geometrical constructions take place—is a direct realization, as it were, of the transcendental unity of apperception *within* our pure form of outer intuition. For this form of intuition originally consists of an aggregate or manifold of possible local spatial perspectives, which the transcendental unity of apperception then transforms into a single, unitary, geometrical (Euclidean) space in the way that I have sketched above. Whereas our original form of outer intuition does not have the (geometrical) structure in question independently of transcendental apperception, it is equally true that no such realization of the latter can arise independently of our original form of outer intuition: this *particular* realization of the unity of apperception can by no means be derived in what Kant calls a manifold of intuition in general.[22]

That there is a uniquely privileged mathematical science, the science of geometry, which establishes universal truths about a special domain of magnitudes (spatial regions as *quanta*) constructible in pure intuition, therefore depends on the existence of *our* pure form of outer intuition. Yet it also depends—mutually and equally—on the action of the transcendental unity of apperception (understood, in the first instance, in terms of a manifold of intuition in general) on this particular form of sensibility. Sensibility does make an independent contribution to the synthetic determination of appearances by the understanding, but it cannot make this contribution, of course, independently of the understanding. In particular, the distinctively geometrical structure realized in *our* pure form of outer intuition, on my reading, is the one and only realization of the unity of apperception in a domain of objects or magnitudes constructible in pure intuition.[23] And the understanding, on my reading, can only subsequently operate on empirical intuition through the *mediation* of the resulting formal intuition of space. The fundamental aim of the understanding, in this context, is to secure the possibility of the modern mathematical

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[22] As we have seen, the pure intellectual concept of magnitudes in general abstracts from the structure of specifically spatial (geometrical) magnitudes and involves only "the synthetic unity of the homogenous in an intuition in general" (B162; compare note 19 above, together with the paragraph to which it is appended).

[23] I am here indebted to a very helpful conversation with Graciela De Pierris concerning the precise connection between the transcendental unity of apperception and geometry in my reading.

science of nature—which, as suggested, essentially involves a greatly expanded domain of physical magnitudes extending far beyond those traditionally considered in geometry.[24] It is in this way, as we shall now see, that we can finally secure the possibility of what Kant calls experience.[25]

4. TIME DETERMINATION, LAWS OF NATURE, AND EXPERIENCE

When discussing the question "How is pure mathematics possible?" in the *Prolegomena to any Future Metaphysics* (1783), Kant isolates three principal mathematical sciences, namely, geometry, arithmetic, and "pure mechanics" (4, 283): "Geometry takes as basis the pure intuition of space. Even arithmetic brings its concepts of numbers into being through the successive addition of units in time; above all, however, pure mechanics can bring its concept of motion into being only by means of the representation of time." This passage suggests that it is pure mechanics, rather than arithmetic, which relates most directly to time. The reason, as Kant explains in the letter to Schultz, is that numbers are not themselves temporal entities. Numbers are only "pure determinations of magnitude," and not, like "every alteration (as a *quantum*)," properly temporal objects (10, 556-557).[26] Whereas all calculation with numbers takes place within our pure intuition of time, the numbers themselves do not relate to parts of time or temporal intervals in the way that the science of geometry—through the construction of figures—relates to the parts of space or spatial regions corresponding to these figures.

Kant added two new sections to the Transcendental Aesthetic in the second edition of the *Critique*: a "transcendental exposition of the concept of space" (§ 3) and a "transcendental exposition of the concept of time" (§ 5). The first argues that space is indeed a pure or a priori intuition by appealing to the synthetic a priori science of

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[24] Compare the paragraph to which note 20 above is appended.

[25] As suggested, I am in agreement with Béatrice Longuenesse concerning fundamental issues surrounding the interpretation of § 26 (see note 18 above). In particular, I agree with her that the figurative synthesis that unifies space and time as formal intuitions is pre-conceptual and thus pre-categorical—and, accordingly, that it proceeds directly from the transcendental unity of apperception without relying on any particular category. Yet the understanding originally affects sensibility, for Longuenesse, in *empirical* rather than *pure* intuition: in the process of "comparison, reflection, and abstraction" by which we ascend from what is sensibly given in perception to form ever more general empirical concepts. For a detailed discussion of the resulting differences between our two readings—which involve, in particular, our differing conceptions of the application of the categories of quantity—see Friedman (2015).

[26] More fully (ibid.): "Time, as you correctly remark, has no influence on the properties of numbers (as pure determinations of magnitude), as [it does], e.g., on the properties of every alteration (as a *quantum*), which is itself only possible relative to a specific constitution of inner sense and its form (time), and the science of number, regardless of the succession that every construction of magnitude requires, is a pure intellectual synthesis, which we represent to ourselves in thought."

geometry. The second, however, introduces the consideration of a new mathematical science—not mentioned in the first edition—which Kant calls the "general doctrine of motion [*allgemeine Bewegungslehre*]":

> Here I may add that the concept of alteration and, along with it, the concept of motion (as alteration of place) is possible only in and through the representation of time: so that, if this representation were not an a priori (inner) intuition, no concept, whatever it might be, could make an alteration—i.e., the combination of contradictorily opposed predicates (e.g., the being and not-being of one and the same thing at one and the same place)—conceivable. Only in time can two contradictorily opposed determinations in one thing be met with, namely, *successively*. Therefore, our concept of time explains as much synthetic a priori knowledge as is set forth in the general doctrine of motion, which is by no means unfruitful. (B48-49)

This strongly confirms the idea that it is the mathematical science of motion ("pure mechanics"), not arithmetic, which relates to time as geometry does to space—as the latter science, in particular, relates to the parts of space or spatial regions corresponding to geometrical figures.[27]

It is in the *Metaphysical Foundations of Natural Science* (1786) that Kant develops in detail what he takes to be the synthetic a priori principles contained in the general doctrine of motion. He explains in the Preface that the natural science for which he is providing a metaphysical foundation is "either a pure or an applied *doctrine of motion* [*reine oder angewandte Bewegungslehre*]" (4, 476). Moreover, he concludes the Preface by saying that he wants to bring his enterprise "into union with the mathematical doctrine of motion [*der mathematischen Bewegungslehre*]" (478) and suggesting that it is

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[27] In the "figurative" representation of time via the pure act of motion "in the *drawing* of a straight line" (B154) we abstract from the representation of *space* and attend only to the act of successive synthesis (in time) by which different *times* are determined as successive (see note 17 above). We thereby arrive at the representation of "time itself" as a single one-dimensional (continuous) ordering—in which any two different times are ordered as successive—by "inferring from the properties of ['a line progressing to infinity'] to all the properties of time, with the exception that the parts of the former are simultaneous while those of the latter are always successive" (A33/B50). In this way, the conclusion that the whole-part structure of time generates what we now call a total (continuous) linear ordering (compare A31-32/B47) corresponds to the conclusion that the parts of space are all contained within a single "all-encompassing" (metaphysical) space and thereby generate the structure of a single (Euclidean) space (A24-25/B39). The crucial difference is that, whereas the "figurative" representation of "time itself" thereby makes possible the science of number or arithmetic (which certainly presupposes, for Kant, the possibility of indefinite succession in time), it does not yet constitute the determination of parts of time as mathematical magnitudes. As explained below, time only acquires what we would now call a *metrical* structure by means of precisely the mathematical theory of motion—where, in particular, we can no longer abstract from space. Here I am especially indebted to comments from Greg Taylor.

Newton's *Principia*, in particular, which he here has in mind.[28] Then, in the first chapter or Phoronomy, Kant characterizes the object of this synthetic a priori science as "the movable in space" (480) and remarks that "this concept, as empirical, could only find a place in a natural science, as applied metaphysics, which concerns itself with a concept given through experience, although in accordance with a priori principles" (482). Nevertheless, he also suggests that he is envisioning a *transition* from what § 24 of the B Deduction will call the pure act of motion of the subject—"as the *describing* of a space" (B155n)—to the motion of an empirically given object (a perceptible body) considered in the *Metaphysical Foundations*. For he begins the Phoronomy by considering moving matter as an abstract mathematical point—whereby "motion can only be considered as the *describing of space*" (489)—and reserves its subsumption under the more empirical concept of an extended (massive) body for later.[29]

The *Metaphysical Foundations* is organized into four main chapters—the Phoronomy, Dynamics, Mechanics, and Phenomenology—in accordance with the four headings of the table of categories (quantity, quality, relation, and modality). In the third chapter Kant formulates his own three "Laws of Mechanics," which he employs in the fourth chapter to determine the true or actual motions in the cosmos from the merely apparent motions that we observe from our parochial position here on the surface of the earth.[30] He thereby shows how we can move from the mere "appearance [*Erscheinung*]" of motion to a determinate "experience [*Erfahrung*]" thereof (554-555). Moreover, whereas Kant's three Laws of Mechanics are derived as more specific realizations or instantiations of the three Analogies of Experience, his procedure for determining true from merely apparent motions involves a more specific realization or instantiation of the three Postulates of Empirical Thought. He determines the true from the merely apparent motions, in other words, by successively applying the three modal categories of possibility, actuality, and necessity. I have argued elsewhere, and in great detail, that

[28] For further discussion of the relationship between Kant's "mathematical doctrine of motion" and Newton's *Principia* see Friedman (2012b).

[29] The quoted passage reads more fully (489): "In phoronomy, since I am acquainted with matter through no other property but its movability, and thus consider it only as a point, motion can only be considered as the *describing of a space*—in such a way, however, that I attend not solely, as in geometry, to the space described, but also to the time in which, and thus to the velocity with which, a point describes a space. Phoronomy is thus just the pure theory of magnitude (*Mathesis*) of motion." Thus it is clear that we attend to *both* space *and* time (and thus to velocity) in this representation of motion. For further discussion of the transition from pure to empirical motion see Friedman (2012b) and (more fully) Friedman (2013).

[30] Kant's three Laws of Mechanics are the conservation of the total quantity of matter, the law of inertia, and the equality of action and reaction; compare the discussion (and illustration) of the synthetic a priori propositions of pure natural science in the Introduction to the second edition of the *Critique* (B20-21). I (briefly) comment on the relationship between these laws and the Newtonian Laws of Motion in Friedman (2012b) and (more fully) in Friedman (2013).

Kant's model for this procedure is precisely Book 3 of the *Principia*, where Newton determines the true motions in the solar system from the initial "Phenomena" encapsulated in Kepler's laws of planetary motion and, at the same time, thereby establishes the law of universal gravitation.[31]

It is especially significant that Kant's Laws of Mechanics are more specific realizations of the Analogies of Experience. For the latter are characterized in the first *Critique* as the fundamental principles for the determination of time:

> These, then, are the three analogies of experience. They are nothing else but the principles for the determination of the existence of appearances in time with respect to all of its three modes, the relation to time itself as a magnitude (the magnitude of existence, i.e., duration), the relation in time as a series (successively), and finally [the relation] in time as a totality of all existence (simultaneously). This unity of time determination is thoroughly dynamical; that is, time is not viewed as that in which experience immediately determines the place of an existent, which is impossible, because absolute time is no object of perception by means of which appearances could be bound together; rather, the rule of the understanding, by means of which alone the existence of the appearances can acquire synthetic unity with respect to temporal relations, determines for each [appearance] its position in time, and thus [determines this] a priori and valid for each and every time. (A215/B262)

Just as we need the transcendental unity of apperception, in connection with the categories of quantity, to secure the application of the mathematical science of geometry to all objects that may be presented within this form, we need the same transcendental unity of the understanding, in connection with the categories of relation, to generate a parallel mathematical structure (for duration, succession, and simultaneity) governing all objects that may be presented to us in time—that is, all objects of the senses whatsoever. And it is only at this point, in particular, that parts of time (temporal intervals) are themselves determined as mathematical magnitudes.[32]

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[31] See, e.g., Friedman (2012c) and (more fully) Friedman (2013).

[32] Compare note 27 above. In applying the Analogies of Experience to the mathematical science of motion, in particular, we determine the magnitudes of temporal intervals by reference to idealized perfectly uniform motions, which then set the standard for correcting the actually non-uniform motions found in nature. In Newton's famous remarks about "absolute, true, and mathematical time" in the *Principia* (1999, 408), for example, we thereby correct the common "sensible measures" of time such as "an hour, a day, a month, a year" (ibid.)—and I argue in Friedman (2013) that Kant takes this procedure as his model for time determination in the above passage from the Analogies (A215/B262). I also argue that Kant has the same procedure in mind in his Second Remark to the Refutation of Idealism, according to which, for example,

But there is a crucial disanalogy between the two cases. The objects or magnitudes (*quanta*) considered in geometry, as explained, can be given or constructed in pure intuition—which, in turn, is the necessary form of all empirical intuition of outer objects. The Axioms of Intuition, therefore, are constitutive of such objects as appearances. The Analogies of Experience, however, as what Kant calls "dynamical" rather than "mathematical" principles, are concerned with "*existence* [*Dasein*] and the *relation* among [the appearances] with respect to [their] existence" (A178/B220). Further, because "the existence of appearances cannot be cognized a priori" (A178/B221), because "[existence] cannot be constructed" (A179/B221), the latter principles, unlike the former, cannot be constitutive of appearances (A180/B222-223): "An Analogy of Experience will thus only be a rule in accordance with which from perceptions unity of experience may arise (not, like perception itself, as empirical intuition in general), and it is valid as [a] principle of the objects (the appearances) not *constitutively* but merely *regulatively*."[33]

I can now delineate more exactly the uniquely privileged role of the mathematical science of geometry in Kant's conception of the possibility of experience. Geometry, for Kant, involves a procedure whereby all the objects of this science—all the figures considered in Euclid's geometry—are constructed step-by-step in pure intuition within space as a singular and unitary formal intuition (metaphysical space). Since all (outer) appearances as empirical intuitions are also given within this space, geometry necessarily applies to all objects of outer sense merely considered as objects of perception or appearance. The mathematical structure of time resulting from the general doctrine of motion, by contrast, can by no means be constructed in pure intuition. It can only arise within the context of the relational categories, and it thus involves a crucial transition from objects of perception or appearance to objects of what Kant calls experience. So we can only determine objects within this structure as objects of experience by beginning our determination in empirical rather than pure intuition.[34]

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we "undertake [*vornehmen*]" such time determination from the observed "motion of the sun with respect to objects on the earth" (B277-278).

33 Although the Analogies of Experience are thus not constitutive of *appearances*, they are (of course) constitutive of what Kant calls "experience." Compare Kant's discussion of this distinction in the Appendix to the Transcendental Dialectic (A664/B692).

34 There is thus a fundamental difference between the "figurative" representation of "time itself" (as a formal intuition) in § 24 of the Deduction and time determination in accordance with the Analogies. While the former takes place in pure intuition and determines time only as a one-dimensional (continuous) ordering (see again note 27 above), the latter takes place in empirical intuition and thereby determines the parts of time as mathematical magnitudes. A transition from the first of these two perspectives to the second is visible in the General Remark to the System of Principles—where, after emphasizing the importance of instantiating the relational categories in *outer* intuition, Kant concludes (B293): "It can

This crucial asymmetry, for Kant, between the mathematical structure of time and that of space sheds further light on the independent contribution of the faculty of sensibility to the determination of the objects of experience by the understanding. I have explained the independent contribution of space as the form of outer sense in terms of the circumstance that geometry is the only mathematical science whose objects (as magnitudes) are determinable in pure intuition. It is now clear, however, that it is only by taking account of the characteristic structure of both space and time—the structure of our *spatio-temporal* sensibility—that we can fully appreciate the way in which our understanding can similarly determine the objects of experience. For the latter objects can only be so determined in empirical rather than pure intuition, and, for this purpose, we need to make a transition from *perception* (in accordance with the mathematical principles) to *experience* (in accordance with the dynamical principles).

The *Metaphysical Foundations*, I have suggested, takes the argument of Book 3 of the *Principia* as its model for determining true from merely apparent motions, and thus for determining "experience" from "appearance." In this procedure Kant substitutes his own Laws of Mechanics for Newton's Laws of Motion, where these Laws of Mechanics, in turn, are more specific realizations or instantiations of the Analogies of Experience. The determination in question, moreover, proceeds in accordance with the modal categories of possibility, actuality, and necessity, and thus by a more specific realization or instantiation of the Postulates of Empirical Thought.[35] So at the end of Kant's procedure, in particular, we have determined the resulting causal interactions between each body and every other body subject to the law of universal gravitation as *necessary* in the sense of the third Postulate (A218/B266): "That whose coherence [*Zusammenhang*] with the actual is determined in accordance with the universal conditions of experience, is (exists as) *necessary*." Indeed, as I have argued in detail elsewhere, it turns out that the law of universal gravitation itself (in sharp contrast with the Keplerian Phenomena from

easily be shown that the possibility of things as *magnitudes*, and therefore the objective reality of the category of magnitude, can also only be exhibited in outer intuition, and only by means of it subsequently applied to inner sense as well." And he remarks that we thereby obtain a "confirmation" of the earlier Refutation of Idealism (ibid.). Indeed, a similar point is already suggested in § 24 of the Deduction—where, towards the end of the paragraph that we have been considering, Kant concludes that "we must always derive the determination of the lengths of time [*Zeitlänge*], or even the places in time for all inner perceptions, from that which outer things present to us as alterable, and we must therefore order the determinations of inner sense as appearances in time in precisely the same way as we order those of the outer senses in space" (B156). But further discussion of the precise relationship between the "figurative" representation of "time itself" and the quite different perspective of the Refutation of Idealism will have to wait for another occasion.

[35] See the paragraph to which note 31 above is appended.

which it is inferred) is thereby determined, at the same time, as a universally valid and necessary *law*—as opposed to a merely inductive regularity or general "rule."[36]

It follows, more generally, that the transition from what Kant calls "perception" to what he calls "experience" is also a transition from that which is merely *actual* (in the sense of the Postulates) to that which is *necessary* (in the same sense). For Kant says of the Postulates as a whole that they "together concern the synthesis of mere intuition (the form of appearance), of perception (the matter of appearance), and of experience (the relation of these perceptions)" (A180/B223). Indeed, in the second edition Kant reformulates the general principle governing all three Analogies so as, in effect, to explain "experience" in terms of such necessity (B218): "*Experience is only possible through the representation of a necessary connection* [*Verknüpfung*] *of perceptions*." He also adds an important footnote to his preliminary discussion of all four sets of principles distinguishing the original "combination [*Verbindung*]" of the understanding into two distinct subspecies (B201n): "All *combination* (*conjunctio*) is either *composition* [*Zusammensetzung*] (*compositio*) or *connection* [*Verknüpfung*] (*nexus*)." The former concerns a synthesis of elements of the manifold that do "*not* belong *necessarily to one another*," as in "the synthesis of the *homogeneous* in all that can be considered *mathematically*" (ibid.). The latter concerns a synthesis of elements in so far as they "belong *necessarily to one another*, such as, e.g., the accidents to any substance, or the effect to the cause" (ibid.).

It lies well beyond the scope of this essay to follow the many complexities in Kant's treatment of the specifically dynamical categories and principles any further. I shall bring my discussion to a close, therefore, by emphasizing the two central points involved in the just-quoted distinction between *composition* and *connection*. In the first place, the former notion designates the composition of the homogeneous considered in the traditional theory of mathematical magnitudes as paradigmatically instantiated in geometry.[37] So the transition from this notion to the latter (from perception to experience) corresponds to the uniquely privileged role of geometry and the categories of quantity in extending this traditional theory to all other mathematical magnitudes—including, in particular, the representation of temporal duration itself as a magnitude. In the second place, the latter notion essentially involves the concept of *necessary*

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[36] Friedman (2012c) is my most recent detailed discussion of this point.

[37] Kant illustrates the first kind of combination by a geometrical example (B201n): "[F]or example, the two triangles into which a square is divided by the diagonal do not necessarily belong to one another in themselves, and of this kind is the synthesis of the *homogeneous* in all that can be considered *mathematically*."

connection in accordance with the Analogies of Experience.[38] So it involves Kant's fundamental differences with Hume concerning laws of nature and the associated causal connections—which differences are more directly and extensively discussed in the *Prolegomena*.

It is well worth noting, therefore, that both of these points are already suggested in the parts of § 26 of the Deduction from which I have already quoted. The first is clearly suggested by the circumstance that, after Kant has referred us back to the Transcendental Aesthetic and emphasized the special role of space and the science of geometry in the attached footnote, the concluding sentence of the main argument introduces the transition from perception to experience associated with the notion of connection (B161): "Consequently all synthesis, even that whereby perception becomes possible, stands under the categories, and, since experience is knowledge through connected [*verknüpfte*] perceptions, the categories are conditions of the possibility of experience, and thus are a priori valid for all objects of experience." The second is suggested by the introductory remarks where Kant announces the goal of the argument to follow: namely, to explain "the possibility of knowing a priori, *by means of categories*, whatever objects *may present themselves to our senses*, not, indeed, with respect to the form of their intuition, but with respect to the laws of their combination—**and thus to prescribe the law to nature and even make nature possible**" (B159; bold emphasis added).

To be sure, Kant does not explicitly mention either Hume or the concept of necessary connection in these introductory remarks. But his discussion here, the remainder of § 26, and the concluding § 27 of the Deduction runs parallel, in several respects, to the corresponding discussion in the *Prolegomena*. Thus, for example, Kant concludes § 36 of the *Prolegomena* with the striking claim (4, 320): "*The understanding does not extract its laws* (*a priori*) *from, but prescribes them to, nature*." The introductory remarks in § 26, as we have just seen, clearly echo this claim. Similarly, § 36 of the *Prolegomena* asks (318): "How is nature possible in the *formal* sense, as the sum total of the rules to which appearances must be subject if they are to be thought as connected [*verknüpft*] in one experience?" The remainder of § 26 investigates "the original ground of [nature's] necessary lawfulness (as *natura formaliter spectata*)" (B165).

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[38] Kant explains that the "second [kind of] combination (*nexus*) is the synthesis of the manifold, in so far as [its elements] belong *necessarily to one another*, . . . and thus [they are] also represented *as inhomogeneous* yet necessarily combined" (B201n). Kant continues (ibid.): "[This] combination, since it is not arbitrary, I therefore call *dynamical*, because it concerns the combination of the *existence* of the manifold."

What is most striking, however, is that the concluding § 27 rejects an alternative "*preformation-system* of pure reason" (B167). Kant's point is that, on this system, we would be left with only a "subjective necessity" attaching to the relation of cause and effect (B168): "I would not be able to say that the effect is combined with the cause in the object (i.e., necessarily), but only that I am so constituted that I can think this representation in no other way than as so connected [*verknüpft*]—precisely that which the skeptic most desires." So Kant here appears to be countering specifically Humean skepticism with his own explanation of the ground of the *objective* necessity that he takes to be involved.[39] Moreover, the discussion of laws of nature in §§ 26 and 27 corresponds to the three main sections (§§ 36-38) at the end of the discussion of the "Second Part of the Main Transcendental Question: *How is pure natural science possible?*" in the *Prolegomena*. In § 37 Kant says that he will illustrate his "seemingly bold proposition"—that the understanding prescribes laws to nature (320)—with "an example, which is supposed to show that laws which we discover in objects of sensible intuition, especially if these laws have been cognized as necessary, are already held by us to be such as have been put there by the understanding, although they are otherwise in all respects like the laws of nature that we attribute to experience" (ibid.). The example of such a law that Kant considers in the following section § 38 is none other than the law of universal gravitation (321): "a physical law of reciprocal attraction, extending to all material nature, the rule of which is that these attractions decrease inversely with the square of the distance from each attracting point."

It is not too far fetched to suppose, therefore, that Newtonian natural science in general and the law of universal gravitation in particular are just as relevant to the "answer to Hume" suggested in the last two sections of the Deduction as they are (explicitly) in the Second Part of the *Prolegomena*.[40] And it is quite clear, in any case, that Kant's treatment of the possibility of experience in the Deduction is just as involved with the question of how pure natural science is possible. The formal intuition of space

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[39] For an illuminating discussion see Pollok (2008), which discusses the B Deduction against the background of both the *Prolegomena* and the lengthy footnote to the Preface of the *Metaphysical Foundations* (4, 474-476) where Kant sketches a revised version of the Deduction already in 1786.

[40] The relevance of the law of universal gravitation, in particular, is suggested in § 19 of the Deduction, which develops an account of the "*necessary unity*" belonging to the representations combined in any judgement as such—"i.e., a relation that is *objectively valid*, and is sufficiently distinguished from the relation of precisely the same representations in which there would be only subjective validity, e.g., in accordance with laws of association" (B142). Kant illustrates his point by the relation between subject and predicate in the judgement "Bodies are heavy" (ibid.). This discussion continues the "answer to Hume" developed in the *Prolegomena*, and the example Kant chooses invokes universal gravitation as discussed in both § 38 of the *Prolegomena* and the *Metaphysical Foundations*. For a detailed discussion see Friedman (2012c).

as a whole highlighted in the footnote to § 26—"[s]pace represented as *object* (as is actually required in geometry)" (B160n)—is the three-dimensional, infinite, "geometrized" space central to the new science of nature. It is that space in which all of nature is contained so as thereby to subject it to a unified system of mathematically formulated universally valid laws. This essentially modern conception of the laws of nature, Kant sees, has been finally successfully realized by Newton, who shows, for the first time, how we can thereby rigorously treat temporal duration as a mathematical magnitude as well. Kant incorporates this insight into his own revolutionary conception of transcendental time determination in accordance with the Analogies of Experience, whereby the universally valid and necessary laws of nature turn out to be prescribed to nature by us. Nature, on this conception, is nothing more nor less than the sum total of sensible objects in space and time, as necessarily subject to the lawgiving activity of the understanding. And it is in precisely this way that nature itself, for Kant, becomes the necessarily correlative object of our (human) experience.

References

Allison, H. E. (1973), *The Kant-Eberhard Controversy* (Baltimore and London: Johns Hopkins University Press).

Cohen, H. (1871), *Kant's Theorie der Erfahrung* (Berlin: Dümmler; 2nd ed., 1885; 3rd ed., Bruno Cassirer, 1918).

Fichant, M. "'L'Espace est représenté comme une grandeur infinie donnée'. La radicalité de l'Esthétique," *Philosophie* 50 (1997), pp. 20-48.

Friedman, M. (1985), "Kant's Theory of Geometry," *Philosophical Review* 94, pp. 455-506; reprinted in Friedman (1992), pp. 55-95.

Friedman, M. (1990),"Kant on Concepts and Intuitions in the Mathematical Sciences," *Synthese* 84, pp. 213-257; reprinted in Friedman (1992), pp. 96-135.

Friedman, M. (1992), *Kant and the Exact Science* (Cambridge, Mass.: Harvard University Press).

Friedman, M. (2000), "Geometry, Construction, and Intuition in Kant and his Successors," in G. Scher and R. Tieszen, eds., *Between Logic and Intuition: Essays in Honor of Charles Parsons* (Cambridge: Cambridge University Press), pp. 186-218.

Friedman, M. (2012a), "Kant on Geometry and Spatial Intuition," *Synthese* 186, pp. 231-255.

Friedman, M. (2012b), "Newton and Kant: Quantity of Matter in the *Metaphysical Foundations of Natural Science*," *Southern Journal of Philosophy* 50, pp. 482-503.

Friedman, M. (2012c), "The *Prolegomena* and Natural Science," in H. Lyre and O. Schliemann, eds., *Kants Prolegomena. Ein kooperativer Kommentar* (Frankfurt am Main: Klostermann), pp. 299-326.

Friedman, M. (2013), *Kant's Construction of Nature: A Reading of the* ***Metaphysical Foundations of Natural Science*** (Cambridge: Cambridge University Press).

Friedman, M. (2015), "Kant on Geometry and Experience," in V. De Risi, ed., *Mathematizing Space: The Objects of Geometry from Antiquity to the Modern Age* (Basel-Boston-Berlin: Birkhäuser), pp. 275-309.

Hegel, G. W. F. (1802), "Glauben und Wissen, oder die Relfexionsphilosophie der Subjektivität in der Vollständigkeit ihrer Formen als Kantische, Jacobische und Fichtesche Philosophie," *Kritisches Journal der Philosophie* 2.1; (Hamburg: Meiner, 1962); translated as *Faith and Knowledge* (Albany: State University of New York Press: 1977).

Heidegger, M. (1929), *Kant und das Problem der Metaphysik* (Bonn: Friedrich Cohen); translated (from the 4th ed., Frankfurt am Main: Klostermann, 1973) as *Kant and the Problem of Metaphysics* (Bloomington and Indianapolis: Indiana University Press).

Heidegger, M. (1977), *Phänomenologische Interpretation von Kant's Kritik der reinen Vernunft* (Frankfurt am Main: Klostermann); translated as *Phenomenological Interpretation of Kant's* ***Critique of Pure Reason*** (Bloomington and Indianapolis: Indiana University Press, 1997).

Henrich, D. (1955), "Über die Einheit der Subjektivität," *Philosophische Rundschau* 3, pp. 28-69; translated in *The Unity of Reason* (Cambridge, Mass.; Harvard University Press, 1994), pp. 17-54.

Henrich, D. (1968-69), "The Proof-Structure of Kant's Transcendental Deduction," *Review of Metaphysics* 22, pp. 640-659.

Longuenesse, B. (1998a), *Kant and the Capacity to Judge: Sensibility and Discursivity in the Transcendental Analytic of the* ***Critique of Pure Reason*** (Princeton: Princeton University Press).

Longuenesse, B. (1998b), "Synthèse et donation. Réponse à Michel Fichant," *Philosophie* 60, pp. 79-91; translated in *Kant on the Human Standpoint* (Cambridge: Cambridge University Press, 2005), pp. 64-78.

Newton, I. (1999). *The* ***Principia****: Mathematical Principles of Natural Philosophy*, translated and edited by I. Bernard Cohen and Anne Whitman, assisted by Julia Budenz (Berkeley and Los Angeles: University of California Press).

Parsons, C. (1992). "The Transcendental Aesthetic," in P. Guyer, ed., *The Cambridge Companion to Kant* (Cambridge: Cambridge University Press), pp. 62-100.

Pollok, K. (2008), "'An Almost Single Inference'—Kant's Deduction of the Categories Reconsidered," *Archiv für Geschichte der Philosophie* 90, pp. 323-345.

Stein, H. (1990). "Eudoxus and Dedekind: On the Ancient Greek Theory of Ratios, and Its Relation to Modern Mathematics," *Synthese* 84, pp. 163-211.

Sutherland, D. (2004a), "The Role of Magnitude in Kant's Critical Philosophy," *Canadian Journal of Philosophy* 34, pp. 411-442.

Sutherland, D. (2004b), “Kant’s Philosophy of Mathematics and the Greek Mathematical Tradition,” *Philosophical Review* 113, pp. 157-201.

Sutherland, D. (2006), “Kant on Arithmetic, Algebra, and the Theory of Proportion,” *Journal of the History of Philosophy* 44, pp. 533-558.