A Kantian Legacy in History and Philosophy of Science [1]

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In this lecture I shall build on the previous lectures to construct what I take to be the most appropriate way of generalizing and transforming Kant's conception of proper natural science within our post-Einsteinian—and post-Kuhnian—intellectual context. In particular, I shall build on Helmholtz's conception of what is lawlike in the phenomena and Cassirer's emphasis on abstract mathematical structures to delineate a replacement for the Kantian notion of schematism: an account of the *application* of mathematical structures to experience in physics. The resulting conception owes much to two contemporary philosophical scholars of the development of modern physics beginning with Newton and extending through the late nineteenth and early twentieth centuries—Howard Stein and George E. Smith. From the former I appropriate a conception of abstract structures "in the phenomena," from the latter a conception of "theory-mediated measurement." My primary focus throughout is on the legacy of Kant's anti-Humean conception of causal necessity.

My previous discussions of Kantian causal necessity have emphasized the role of the category of necessity in the Postulates of Empirical Thought **[2]**. I have argued, against the views of Gerd Buchdahl and others, that Kant does have an anti-Humean conception of necessary causal connections and necessary empirical causal laws, and that this kind of necessity involves an essential *constitutive* contribution from the faculty of understanding over and above all merely *regulative* contributions from the faculties of reason and/or reflective judgement. For the category of necessity, in particular, is precisely a pure concept of the understanding. I have also argued that the further articulation of Kant's conception of causal necessity from the first edition of the *Critique of Pure Reason* (1781) through the *Prolegomena* (1783) and *Metaphysical Foundations of Natural Science* (1786) to the second edition (1787) shows that the Newtonian law of universal gravitation is paradigmatic, for Kant, of a necessary but still empirical causal law.

I am now further developing and extending this interpretation by focussing on the category of causality and, in particular, on the way in which this category is related to the "predicable" (derivative category) of force. I argue that Kant's conception of force is more indebted to the Newtonian mathematical concept of impressed force than I had previously emphasized, and that what Kant takes to be necessary and strictly universal empirical causal laws—such as, paradigmatically, the law of gravitation—thereby acquire a more than inductive, as well as more than hypothetical, epistemic status. Newton's methodology of "deduction from phenomena" thus turns out to be central to Kant's conception of causal necessity. I then explore the legacy of this conception: the prospects for extending it into the future in the context of post-Newtonian physical theories such as Einstein's theory of relativity.

1. Force in Kant and Newton

I begin with the fourth paragraph of the Preface to the *Metaphysical Foundations*, where, after introducing the notion of *proper* natural science in the third paragraph, Kant explains that this kind of science depends on its pure part (*MF* 4: 468-9) **[3]**: "Since the word nature already carries with it the concept of laws, and the latter carries with it the concept of the *necessity* of all determinations of a thing belonging to its existence, one easily sees why natural science must derive the legitimacy of this title only from its pure part—namely, that which contains the a priori principles of all other natural explanations—and why only in virtue of this pure part is natural science to be proper science." Kant thus indicates that "proper" natural science—paradigmatically physics—is wider than "pure" natural science: the latter is strictly a priori, and it makes possible empirical natural laws, like the law of universal gravitation, which are necessary (in the sense of the category of necessity) but *not* a priori.[3] **[4]** In the case of physics, then, the a priori principles in question belong to *pure* natural science, but there are also empirical natural laws, like gravitation, which still count as necessary and belong to *proper* natural science.

All demonstrated Propositions in the *Metaphysical Foundations*—especially Kant's three Laws of Mechanics **[5]** (conservation of quantity of matter, inertia, action equals reaction)—are strictly synthetic a priori. But the law of universal gravitation is not demonstrated a priori, for Kant, but is inferred from Kepler's so far merely inductive "rules" by what Newton calls "deduction from phenomena." **[6]** The law of universal gravitation—and thereby Kepler's laws as well (as suitably *corrected* by Newton)—now acquire a more than merely inductive status. I elaborate upon this status in what follows.

Consider a passage from the second remark to Proposition 7 of the Dynamics where Kant appears sharply to differ with Newton concerning gravity as an immediate action at

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[3] Kant's clearest statement in the *Metaphysical Foundations* occurs in the General Remark to Dynamics (534): "[N]o law of either attractive or repulsive force may be risked on a priori conjectures. Rather, everything, even universal attraction as the cause of weight [*Schwere*], must be inferred, together with its laws, from data of experience."

a distance (515) **[7]**: "[I]t is clear that the offense taken by his contemporaries, and perhaps even by Newton himself, at the concept of an original attraction set him at variance with himself: for he could absolutely not say that the attractive forces of two planets, e.g., of Jupiter and Saturn, manifested at equal distances of their satellites (whose mass is unknown), are proportional to the quantity of matter of these heavenly bodies, if he did not assume that they attracted other matter merely as matter, and thus according to a universal property of matter." At issue is the method Newton develops in Book 3 of the *Principia* for measuring the masses of primary bodies by the accelerations produced in their satellites **[8]** (a method that is paradigmatic, for me, of what George Smith calls "theory-mediated measurement"). Newton, according to Kant, must assume the immediacy of the gravitational interaction betewen Jupiter and Saturn, independently of any intervening matter surrounding them, in order to infer their masses from the accelerations of their moons. **[9] [10]**

Is Kant saying that Newton must accept the existence of real action at a distance despite the qualms vigorously expressed by contemporary representatives of the mechanical philosophy such as Huygens and Leibniz? This question is more subtle than I had previously thought. For consider what Kant says before the just-quoted passage (515; emphasis added) **[11]**: "[Newton] *rightly* abstracted from all hypotheses purporting to answer the question as to the cause of the universal attraction of matter, for this question is physical or metaphysical, but not mathematical." Here Kant is endorsing the abstract treatment of impressed force as a measurable quantity that Newton has been articulating since Proposition 1 of Book 1 **[12]**, which demonstrates that Kepler's area law for a given trajectory is mathematically equivalent to generation by a centripetal force (always directed towards a single center), whatever its deeper explanation might be.[7] The aim is to determine the mathematical properties of this force **[13]**, and then to use these properties to support inferences from empirical (albeit mathematically expressed) propositions like Kepler's "rules" to further mathematical properties of the force in question (such as the inverse-square law for gravitational force). We thereby open the way to a continued series of empirical inferences that progressively correct the mathematical laws obtained at one stage by considering ever-more subtle effects of gravitational force, which both further support the earlier-obtained laws while also revealing their fundamentally approximate character.

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[7] The treatment is thus "abstract" in the sense of *abstracting from* any such explanation; it does not mean that the force is an abstract mathematical entity in the sense of contemporary mathematical Platonism (more on this below).

Thus, for example, Newton's derivation (from Phenomenon 6) in Proposition 3 of Book 3 **[14]** of an inverse-square acceleration of the moon towards the earth involves a promissory note concerning the perturbative effect of the sun on the moon. But the note only begins to be redeemed, later in this Book, with Newton's detailed attack on the three-body problem, and it is only fully redeemed, after Newton's death, by Euler's and Clairault's first successful treatment of the motion of the moon in the 1740s. A similar sequence of increasingly accurate approximations leads (as already suggested) from Kepler's "rules" assumed as exact to corrections of them based on the perturbative effects of bodies other than the sun, and so on. This procedure of approximative correction and theory-mediated measurements (here of the relevant accelerations) continues with ever-increasing success throughout the nineteenth century, the most spectacular being the well-known discovery of Neptune by Adams and Leverrier in 1846. Decisive obstacles arose only at the turn of the twentieth century, when minute deviations in the predicted advance of the perihelion of Mercury eluded all attempts at solution within Newtonian theory. The first successful solution then became one of the triumphs of Einstein's general theory of relativity, a fundamentally revised theory of gravitation, in 1915. I shall return to this case below, in discussing the legacy of Kant's conception in our post-Newtonian context.

But what is the evidence for attributing the Newtonian conception of impressed force—which abstracts from all hypotheses concerning the underlying mechanism of gravitational interaction—to Kant? To begin with, Kant was undoubtedly familiar with Newton's *hypotheses non fingo passage* in the General Scholium to the *Principia* (1999, 943) **[15]**: "I have not as yet been able to deduce from phenomena the reason for these properties of gravity, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy." Newton is here defending his mathematical law of gravitation from criticisms raised by mechanical philosophers (especially Huygens and Leibniz), who insisted on the priority of action by contact in order to explain this law, and Kant develops a parallel response to the mechanical philosophy on behalf of what he calls a "metaphysical-dynamical mode of explanation" in the General Remark to Dynamics.

The most significant evidence, however, is found in the discussion following the fourth paragraph of the Preface (468-9). The fifth paragraph explains that the *pure* part of proper natural science consists of two further parts (469) **[16]**: "pure rational cognition from mere *concepts*" or "metaphysics," and "that which grounds its cognition only on the *construction* of concepts" or "mathematics." He asserts that we must **[17]** presuppose, in the first place, "metaphysics of nature"—"[f]or laws, that is principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be constructed, since existence cannot be presented a priori in any intuition" (ibid.). But "metaphysics of nature," according to the sixth paragraph (469-70), again consists of two further parts: a metaphysics of nature *in general*, which considers only the pure concepts of the understanding, and a *special* metaphysics of nature—which, for example, considers the empirical concept of matter as a particular instantiation of the pure concepts of the understanding.

There follows a notoriously difficult (seventh) paragraph, according to which special, unlike general, metaphysics *requires* the application of mathematics. The crux of the difficulty involves the role of mathematical construction (470) **[18]**: "[T]o cognize something a priori means to cognize it from its mere possibility. But the possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). Hence, in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the *intuition* corresponding to the concept be given a priori, that is, that the concept be constructed." Is Kant saying that special metaphysics—in particular the special metaphysics of corporeal nature—proceeds by mathematically constructing its concept of matter in pure intuition? This cannot be right. For the concept of matter is an *empirical* concept, and Kant is clear—e.g., in the Postulates of Empirical Thought—that empirical concepts as such cannot be constructed in pure intuition.

More telling in the present context, however, is the very passage from the Preface that we are considering. The possibility that needs to be demonstrated a priori is that "of the object [of the concept], as a natural thing that can be given outside the concept (*as existing*)" **[19]** (470; emphasis added). And Kant has initiated this discussion by explaining the need to begin with metaphysics of nature—as opposed to mathematics—because, in this context, we **[20]** "are concerned with a concept that cannot be constructed, since *existence* cannot be presented a priori in any intuition" (469; emphasis added). Moreover, according to Kant's discussion of the Analogies of Experience in the first *Critique*, the (general) metaphysical principles governing **[21]** the *existence* of things are precisely these Analogies.

So Kant is not saying that the empirical concept of matter can be constructed in pure intuition, independently of both general metaphysics and experience. The point, rather, is that, assuming general metaphysics as our background, we must articulate the *empirical* concept of matter as an instantiation of the categories of the understanding, and this must be a *mathematical* instantiation, in particular, of the categories of substance, causality, and community. Thus, the most important principles of the special metaphysics of corporeal nature **[22]** are Kant's three Laws of Mechanics, which instantiate the corresponding categories by *applying* mathematics to the empirical concept of matter (and thereby *articulating* this concept mathematically). We thereby obtain a mathematically precise notion of quantity of matter (and thus quantity of substance), a precise notion of force or causal action (governed by the law of inertia), and a precise notion of mutual action or interaction (governed by action equals reaction).

It is no wonder, then, that Kant proposes precisely these Laws of Mechanics in the Introduction to the second (1787) edition of the first *Critique* as indisputable examples of synthetic a priori propositions of "pure natural science." **[23]** It is also no wonder that, when discussing the a priori concepts of substance, causality, action, and force in the Second Analogy, Kant appeals to the example of *moving forces* to provide the particular content of an alteration of state (A207/B252) **[24]**: "For this acquaintance with actual forces is required, which can only be given empirically, e.g., the moving forces, or, what is the same, certain successive appearances (as motions), which indicate such forces." And, since these forces give rise to law-governed motions (accelerations) conceived as alterations of state, the law of inertia is also necessarily involved. **[25]**

Before turning to the legacy of Kant's conception of force and causal necessity in post-Newtonian physical science, I shall summarize and clarify the relationship between this conception and Newton's. The first point is **[26]** that Kant accepts Newton's abstract conception of force as a measurable mathematical magnitude, considered independently of all merely hypothetical elements. This does not mean that force is either what we would call an abstract mathematical entity or what we would call an observable as opposed to theoretical entity—one reducible to purely observational, theory-neutral concepts. On the contrary, the fundamental concepts of mass, force, and acceleration function as primitive terms in Newton's physical theory, where they are related to one another mathematically (and thereby *articulated* mathematically) via Newton's Axioms or Laws of Motion. The latter (**[27]** and this is the second point) serve to enable *theory-mediated* measurements of these quantities, including the quantity of duration (elapsed time). Thus, for example, given the observed Keplerian orbits, Proposition 1 of Book 1 provides an empirical measure of duration in terms of geometrical areas; later propositions of the *Principia* provide an empirical inverse-square estimation of the centripetal force in question; and, as we have seen, still later propositions successively build on these results **[28]** to provide a universal common measure of mass (third point) or quantity of matter for the primary bodies in the solar system. When I speak of abstract structures *in the phenomena* (**[29]** fourth point), therefore, I am referring to systems of empirically determinable lawlike relations between empirically measurable (albeit still theoretical) quantities. Such systems of empirically determinable relations are to be contrasted with the merely hypothetical postulation of "theoretical entities" that may not have, at least at present, any such means of empirical determination. Thus, for example, while contemporary representatives of the mechanical philosophy insisted on the need for vortex models of planetary motion to explain Newton's results, he was well within his rights to reject this insistence on behalf of his so far not further explained *law* of universal gravitation—since the vortical motions in question lacked all means, at the time, of robust empirical determination.

The possibility of robust empirical determinations of the relevant physical quantities provides Newton's theory with a more secure epistemic status than either mere inductive inferences (curve-fitting) or mere hypothetico-deductive arguments (inference to the best explanation). The relative weakness of mere hypothetico-deductive arguments is well illustrated by the example of Newton's confrontation with the mechanical philosophy over its insistence on vortex models of planetary motion. For the relative weakness of mere inductive inferences, however, consider the relationship between Newton's argument for the law of gravitation and Kepler's (so far) merely inductive "rules." To be sure, Newton begins his argument from the observable Keplerian phenomena, taken provisionally as well-supported universal claims. The crucial point, however, is that Newton's argument involves an open-ended sequence of *corrections* to Kepler's "rules" taking increasingly accurate account of gravitational perturbations. Indeed, these corrections lead to the conclusion that the "orbits" in question are not even closed curves (due to perturbatively induced precession)—a conclusion that could not be effectively reached by the collection of further data points and curve-fitting. It is precisely here, in my view, that the greater strength of the characteristically Newtonian method of reasoning *from phenomena* becomes fully clear.[18] And this greater epistemic strength **[30]**, on my reading, is a centrally important constituent of Kant's notion of causal necessity (fifth point).

[18] An important advantage of this method is that Newton could settle the choice between geocentric and heliocentric world-systems *empirically* by theory-mediated measurements of the masses of the primary bodies in the solar system: the center of mass of the system (which is always very close to the center of the sun) is thereby determined as the preferred dynamical center for describing the true motions. A merely kinematical description, by contrast, necessarily involves an entirely arbitrary choice of reference frame.

2. The Legacy of Kant's Conception in Post-Newtonian Physics

The original Newtonian method breaks down, as suggested, in the case of minute deviations from the predicted perturbations of the orbit of Mercury, which are only satisfactorily explained on the basis of Einstein's theory of relativity developed in the years 1905-1915. At this point our conception of gravitation takes a completely unexpected direction by now considering a non-Newtonian causal action exerted by the variable non-Euclidean curvature of space-time (sixth point **[31]).** And it is at precisely this point that the original Kantian conception of force and causal necessity also breaks down. (seventh point **[32]**) Kant does have room for the corrections to the planetary orbits that can be accounted for within the Newtonian theory of gravitation—that is, the Newtonian gravitational perturbations.[19] **[33]** But he has no room for causal actions **[34]** exerted by a non-Euclidean variation of curvature in our form of outer (spatial) intuition. So the question of the legacy of Kant's conception becomes that of the precise way in which it might now be appropriately extended and generalized.

I shall approach this question, as before, by focussing on the concept of force—and, more generally, of interaction. Einstein's **[35]** general theory of relativity is, like Newton's theory, a theory of specifically gravitational interaction. In order properly to appreciate Einstein's theory, however, we must first gain an appreciation of Einstein's earlier (1905) special theory of relativity, which articulates a post-Newtonian spatio-temporal framework for considering electro-magnetic force—the second of what we now take to be the four fundamental forces of nature, beginning (in order of discovery) with gravitational force.

An adequate mathematical description of what we now call the electro-magnetic field **[36]** was achieved by Maxwell in the second half of the nineteenth century, in terms (as we now understand it) of electric and magnetic force vectors continuously distributed over space. This field of force **[37]**, like Newtonian gravitational force, is a mathematically described physical magnitude. And, as such, it can be studied in abstraction from (independently of) any deeper physical realizations—involving, for example, underlying motions in mechanical, fluid, or elastic aethers. Unlike Newtonian gravitational force, however, the electro-magnetic field is *dynamical*: it is propagated

[19] Kant acknowledges such corrections in the Appendix to the Dialectic (A663/B691): "[W]e arrive at unity of the genera of these paths according to their form [at circular, elliptic, parabolic, and eventually hyperbolic orbits]; and we thereby further arrive, however, at unity of the cause of all the laws of their motion (i.e., gravitation). From there we afterwards extend our conquests further, seeking also to explain all variations and apparent deviations from these rules from the same principle."

through space with a definite finite velocity *c* (the velocity of light). It is thus, as Howard Stein (1970) has emphasized, an actor in the physical world along with the matter upon which it acts: there are energy and momentum exchanges between matter and the field, for example, not only among the material bodies themselves.

Light waves **[38]**, on Maxwell's theory, are propagations in the electro-magnetic field confined to wave-lengths within the visible spectrum. But the electro-magnetic spectrum extends beyond visible light on both ends: with successively shorter wave-lengths, as in ultraviolet, X-rays, and gamma rays, and successively longer wave-lengths, as in infrared, microwave, and radio waves. Heinrich Hertz's celebrated experiments towards the end of the nineteenth century **[39]** involved the first generation and reception of electro-magnetic waves by means of what we would now call a radio transmitter and receiver located several meters apart. Hertz could thereby measure the velocity of transmission (which turned out, as predicted, to be the same as the velocity of light)) and compare the frequencies of oscillations in the transmitter and antenna receiver (which turned out, as predicted, to be the same). Perhaps most importantly, as Stein (1970) has emphasized, Hertz thereby effected an experimental realization and clarification of the characteristically Maxwellian displacement current (the apparent flow of a current where this is no conductor). Hertz's experiments, in this sense, empirically realized the Maxwellian electro-magnetic field and established its mathematical properties, just as those of gravitational force had been established two centuries earlier by Newton's inferences from phenomena.

In both cases, however, "established" does not mean proved to be correct with no possibility of revision. Rather, Newton's inferences from phenomena and Hertz's experiments provided the corresponding forces with observationally determinable empirical reality (objective reality in Kant's sense). They thereby established the possibility of continuing progressive empirical inquiry into these forces—a process which, as we saw, is always open to further correction as our empirical determinations of the relevant quantities become increasingly precise. This kind of reasoning provides such empirical determinations with greater epistemic security than either merely inductive reasoning or hypothetico-deductive reasoning. Thus, just as in the case of Newton and the mechanical philosophy, the hypothetical aether models that were originally proposed for both the wave theory of light and electro-magnetic theory were considerably more problematic than Hertz's experimental investigations into the mathematical properties of the Maxwelleian electro-magnetic field—which could thus be empirically and mathematically investigated independently of any such model.

We can now appreciate how Einstein's general theory of relativity is bound up with his special theory by observing that there is a fundamental tension between the kind of causality exemplified by the electro-magnetic field and that of Newtonian universal gravitation. The latter is propagated instantaneously **[40]**, along "planes of absolute simultaneity" in the Newtonian spatio-temporal background structure, while the former (the Maxwell field) is always propagated with finite velocity c. But no finite velocity can be the same in two different inertial frames of reference in a Newtonian spatio-temporal structure—where an inertial frame is one in which the law of inertia is valid (together with the other Newtonian Laws of Motion). Only the planes of absolute simultaneity themselves, representing infinitely fast or instantaneous causal actions, are invariant in such a structure. The special theory of relativity then addressed this tension by changing **[41]** the spatio-temporal background to what we now call "Minkowski space-time"—wherein the mathematical properties of Maxwellian (finitely propagated) causality are taken to be those of the spatio-temporal background structure as well. In particular, the simultaneity relation is *relativized* so that there are different planes of simultaneity through a given point-event relative to different inertial frames. The "light cone" determined by all spatio-temporal trajectories with the given finite velocity through a given point-event now replaces the plane of absolute simultaneity (through this point-event) as fundamental spatio-temporal invariant.

Interferometer experiments by Michelson, Morley **[42]**, and others at the end of the nineteenth and beginning of the twentieth century securely established these mathematical properties of the spatio-temporal background relative to those of the electro-magnetic field—that is, the properties of what we now call Minkowski space-time—in the same sense that the mathematical properties of the electro-magnetic field were previously established by Hertz. By contrast, appealing instead to hypothetical physical mechanisms generating contractions of rods and retardations of clocks while retaining the Newtonian spatio-temporal background goes well beyond what can be empirically established (at least so far) in the same way that Newton's opponents within the mechanical philosophy appealed to (otherwise empirically undeterminable) vortices in the aether to explain the motions of the planets.

The crucial question for Einstein's general theory of relativity was then how to represent the gravitational field within the background structure of Minkowski space-time—where this field must now be finitely propagated like the Maxwell field. And the solution Einstein arrived **[43]** at was to retain the structure of Minkowski space-time—and thus the light-cone structure—as far as possible, while simultaneously introducing a *variable curvature* into the underlying space-time manifold upon which the light-cones are defined: in particular, the orientations and dimensions of the light-cones can vary from point to point depending on the space-time curvature. **[44]** This variable light-cone structure then determines the two possible types of (four-dimensional) trajectories representing geodesic motions in a gravitational field—that is, *straightest possible* (four-dimensional) curves within the geometry in question. In particular, the time-like geodesics always lie within the light-cones and represent freely-falling (massive) bodies subject to none but gravitational interactions, while the light-like geodesics lie on the surfaces of the light-cones themselves.

The resulting four-dimensional space-time geometry has implications for three-dimensional, purely spatial geometry. Indeed, as we saw in the last lecture, the three-dimensional spatial geometry of the solar system can be visualized in two dimensions **[45]** as circularly symmetric with increasing negative curvature radially approaching the central sun. And this representation allows one to visualize Einstein's prediction of the deflection of light in the gravitational field of the sun that was tested during eclipse observations in 1919. To fully appreciate the import of these observations, however, one needs to bear in mind that the Einsteinian prediction was being tested against a corresponding Newtonian prediction (going back to the turn of the nineteenth century) in which light was represented as a very small (and very light) massive particle moving at the (already known) velocity *c*. In particular, the Newtonian predicted deflection turned out to be one-half the value of Einstein's. Thus, the observations and photographic measurements of the full Einsteinian deflection in 1919 **[46]** empirically established a "structural" relationship between Newtonian and Einsteinian gravity (where gravity is represented as a curvature imparted to Minkowski space-time rather than as a Newtonian impressed force), just as the previous observations and experiments of Newton, Hertz, and Michelson-Morley *et al*. established the mathematical "structural" properties of the different forms of causal interaction at issue in these cases.[26]

The continuity in method and resulting epistemic status involved in all three cases captures the sense in which Kant's conception of causal necessity can indeed be extended into the post-Newtonian period. Of course the nature of the causal relation undergoes important changes. In the Newtonian case the relevant force—the gravitational field—mediates the interactions between material substances without being an actor in turn. The only substances, properly speaking, are massive material bodies, and they do not exchange momentum, in turn, with the gravitational field. This situation fundamentally changes with the electro-magnetic field, which is continuously propagated with a finite

[26] See Kennefick (2009) for an excellent discussion of the 1919 observations.

velocity rather than instantaneously: it thereby exchanges both momentum and energy with the relevant (charged) bodies upon which it acts and is thereby acted upon. The situation changes even more radically in the general theory of relativity, where the gravitational field is not only propagated continuously but is now a field of geometrical curvature in the fabric of space-time itself. And this transformation of geometry—including even the three-dimensional geometry of space—into a physically acting and interacting field of curvature implies that Kant's conception of space as the a priori form of outer intuition has most definitely broken down. Nevertheless, a central element in his conception of causal necessity—its more than inductive and more than merely hypothetical epistemic status—is in fact preserved, relatively straightforwardly, even here.

The fate of Kant's conception of pure intuition is less straightforward, but there is a natural way to *generalize* his conception of space as an a priori form of intuition so as to embrace all the post-Newtonian developments we have considered. Instead of viewing this form as reaching globally into the cosmos—so as thereby to enable Newton's Euclidean treatment of the motions of the heavenly bodies within the solar system (and, for Kant, throughout the rest of the cosmos as well)—we view it as only reaching locally into a very small region (measured in cubic meters) embracing the laboratory and/or observatory spaces of the relevant experiments. We see this, in fact, in all of our post-Newtonian cases: **[47]** Hertz's experiments with radio waves, the Michelson, Morley, *et al*. **[48]** interferometer experiments, and the photographic observations and measurements verifying Einstein's prediction for the deflection of light **[49]**. In the last case, in particular, the space of the observatory is so small relative to the corresponding space of the cosmos (here extending well beyond the solar system) that it is, for all intents and purposes, *infinitesimally* small; so its geometry, even according to the general theory of relativity, remains Euclidean.[27] In this sense, it is still reasonable to view the use of Euclidean geometry as an a priori constitutive presupposition of the empirical observations that serve as tests (and empirical realizations) of the theory, even though the global (cosmic) space employed by the theory is measurably non-Euclidean.

3. The Legacy of Kant's Metaphysics of Experience [50]

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[27] For example, the 1919 observatory at Sobral in Brazil was less than 10 cubic meters in volume; the stars observed belong to the Hyades cluster, the center of which is approximately 150 light-years from the earth. In general, in any Riemannian manifold of any curvature and dimension, geometry in the infinitesimally small (of the tangent space at any point) is nonetheless flat or Euclidean.

I have argued that Kant's conception of causal necessity centrally involves the Newtonian method of deduction from phenomena—which, in particular, depends on a mathematical description of force that abstracts from hypotheses not yet determinable from phenomena. I have also argued that this method can be seen as continuing into the post-Newtonian period in the work of Maxwell, Hertz, and Einstein. By emphasizing the mathematical and methodological aspects of Kant's conception, however, I have so far said relatively little about its metaphysical aspects. Yet it is clear that Kant's "metaphysics of experience"—based on the pure categories of the understanding and pure forms of sensibility—plays a central role in his conception. In particular, the categories of substance, causality, and community, together with the three corresponding Analogies of Experience, are fundamental. This aspect of Kant's conception represents an evident divergence from Newton's more guarded attitude towards metaphysics, and so it is necessary to consider it further here. Otherwise, it might appear that what I have said about the legacy of Kant's conception concerns the legacy of *Newton* rather than Kant.

The eleventh paragraph of the Preface to the *Metaphysical Foundations* is of particular interest in this connection, because there Kant emphasizes a significant difference between his approach and Newton's. Kant begins by describing what he takes to be the typical attitude of mathematically inclined natural philosophers **[51]** (472): "Hence all natural philosophers who have wished to proceed mathematically in their occupation have always, and must have always, made use of metaphysical principles (albeit unconsciously), even if they themselves solemnly guarded against all claims of metaphysics upon their science." Yet these natural philosophers have understood metaphysics in the wrong way **[52]** (ibid.): "Undoubtedly they have understood by the latter the folly of contriving possibilities at will and playing with concepts, which can perhaps not be presented in intuition at all, and have no other certification of their objective reality than that they merely do not contradict themselves." Thus, the relevant "mathematical physicists" are dissatisfied with the merely hypothetical "fabrications" characteristic of the mechanical philosophy, and, apparently following Newton, they want to leave aside *all* "hypotheses," whether physical *or* "metaphysical."

Kant responds, then, with his own conception of "*true* metaphysics" **[53]** (ibid.): "All true metaphysics is drawn from the essence of the faculty of thinking itself, and it is in no way fabricated [*erdichtet*] on account of not being borrowed from experience. Rather, it contains the pure actions of thought, and thus a priori concepts and principles, which first bring the manifold of *empirical representations* into the law-governed connection through which it can become *empirical* **cognition**, that is, experience." For Kant, therefore, "true metaphysics" refers, in the first instance, to the *general* metaphysics articulated in the first *Critique*, which is concerned with the entirely *pure* "a priori concepts and principles" at the basis of the possibility of *all* "*empirical* **cognition**, that is, experience."

At the end of the paragraph, however, Kant turns to the *special* metaphysics of (corporeal) nature now to be articulated in the *Metaphysical Foundations* **[54]** (ibid.): "Thus these mathematical physicists could in no way avoid metaphysical principles, and, among them, also not those that make the concept of their proper object, namely matter, a priori suitable for application to outer experience, such as the concept of motion, the filling of space, inertia, and so on. **[55]** But they rightly held that to let merely empirical principles govern these concepts would in no way be appropriate to the apodictic certainty they wished their laws of nature to possess, so they preferred to postulate such [principles], without investigating them with regard to their a priori sources." Thus, the "metaphysical principles" in question are precisely those articulated in the body of the *Metaphysical Foundations*, and Kant's investigation of their "a priori sources" is intended to fill in the lacuna in the purely postulational approach favored "mathematical physicists." There can be very little doubt, therefore, that Newton is paradigmatic of the "mathematical physicists" in question, and, for Kant, the task left open for his metaphysical treatment, among other things, is that of explaining the a priori character (by Kant's lights) of **[56]** the "Axioms or Laws of Motion" initiating the *Principia*.

The required metaphysical explanation, as we have seen, proceeds by showing how the fundamental empirical concepts of Newtonian physics—such as mass, force, motion (acceleration), and interaction—are mathematically precise instantiations of the categories of the understanding, in particular, the categories of substance, causality, and community. **[57]** The categories of the understanding, in turn, derive from the logical structure of the pure intellect, and, when applied to the spatio-temporal structure of our pure intuition via schemata, they result in the a priori principles of the understanding, in particular, the Analogies of Experience. It is in this way, as I have argued elsewhere, that Kant thereby achieves a fruitful synthesis **[58]** of Newtonian physics and Leibnizean metaphysics, where the latter has now been transformed (with the help of Newton) into Kant's own revolutionary *metaphysics of experience*. **[59]** However, just as physics has changed fundamentally since the time of Newton and Leibniz, metaphysics or philosophy in the tradition initiated by Kant has also changed. So it is necessary to explore both sets of changes together in order fully to appreciate the legacy of Kant's achievement.

My own work in this direction began with a neo-Kantian reinterpretation of the a priori in **[60]** Reichenbach (1920), which distinguished between a priori in the sense of necessary unrevisability and a priori in the sense of constitutive of the object of (scientific) knowledge. The lesson of the theory of relativity, for Reichenbach, was that, while Kant himself had equated these two notions of the a priori, Einstein had shown that they must come apart. In particular, while Euclidean geometry and the Newtonian Laws of Motion are indeed constitutive, *relative* to this stage in the development of physical theory, the fundamental mechanical laws (characteristic of the Newtonian inertial frames) are changed in Einstein's special theory to those of relativistic mechanics (characteristic of the differently related inertial frames of special relativity). Moreover, even physical geometry is fundamentally changed in general relativity, so that we cannot presuppose in advance any particular (metrical) geometry at all, but only the more general structure of a four-dimensional (semi-)Riemannian manifold (of Lorentz signature).

My **[61]** *Dynamics of Reason* (2001)—which was intended, as we have seen, as both an appreciation of and corrective to Kuhn's theory of scientific revolution—basically agreed **[62]** with Reichenbach on this point, although some of the details varied. What is most important, however, is that I there added an historical account of parallel developments in **[63]** scientific philosophy between Kant and Reichenbach, including such figures as Helmholtz, Mach, and Poincaré. Since then, as we have also seen, I have been considering, in addition, the *Naturphilosophie* of Schelling and Hegel, as well as the academic neo-Kantianism of the Marburg School, culminating in Cassirer. The idea, in general, is thereby better to appreciate exactly how such philosophical developments were intertwined with the scientific developments leading from Newton to Einstein (and, I hope, beyond).

It is precisely this intertwined set of developments, in my view, that provides the post-Kantian philosophical explanation **[64]** of the "a priori sources" of the constitutive principles in question. Such principles are a priori (and indeed *synthetic* a priori)—in the relativized sense—rather than empirical, because they are what I have called enabling conditions for a procedure of theory-mediated measurement capable of bestowing a more than inductive and more than hypothetical epistemic status on properly empirical laws of nature. (This, as we have seen, is my replacement for the Kantian notion of schematism.) Unlike in Kant, however, insight into the "a priori sources" of these constitutive principles is now provided by an explicitly *historical* account of the relationship between the relevant developments in the physical sciences and the corresponding developments in scientific philosophy—beginning with **[65]** Kant's metaphysical foundation for Newtonian physics and continuing with the transformation of Kant's approach at the hands of **[66]** Schelling, Hegel, Helmholtz, **[67]** Mach, Poincaré, the Marburg School, and **[68]** (yes) Thomas Kuhn. Kant's metaphysics of experience is thereby relativized and historicized, along with the a priori itself.