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Helmholtz's Zeichentheorie *and Schlick's* Allgemeine Erkenntnislehre: *Early Logical Empiricism and Its Nineteenth-Century Background*[1]

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The year 1921 marked the one-hundredth anniversary of the birth of Hermann von Helmholtz, one of the most remarkable minds of his era, who made fundamental contributions to energetics, physiological psychology, the foundations of geometry, electrodynamics, and epistemology. In philosophy, in particular, he became identified as a leader of the scientifically oriented "back to Kant!" movement and, as such, has a claim to be one of the principal founders of the discipline we now call philosophy of science.[2] When he died in 1894 Helmholtz was universally recognized as the greatest German scientific thinker of the nineteenth century. It is no wonder, then, that the centenary year 1921 was distinguished by a variety of memorial lectures, journal issues, monographs, and the like,[3] including a collection of Helmholtz's *Epistemological Writings* edited by Moritz Schlick and the physicist Paul Hertz.[4] Schlick himself was then identified as perhaps the most important of the small group of scientifically minded philosophers inspired by the revolutionary import of Einstein's theory of relativity. He had published a substantial paper on the special theory of relativity in 1915,[5] followed by an extremely influential exposition of the general theory, *Space and Time in Contemporary Physics,* first appearing in 1917 and going through four editions by 1922.[6] And his full-scale treatise on scientific epistemology, *General Theory of Knowledge,* had meanwhile appeared in 1918.[7] On the strength of these achievements, especially his work on the philosophical significance of the theory of relativity, Schlick was named in 1922 to the Chair for the Philosophy of the Inductive Sciences previously occupied by Ernst Mach and Ludwig Boltzmann at the University of Vienna, where he became the leader and guiding spirit of what we now know as the Vienna Circle.[8]

In the centenary collection of Helmholtz's *Epistemological Writings,* Schlick contributed extensive explanatory notes to the more explicitly philosophical pieces, where, among other things, he attempted to appropriate Helmholtz's ideas on behalf of his own developing version of scientific empiricism—in explicit opposition, for example, to parallel attempts at appropriation by neo-Kantian philosophers.[9] Of particular importance here is Schlick's attempt to assimilate Helmholtz's celebrated *Zeichentheorie* of perception, according to which our sensations must be viewed as *signs* [*Zeichen*], rather than pictures or *images* [*Bilder*] of external objects, to his own characteristic conception, developed at great length in *General Theory of Knowledge,* of the essence of knowledge and truth as consisting solely in a relation of *coordination* [*Zuordnung*] or *designation* [*Bezeichnung*] between our system of concepts and judgments and its objects.

In *General Theory of Knowledge* itself, Helmholtz is not explicitly invoked in this context, but we do find a parallel rejection of the "popular" or "naive" conception of knowledge as some kind of picturing or imaging of reality:

> Thus the concept of agreement [*Übereinstimmung*] melts away under the rays of analysis, insofar as it is supposed to mean sameness or similarity, and what remains is only the univocal coordination. In this consists the relation of true judgments to reality, and all those naive theories, according to which our judgments and concepts could somehow "picture [*abbilden*]" reality, are in principle destroyed. There remains no other meaning for the word agreement than that of univocal coordination.
>
> Univocality [*Eindeutigkeit*] is the sole essential virtue of a coordination, and, since truth is the sole virtue of judgment, truth must consist in the univocality of the designation for which the judgment is supposed to serve.[10]

And Schlick makes it amply clear, moreover, that concepts so conceived in their purely designative function operate merely as *signs* [*Zeichen*] and not as pictorial images [*Vorstellungen*] (§§ 5, 8).

These passages from *General Theory of Knowledge* parallel Helmholtz's classic statement of his *Zeichentheorie* in the most extensive presentation of his epistemological position, "The Facts in Perception," in 1878:

> Insofar as the quality of our sensation gives us a report of the character of the external influence through which it is excited, it may count as a *sign* [*Zeichen*] of the latter but not as an *image* [*Abbild*]. For of an image one requires some kind of sameness with the pictured [*abgebildeten*] object, of a statue sameness of form, of a delineation sameness of perspective projection in the visual field, of a painting also sameness of color. But a sign needs to have no kind of similarity at all with that of which it is a sign. The relation between the two is limited to the fact that the same object, exerting an influence in the same circumstances, calls forth the same sign, and thus that different signs always correspond to different influences.
>
> To the popular opinion, which accepts in good faith the full truth of images [*Bilder*], this remainder of similarity, which we do recognize, may appear very insignificant. In reality it is not so; for with it a matter of the very greatest importance can still be achieved, namely, the picturing [*Abbildung*] of the lawlikeness in the processes of the actual world.[11]

In his explanatory notes of 1921, Schlick then comments on this famous passage as follows:

> An attempt is made to show that such a picturing [*Abbildung*] of the lawlikeness of the actual with the help of a sign system constitutes, in general, the essence of all knowledge, so that our knowing can fulfill its task in no other way, and needs no other method for this purpose, in Schlick, *General Theory of Knowledge,* Part I.[12]

In this way, the assimilation of Helmholtz's *Zeichentheorie* of perception to the most characteristic doctrines of *General Theory of Knowledge* is made perfectly explicit and complete.[13]

I will here explore the relations between Helmholtz's *Zeichentheorie* and Schlick's conception of coordination or designation in some detail. We will see that, although it is certainly legitimate to view Helmholtz's *Zeichentheorie* as a philosophical precursor of Schlick's conception, there remain deep and fundamental differences between the two—differences directly connected with the very substantial changes in both the relevant sciences and in scientific epistemology that have since taken place in the interim. Indeed, radically new developments in the foundations of geometry, in particular, make it entirely impossible for Schlick to embrace Helmholtz's *Zeichentheorie* in its original form, and the most that we can say, in the end, is that Schlick's conception constitutes a radical transformation or transmutation of the former theory.[14] By thus tracing out some of the fundamental divergences between the two theories, we will gain additional insight, more specifically, into the intimate relationship between early-twentieth-century scientific epistemology and its contemporary scientific context. We will also increase our appreciation, more generally, of the subtle and imaginative strategies—which, nonetheless, are often extraordinarily misleading from a historical point of view—employed by philosophers in appropriating their philosophical past.

I

At the center of Schlick's epistemological conception is a sharp and pervasive distinction between conceptual knowledge, on the one side, and intuitive acquaintance, on the other. Indeed, the failure sharply to distinguish knowledge [*Erkennen*] from acquaintance [*Kennen*] constitutes, for Schlick, the root of all philosophical evil and leads, for example, to such impossible notions as "intuitive knowledge," "knowledge by acquaintance," and so on (§ 12). To know an object is to succeed in (univocally) designating it via concepts, and knowledge is therefore essentially mediated by conceptual thought. To be acquainted with an object, by contrast, is simply to experience it [*Erleben*], as we experience the immediately given data of our consciousness independently of all conceptual thought: "[i]n intuitive experiences [*anschaulichen Erlebnissen*], the immediate data of consciousness, e.g., pure sensations, we find pure facts that are independent of all thinking."[15] Such immediate data thus constitute the objects of intuition; they are directly given realities that are simply "there" present to consciousness; but they are not in the first instances objects of knowledge: "sensation gives us no knowledge, but only an acquaintance with things."[16]

This sharp distinction between knowledge and acquaintance is associated with a parallel distinction between concepts [*Begriffe*] and ideas or images [*Vorstellungen*] (§§ 4, 5). Ideas or images are elements of our conscious mental life (e.g., memory images). They belong to the immediately given data constituting the stream of our consciousness and, as such, share in the privacy, subjectivity, fleeting and transitory character, and lack of precisely defined boundaries peculiar to all such immediate data. Concepts, by contrast, are supposed to be public or objective representations with perfectly precise boundaries. Concepts and conceptual thought make possible genuinely scientific knowledge, a knowledge which is both objective and perfectly exact. But how are such representations themselves possible, and how do they acquire their exact and objective meaning? Concrete or ostensive definitions would be quite inadequate for this purpose, for such definitions inevitably lead back to the immediately given data of consciousness characterized precisely by their inexactness and subjectivity (§ 6). Following the tradition of Moritz Pasch and David Hilbert in the foundations of geometry, however, we see that there is an essentially different species of definition, so-called *implicit definition,* whereby concepts are fully and precisely specified through their mutual relationships with one another in a rigorous axiomatic system, without ever being related to intuitively given objects existing outside the system (§ 7).

Hilbert-style axiomatic systems are thus paradigmatic of objective scientific conceptualization and objective scientific knowledge. And the sharp distinction, pertaining to such systems, between the formal-logical structure expressed in the axioms and their manifold possible interpretations (spatial, numerical, and so on) is mirrored in Schlick's central distinction between conceptual knowledge and intuitive acquaintance. Just as the Hilbertian focus on formal-logical structure is intended to purge geometrical deduction from possibly misleading reliance on spatial intuition—so as, in particular, to allow the logical relations of dependence between geometrical propositions to stand out more clearly—Schlick's theory of scientific conceptualization is intended to free it once and for all from all vagaries of intuitive representation by allowing us to characterize scientific concepts in general solely in terms of their formal-logical relations to one another. In this way, the Pasch-Hilbert distinction between a formal axiom system for geometry (what we would now call an uninterpreted formal system), on the one side, and a possible interpretation for such a system via intuitive spatial forms, on the other, provides Schlick with the primary model for his own distinction between knowledge [*Erkennen*] and experience or acquaintance [*Erleben, Kennen*].

The model of a Hilbert-style axiomatic system also leads Schlick to the central problem of his early epistemology: elucidating the relation of such a system to the objects or realities that are now supposed to be known thereby. The problem is that a Hilbert-style axiomatic system, precisely in its purely formal-logical, essentially uninterpreted character, is deliberately and self-consciously divorced from all contact with reality:

> Implicit definition, by contrast [with concrete or ostensive definition], never stands in community or connection with reality, it denies this intentionally and in principle, it remains in the realm of concepts. A framework of truths constructed with the help of implicit definition never rests on the ground of reality, but, as it were, floats free, bearing, like the solar system, the guarantee of its stability within itself. None of the concepts appearing therein designate, in the theory, a real thing [*ein Wirkliches*]; rather, they mutually designate one another in such a way that the meaning of one concept consists in a determinate constellation of a number of the others.[17]

To now explain how knowledge of reality is possible—that is, in his own terms, how we can nevertheless set up a relation of designation or coordination between a Hilbert-style uninterpreted axiom system and some domain of real objects—is thus the sine qua non of Schlick's early scientific epistemology.

His explanation, appropriately enough, is inspired by the second great advance in the foundations of geometry of his time, the application of non-Euclidean geometry to nature effected by Einstein's general theory of relativity.

Reality, for Schlick, includes, paradigmatically, the domain of our private, immediately given data of consciousness, which constitute the totality of intuitive objects of acquaintance. These data are characterized by both intuitive temporality and intuitive spatiality, in that there are immediately given temporal relations between them, and some of them (visual fields, for example) exhibit intuitive spatial extendedness as well. Such intuitive spatiality and temporality are just as subjective as the immediately given data of consciousness themselves, and, in this sense, it is perfectly correct to say that (intuitive) space and time are subjective (§§ 27–29 [§§ 28–29]). But this realm of intuitively given data is not the only reality, and the domain of intuitive spatiality and temporality, in particular, is not the only spatiotemporal reality. On the contrary, there is also an objective spatiotemporal reality, described, paradigmatically, by modern mathematical physics, which includes a great wealth of "transcendent" objects that are not intuitively given (electromagnetic fields and the like) and that extend far (in objective space and time) beyond the meager domain of realities immediately present to our consciousness. In this sense, the domain of immediate acquaintance is only a small fraction of existing reality.

Moreover, in this connection, especially, the failure sharply to distinguish between conceptual knowledge and intuitive acquaintance has produced serious philosophical confusion and has led to the temptation, specifically, to restrict the domain of reality to the immediately given data of consciousness. Once we see, however, that knowledge means designation by concepts, and thus in no way requires intuitive acquaintance, we are in the best possible position definitively to resist this temptation:

> The intuition [*Schauen*] of things is not knowing and also not a precondition of knowing. The objects of knowledge must be *thinkable* without contradiction, that is, allow of a univocal designation via concepts, but they do not need to be intuitively representable [*anschaulich vorstellbar*].[18]

So the way is now open, in particular, to reject the subjective idealist "philosophy of immanence" on behalf of a fully robust scientific realism (§§ 24–25 [§§ 25–26]).

Indeed, Schlick's conception of knowledge as designation via concepts actually leads to an even stronger result, for it turns out that those objects that, in the first instance, are capable of such designation are not the intuitively spatial and temporal realities of immediate acquaintance, but are rather precisely the objective "transcendent" realities described by modern mathematical physics as existing in objective, mathematical-physical space and time outside of our consciousness. It is precisely the latter realities, *rather than* the former, which constitute the proper objects of knowledge. This result already follows from Schlick's model of objective conceptual thought as given by mathematically precise axiomatic systems in which concepts are exactly specified, by means of implicit definitions, through their formal-logical relations to one another. (No such system is available, for example, for the domain of introspective psychology.) But it follows equally from Schlick's detailed explanation of how an abstractly specified system of implicit definitions acquires a relation of designation or coordination to the realities that are supposed to be known thereby, for this explanation yields a parallel and complementary emphasis on quantitative as opposed to qualitative knowledge.

Schlick explains how we set up the crucial relation of designation or coordination between our system of concepts and reality in section 30 [31], entitled "Quantitative and Qualitative Knowledge." We begin, to be sure, with the intuitive spatiotemporal ordering of the immediately given data of consciousness, since our construction of the objective or "transcendent" spatiotemporal ordering is based upon this subjective ordering:

> The ordering of our contents of consciousness in space and time is likewise the means by which we learn to determine the transcendent ordering of things outside our consciousness, and the latter ordering is the most important step towards their cognition.
>
> The problem now is to become clear how one proceeds from the intuitive spatiotemporal ordering to the construction of the transcendent ordering. This always occurs by the same method, which we can designate as the *method of coincidences*. It is epistemologically of the very highest importance.[19]

It turns out, however, that what is primarily knowable by this process is the quantitative structure of the "transcendent" ordering thereby effected. The qualitative structure of the immediately given data of consciousness with which we begin can itself only become known *after* we have fully articulated the objective ordering.

We construct the "transcendent" ordering, more specifically, on the basis of singularities or coincidences in our various intuitively given sensory fields. For example, I see the tip of my pencil touch my finger in my visual field and, at the same time, feel its touch on my finger in my tactile field. The intuitive spatiality of these two sensory fields is entirely different in the two cases, and they have, as such, no intuitive spatial relations to one another. I then bring them into relation by constructing a single, nonintuitive spatial ordering containing both the pencil and my finger, where a single point in objective space (the coincidence of my finger with the pencil tip) corresponds to both singular points in the two previously independent sensory fields. In this procedure I abstract completely from the qualitative peculiarities of my sensory fields (color, tactile quality, and so on) and concentrate solely on their purely topological properties—the presence or absence of a singularity. And this focus on singularities or coincidences is also crucial from a scientific point of view, for it is precisely on the basis of such coincidences that the technique of numerical measurement now proceeds. We measure objective spatial intervals by observing the coincidences of the endpoints of a measuring rod with points on a measured object; we measure objective temporal intervals by observing coincidences between events in a given natural process and pointer positions on a clock; and so on: "all measurement, from the most primitive to the most sophisticated, rests on the observation of spatiotemporal coincidences."[20]

In the method of coincidences, then, I construct a numerical model, as it were, for an abstractly specified axiom system for mathematical physics[21] by carrying out measurements (of objective spatial and temporal intervals, but also of various objective physical magnitudes, such as the electromagnetic field) based on my perceptions of measuring instruments and thus, in the end, on immediately given coincidences or singularities in my intuitive sensory fields. In this way, an abstractly specified axiom system acquires a relation of designation to quantitatively structured objective reality *by way of* the immediate data of consciousness, and the objective or "transcendent" spatiotemporal ordering of realities described by modern mathematical physics thereby becomes a genuine object of knowledge. It does not follow, however, that the purely qualitative data immediately present to consciousness themselves become objects of knowledge as well. On the contrary, precisely because they are not yet describable in truly quantitative fashion, such purely qualitative intuitive data are not yet objects of knowledge. They will only acquire this status, in fact, when they, too, are described in exact mathematical-physical fashion: "[t]he life of consciousness is thus only completely knowable insofar as we succeed in transforming introspective psychology into a physiological, natural-scientific psychology, ultimately into a physics of brain processes."[22]

We have not yet drawn a connection between the objective spatiotemporal ordering known via the method of coincidences and the new conceptions of space and time due to the general theory of relativity. Nor does Schlick himself make this connection explicit in the relevant parts of *General Theory of Knowledge.* Rather, it is in *Space and Time in Contemporary Physics,* written virtually simultaneously, that this crucial step in Schlick's reasoning is explained. The most important chapter of the latter work, in this connection, is entitled "The General Postulate of Relativity and the Metrical Determination of the Space-Time Continuum," where Schlick draws a fundamental contrast between general relativity and both Newtonian physics and special relativity. In both of the latter two theories, he explains,

> [space] still preserved a certain objectivity, so long as it was still tacitly thought as equipped with completely determined metrical properties. In the older physics one based every measurement procedure, without hesitation, on the idea of a rigid rod, which possessed the same length at all times, no matter at which place and in which situation and environment it may be found, and, on the basis of this thought, all measurements were determined in accordance with the precepts of Euclidean geometry. . . . In this way, space was still left with a "Euclidean structure," as a separate and independent [*selbständig*] property, as it were, for the result of these metrical determinations was thought to be entirely independent of the physical conditions prevailing in space, e.g., of the distribution of bodies and their gravitational fields.[23]

But this is emphatically not the case in Einstein's new theory:

> If we want, therefore, to maintain the general postulate of relativity in physics, we must refrain from describing measurements and situational relations in the physical world with the help of Euclidean methods. However, it is not that, in place of Euclidean geometry, a determinate other geometry—e.g., Lobachevskian or Riemannian—would now have to be used for the whole of space, so that our space would be treated as pseudospherical or spherical, as mathematicians and philosophers are accustomed to imagine this. Rather, the most various kinds of metrical determinations are to be employed, in general, different ones at each position, and what they are now depends on the gravitational field at each place.[24]

Space-time in general relativity now has no background geometry at all—neither Euclidean nor non-Euclidean—that would be determined independently of the distribution of matter therein; and, according to the general postulate of relativity (the principle of general covariance), the only background that remains is the topological or manifold structure of number quadruples, that is, the space-time coincidences, so that "the whole of physics can be conceived as a totality of laws in accordance with which the occurrence of these space-time coincidences takes place."[25]

In the final chapter, entitled "Relations to Philosophy," Schlick then explains the significance of Einstein's new view of space and time for epistemology. He points out that the objective spatial structure employed by physics is not intuitively given, but is rather a *"conceptual construction,"* that is, a "nonintuitive ordering, which we then call objective space and conceptually grasp through a manifold of numbers (coordinates)."[26] Yet this objective conceptual construction proceeds, just as in *General Theory of Knowledge,* on the basis of the subjective spatiotemporal coincidences present in various sensory fields of various individuals:

> In order to fix a point in space, one must somehow, directly or indirectly, *point* to it, . . . that is, one establishes a spatiotemporal coincidence of two otherwise separate elements. And it now turns out that these coincidences always occur in agreement for all intuitive spaces of different senses and all individuals; precisely so is an objective "point," independent of individual experiences and valid for all, thereby defined. . . . By closer consideration one easily finds that we attain to the construction of physical space and time exclusively through this method of coincidences and in no other way. The space-time manifold is nothing else than the totality of objective elements defined through this method. That it is precisely a four-dimensional manifold results from experience by the execution of this method itself.
>
> This is the result of the psychological-epistemological analysis of the concepts of space and time, and we see that we encounter precisely *that* meaning for space and time which Einstein has recognized as alone essential for physics, where he has shown it to best advantage. For he rejected the Newtonian concepts, which denied the origin we have described, and based physics instead on the concept of the coincidence of events. So here physical theory and epistemology extend their hands to one another in a beautiful alliance.[27]

The connection between Schlick's epistemological method of coincidences and the general theory of relativity could not be stated more clearly.

II

A full forty years separate *General Theory of Knowledge* from Helmholtz's mature presentation of his *Zeichentheorie* in "The Facts in Perception." These forty years, as suggested above, were quite extraordinarily eventful, being distinguished, in particular, by the development of the new formal-axiomatic point of view in the foundations of geometry by Pasch and Hilbert, on the one hand, and by Einstein's physical application of a non-Euclidean geometry of variable curvature, dynamically linked to the distribution of matter, on the other. It is to Schlick's enduring credit that he clearly recognized the radical philosophical implications of these new ideas and, accordingly, made them the linchpins of his early scientific epistemology. Helmholtz, however, worked in a quite different scientific environment and had, for his part, no inkling of these particular developments in the foundations of geometry that were still to come. Instead, Helmholtz's articulation of the *Zeichentheorie* of perception was framed by three fundamental contributions of his own scientific work: his conception of the principle of causality or the lawlikeness of nature as a basic essential principle of physical science, his defense of "empiricist" over "nativist" theories in the psycho-physiology of sense perception, and his own work in the foundations of geometry on what we now call the Helmholtz-Lie space-problem. The *Zeichentheorie* of perception, as we shall see, is a synthesis and philosophical elaboration of these elements—which, as we shall further see, is quite incompatible with the scientific epistemology of *General Theory of Knowledge.*

Helmholtz's characteristic emphasis on the principle of causality or the lawlikeness of nature makes its first appearance in his great monograph on the conservation of energy, published in 1847. In the introduction to that work he describes "the ultimate and proper goal of the physical natural sciences as such" as beginning with an "experimental part," where one seeks to describe "the individual natural processes" by "general rules . . . which are obviously nothing but universal generic concepts though which all of the appearances belonging thereto are comprehended," and proceeding to a "theoretical part . . . which seeks, by contrast, to find the unknown causes of the processes from their visible effects; it seeks to conceptualize them in accordance with the law of causality."[28] This procedure leads us eventually to the "ultimate unalterable causes" lying at the basis of all appearances:

> We are compelled and justified in this task by the principle that every alteration in nature *must* have a sufficient cause. The proximate causes that underlie the appearances of nature can themselves be either unalterable or alterable; in the latter case the same principle compels us to seek for other causes of this alteration in turn, and so on, until we finally arrive at the ultimate causes that act in accordance with an unalterable law, and which, therefore, bring about at every time, under the same external relations, the same effect. The final end of the theoretical natural sciences is thus to discover the ultimate unalterable causes of natural processes.[29]

Such ultimate unalterable causes turn out to be the mass points of analytical mechanics, interacting with one another solely through time-independent (constant or "unalterable") central forces of attraction and repulsion depending only on the distances between the points in question. The possibility of reducing all of the appearances of nature to this basis is thus "the condition for the complete conceptualizability of nature."[30] And the main burden of the monograph that follows is then to contribute to this goal by showing that the phenomenological principle of the conservation of energy—the principle, as Helmholtz's phrases it, that a perpetual motion machine (of the first kind) is impossible—is equivalent to the theoretical principle that all actions in nature are in fact reducible to ultimate forces of attraction and repulsion in this way.

The principle of causality therefore functions here as a bridge between the observable world of appearances and the ultimate unobservable causes that are thought to underlie and explain the appearances. The principle serves a parallel function, in the context of the psycho-physiology of perception, in Helmholtz's celebrated lecture on human vision of 1855 (see note 2 above), where its role is extended to justify even our inference that there is a world of external objects in the first place:

> [W]e never immediately perceive the objects of the external world, but we only perceive the effects of these objects on our nervous apparatus, and this has been so from the first moment of our life onwards. In what way, then, have we first reached across from the world of the sensations of our nerves into the world of actuality? Obviously, only through an inference; we must presuppose the presence of the external object as *cause* of our nerve-excitation; for there can be no effect without a cause.[31]

The principle of causality, for precisely this reason, cannot be an empirical proposition, for it is thus required before there can be any experience of objective external things at all: "[h]ence the investigation of sense perception leads us on also to that knowledge already found by Kant, that the proposition, 'no effect without a cause,' is a law of our thinking given prior to all experience."[32]

At this stage in Helmholtz's development, then, the principle of causality is embedded in a version of a classical causal realist theory of perception. Behind the veil of perception of our sensations is a world of external objects in space, a world we can only reach epistemically via an inference from observed effects to unobserved causes. By the time of "The Facts in Perception," however, Helmholtz has modified this picture considerably. He characterizes *both* causal realism and subjective idealism as "metaphysical hypotheses" and asserts, in a well-known phrase, that "[w]hat we can find unambiguously and as fact, however, without hypothetical interpolation, is the lawlike in the appearance."[33] The "law of causality" continues to be "an a priori given, a transcendental law," because lawlikeness continues to be the condition for "the conceptualizability of the appearances of nature."[34] But this principle can no longer serve to underwrite causal realism; it can no longer transport us behind the veil of appearances. Thus, when Helmholtz adds notes to his monograph on the conservation of energy in 1881, he corrects the above-cited sentence, where the law of causality is said to lead us to "unknown causes" from their "visible effects," accordingly:

> 1) *To page 13. The philosophical discussion in the introduction* is more strongly influenced by Kant's epistemological views than

> I would now like to recognize as correct. I only made it clear to myself later that the principle of causality is actually nothing other than the presupposition of the lawlikeness of all the appearances of nature.[35]

The principle of causality cannot serve, as Helmholtz understood Kant in 1855, as an a priori justification for causal realism—for the postulation of otherwise unknown objects behind the veil of appearances. On the contrary, lawlikeness is now a fundamental principle governing the appearances themselves, as something we "find unambiguously and as fact . . . *in* the appearance."

This important shift in Helmholtz's thinking was mediated by the further articulation of what he called an "empiricist" theory of visual perception in his monumental *Handbook of Physiological Optics,* first appearing between 1856 and 1867.[36] Although he had declared his allegiance to such a theory, and his opposition to "nativism," in 1855, the theory did not acquire a clear articulation until 1865–66.[37] The basic idea is that our ability to see objects around us in space, as localized at particular places therein, is not an innate capacity of either our consciousness or our nervous apparatus, which would be somehow built in prior to all particular experiences. Rather, this ability is itself gradually learned or acquired—as we learn or acquire our native language, for example—by a process of "unconscious inductive inference" based on regularities or associations among our sensations. For example, my ability to localize a perceived table in three-dimensional space is in no way directly given by simple visual or tactile sensations, for "[a] direct image [*Bild*] of a three dimensional extended spatial magnitude is given by neither the eye nor the hand." Such a perception of a three-dimensional spatial object instead requires (unconscious) knowledge of a large number of regularities among such simple sensations, generated as I move around the object, reach out and touch it, and so on:

> The representation of a spatially extended body, e.g., a table, includes a mass of individual observations. There lies comprised therein the entire series of images that this table would provide me if I were to consider it from various sides and from various distances, if I were to lay my hands successively on the various points of its surface. Such a representation of a particular individual body is thus actually already a *concept,* which comprehends under itself an infinite number of particular intuitions following one another in time, all of which can be derived from it, just as the generic concept 'table' in turn comprehends within itself all particular tables and expresses their common characters.[38]

In this sense, the ability to see objects in space is primarily an affair of the understanding, and "[t]he fundamental principle of the empiricist view is [that] *sensations are signs for our consciousness, where learning to understand their meaning is left to our understanding.*"[39]

By the same token, therefore, the process of learning to localize objects in space is closely analogous to the conscious procedure of inductive inference characteristic of natural science:

> Of the greatest importance, finally, for the fixity of our conviction in the correctness of our sensory perceptions are the tests that we undertake by means of the optional motions of our body. There thereby arises the same kind of fixed conviction, relative to merely passive observation, that we gain in scientific investigations through the experimental method. The proper ultimate ground, through which all our consciously executed inductions receive the power of conviction, is the law of causality.[40]

Indeed, except for the fact that the former inferences are unconscious, the analogy is perfectly exact:

> Now the same great significance that experiment has for the security of our scientific convictions, it has also for the unconscious inferences of our sensory perceptions. Only insofar as we bring our sense organs, in accordance with our own willing, into various relations to the objects, do we learn securely to judge about the causes of our sensations, and such experimenting takes place from the earliest childhood on, without interruption, throughout the whole of life.[41]

We thus learn or acquire the complicated system of regularities among initially isolated and fragmentary sensations, which, as a system, first constitutes the perception of an object in space, by the very same procedure, and in accordance with the very same causal or inductive principle, that we self-consciously employ in scientific inference. Hence, since the primary role for the causal or inductive principle here is now precisely to secure our grasp of regularity or lawlikeness on the side of our perceptions, it no longer functions as a bridge to another realm existing *behind* our perceptions.

To be sure, some of Helmholtz's language still suggests a version of causal realism. For example, he above speaks of judgments about the "*causes* of our sensations," and elsewhere, even more strikingly, he asserts that "we can never emerge from the world of our sensations to a representation of an external world except through an inference from the changing sensation to external objects as the causes of this change."[42] In other passages, however, Helmholtz clearly and explicitly excludes causal realism:

> I believe, therefore, that there can be no possible sense at all in speaking of any other truth for our representations except a *practical* [truth]. Our representations of things *can* be nothing else at all except symbols, naturally given signs for things, that we learn to use for the regulation of our motions and actions. When we have correctly learned to read such a symbol, we are then capable of so adjusting our actions with its help that they have the desired result, that is, the expected new sensations

> occur. Another comparison between representations and things not only fails to exist in *actuality*—here all schools agree—but any other kind of comparison is in no way *thinkable* and has no sense at all.[43]

> Thus representations of the external world are images [*Bilder*] of the lawlike temporal succession of natural events, and if they are correctly formed [*gebildet*] in accordance with the laws of our thinking, and we are able correctly to translate them back again into actuality though our actions, then the representations that we have are also the *uniquely true* [ones] for our faculty of thought; all others would be false.[44]

Here Helmholtz comes very close indeed to the view that lawlike relations among our sensations—arrived at by inductive inferences in accordance with the principle of causality or the lawlikeness of nature—are *constitutive* of their relationship to an external world.

The final piece of the puzzle, which put Helmholtz in a position definitively and unambiguously to reject causal realism, and which paved the way, accordingly, for "The Facts in Perception," was his own fundamental mathematical contribution to what we now call the Helmholtz-Lie space-problem, developed in the years 1866–70. The upshot of this contribution was to show that the same regularities in our sensations, on the basis of which we acquire the ability to localize objects in space, also give rise to the representation of space itself. The voluntary actions of our bodies, which allow us to localize objects by moving toward, away, and around them, make possible, in addition, a precise mathematical construction of the very three-dimensional space within which this process of localization takes place. In this way, space does not serve, as it were, as the home of "external" objects that exist behind the veil of perception, but rather as a "subjective *form of intuition*" in the sense of Kant—so that "space will also appear to us sensibly, clothed with the qualities of our sensations of motion, as that through which we move, through which we can gaze forth." Space is thus the "*necessary* form of our external intuition . . . because we comprehend precisely that which we observe as spatially determined as the external world."[45] The external world of objects in space is therefore a construction, erected entirely on the basis of our acquired ability to localize objects therein.

Helmholtz begins his first mathematical paper, in 1866, by explaining that his "investigations into the manner in which localization in the visual field takes place" stimulated him "to consider also the origins of general spatial intuition as such."[46] As in Bernhard Riemann's great lecture, "On the Hypotheses Which Lie at the Basis of Geometry," Helmholtz sets up the problem mathematically by asking after those analytic conditions which distinguish our actual three-dimensional space from other continuous manifolds, such as the manifolds of colors, sounds, and so on.[47] Unlike Riemann, however, Helmholtz is not content simply to postulate, in answer to this question, the existence of what is now known as a Riemannian metric. He looks rather for "truths of factual significance"[48]—facts, it emerges, of our perceptual experience—from which Riemann's "hypotheses" can then be derived. And he finds this factual basis, as is well known, in the free mobility of rigid bodies throughout the manifold: the possibility of moving rigid bodies without distortion from any one place and situation to any other, so as thereby to perform measurements and, in particular, to determine congruence. This possibility expresses a lawlike feature of our experience, so that "[l]ike every physical measurement, that of space must also rest on an unalterable law of uniformity in the appearances of nature."[49]

What is not quite so well known is that Helmholtz did not just attempt, in accordance with what we now call the Helmholtz-Lie theorem, to derive the existence of a Riemannian metric—which then must have constant curvature as well—from his factual basis of free mobility.[50] He also attempted to derive the specifically Euclidean character of our "in fact existing space" from a similarly factual basis. In particular, by overlooking the existence of spaces of constant negative curvature described by Bolyai-Lobachevsky geometry, he argued that the specific character of our actual space could be derived from three-dimensionality, free mobility, and infinity—where this last condition rules out the case of constant positive curvature. Only after he had become acquainted with Eugenio Beltrami's work on models for Bolyai-Lobachevsky geometry in 1868 was Helmholtz able to correct this mistake, in footnotes to his "On the Facts Lying at the Basis of Geometry" (in which the main text still repeats the earlier mistaken claim). From our present point of view, however, it is highly significant that Helmholtz had begun his mathematical project, in 1866, by thinking that he could construct all of the properties of three-dimensional Euclidean space from a factual basis of lawlike regularities in our perceptual experience.

In any case, Helmholtz soon corrected his mistake, and this brought about another important shift in his thinking.[51] For it was now clear that the specifically Euclidean character of space is not a consequence of the factual basis invoked by Helmholtz to distinguish our space from all other types of manifolds: free mobility (together with three-dimensionality and infinity) does not yield specifically Euclidean geometry. Thus, Euclidean geometry is not built into the essential or necessary character of space, through which "the system of spatial measurement must presuppose those conditions under which alone we can speak of determination of congruence."[52] The particular propositions of Euclidean geometry, as Helmholtz now argues explicitly in 1870,

> are not included in the general concept of a three-dimensional extended magnitude and the free mobility of the bounded struc-

> tures contained in it. They are not *necessities of thought,* which flow from the concept of such a manifold and its measurability, or from the most general concept of a rigid structure contained therein and its freest mobility.[53]

Indeed, we can now make an even stronger claim. For it follows from Helmholtz's theory of the origin of spatial intuition in our experience of bodily motion that the particular propositions of Euclidean geometry are also not necessities of intuition. In particular, we can now imagine the series of sensations we would have if we were to find ourselves moving around in one of Beltrami's pseudospherical models for Bolyai-Lobachevsky geometry, and such a series of sensations, by Helmholtz's theory, would *be* an intuition of space:

> We can picture to ourselves the view of a pseudospherical world going in all directions, just as well as we can develop its concept. We can therefore also not admit that the axioms of our geometry are grounded in the given form of our faculty of intuition, or are in any way connected with such [a form].[54]

Since the axioms of Euclidean geometry are not built into the most general necessary conditions underlying our spatial intuition, Kant's theory of the origin of these axioms in our "necessary" and "transcendental" intuition of space is incorrect, and they emerge rather as *merely* empirical facts—facts about the actual behavior of measuring instruments in our actual world.

Nevertheless, Kant's insight that space itself is a "subjective form of intuition," rather than an ordering of things in themselves existing behind the veil of appearances, continues to be correct; for, as Helmholtz conceives it, "the most essential features of spatial intuition"—including free mobility and therefore constant curvature—are derived from the same original lawlike experiences of bodily motion on which our ability to localize objects in space depends.[55] In this sense, as Helmholtz famously puts it, "space can be transcendental without the axioms [i.e., the axioms of specifically Euclidean geometry] being so."[56] And it follows, just as it does for Kant, that we can now give an answer to the fundamental question of epistemology—"What is truth in our intuition and thought? In what sense do our representations correspond to actuality?"[57]—that does not involve a relation of correspondence or representation between our perceptions, on the one side, and "external" objects existing behind our perceptions, on the other. For that to which our representations finally correspond are lawlike patterns taking place *within*—and indeed *constituting*—the space of our form of intuition:

> I return to the discussion of the first original facts of our perception. We have, as we have seen, not only changing sense impressions that come upon us without our doing anything for this purpose, but we observe during our own continuing activ-

> ity, and we thereby achieve an acquaintance with the *enduring existence* [*Bestehens*] of a lawlike relation between our [motor] innervations and the becoming present of various impressions from the current range of presentables. Each of our optional motions, whereby we modify the manner of appearance of the object, is to be considered as an experiment, through which we test whether the lawlike behavior of the appearance lying before us—that is, its displayed enduring existence in a determinate spatial ordering—has been correctly apprehended.[58]

In this way, the correspondence of our sensations to enduring external objects in space is equated with their characteristic lawlikeness:

> What we can find unambiguously and as fact, however, without hypothetical interpolation, is the lawlike in the appearance. From the first step on, when we perceive the objects lingering before us distributed in space, this perception is the recognition of a lawlike connection between our motions and the sensations occurring thereby.[59]

This is what remains as fact, that is, in contrast to the merely hypothetical character of both causal realism and subjective idealism.

Helmholtz expresses the Kantian contrast, as he understands it, between appearances and things in themselves as that between the *actual* and the *real:*

> We have in our language a very happy designation for that which influences us [*auf uns einwirkt*], standing behind the change of appearances, namely "the actual [*das Wirkliche*]." Here only the action [*das Wirken*] is expressed; it lacks the secondary reference to enduring existence as substance that the concept of the real [*das Reelen*] includes.[60]

As we have seen, however, we only know enduring existence as *law,* in that "[t]he lawlike is thus the essential presupposition for the character of the actual":

> I do not need to explain to you that it is a contradiction in terms to represent the real or Kant's "thing in itself" via positive determinations, but without taking it up into the form of our representing. This is often discussed. But what we can achieve is an acquaintance with the lawlike ordering in the realm of the actual, to be sure only presented in the sign system of our sense impressions.[61]

Our sensations count as signs of external objects, therefore, not in virtue of a relation of correspondence or representation to otherwise unknown things in themselves existing behind our perceptions, but rather because they are initially isolated and fragmentary products of nervous excitation, which, as such, only *become* perceptions of external objects in space via a gradually learned process of *interpretation:*

> [Sensations] are signs, which we have *learned to read*; they are a language, given to us along with our organization, in which external objects speak to us. But we must learn to understand this language through practice and experience, just as much as our native language.[62]

And the main point of Helmholtz's *Zeichentheorie,* in the end, is to emphasize precisely this process of interpretation—a process in which spatially located objects, together with space itself, are constituted by lawlike relations among our sensations.

III

For Schlick, as we have seen, concepts and conceptual thought are rigidly separated from intuition. Concepts are identified with uninterpreted symbols in a Hilbert-style axiomatic system or formal calculus, and conceptual thought is identified with the procedure of operating such a calculus by purely formal rules independently of all possible interpretations or applications. Standing over against this purely formal system of concepts are then the immediately given realities of intuition constituting our consciousness, on the one hand, and the objective "transcendent" realities existing outside of our consciousness, on the other. In the case of a formal axiomatic system for geometry, in particular, there are both the intuitively spatial forms and qualities immediately present in our various sensory fields, and the objective spatial ordering to which our various sensory fields (including the sensory fields of different individuals) are all coordinated. And conceptual thought acquires a relation of coordination or designation to reality, in virtue of which it thereby counts as knowledge, through the method of coincidences. In the case of geometry, in particular, we construct a numerical assignment of coordinates on the basis of subjectively given singularities or coincidences in our various sensory fields, resulting in an objective numerical model for our initially uninterpreted axioms. A purely formal system of concepts thereby acquires an interpretation or application in the nonintuitive, objective, or "transcendent" realm by way of a projection, as it were, from the immediately given, subjective domain of intuitive consciousness.

It is central to Schlick's epistemological conception that we enforce a similarly rigid separation between subjective, intuitive, or psychological space, on the one side, and objective, "transcendent," or physical space, on the other. The former is an indefinable, purely qualitative object of immediate acquaintance given prior to all conceptualization and thought; the latter is a quantitative, nonintuitive ordering that is conceptually thinkable and representable through an abstract formal system of judgments. Helmholtz's attempt to defend a modified version of the Kantian conception of space as a subjective form of intuition, in which all objects of external experience are nevertheless embedded, must therefore, from this point of view, appear as a fundamental confusion, and Schlick himself makes this amply clear in his note to the above-cited passage from "The Facts in Perception" (note 45 above) where Helmholtz introduces the idea:

> Here again, as above, the expression "form of intuition" is used in a completely different sense than in Kant (cf. note 16). But Helmholtz is entirely correct to align the spatial intuition he describes with the sensory qualities, for it is a matter in both cases of subjective, psychical contents. What he describes, namely, in what precedes and what follows, is *psychological* space (or, properly, psychological spaces; for one must separate the spatial data of, e.g., sensations of motion from those of visual or tactile perceptions, as something wholly different, even though they are all connected through close associations), not physical-geometrical space. The latter is a non-qualitative, formal, conceptual construction; the former, as an intuitive given, is clothed, according to Helmholtz's words, with the qualities of sensations, and is just as subjective as they are.[63]

Indeed, it follows from this passage that Helmholtz's entire theory of space-perception has only to do, from Schlick's point of view, with what Schlick calls psychological or intuitive space; it remains wholly within the purely subjective realm and never touches at all on the objective physical world.

Yet Schlick's sharp distinctions between intuition and conceptual thought, and between subjective psychological and objective physical-geometrical space, are completely foreign to Helmholtz's *Zeichentheorie.* Indeed, as we have seen, one of the main pillars of this theory is the idea that what Schlick calls objective physical-geometrical space can be constructed or generated from what he calls subjective psychological space. And it is generated by a procedure, moreover, in which conceptual elements belonging to what Helmholtz calls the realm of thought are inextricably mixed with intuitive elements:

> From the first step on, when we perceive the objects lingering before us distributed in space, this perception is the recognition of a lawlike connection between our motions and the sensations occurring thereby. Thus the first elementary representations already contain thinking and proceed in accordance with the laws of thinking. Everything in intuition that is added to the raw material of sensations can be resolved into thinking, if we take the concept of thinking as so extended as has been done above.[64]

The extension in question is given by Helmholtz's theory of unconscious inductive inferences—which, accordingly, is anathema to Schlick.[65] Schlick does not fully appreciate, however, the extent to which Helmholtz's view of conceptual thought is also quite different from his own. For Helmholtz, conceptual thinking does not involve manipulating a Hilbert-style uninterpreted calculus in accordance with purely formal rules. Rather, following J. S. Mill's interpretation of traditional syllogistic logic, Helmholtz sees the essence of conceptual thinking in the procedure of forming inductive inferences (which would yield major premises for syllogisms), whereby a variety of particular instances are associated together under a single concept.[66] Conceptualization [*Begreifen*], for Helmholtz, just is such a process of inductive association, and this is precisely why he views the principle of causality or the lawlikeness of nature as the fundamental principle of thought: "[t]he first product of the thoughtful conceptualization of appearance [*des denkenden Begreifens der Erscheinung*] is the *lawlike*."[67]

What Helmholtz calls spatial intuition is thus a product of conceptual thinking in his sense. Unlike the initial sensory qualities themselves ("the raw material of sensations"), which are given as direct outputs of elementary nervous excitations, and which, according to Helmholtz's "empiricist" theory, do not yet display spatial extendedness at all, spatial intuition is a learned or acquired *ordering* of such qualities in a lawlike structure. In this respect, Helmholtz's conception of spatial intuition is indeed similar to Kant's, although, as Helmholtz himself points out, his theory, in contrast to Kant's, resolves what the latter calls intuition into a process of thought:

> In that which has always seemed to me to be the most essential advance in Kant's philosophy, we still stand upon the ground of his system. In this sense I have also, in my previous works, frequently emphasized the agreement of recent sensory psychology with Kant's doctrines, although this certainly does not mean that I had to swear by the master's words in all subordinate points as well. I believe that the resolution of the concept of intuition into elementary processes of thought must be considered as the most essential advance of recent times—which resolution is still lacking in Kant, and through which [lack] his conception of the axioms of geometry as transcendental propositions is conditioned.[68]

Viewing spatial intuition in terms of immediately given sensory presence is the source, according to Helmholtz, of the mistaken view that the axioms of specifically Euclidean geometry have their origin there. Once we view it as a temporally extended process essentially linked to motion, by contrast, we see that only free mobility in general, and not specifically Euclidean geometry, is a "transcendental" condition of the possibility of spatial intuition as such.[69]

By the same token, however, once we see that free mobility allows us to construct or generate the three-dimensional ordering of geometrical-physical space from lawlike associations among our sensations, the way is also now open for viewing the underlying external actuality to which these sensations correspond as simply such lawlike associations themselves:

> That which remains the same, without dependence on anything else, in all change in time, we call *substance*; the relationship that remains the same between variable quantities, we call the *law* connecting them. What we directly perceive is only the latter. . . . The first product of the thoughtful conceptualization of appearance is the *lawlike*. If we have separated it out so purely and conceived its conditions so completely and securely delimited, and also so universally, that for all possible occurring cases the result is uniquely determined, and we simultaneously attain the conviction that it has held good, and will continue to hold good, at all times and in all cases, then we recognize it as an enduring existence independent of our representations and call it the *cause*, that is, something that originally remains and continues to exist behind the change [*das hinter dem Wechsel ursprünglich Bleibende und Bestehende*]; only in this sense, in my view, is the application of the word justified, even though common usage applies it in a vague and washed-out manner for antecedent or occasion in general.[70]

The idea that the external cause standing behind the play of our sensations just is a lawlike relation governing them could hardly be more clearly expressed.

Schlick, for his part, is scandalized by this idea. In his notes to the above passage, for example, he rejects the claim that we can "directly perceive" a law and protests with particular vehemence against Helmholtz's use of the notion of cause here.[71] But the most revealing passage occurs in *General Theory of Knowledge,* in the course of an extended polemic against "the philosophy of immanence." He there explicitly criticizes attempts to reduce external things to lawlike relations among our sensations, as in Mill or Mach, say, on the grounds that they thereby inadmissibly "hypostasize" such relations; and what is especially revealing, in the present context, is that this polemic provides Schlick with a rare occasion (in *General Theory of Knowledge*) to refer to Helmholtz:

> Whoever says here that a thing in the external world *is* a lawlike connection of elements that also continues to exist when the elements are not given, and then believes to have thereby ascribed to the things the same reality that a sense-datum, for example, also possesses, has thereby *reified* the law, and his concept formation is identical with the concept of *force,* as it used to be dominant in a phase of natural science that has now been overcome. The lawlikeness of the connection has now in fact become a power for him, which simply *generates* certain elements. "The law recognized as objective power we call force" wrote Helmholtz in the year 1881 (in the notes to his treatise on

> the conservation of force). What is thought in the concept of the permanent possibilities of sensations or in the "objectively existing law" is entirely and precisely the same as what one otherwise used to think under the concept of force.[72]

So Helmholtz, by implication, is guilty of the same confusion that generally vitiates all "immanence philosophy."[73]

The mistake here, from Schlick's point of view, rests on a failure clearly and sharply to distinguish between abstract concepts and concrete realities—the very failure to which Schlick traces virtually all of the confusions of traditional philosophy. Paradigmatic of existence, reality, or actuality (among which Schlick does not distinguish) are the immediately given data of consciousness themselves. It is here, and here alone, that we first form the concept of reality. This concept can, and indeed must, then be extended to the "transcendent" realities existing outside of consciousness as well. In neither case, however, should the concrete existence of realities be confused with the abstract concepts we use to designate them:

> The concept of the real cannot be reduced to unreal [*unwirkliche*] concepts; it must be taken from experience [*Erleben*]. Concepts and realities, by their nature, are incomparably different and cannot be transformed into one another. Only the recognition of this distinction makes logical thinking possible, and any blurring of the distinction leads to the great mistakes of the historical metaphysical systems.[74]

Thus, since laws are simply abstract concepts used to designate realities, they themselves cannot be identified with realities. Just as Schlick diverges fundamentally from Helmholtz in his view of conceptual thought as the manipulation of an abstract formal system independently of all interpretation, he similarly diverges from Helmholtz in his conception of laws and their relation to actuality.[75]

Yet Schlick does not appreciate, in this connection, the characteristic way in which Helmholtz's own conception of laws and their relation to actuality diverges radically from more typical forms of "immanence philosophy," such as those found in Berkeley, Mill, or Mach. The heart of the matter, for Helmholtz, is that geometrical-physical space is itself a system of lawlike relations among our sensations, so that external physical objects then emerge as lawlike patterns *within* geometrical-physical space. This conception of intuitive or psychological space as, at the same time, an objective, conceptually determined, geometrical-physical space is, as we have seen, entirely alien to Schlick's way of thinking. Indeed, the whole point of Schlick's method of coincidences is to effect a relation of correspondence or coordination between two essentially distinct space-time structures, the subjective-intuitive and the objective-conceptual, and it is precisely this feature of his conception, moreover, that allows him to defend a version of causal realism against traditional phenomenalism.[76] For Helmholtz, by contrast, the conception of space as a "transcendental form of intuition" on the basis of which alone objective external experience is possible—which, as we have also seen, aligns Helmholtz most closely with the philosophy of Kant—finally enables Helmholtz to stake out an entirely original position on the question of our perception of the external world, which, like Kant's own, avoids both traditional causal realism and traditional phenomenalism.[77]

It is at first sight quite surprising that Schlick never once mentions Helmholtz's construction of geometrical-physical space on the basis of the Helmholtz-Lie theorem. This is an extremely striking omission of Helmholtz's own fundamental contribution to the foundations of geometry—which, from our present point of view, is absolutely central to his scientific epistemology as well. With the benefit of hindsight, however, this very striking omission turns out to be not so surprising at all. For the plain fact is that such a construction of geometrical-physical space is quite incompatible with the new conceptions of space and time due to the general theory of relativity. The Helmholtz-Lie theorem necessarily yields manifolds of constant curvature, whereas the general theory of relativity is based on a manifold of variable curvature. Free mobility, and the consequent determination of congruence via transported rigid rods, therefore fails in the space(time) employed by that theory; and so this particular space(-time), characterized by a variable curvature essentially linked to the distribution of matter therein, *cannot* be the outcome of a construction based on the Helmholtz-Lie theorem.[78]

We have seen that Schlick, in *Space and Time in Contemporary Physics*, clearly recognizes that general relativity has decisively broken with the measurement procedures of the "older physics," insofar as these were based, "without hesitation, on the idea of a rigid rod, which possessed the same length at all times, no matter at which place and in which situation and environment it may be found." Schlick also clearly recognizes, accordingly, that the geometrical structure of space can no longer be conceived as "a separate and independent property . . . entirely independent of the physical conditions prevailing in space, e.g., of the distribution of bodies and their gravitational fields."[79] Thus measurement must now be based on the "method of coincidences," where we begin with an assignment of numbers or coordinates having no initial metrical significance; properly metrical determinations are only then possible by taking account of the empirical distribution of matter (and hence curvature) against this so far merely topological background.[80] We have also seen that Schlick explicitly equates this general relativistic "method of coincidences" with his own epistemological solution to the problem of the external world; for it is through this method, and this method alone, that we construct the required bridge between the purely subjective space and time of our intuition, on the one side, and the objective, geometrical-physical space-time ordering, on the other. And Schlick is thereby ultimately committed, by this essentially dualistic conception, to a version of just the kind of causal realism Helmholtz thought he had finally overcome in his *Zeichentheorie*. That the realism in question is a sophisticated and abstract version, based on physics rather than sensory psycho-physiology, goes a long way toward explaining how Schlick could have given such an unexpected twist to Helmholtz's original theory, but, in the end, it does nothing to diminish the highly ironical character of this particular instance of philosophical appropriation or transmutation.

NOTES

1. Versions of this paper were presented at a meeting of the History of Philosophy of Science Working Group (HOPOS) at Notre Dame and at the London School of Economics, both in March 1998. I am particularly indebted to Thomas Ryckman for carefully reading several earlier drafts and making a variety of penetrating suggestions.
2. Helmholtz's celebrated address, "Über das Sehen des Menschen," delivered at the dedication of a monument to Kant in Königsberg in 1855, condemns the then current rift between philosophy and natural science due, in Helmholtz's opinion, to the entirely speculative *Naturphilosophie* of Schelling and Hegel and announces a new project of cooperation between the two disciplines in the spirit of Kant. This address became a model for philosophers who wished to turn away from the "metaphysics" of post-Kantian absolute idealism to a new type of scientific "epistemology"—particularly for those in what then became the tradition of neo-Kantianism. For a paradigmatic instance of this idea, see E. Cassirer, *The Problem of Knowledge: Philosophy, Science, and History since Hegel,* trans. W. Woglom and C. Hendel (New Haven, Conn.: Yale University Press, 1950), 3–5. See also K. Köhnke, *The Rise of Neo-Kantianism: German Academic Philosophy between Idealism and Positivism,* trans. R. Hollingdale (Cambridge: Cambridge University Press, 1991), 96–107.
3. A special issue of *Die Naturwissenschaften* 9 (1921), contained J. von Kries, "Helmholtz als Physiolog," W. Wien, "Helmholtz als Physiker," W. Nernst, "Die electrochemischen Arbeiten von Helmholtz," A. Riehl, "Helmholtz als Erkenntnistheoretiker," and E. Goldstein, "Errinerungen eines Laboratoriumspraktikanten." In the *Abhandlungen der preussischen Akademie der Wissenschaften, philosophisch-historische Klasse*, Jahrgang 1921, B. Erdmann, *Die philosophischen Grundlagen von Helmholtz' Wahrnehmungstheorie,* appeared as no. 1. Three lectures were held at a meeting of the physical, physiological, and philosophical societies of Berlin—E. Warburg, "Helmholtz als Physiker," M. Rubner, "Helmholtz als Physiologe," M. Schlick, "Helmholtz als Erkenntnistheoretiker"—which then appeared together as *Helmholtz als Physiker, Physiologe und Philosoph* (Karlsruhe: C. F. Müllersche Hofbuchhandlung, 1922).
4. P. Hertz and M. Schlick, eds., *Hermann v. Helmholtz: Schriften zur Erkenntnistheorie* (Berlin: Springer, 1921); English edition, trans. M. Lowe, ed. R. Cohen and Y. Elkana, *Hermann von Helmholtz: Epistemological Writings* (Dordresht: Reidel, 1977).
5. M. Schlick, "Die philosophische Bedeutung des Relativitätsprinzip," *Zeitschrift für Philosophie und philosophische Kritik* 159 (1915): 129–75; English edition, trans. P. Heath, in H. Mulder and B. van de Velde-Schlick, *Moritz Schlick: Philosophical Papers* (Dordrecht: Reidel, 1978), vol. 1.

6. M. Schlick, *Raum und Zeit in der gegenwärtigen Physik* (Berlin: Springer, 1917 [first ed.], 1919 [second ed.], 1920 [third ed.], 1922 [fourth ed.]); English version of the third edition, trans. H. Brose (Oxford: Oxford University Press, 1920), together with revisions of fourth edition, trans. P. Heath, in H. Mulder and B. van de Velde-Schlick, *Moritz Schlick: Philosophical Papers.*
7. M. Schlick, *Allgemeine Erkenntnislehre* (Berlin: Naturwissenschaftliche Monographien und Lehrbücher I, 1918 [first edition], 1925 [second edition]); English version of second edition, trans. A. Blumberg (Vienna: Springer, 1974; reprint, La Salle, Ill.: Open Court, 1985).
8. Schlick's invitation to Vienna was engineered by the mathematician Hans Hahn, then a professor at the university, probably with help from his friend Philipp Frank, Einstein's successor in theoretical physics at Prague. It also appears likely that Einstein himself may have aided in this effort. In any case, Einstein was most impressed by Schlick's work on relativity, and, after visiting Schlick at Rostock in 1919, he wrote to Max Born of the need to find Schlick a professorship. See, e.g., F. Stadler, *Vom Positivismus zur "Wissenschaftlichen Weltauffassung"* (Wien: Löcker, 1982), 117–20.
9. Thus, in the centenary contributions cited in n. 3 above, Alois Riehl attempts to appropriate Helmholtz for neo-Kantianism, just as Schlick, for his part, attempts to appropriate him for empiricism. Riehl serves as a target for Schlick in his explanatory notes to Helmholtz's epistemological writings as well.
10. *Allgemeine Erkenntnislehre* (1918), 57–58; *General Theory of Knowledge* (1985), 61–62. Section numbers given parenthetically in the text refer to this work; where the section numbers of the two editions diverge, I give those of the later edition in brackets. All translations from the German, for both Schlick and Helmholtz, are my own.
11. *Schriften zur Erkenntnistheorie* (1921), 115; *Epistemological Writings* (1977), 121–22.
12. "The Facts in Perception," 156n. 15; *Schriften zur Erkenntnistheorie*; *Epistemological Writings,* 166n. 15.
13. The reader will notice that, although both Helmholtz and Schlick deny a relation of "picturing [*Abbildung*]" in general, they both affirm such a relation in the special case of "lawlikeness [*Gesetzmäßigkeit*]." We will come back to this point in n. 75 below.
14. Commentators have been far too quick, in my view, simply to take Schlick's attempt at appropriation at face value. Thus, J. A. Coffa, for example, in *The Semantic Tradition from Kant to Carnap: To the Vienna Station* (Cambridge: Cambridge University Press, 1991), introduces the Schlick of *General Theory of Knowledge* as "the first to attempt a systematic formulation of the picture of knowledge implicit in Helmholtz's writings," and describes this work as an "elaboration of Helmholtz's picture of the link between knowledge and reality" (171–72). T. Ryckman, "*Conditio Sine Qua Non? Zuordnung* in the Early Epistemologies of Cassirer and Schlick," *Synthese* 88 (1991): 57–95, provides a detailed and nuanced examination of the role of this concept in the scientific thought of the period, paying special attention to the crucial importance of Helmholtz's *Zeichentheorie,* which, however, is portrayed as in close agreement with Schlick's particular version of epistemological realism; see esp. 73, 76, 80, and also 86n. 40—which takes the above-cited n. 15 to "The Facts in Perception" as evidence for the close agreement in question.
15. *Allgemeine Erkenntnislehre,* 311; *General Theory of Knowledge,* 365.
16. *Allgemeine Erkenntnislehre,* 72; *General Theory of Knowledge,* 87.
17. *Allgemeine Erkenntnislehre,* 35; *General Theory of Knowledge,* 37.
18. *Allgemeine Erkenntnislehre,* 199; *General Theory of Knowledge,* 232.
19. *Allgemeine Erkenntnislehre,* 233–34; *General Theory of Knowledge,* 272.
20. *Allgemeine Erkenntnislehre,* 236; *General Theory of Knowledge,* 275.
21. Paradigmatic of such systems, for Schlick, are Maxwell's equations for the electromagnetic field and Einstein's field equations for gravitation; see *Allgemeine Erkenntnislehre,* 207; *General Theory of Knowledge,* 242–43. In this way, such well-known slogans as

"Maxwell's theory is Maxwell's equations," in the context of the Pasch-Hilbert tradition in geometry, constitute the immediate background to Schlick's epistemological conception.

22. *Allgemeine Erkenntnislehre,* 247; *General Theory of Knowledge,* 288.
23. *Raum und Zeit in der gegenwärtigen Physik* (1917), 32; *Space and Time in Contemporary Physics,* in Mulder and van de Velde-Schlick, *Moritz Schlick: Philosophical Papers,* 238–39.
24. *Raum und Zeit,* 33; *Space and Time,* 240.
25. *Raum und Zeit,* 35; *Space and Time,* 241. In my "Critical Notice" of Mulder and van de Velde-Schlick, *Moritz Schlick: Philosophical Papers,* in *Philosophy of Science* 50 (1983): 498–514, I was too quick to equate this passage with an endorsement of a version of verificationism, according to which only *observable* space-time coincidences are real (504–5)—and a parallel error occurs in my *Foundations of Space-Time Theories: Relativistic Physics and Philosophy of Science* (Princeton: Princeton University Press, 1983), 22–25. For in my "Critical Notice," I overlooked the central role of section 30 [31] in *General Theory of Knowledge,* and thereby overlooked the extent to the which the explicitly *realist* position of that work is intimately connected with Schlick's reading of general relativity (contrary to 500n. 3). At the same time, by missing the centrality of section 30 [31], I also did not see that Schlick, in *General Theory of Knowledge,* has a definite way of bridging the gap between purely formal systems of implicit definitions and the realities they are supposed to designate (507–8).
26. *Raum und Zeit,* 53–54; *Space and Time,* 260.
27. *Raum und Zeit,* 58; *Space and Time,* 263. The final sentence is omitted in the later edition, which instead adds some comments on the "paradoxes" of relativity.
28. Helmholtz, "Über die Erhaltung der Kraft," in *Wissenschaftliche Abhandlungen* (Leipzig: Johann Ambrosius Barth, 1882), 1: 12–13; English edition, trans. R. Kahl, in *Selected Writings of Hermann von Helmholtz* (Middleton, Conn.: Wesleyan University Press, 1971), 3.
29. *Wissenschaftliche Abhandlungen,* 1: 13; *Selected Writings,* 4.
30. *Wissenschaftliche Abhandlungen,* 1: 16; *Selected Writings,* 6.
31. Helmholtz, "Über das Sehen des Menschen," in *Vorträge und Reden* (Braunschweig: Friedrich Vieweg und Sohn, 1903), 1: 115–16.
32. Ibid., 116.
33. *Schriften zur Erkenntnistheorie,* 130; *Epistemological Writings,* 138.
34. *Schriften zur Erkenntnistheorie,* 133–134; *Epistemological Writings,* 142.
35. *Wissenschaftliche Abhandlungen,* 1: 68; *Selected Writings,* 49 (in the latter, however, the first sentence is grossly mistranslated so as to have Helmholtz saying that he *still* considers "Kant's epistemological insights" to be correct). The note goes on to suggest that Helmholtz was also earlier mistaken in separating *matter* too sharply from the *forces* or *laws* in accordance with which it acts—where Helmholtz's conception of matter in 1847 parallels Kant's conception, in the *Metaphysische Anfangsgründe der Naturwissenschaft* (1786), which also depicts matter as acting only in accordance with the "fundamental forces" of attraction and repulsion. It seems likely that Helmholtz's movement away from this earlier Kantian position was closely connected with his realization that *electromagnetic* forces cannot be assimilated to "unalterable" central forces in this sense, and thus that the fundamental equivalence he had tried to set up between phenomenological energy conservation, on the one hand, and a reduction to mass points and central forces, on the other, does not in fact hold good (see nn. 2–4 from 1881). This particular route from phenomenological lawlikeness to "ultimate unalterable causes" lying behind the appearances was therefore closed.
36. Helmholtz, *Handbuch der physiologischen Optik* (Leipzig: Leopold Voss, 1857–67 [first edition], 1896 [second edition], 1910 [third edition]); English version of third edition, trans. J. Southall (Menasha Wisc.: George Banta, 1925). The most extended defense of

Helmholtz's "empiricist" theory is found in § 26, "Von den Wahrnehmungen im allgemeinen." The second edition drastically revises this section by incorporating much of the content of "The Facts in Perception." The third edition is a posthumous reprinting of the first. I will cite the third edition, which gives the page numbers of the first edition in the margins.

37. For the development and basic ideas of Helmholtz's theory I largely follow the lucid and balanced account in G. Hatfield, *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz* (Cambridge, Mass.: MIT Press, 1990), chap. 5. Hatfield is particularly careful to distinguish the "empiricist"/"nativist" debate in the psycho-physiology of perception from the traditional "empiricist"/"rationalist" debate in modern epistemology; see Appendix A. I also agree with Hatfield in seeing a gradual "retreat" from the early causal realism of 1855, culminating in "The Facts in Perception," as Helmholtz further develops his theory; see 208–14.
38. *Handbuch* (1910), 3: 21; *Handbook* (1925), 3: 23.
39. *Handbuch,* 3: 433; *Handbook,* 3: 533.
40. *Handbuch,* 3: 26; *Handbook,* 3: 29.
41. *Handbuch,* 3: 28; *Handbook,* 3: 30–31.
42. *Handbuch,* 3: 29; *Handbook,* 3: 32.
43. *Handbuch,* 3: 18; *Handbook,* 3: 19.
44. *Handbuch,* 3: 22; *Handbook,* 3: 24. In an unpublished note reproduced in L. Königsberger, *Hermann von Helmholtz* (Braunschweig: Friedrich Vieweg und Sohn, 1903), Helmholtz writes, similarly, that "representations are signs, which are translatable back into actuality through motions" (2: 159).
45. "The Facts in Perception," *Schriften zur Erkenntnistheorie,* 117; *Epistemological Writings,* 124.
46. *Wissenschaftliche Abhandlungen* (Leipzig: Johann Ambrosius Barth, 1883), 2: 610.
47. Riemann's work was first presented as an Habilitation lecture in 1854 and only published in 1867. As Helmholtz explains, he first became aware of Riemann's work after he had already begun his own investigations.
48. *Wissenschaftliche Abhandlungen,* 2: 610.
49. Ibid., 616.
50. Helmholtz's mathematical results were rigorously proved with the resources of the new theory of continuous groups (what we now call Lie groups) by Sophus Lie in his *Theorie der Transformationsgruppen* of 1873. The basic content of Lie's theorem, as we would now formulate it, is as follows: Given a differentiable manifold, and a Lie group of differentiable transformations on this manifold acting freely transitively, there is a unique (up to a scale factor) Riemannian metric on the manifold whose isometries are given precisely by the Lie group in question. Since the metric thereby constructed has a nontrivial group of isometries or rigid motions, it must then have constant curvature. (To say that the group of transformations acts "freely transitively" is to say, intuitively, that given any two points on the manifold, together with two "observers" or "points of view" having definite situations and orientations at these points, there is exactly one transformation in the group mapping one onto the other.)
51. The importance of Helmholtz's earlier mistaken claim, its connection to his "empiricist" theory of spatial perception, and the shift that resulted when he corrected this claim, are all clearly and perceptively explained in J. Richards, "The Evolution of Empiricism: Hermann von Helmholtz and the Foundations of Geometry," *British Journal for the Philosophy of Science* 28 (1977): 235–53.
52. *Wissenschaftliche Abhandlungen,* 2: 614.
53. "On the Origin and Significance of the Axioms of Geometry," *Schriften zur Erkenntnistheorie,* 17; *Epistemological Writings,* 17.
54. Ibid., *Schriften zur Erkenntnistheorie,* 22; *Epistemological Writings,* 23.

55. See "The Facts in Perception," *Schriften zur Erkenntnistheorie,* 117–21; *Epistemological Writings,* 124–28. Strictly speaking, to make good on this claim we need to connect the bodily motions of the *perceiver* with the motions of *measuring instruments* definitive of congruence. Helmholtz himself makes the connection by viewing measurement as a refinement and specification of ordinary bodily motion; see *Schriften zur Erkenntnistheorie,* 18; *Epistemological Writings,* 19. In the group-theoretical formulation due to Sophus Lie (n. 50 above), however, the connection is transparent. We start with an abstract group of transformations, which can then be taken simply *to be* the group of bodily "displacements" of the perceiver, and this last approach is explicit in Henri Poincaré's work on the philosophical foundations of geometry. For discussion of Poincaré in relation to Helmholtz (and Kant), see my "Geometry, Construction, and Intuition in Kant and his Successors," in *Between Logic and Intuition: Essays in Honor of Charles Parsons,* ed. G. Scher and R. Tieszen (Cambridge: Cambridge University Press, forthcoming).

56. This is the title of Appendix 2 to "The Facts in Perception," *Schriften zur Erkenntnistheorie,* 140–42; *Epistemological Writings,* 149–52. In the body of the paper, after explaining how "the most essential features of spatial intuition" can be derived from our experience of bodily motion, Helmholtz puts the distinction this way: "[i]t is well known that Kant already assumed, not only that the general form of spatial intuition is transcendentally given, but also that it contains in advance, and prior to all possible experience, certain narrower determinations, as they are expressed in the axioms of geometry." He proceeds to give axioms for specifically Euclidean geometry, and, after explaining that "those reasons that allow us to infer that the form of intuition of space is transcendental, do not yet necessarily suffice to prove at the same time that the axioms are also of transcendental origin," he points out again that this conflicts with Kant's conception of spatial intuition, according to which "spatial relations that contradicted the axioms of Euclid could not at all even be represented" (*Schriften zur Erkenntnistheorie,* 121–22; *Epistemological Writings,* 128–29). These passages, in connection especially with those from "On the Origin and Significance of the Axioms of Geometry" cited in the text immediately above, strongly support the idea that Helmholtz includes free mobility (and thus constant curvature) in the "general transcendental form of spatial intuition," and thus excludes the "axioms" understood as specifically Euclidean. Schlick, in his nn. 32 and 33 to these passages from "The Facts in Perception," suggests that "the most essential features of spatial intuition" comprise those of a "three-dimensional continuous manifold . . . in which the comparison of magnitudes is possible"—so that, for Helmholtz, free mobility is therefore included among them. In considering what "the general form of spatial intuition" might be, however, Schlick distinguishes two possibilities: it could comprise precisely these "most essential features," or it could comprise only the purely qualitative "extendedness" provided by "that indescribable psychological moment of spatiality adhering to sense perceptions." Because the former alternative would still lead to a description in terms of *some or another* axioms, Schlick himself favors the latter. See *Schriften zur Erkenntnistheorie,* 161–62; *Epistemological Writings,* 172–75.

57. *Schriften zur Erkenntnistheorie,* 111; *Epistemological Writings,* 117.

58. *Schriften zur Erkenntnistheorie,* 128; *Epistemological Writings,* 135–36.

59. *Schriften zur Erkenntnistheorie,* 130; *Epistemological Writings,* 138.

60. *Schriften zur Erkenntnistheorie,* 132; *Epistemological Writings,* 140.

61. *Schriften zur Erkenntnistheorie,* 132; *Epistemological Writings,* 140–41.

62. "The Aim and Progress of Physical Science" (1869), *Vorträge und Reden,* 1: 393; *Selected Writings,* 242. Compare n. 44 above.

63. *Schriften zur Erkenntnistheorie,* 157n. 20; *Epistemological Writings,* 167n. 20.

64. *Schriften zur Erkenntnistheorie,* 130; *Epistemological Writings,* 138.

65. See especially n. 44, where Schlick explains that "modern psychology rejects the concept of unconscious inference energetically, because it rightly considers thinking, the logical process, exclusively as a function of *consciousness*" (*Schriften zur Erkenntnistheorie,* 165; *Epistemological Writings,* 176).

66. For Helmholtz's use of Mill here, see, e.g., *Handbuch,* 3: 23–24; *Handbook,* 3: 24–26. A particularly thorough discussion of this aspect of Helmholtz's theory is given by Erdmann, *Die philosophischen Grundlagen von Helmholtz' Wahrnehmungstheorie,* 31–37 ("Die logischen Annahmen").

67. *Schriften zur Erkenntnistheorie,* 131; *Epistemological Writings,* 139.

68. *Schriften zur Erkenntnistheorie,* 134; *Epistemological Writings,* 143.

69. See n. 56 above. All commentators known to me have missed the fundamental difference, in this regard, between the original sensory qualities themselves and the lawlike spatial ordering in which they are embedded. Accordingly, they have also been unable to see that, whereas the former are indeed purely subjective, the latter has a necessary "objectifying" role, as a condition of the possibility of objective external experience, having its own necessary structure given by the condition of free mobility. As a consequence, such commentators have also been unanimous in finding a basic confusion in Helmholtz between the subjectivity of the sensory qualities, on the one hand, and the idea that space is a subjective form of intuition, on the other—a confusion due, supposedly, to Helmholtz's assimilation of the latter to the former. We find just this charge in the passage from Schlick's n. 20 cited above in my n. 63, where the misunderstanding is only further compounded by the identification of what Schlick calls "psychological space" with intuitively given, purely qualitative spatiality—which, according to Schlick's n. 33, is an "indescribable . . . purely qualitative 'extendedness'" (n. 56 above). Schlick thereby fails to appreciate that one of the main points of Helmholtz's "empiricist" theory of space is *to deny* the existence of any originally given, purely qualitative "extendedness." Even Hatfield, who is otherwise extremely helpful and clear about this last feature of Helmholtz's theory, nevertheless repeats the traditional complaint against "Helmholtz's assimilation of Kantian subjectivity of space to the accepted notion of the subjectivity of color" (*The Natural and the Normative,* 224), and what is missing here, once again, is an appreciation of how Helmholtz's own work in the foundations of geometry still yields a necessary and objective structure (but a structure more general than the Euclidean one endorsed by Kant) for the *ordering* of sensory qualities in what Helmholtz calls "the subjective form of intuition." In this sense, space is not only subjective, but also "transcendental." And it is for this reason that Helmholtz applies the latter term *only* to space, and *not* to the sensory qualities thereby ordered in space.

70. *Schriften zur Erkenntnistheorie,* 131; *Epistemological Writings,* 139.

71. *Schriften zur Erkenntnistheorie,* 168–69nn. 54, 55; *Epistemological Writings,* 179nn. 54, 55.

72. *Allgemeine Erkenntnislehre,* 183; *General Theory of Knowledge,* 215.

73. In his centenary lecture of 1921 (n. 3 above) Schlick presents a more nuanced and (from his own point of view) more charitable picture of Helmholtz's relation to "immanence philosophy." On the basis of the passages we have emphasized from "The Facts in Perception," where Helmholtz distances himself from both realism and idealism, Schlick applauds Helmholtz's rejection of a "realist metaphysics" in favor of the view "that we have in signs a means for depicting [*abzubilden*] the *lawlikeness* of the actual; nothing further than this is asserted or required—but this requirement or assertion is independent of every metaphysical hypothesis" ("Helmholtz als Erkenntnistheoretiker," 34–35; trans. in Mulder and van de Velde-Schlick, *Moritz Schlick: Philosophical Papers,* 338–39). Schlick then approvingly associates Helmholtz with the "program for a positivistic interpretation of physics" of Gustav Kirchhoff. Does this mean that Schlick is now rejecting his own scientific realism in *General Theory of Knowledge*? No; for Schlick also endorses Kirchhoff in an analogous context in *General Theory of Knowledge* (§ 11 [12]), where metaphysical views (such as those of Bergson and Husserl, for example) that result from the confusion of knowledge with acquaintance are rejected. In general, the main point of "Helmholtz als Erkenntnistheoretiker" is to portray Helmholtz as a paradigm of *empiricism,* against both Kantian and metaphysical interpretations of science. In the immediately following passage, for example, Schlick invokes the *Zeichentheorie,* not to reject the scientific realism of *General Theory of*

Knowledge, but rather to reject the traditional conception of an underlying *substance* existing behind natural actions or effects.

74. *Allgemeine Erkenntnislehre,* 184; *General Theory of Knowledge,* 215.

75. This divergence shows itself in the very different ways in which Helmholtz and Schlick conceive our ability to "picture" the lawlikeness of reality (see n. 13 above). For Helmholtz, we apprehend lawlikeness in the temporal succession of objective events through the similarly lawlike temporal succession of the sensations to which they "correspond" (i.e., of which they are interpretations). Since both "sides" of the relationship are temporal, we have genuine similarity and thus "picturing." For Schlick, by contrast, there can be no such similarity between the subjective and the objective, and the sense in which our system of concepts "pictures" the lawlikeness of reality is solely that of an abstract mapping or coordination: "One will have to describe the 'universe in itself' as a manifold of infinitely many different qualities which are so interwoven and interdependent that they can be designated by the quantitative conceptual systems of the natural sciences. Through these [systems] the lawlikeness of their coming to be and ceasing to be is reproduced (where the words 'becoming' and 'ceasing' are to be taken in a transferred sense, for it is not a matter of alterations in intuitive time, but rather of places in the objective ordering). To each of the qualities in the external world one can coordinate a concept that is constructed out of a combination of concepts of other qualities: in precisely this way the lawlikeness of the all-encompassing interconnection [of realities] expresses itself, for it is first through it that this kind of coordination becomes possible. To discover this lawlikeness is to know the external world, for with it the most general is found again in the individual, and the latter is thereby known" (*Allgemeine Erkenntnislehre,* 244; *General Theory of Knowledge,* 284).

76. An important aspect of Schlick's correspondence-theoretic conception of the objective external world is that the relation of coordination that projects the subjective-psychological realm onto this objective realm turns out to be highly *non*-univocal. In § 30 [31] of *General Theory of Knowledge* Schlick makes a prima facie claim to such univocality: "Two perceptual objects that touch one in another in visual or tactual space . . . must correspond to transcendent things that have a 'point' in common in the objective order-schema, for otherwise two places of the transcendent space would be coordinated to one and the same place in a perceptual space, which would contradict univocality" (*Allgemeine Erkenntnislehre,* 235; *General Theory of Knowledge,* 274). In the second edition, however, Schlick adds a paragraph pointing out that such prima facie univocality of coordination is often misleading, because objects that touch one another in the visual field, for example, are often quite distant in objective space. Thus, "objective coincidences are never experienced directly," but are rather "inferred" from immediate experience on the basis of "rules . . . [that] are treated in more detail in the philosophy of science" (*General Theory of Knowledge,* 274). For Helmholtz, by contrast, the construction of geometrical-physical space from lawlike relations among our sensations entails, via what we now know as the Helmholtz-Lie theorem (n. 50 above), that the former is *uniquely* determined by the latter.

77. Kant's own solution to the problem of the external world, on the basis of a rejection of causal realism in favor of the idea that we directly perceive objects outside us in space, is found in the Fourth Paralogism in the first edition of the *Critique of Pure Reason* and the Refutation of Idealism in the second, where both arguments presuppose from the beginning the doctrine of space as a subjective, "transcendentally ideal" form of intuition. The widespread failure (see n. 69 above) to appreciate the close remaining kinship between this doctrine and Helmholtz's theory of space among commentators, Schlick included, is responsible, in my opinion, for the similarly widespread failure to appreciate the close remaining kinship between Helmholtz's and Kant's solutions to the problem of the external world.

78. In general relativity we have approximately rigid rods and approximately free mobility, in the sense that both exist in infinitesimal neighborhoods. In this very same sense, however, we also have approximately *Euclidean* geometry. What we do not have is a situation where constant curvature is fixed antecedently, as it were, and then experience is

called in to select one of the three classical geometries of constant curvature. But it is precisely this view of the situation, as we have seen, that frames Helmholtz's own empiricist conception of geometry, according to which the "axioms" of [specifically Euclidean] geometry now emerge as merely empirical facts. In "Helmholtz als Erkenntnistheoretiker" especially (n. 73 above) Schlick fastens on this last feature of Helmholtz's conception to portray him as a defender of empiricism against Kantianism, and to claim that general relativity has "brilliantly confirmed" Helmholtz's conception ("Helmholtz als Erkenntnistheoretiker," 36–38; Mulder and van de Velde-Schlick, *Moritz Schlick: Philosophical Papers,* 339–41). Schlick thereby misses once again the sense in which free mobility gives Helmholtz an analogue of Kant's doctrine of space as a "transcendental" form of intuition, within which all properly empirical discoveries (including the discovery of a particular value for the fixed constant curvature of space) are then made possible (see again n. 69 above).

79. See n. 24 above.

80. In his n. 39 to "On the Origin and Significance of the Axioms of Geometry," where Helmholtz say, that "we may not forget here that all geometrical measurements rest finally on the principle of congruence" (which, for Helmholtz, clearly involves free mobility), Schlick, executing a most remarkable transposition, comments as follows:

> "Congruence" is established through observation of the coming together [*Zusammenfallens*] of material points. All physical measurements can be reduced to this same principle . . . Helmholtz's proposition can therefore be extended to the truth that, in general, no other events are physically ascertainable than meetings of points, and Einstein has logically drawn the conclusion from this that all physical laws may in principle contain only assertions about such coincidences. The following paragraphs of Helmholtz's lecture contain statements that move in the same direction. (*Schriften zur Erkenntnistheorie,* 321; *Epistemological Writings,* 33–34)