Modal Science

[to appear in *Canadian Journal of Philosophy*, 46 (2016), supplementary issue, and in Mark McCullagh and Juhani Yli-Vakkuri (eds.), *Williamson on Modality* (London: Routledge); penultimate draft]

Timothy Williamson

Too often, philosophers have discussed 'metaphysical' modality — possibility, contingency, necessity — in isolation. Yet metaphysical modality is just a special case of a broad range of modalities, which we may call 'objective' by contrast with epistemic and doxastic modalities, and indeed deontic and teleological ones (compare the distinction between objective probabilities and epistemic or subjective probabilities). Thus metaphysical possibility, physical possibility, and immediate practical possibility are all types of objective possibility. We should study the metaphysics and epistemology of metaphysical modality as part of a broader study of the metaphysics and epistemology of the objective modalities, on pain of radical misunderstanding. Since objective modalities are in general open to, and receive, natural scientific investigation, we should not treat the metaphysics and epistemology of metaphysical modality in isolation from the metaphysics and epistemology of the natural sciences.

In what follows, section 1 gives a preliminary sketch of metaphysical modality and its place in the general category of objective modality. Section 2 reviews some familiar forms of scepticism about metaphysical modality in that light. Later sections explore a few of the many ways in which natural science deals with questions of objective modality, including questions of quantified modal logic.

1. The category of objective modality

Let 'n' name the actual number of inhabited planets. There are exactly n inhabited planets, as our stipulation guarantees. Since our planet is inhabited, we know that $n \geq 1$. However, even though we know for sure that there are no fewer than n inhabited planets, there could have been fewer than n, because there could have been *no* inhabited planets. Such a sense in which things could have been otherwise is *objective* rather than *epistemic*. It is not a matter of what any actual or hypothetical agent knows, or believes, or has some other psychological attitude to; nor is it a matter of what any actual or hypothetical agent *ought* to be or do, either morally or in order to achieve a given purpose. Conversely, some epistemic possibilities are not objective possibilities of any kind. For instance, since we do not know whether other planets are inhabited, it is in some sense both epistemically possible for us that $n \geq 2$ and epistemically possible for us that $n < 2$. But the ordering of the natural numbers is non-contingent: either $n \geq 2$ and there is no objective possibility of any kind that $n < 2$, or else $n < 2$ and there is no objective possibility of any kind that $n \geq 2$. Objective modalities also differ in conception from *deontic* and *teleological* modalities, which concern how things ought or may permissibly be, either absolutely or for a given purpose; but since there is much less danger of confusion in that respect, no more need be said about deontic and teleological modalities.

Objective modalities are non-epistemic, non-psychological, non-intentional. Thus they are not sensitive to the guises under which the objects, properties, relations, and states of affairs at issue are presented. For instance, suppose that there are in fact exactly 29 inhabited planets; we may read 'n' and '29' as simply two directly referential names of the same object. Then, for each kind of objective necessity, since it is necessary that $29 = 29$, it is necessary that $n = 29$; that is just the same objective necessity in other words. By contrast, epistemic modalities presumably *are* sensitive to guise. For instance, it is epistemically necessary that $29 = 29$, but not epistemically necessary that $n = 29$. The matter is admittedly delicate, because on coarse-grained views of the individuation of propositions the proposition that $29 = 29$ just *is* the proposition that $n = 29$, so the latter proposition is epistemically necessary if the former is. But even such coarse-grained views have to make the relevant epistemic distinctions *somehow*. In some sense, the identity is epistemically necessary under the guise of the sentence '29 = 29' but not under the guise of the sentence 'n = 29'. Then the pertinent contrast is that objective modality admits no such non-trivial relativization to guises. The identity is simply objectively necessary irrespective of the guise under which it is presented.

In linguistics, the distinction between epistemic and non-epistemic modality is widely taken to be fundamental to the taxonomy of modal constructions in natural languages, such as the almost omnipresent auxiliary verbs like 'can' and 'must'.[1] The present category of objective modality corresponds roughly to Angelika Kratzer's 'root' or 'circumstantial' modals and to Paul Portner's 'dynamic' modals. Although our present concern is primarily with the objective modalities themselves, rather than the semantic means of expressing them in natural languages, the theoretical significance of the category gains some defeasible support from its apparently universal recognition in human thought and talk. Similarly, although the separation of objective probabilities from epistemic probabilities is primarily motivated by theoretical concerns in the philosophy of probability, its affinity with distinctions marked in natural languages gives further support to its significance (Kratzer 2012, p. 61). Of course, someone might argue that the distinction between 'objective' and 'non-objective' modalities is less clear than it looks, but how many useful distinctions are perfectly clear?

Objective possibility and necessity come in many varieties. I could easily type slightly faster than I do; it would be harder but not physically impossible for me to type much faster than I do. As I will use the terms, a proposition is *metaphysically possible* if and only if it has at least one sort of objective possibility. A proposition is *metaphysically necessary* if and only if its negation is not metaphysically possible, that is, if and only if it has every sort of objective necessity. In given circumstances, a proposition is *nomically possible* if and only if it is metaphysically compossible with what, in those circumstances, are the laws of nature (their conjunction is metaphysically possible). A proposition is *nomically necessary* if and only if its negation is not nomically possible, that is, if and only if it is a metaphysically necessary consequence of what, in the circumstances, are the laws of nature. Both metaphysical and nomic modalities are objective. Natural science studies nomic possibility, impossibility, and necessity (amongst many other things). Philosophy, especially metaphysics, studies metaphysical possibility, impossibility, and necessity (amongst many other things). Of course, in everyday speech modal words such as 'can' and 'can't' are typically used to speak about much more restricted kinds of possibility and necessity. Right now, I can reach my keyboard, but I can't reach my bookshelves, even though the laws of physics do not preclude my reaching them. In such examples, the modal words still express objective possibilities or impossibilities, but ones that hold fixed my current circumstances — the position of the chair in which I am sitting, the length and inelasticity of my arms, and so on.

Many linguists use Kratzer's term 'circumstantial modality' in a sense similar to my sense of 'objective modality'. It is particularly appropriate for modalities conditioned on the specific circumstances at hand. It is less appropriate when 'could' is used to express an equally objective modality that generalizes away from all circumstances. Suppose that the universe has always been k-dimensional. On some reasonable views, however, it could have always been (k–1)-dimensional. Although 'could' is the past tense of 'can', English permits us to recruit the past tense to express something more purely modal. When we say 'The universe could have always been (k−1)-dimensional', we are not saying, absurdly, that at some past time (when, by hypothesis, the universe was already k-dimensional) circumstances then permitted the universe to have always been (k−1)-dimensional. We need not be conditioning on any circumstance at all. For our purposes, 'objective' is a more suitable word than 'circumstantial' because it encourages a broader reading that need not be circumstance-bound, and suggests a relevant analogy with objective probabilities.

The class of objective modalities is plausibly taken to be unified by its closure under various operations. For instance, if $\Box_1$ and $\Box_2$ express types of objective necessity, and $\Diamond_1$ and $\Diamond_2$ express the dual types of objective possibility, then $\Box_1\Box_2$ also expresses a type of objective necessity, and $\Diamond_1\Diamond_2$ the dual type of objective possibility. In order to articulate such principles more systematically, it is helpful to use a framework of propositions, at least as a first approximation. It enables us to avoid starting with a framework of possible worlds, which would be problematic because the term 'possible' takes for granted the distinction we are trying to explain. Equally problematic would be an indiscriminate framework of worlds, some of which may turn out to be impossible, others possible, for such worlds are standardly treated as linguistic constructs, which are unsuitable for an account of the modalities themselves rather than their linguistic expressions. Instead, we use a simple framework of non-linguistic propositions (see also Williamson 2013a, pp. 103-4).

We assume that propositions form a Boolean algebra under negation (~), conjunction ($\wedge$), and disjunction ($\vee$); 1 is the top element (the tautology), 0 the bottom (the contradiction). The algebra is complete: any set of propositions, finite or infinite, has a conjunction and a disjunction. For present purposes, we need not assume the algebra to be atomic, where atoms play the role of worlds.

Since the propositions satisfy the usual Boolean equations, they are more or less coarse-grained. For instance, $(p \wedge q) \vee p$ is the same proposition as p. The coarse-graining brings out key structural points most clearly. If we wanted, we could develop a similar theory of objective modalities in terms of more fine-grained propositions.

Let O be the family of objective necessity operators. They map propositions to propositions. For $L_i \in O$, M_i is $\sim L_i \sim$, the dual objective possibility operator. The only special assumption we make about the individual behaviour of objective possibility operators is that each of them commutes with conjunction. In other words, if $L_i \in O$ then $L_i \wedge_{p \in X} p = \wedge_{p \in X} L_i p$ for any set X of propositions: a conjunction is necessary$_i$ just in case all its conjuncts are necessary$_i$. As a special case when P = {}, this implies that L1 = 1, for the conjunction of the empty set of propositions is the tautology, since its conjunction with any proposition is that proposition.

For each objective necessity operator $L_i \in O$ we can define a propositional modal logic S_i thus. The language is a standard one for propositional modal logic. A *faithful interpretation* of the language is any mapping I of all formulas to propositions that treats the truth-functions in the obvious way and maps $\Box$ to L_i: in other words, where &, ¬, and $\Box$ are the signs for conjunction, negation and necessity in the language and α and β are any formulas, $I(\alpha \,\&\, \beta) = I(\alpha) \wedge I(\beta)$, $I(\neg\alpha) = \sim I(\alpha)$, and $I(\Box\alpha) = L_i I(\alpha)$. The formula α is a theorem of the logic just in case $I(\alpha) = 1$ for all faithful interpretations I (see also Williamson 2013a, p. 106). One can show that S_i is automatically a normal modal logic in the usual sense, by the assumption that L_i commutes with conjunction. That is, all truth-functional tautologies are theorems, as are all formulas of the form $\Box(\alpha \rightarrow \beta) \rightarrow (\Box\alpha \rightarrow \Box\beta)$ (axiom schema K), and the theorems are closed under the rules of modus ponens, uniform substitution, and necessitation (whenever α is a theorem, so is $\Box\alpha$).

We now postulate four closure principles about O:

(1) **Identity**: There is an $L_1 \in O$ for which $L_1p = p$ for all propositions p.

L_1 is in effect a propositional truth operator, or double negation; we can consider it as an objective necessity operator by recalling that it is the limiting case of the successive application of *n* objective necessity operators for *n* = 0 (these are all the same function on propositions). Trivially, L_1 commutes with conjunction. On a corresponding account in terms of Kripke models, the accessibility relation for L_1 would simply be the identity relation between worlds.

(2) **Composition**: For all $L_i, L_j \in O$ there is an $L_{ij} \in O$ for which $L_{ij}p = L_iL_jp$ for all propositions p.

Composing a pair of objective necessity operators yields an objective necessity operator. $L_{ij}p$ commutes with conjunction because L_i and L_j do. In terms of Kripke models, the accessibility relation for L_{ij} would be the composite of the accessibility relations for L_i and L_j. The joint effect of (1) and (2) is that composing a sequence of length *n* of objective necessity operators yields an objective necessity operator for any natural number *n*.

(3) **Conjunction**: For all $O' \subseteq O$ there is $L_{O'} \in O$ for which $L_{O'}p = \wedge_{i \in O'} L_ip$ for all propositions p.

The conjunction of any objective necessity operators is itself an objective necessity operator. It commutes with conjunction because each of the conjoined operators does so. In terms of Kripke models, the accessibility relation for $L_{O'}$ would be the union (disjunction) of the accessibility relations for each operator $L_i \in O'$. One might also conjecture that the intersection (conjunction) of the accessibility relations for a set of objective necessity operators would itself be the accessibility relation for an objective necessity operator, but that conjecture is not needed for what follows.

(4) **Reversal**: For all $L_i \in O$ there is an $L_{/i} \in O$ for which p entails $L_iM_{/i}p$ for all propositions p.

Here p entails q just in case $p \wedge q = p$. Of the four postulates, (4) has the least claim to obviousness. The claim is that if p holds, then it must$_i$ be possible$_j$ in some objective sense j for p to hold. A helpful analogy is with the relations between past and future operators: if p holds then it will always hold that p once held, and it always held that p would one day hold. Thus we might add to the consequent of (4) that p entails $L_{/i}M_ip$, although that conjecture is not needed for what follows. In terms of Kripke models, the accessibility relation for $L_{/i}$ would be the converse of the accessibility relation for L_i, and *vice versa*.

By postulate (3), there is a *strongest* objective necessity operator L_O, the conjunction of all objective necessity operators (let O' = O). Hence $L_O p$ entails $L_i p$ for every proposition p and objective necessity operator L_i. We identify L_O with *metaphysical necessity*. Thus metaphysical necessity implies every objective kind of necessity, and dually every objective kind of possibility entails metaphysical possibility. Then, given postulates (1), (2), and (4), the logic S_O of metaphysical necessity is at least, and presumably exactly, S5.[2] For we can show that the characteristic axiom schemas of S5 belong to S_O thus:

Axiom T $\square\alpha \rightarrow \alpha$ (what is necessarily so is so)

For since L_O is maximal in O, and $L_1 \in O$ by (1), $L_O p$ entails $L_1 p$, which is p. In terms of Kripke models, the accessibility relation for L_O is reflexive because it includes identity. We could have imposed satisfaction of axiom T as a constraint on *all* objective necessity operators, and dispensed with postulate (1), although that strategy would have required a restriction on postulate (3) to nonempty sets of operators, since $L_{\{\}}p = 1$ for all propositions p, including false ones. However, there is no clear need to impose T on all objective necessity operators, so for the sake of generality it is better not to. Thus an objective necessity operator might range only over some counterfactual possibilities.

Axiom 4 $\square\alpha \rightarrow \square\square\alpha$ (what is necessarily so is necessarily necessarily so)

For since L_O is maximal in O, and $L_{OO} \in O$ by (2), $L_O p$ entails $L_{OO} p$. In terms of Kripke models, the accessibility relation for L_O is transitive because it includes its composition with itself. It would not have been plausible to impose satisfaction of axiom 4 as a constraint on all objective necessity operators, for local forms of objective necessity concern only close worlds, and closeness is not transitive.

Axiom B $\alpha \rightarrow \square\Diamond\alpha$ (what is so is necessarily possibly so)

For since L_O is maximal in O, and $L_{/O} \in O$ by (4), $L_O p$ entails $L_{/O} p$, so $M_{/O} p$ entails $M_O p$, so $L_O M_{/O} p$ entails $L_O M_O p$ by normality, but p entails $L_O M_{/O} p$ by (4), so p entails $L_O M_O p$. In terms of Kripke models, roughly, if the accessibility relation for L_o contains its own converse then it is symmetric. Consequently, such accessibility is an equivalence relation. It would not have been plausible to impose satisfaction of axiom B as a constraint on all objective necessity operators: for instance, even if I shall in fact cross the bridge tomorrow, in some objective sense there may still be a practical possibility of its being destroyed later today, making it practically impossible for me to cross it tomorrow.

Thus, even though some objective necessity operators satisfy only much weaker logics, metaphysical necessity itself satisfies the strong modal logic S5, thanks to its role as the strongest objective necessity operator combined with plausible closure principles on the family of objective necessity operators. The result is a tidy picture of metaphysical modality: the modal status of a proposition as metaphysically necessary, metaphysically contingent, or metaphysically impossible is never itself metaphysically contingent.

An attractive conjecture consistent with the foregoing account, including the coarse-grained view of propositions, is that the metaphysical necessity operator is simply defined by these equations: $L_O p = 1$ if $p = 1$; $L_O p = 0$ otherwise. We already know that the first half of that holds because it does so for all objective necessity operators, as we saw; the crux is that $L_O p = 0$ whenever $p \neq 1$. If we assume that commuting with conjunction is sufficient as well as necessary for being an objective necessity operator, the conjecture follows, for the operator defined by those equations can easily be shown to commute with conjunction, and to be the strongest operator to do so.

The schematic characterization of metaphysical modality as the maximal objective modality leaves many questions about it unanswered. In particular, how far does it satisfy Kripke's seminal account (Kripke 1980)? Since the guise under which an object is presented has no bearing on an objective modality, we presumably have the necessity of identity: $x = y \rightarrow \Box\, x = y$.[3] Given that metaphysical modality obeys the principles of S5, we can thence derive the necessity of distinctness: $x \neq y \rightarrow \Box\, x \neq y$ (Prior 1956). These principles already suffice for some distinctive examples of the necessary *a posteriori*, as we should expect of an objective modality. For instance, it is metaphysically necessary, but not knowable *a priori*, that Socrates is distinct from Plato. But such general structural principles are typically neutral with respect to specific essentialist claims.

Nathan Salmon has argued in detail on essentialist grounds that the 4 axiom schema fails for metaphysical modality (Salmon 1982, 1989, 1993). If the essence of an artefact permits small but not large variations in its original constitution, then we should expect the accessibility relation for metaphysical modality to be non-transitive, because many small differences can add up to a large one. Under the reading of 'metaphysical modality' as meaning the maximal objective modality, Salmon's argument against the 4 axiom must fail. He rejects that reading, and indeed his argument may be sound under some alternative readings of 'metaphysical modality' as meaning various non-maximal objective modalities with non-transitive accessibility relations. One might even use Kripkean claims as to how different a given object could or could not have been as paradigms with which to explain an alternative sense for 'metaphysical modality'. For present purposes, however, the question is where Salmon's argument goes wrong when read, contrary to his intentions, with respect to the maximal objective modality. One option is to go for a much stricter form of essentialism, on which ordinary claims that an artefact could have had a slightly different original constitution are dismissed as loose talk (Chisholm 1973). Another option is to go for a much less strict form of essentialism, on which ordinary claims that an artefact could not have had a very different original constitution are interpreted as concerning only quite restricted types of possibility (Mackie 2006). There are also intermediate options (Williamson 2013b, pp. 126-43). We need not decide between these options here.

More generally, the conception of metaphysical modality as the maximal objective modality leaves open a wide range of theoretical options. At one extreme, metaphysical modality might involve only a bare minimum of structural constraints, such as the principles of S5 and the necessity of identity and distinctness. At the other extreme, it might involve rich essentialist constraints. It has not even been excluded that metaphysical modality coincides with nomic modality. These questions should be decided by detailed theoretical investigation, not by stipulation. Our present interest is in the whole range of objective modalities, not just in their maximum.

2. Scepticism about objective modalities

Objective modalities are envisaged as out there in the world, independently of us. There is a long philosophical tradition of scepticism about such modalities. Its patriarch is of course David Hume. He is usually, and most interestingly, interpreted as calling into question the very idea of objective necessity. On this reading, he denies not merely that we can know that billiard balls *must* rebound from the cushion as they do, but even that we can use the word 'must' to express any idea of objective necessity — as opposed to something psychological in ourselves, such as an expectation that they will so behave. Of course, since correlative types of objective necessity and possibility are interdefinable duals, he is calling into question the idea of objective possibility just as much as the idea of objective necessity. Hume's scepticism targets not only metaphysical modality; his arguments are just as relevant to more restricted objective modalities, such as nomic modality, which may be more appropriate to the motion of billiard balls.

Humean arguments remain surprisingly influential in the philosophy of modality, despite (or even because of) their seeming reliance on *a priori* crudely empiricist assumptions. In particular, it is often still taken for granted that the contents of perception are non-modal. Yet, as I write, I can see that, from where I sit, I *can* reach the computer screen but not the window. Of course, such modal contents might conceivably be the conclusions of inferences from the contents of one's perception in some stricter sense to be explained, combined with background beliefs about one's body, but such an interpretation is not obviously correct or even very plausible. A responsible empiricist should at least consider scientific alternatives such as Gibson's theory of perceptual affordances (1979) and its recent successors, on which sense perception has inherently modal contents.[4] After all, from an evolutionary perspective, it would be highly adaptive for perception to present such information about possibilities for action directly to us, rather than leaving us to get there through such time-consuming and troublesome inferences as occur to us. Fast reactions to new perceptual information are often crucial to the success of action, and to survival.

Recent metaphysics has witnessed a less extreme critique of objective modality, which concedes its intelligibility but denies its fundamentality. Where Quine (1953) dismissed quantified modal logic as incoherent, his student David Lewis found a way of interpreting it more charitably within a similar broadly Humean framework, using counterpart theory on the assumption that there are many concrete worlds (maximal connected spatiotemporal systems) other than our own (1968). Lewis formulates counterpart theory in a first-order non-modal language to which Quineans can hardly object, even though they may of course deny that there is more than one world in Lewis's sense. But although his counterpart-theoretic translation presents the quantified modal language as *meaningful*, it also presents it as far from *perspicuous*. The messy complexities of Lewis's translation scheme make the surface forms of quantified modal sentences a misleading guide to their underlying logical relations. From this perspective, it is better to do one's theorizing in the language of counterpart theory itself, free of modal operators, since the latter tend to obscure the deep structure of the metaphysical issues. In his later work, Lewis explicitly did just that, bypassing modal formulations to work directly in the language of counterpart theory (1986). So-called modal realism may just as well be regarded as a form of anti-realism about the modal. Of course, Lewis-style modal realism has never been a majority position. Nevertheless, it has encouraged the tendency not to take modal distinctions at face value, but instead to suspect them of distracting attention from deeper issues.

A more recent motive for downgrading modality is less Humean than Aristotelian in spirit. Disappointingly, distinctions drawn in modal terms have often turned out to be too coarse-grained to do the metaphysical work initially hoped of them. A prominent case was the attempt to use the notion of supervenience to explain the relation between the mental and physical: no mental difference without a physical difference (Davidson 1970). Despite Davidson's Quinean qualms about modality, supervenience is naturally defined in modal terms (no *possible* mental difference without a physical difference), although making it precise reveals that many subtly inequivalent modal definitions are available. However, on all the most attractive modal definitions, supervenience is not an asymmetric relation, and even one-way supervenience may hold between families of properties that seem to be 'on a level' with each other. Moreover, where supervenience does hold, one wants to know *why* it holds; the suspicion is that the real metaphysical action will be in answering the latter question. Thus just saying in a precise modal sense that the mental supervenes on the physical, even if true, clarifies the dependence of the mental on the physical much less than had been hoped (see Kim 1993 for discussion). Similarly, following Kripke (1980), it was widely accepted that what it is for a property to be essential to an object can be explained in modal terms: necessarily, if the object exists then it has the property. But Kit Fine (1994) argued persuasively that any such modal definition of essence is too coarse-grained to capture the difference between essential and accidental properties. Such disappointments have contributed to a view of modal distinctions as shallow and inadequate substitutes for metaphysically deeper distinctions concerning essence, grounding, fundamentality, naturalness, constitution, real definition, ontological explanation, or whatever it may be, even if the modal distinctions have some practical utility as a stopgap convenience (Kment 2014 develops a view of this sort).

The focus of such criticism is often specifically on *metaphysical* modality, for instance on whether it is metaphysically fundamental and unified or just a miscellaneous ragbag of disparate elements (Sider 2011). However, metaphysical modality is only one member of the extended family of objective modalities. Arguments for scepticism about metaphysical modality tend to generalize to other objective modalities, irrespective of the theorist's intentions. For instance, the epistemological challenge 'If something is non-actual, how do you know whether it is possible?' arises for *any* non-trivial objective modality, not just for the metaphysical sort — which is not to say that the challenge cannot be met. Likewise, Quine's logical qualms about quantifying into the scope of modal operators do not depend on whether those modal operators are interpreted as metaphysical, nomic, or practical. If Lewis's counterpart theory is used to interpret metaphysical modality, it should also be used to interpret the other non-trivial objective modalities, which are restrictions of metaphysical modality. Similarly, substituting another objective modality for metaphysical modality just exacerbates the problem of coarse-graining, for the trouble is that metaphysical necessity comes too cheap, and other forms of objective necessity come even cheaper.

Suppose that *some* non-trivial objective modality, A-modality, has the virtues critics deny to metaphysical modality. It is intelligible, but not to be explained in counterpart-theoretic terms, our knowledge of it is reasonable though far from complete, it cuts at a joint, and so on. Perhaps we should not *identify* metaphysical modality with A-modality, because some more general objective modality has all those virtues too. Still, given the virtues of A-modality, standard critiques of metaphysical modality are clearly missing something of crucial importance. Those who seek to disarm metaphysical modality had better disarm the whole family of objective modalities, lest other family members exact their revenge.

One response is that *nomic* modality is a non-trivial objective modality that does not stand or fall with metaphysical modality, because it can be independently explained in terms of natural science: to be nomically possible is to be consistent with the laws of nature, and natural science is our best source of knowledge about those laws.

What does 'consistent' mean there? It might mean *logically consistent*. But 'Hesperus ≠ Phosphorus' is logically consistent with the laws of nature, for their formulation involves nothing as parochial as the names 'Hesperus' and 'Phosphorus', and 'Hesperus = Phosphorus' by itself is no truth of logic. For the same reason, 'Hesperus is a quark' is also logically consistent with the laws of nature. On the proposed account, therefore, it is nomically possible that Hesperus is distinct from Phosphorus and nomically possible that Hesperus is a quark. That is not an attractive view of nomic possibility. Indeed, the overall view is not even consistent. For it entails that nomic modalities are objective modalities, and one mark of objective modality is that it does not block the substitution of co-referring names. Thus if it is nomically possible that Hesperus is distinct from Phosphorus, it is also nomically possible that Hesperus is distinct from Hesperus, in which case the view requires 'Hesperus ≠ Hesperus' to be logically consistent with the laws of nature, which it is not, because it is not even logically consistent with itself. The difference between the names 'Hesperus' and 'Phosphorus' is a difference in our representations that corresponds to no difference in the states of affairs represented or their objective modal status. The same problems arise if one appeals to 'conceptual consistency' (whatever that is) instead of logical consistency. To avoid the problems, in defining nomic possibility one would have to conjoin the laws of nature with all true claims of identity and distinctness, such as 'Hesperus = Phosphorus', and true claims of kind membership and non-membership, such as 'Hesperus is a planet' and 'Hesperus is not a quark'. But those are just the sorts of move philosophers make in trying to reductively define metaphysical modality itself. It is an illusion that one can define a nomic objective modality without running into the issues that beset metaphysical modality. One might as well admit that nomic possibility is metaphysical compossibility with the laws of nature, as suggested earlier. If metaphysical modality is in trouble, so is nomic modality. Like other objective modalities, it depends on metaphysical modality. Indeed, for many purposes, though presumably not all, we may even be able to work with the hypothesis that metaphysical modality coincides with nomic modality. At least, nomic modality is a good approximation to metaphysical modality.

The sceptic may respond: if the other objective modalities depend on metaphysical modality, so much the worse for them, and in particular for nomic modality. Even though the appeal to natural science is necessary for nomic modality, it is not sufficient. According to such a sceptic, natural science can in principle be done without reliance on objective modalities; the science does not vindicate their specifically modal aspect. In a scientistic climate, such an assumed lack of connection to natural science makes the objective modalities look suspiciously ill-grounded.

It is notable how minor a role natural science plays in current discussion of the epistemology of modality. The main emphasis is on folk methods of knowing whether something is possible, perhaps by imaginative means, described in one way or another (Williamson 2007a). One would expect such folk methods to be primarily geared to quite restricted forms of practical modality, though philosophers usually want to discuss knowledge of metaphysical modality. One might get the impression that philosophers have taken some practically convenient everyday ways of thinking ('can'), drastically generalized them ('metaphysically possible'), perhaps far beyond their domains of reliability, and on those tenuous foundations erected a shaky castle of philosophical theory. Implicit in this picture is that science itself has no essential objective modal aspect, so its track record of success offers no support to the enterprise of objective modal theorizing. For instance, Ted Sider claims that 'modality is unneeded for the most fundamental inquiries' (2011, p. 267).

There is no need to disparage folk methods of gaining knowledge about various types of objective possibility and necessity, perhaps including metaphysical possibility and necessity. Such methods are easy to underestimate. But they are not the focus of this chapter. Rather, its concern will be with more scientific methods of learning about objective modalities. Of course, logic is itself a science, in some ways the most rigorous science of all, and the study of quantified modal logic with respect to objective interpretations of the modal operators is a branch of that science. Arguably, it is best pursued by abductive methods of theory choice similar to those used in the natural sciences (Williamson 2013a). But this chapter does not take that view of modal logic for granted. Instead, it asks to what extent there is an implicit (or explicit) objective modal dimension to what are ordinarily counted as natural sciences, and to ordinary mathematics as applied in those sciences.

The mere definition of 'nomic possibility' as compossibility with the laws of nature poses no threat to the picture of natural science itself as non-modal, since it does not imply that the idea of nomic possibility plays any essential role in scientific attempts to identify the laws of nature. For all that the definition shows, modal ideas might be merely epiphenomenal in the scientific process. Likewise with the deduction of nomic possibility from actuality: if natural science discovers that there are black holes, we can of course deduce that it is nomically possible for there to be black holes, but that offers natural scientific help only where it is least needed, since the hard question is how far the possible extends *beyond* the actual.

We could go through numerous articles in journals of natural science and list all the places where modal expressions are used, in plainly objective senses, but we are unlikely to achieve much just by doing so. For such articles are written in quasi-natural language, and one can expect authors often to fall into such everyday ways of expressing themselves, even where they are not strictly needed. How might objective modality play a more essential role in natural science?

An apparent reason for pessimism is the increasing extent to which, as natural scientific theories become more rigorous, their core is expressed in equations or other mathematical formulas. For the language of mathematics is non-modal. It does not contain symbols like □ and ◊, at least not to mean necessity and possibility.[5] Of course, the absence of modal expressions *within* the formulas does not preclude us from ascribing nomic or metaphysical necessity to them from the outside. If '$5 + 7 = 12$' and '$E = mc^2$' are purely non-modal statements, we may still affirm that it is metaphysically necessary that $5 + 7 = 12$ and nomically necessary that $E = mc^2$. But the danger is that such modal claims are merely philosophers' exogenous honorific glosses, functionless within the science itself. It is as if a philosopher went round sticking gold labels on his favourite machines, reading 'This machine has been approved by a qualified metaphysician'. The label may look good, but it makes no difference to the working of the machine. More specifically, if the modal glosses are merely external to the science, then they draw no abductive support from the explanatory successes of the science. Can we find cases where instead the modal glosses reflect some endogenous need of the science? The next section starts to come to grips with that question.

3. Laws support counterfactuals

One obvious starting-point is the near-platitude in the philosophy of science that laws support counterfactuals. If it *is* a law that all Fs are Gs, then if there had been an F, there would have been a G. Even if the universal generalization 'All Fs are Gs' itself contains no modal element, in claiming that it is a law we licence its application to (at least some) counterfactual circumstances. Surely we want to use our scientific theories, including our mathematical theories, in reasoning about how things could have been, as well as about how they are. In such reasoning, we engage a specifically objective modal dimension. For those purposes, the core theory itself need not be cast in modal terms. It is enough that sometimes our legitimate applications of it assign it a modal status. What's the problem?

Not all reasoning from a false hypothesis is counterfactual in the sense relevant to objective modality. To use a standard example, the uncontentious truth 'If Oswald didn't shoot Kennedy, someone else did' is an ordinary indicative conditional, even though Oswald did in fact shoot Kennedy. It is uncontentious because, for sure, *someone* shot Kennedy, so if it wasn't Oswald, it was someone else. It is not equivalent to the so-called subjunctive conditional 'If Oswald hadn't shot Kennedy, someone else would have', which suggests another back-up conspirator lying in wait. Unlike the subjunctive conditional, the indicative conditional does not involve an objective modality. The indicative conditional, but not the subjunctive conditional, can be reasonably inferred from the non-modal statement 'Someone shot Kennedy'. Indicative and subjunctive conditionals interact differently with modally rigidifying devices, such as 'actually' and 'in this world' (Williamson 2006). 'If Oswald didn't shoot Kennedy, someone else did' is equivalent to 'If Oswald didn't shoot Kennedy, someone else did in this world'. By contrast, 'If Oswald hadn't shot Kennedy, someone else would have' is not equivalent 'If Oswald hadn't shot Kennedy, someone else would have in this world', since the former is true and the latter false in the scenario where Oswald's shot pre-empts an efficient backup assassin. In that respect, subjunctive conditionals pattern like objective modals while indicative modals do not: although the epistemic modal sentences 'Oswald may have missed' and 'Oswald may have missed in this world' are more or less equivalent, the objective modal sentences 'Oswald could have missed' and 'Oswald could have missed in this world' are not; indeed, the former is true and the latter false (where in both cases 'in this world' is read as within the scope of the modal verb). In considering applications of scientific theories (including mathematical theories) to hypothetical situations, we must be careful about whether they really require subjunctive conditionals rather than indicative ones, even when it is natural for us to articulate them in terms of subjunctive conditionals.

Imagine that we are assessing a plan A for building a bridge. We ask 'What would happen if we were to build the bridge according to plan A?'. We apply our relevant theories, and come to the conclusion 'If we were to build the bridge according to plan A, it would fall down'. Consequently, we decide not to build the bridge according to plan A (so the antecedent of the conditional is false). We reasoned with subjunctive conditionals, and it was quite natural to do so. But there was no real need to do so. We could just as well have reasoned with indicative conditionals, asking 'What will happen if we build the bridge according to plan A?', applying our relevant theories as before, and concluding 'If we build the bridge according to plan A, it will fall down'. If we know that indicative conditional in those circumstances, we have reason enough not to build the bridge according to plan A. It can be natural to articulate an application of a theory in subjunctive terms even when there is no real need to do so. One might wonder for a moment whether an objective modal dimension, by contrast with an epistemic modal dimension, is ever really needed in practical applications.

Consider learning from mistakes. We see a bridge fall down. We ask the subjunctive conditional question 'What would have happened if the bridge had been built according to plan B?'; we apply our relevant theories, and come to the subjunctive conditional conclusion 'If the bridge had been built according to plan B, it would not have fallen down'. Consequently, we do better ourselves next time we have to build a bridge. In this case, indicative conditionals will not do just as well. We do not assert 'If the bridge was built according to plan B, it did not fall down', for we know for sure that it did fall down, whether or not it was built according to plan B. Even if we treat the indicative conditional as truth-functional, true simply because its antecedent is false (we have concluded that the bridge was not built according to plan B), we should regard it as too conversationally misleading to assert. It is the subjunctive conditional that carries the relevant information. For such applications, we need an objective modal dimension.

Causal hypotheses are also far more strongly connected to subjunctive conditionals than to indicative conditionals, even though it may well be over-optimistic to expect strictly necessary and sufficient conditions for causal hypotheses in terms of subjunctive conditionals, or strictly necessary and sufficient conditions for subjunctive conditionals in terms of causal hypotheses.

Here is a toy example. Suppose that we are wondering whether there is a causal relationship between the variable X, whose value we set at time t, and the variable Y, whose value we observe at time t+1. We set $X = 1$ and observe $Y = 0$. Clearly, just on that basis, we are not in a position to conclude that $Y = 0$ (causally) *because* $X = 1$. But suppose that a wellconfirmed scientific theory, formulated in austerely mathematical, non-modal and non-causal terms, entails that $Y = 1 - X$. That surely does much to confirm the causal hypothesis. A reasonable story about how it does so is that the theory supports subjunctive conditionals, and so in particular the subjunctive conditional $X \neq 1 > Y \neq 0$ (if X had not been 1, Y would not have been 0): without the putative cause, there would not have been the putative effect. Varying the antecedent, we can derive other subjunctive conditionals similarly. Such patterns of counterfactual dependence are closely connected to causal hypotheses, even if the connection falls short of strict implication (see Woodward 2003 for one discussion; the literature is vast).

The material conditional $X \neq 1 \rightarrow Y \neq 0$ is obviously no substitute for the subjunctive conditional, since we already had the material conditional just from the initial observations. However, it is also inappropriate to assert the indicative conditional 'If $X \neq 1$ then $Y \neq 0$' in the absence of the theory, since even if we are wrong about how we set X, that by itself casts no doubt on our observation of Y. To eliminate this feature of the example, tweak it so that we no longer observe Y directly; instead, we set a reliable alarm to go off if and only if $Y = 1 - X$. In both cases, the alarm goes off, so we can assert the indicative conditional without need of the theory. It is the subjunctive conditional that makes the difference.

Thus, even if we start with an austerely formulated scientific or mathematical theory, free of modal and causal vocabulary, applying the theory for practical purposes or to reach causal conclusions often depends on its supporting subjunctive conditionals. When those applications are successful, part of what their success abductively supports are those subjunctive conditionals.

Notoriously, the subjunctive conditional $\alpha > \beta$ is in general much weaker than the corresponding objective strict conditional $\Box(\alpha \rightarrow \beta)$, where $\Box$ expresses metaphysical necessity. In some sense of 'nearby', if α is true at some nearby possible worlds, and β is true at all nearby possible worlds at which α is true, but α is true and β false at some more distant possible world, then $\alpha > \beta$ is true and $\Box(\alpha \rightarrow \beta)$ false. One might therefore suspect that a theory can support subjunctive conditionals without supporting attributions of objective necessity. Fortunately, that danger is largely avoided.

Let us be a little more precise. We start with a scientific theory T in a non-modal language L with at least the truth-functors $\neg$ and $\rightarrow$ (the material conditional) obeying the usual classical laws. We are supposing that T supports counterfactuals. Even if T *entails* only formulas of L, T may *support* formulas of a modal extension L^+ of L. L^+ includes the subjunctive conditional >, and dual operators $\Box$ and $\Diamond$ for metaphysical necessity and possibility and $\Box_N$ and $\Diamond_N$ for nomic necessity and possibility. A reasonable assumption is that what T supports is closed under entailment. That is, if a formula α of L supports a formula β of L^+, and β entails another formula γ of L^+, then α also supports γ.[6] A simple interpretation of 'T supports subjunctive conditionals' is simply this:

(S1) Whenever T entails $\alpha \rightarrow \beta$, T supports $\alpha > \beta$.

Note that $\alpha \supset \beta$ is a formula of both L and L^+, $\alpha > \beta$ a formula only of L^+. But from (S1), we can derive (S2):

(S2) Whenever T entails α, T supports $\Box\alpha$.

Thus T supports claims of metaphysical necessity after all. As a special case, when T entails a material conditional, it supports the corresponding strict conditional as well as the subjunctive conditional.

The argument from (S1) to (S2) is simple. Suppose that (S1) holds and that T entails α. By classical propositional logic, α entails $\neg\alpha \rightarrow \alpha$. Hence, by the transitivity of entailment, T entails $\neg\alpha \rightarrow \alpha$. Therefore, by (S1), T supports $\neg\alpha > \alpha$. But $\neg\alpha > \alpha$ surely entails $\Box\alpha$, for a formula subjunctively implies its own contradictory only if it does so vacuously; if $\neg\alpha$ were metaphysically possible, it would not be that if $\neg\alpha$ were true, so would be α.[7] Thus, by the principle that what T supports is closed under entailment, T supports $\Box\alpha$.

The conclusion (S2) is reasonable when T is a theory of pure mathematics. In effect, it tells us that we cannot use mathematics freely in subjunctive reasoning unless mathematics is assumed to be metaphysically necessary. But for most theories T in the natural sciences, (S2) looks too strong. Plausibly, the most to be claimed for them is nomic rather than metaphysical necessity, given that the latter modality is stronger than the former.[8] In such cases, the problem is not with the reasoning from (S1) to (S2) but with (S1) itself. On the assumption that α is nomically impossible, we should not expect T to yield correct information about what would happen if α were true; T is only concerned with the realm of the nomically possible. In such cases, we should expect exceptions to (1). Thus, for a theory T of natural science, we should weaken (S1) by a restriction to nomic possibility:

(S1*) Whenever T entails $\alpha \rightarrow \beta$, T supports $\Diamond_N\alpha \rightarrow (\alpha > \beta)$.

The reasonable assumption underlying (S1*) is that, for nomic possibilities, if they were to obtain, they would obtain in (actually) nomically possible ways, even if it is also metaphysically possible for them to obtain in (actually) nomically impossible ways: nomically possible worlds are 'closer' than nomically impossible ones to the actual world.[9] From (S1*) we can derive (S2*):

(S2*) Whenever T entails α, T supports $\Box_N\alpha$.

The argument is a modification of that from (S1) to (S2). For suppose that (S1*) holds while T entails α. As before, T entails $\neg\alpha \rightarrow \alpha$, so T supports $\Diamond_N\neg\alpha \rightarrow (\neg\alpha > \alpha)$ by (S1*). But $\neg\alpha > \alpha$ surely entails $\neg\Diamond_N\neg\alpha$; if $\neg\alpha$ were nomically possible, it would not be that if $\neg\alpha$ were true, so would be α. Hence $\Diamond_N\neg\alpha \rightarrow (\neg\alpha > \alpha)$ entails $\neg\Diamond_N\neg\alpha$, which is equivalent to $\Box_N\alpha$. Thus, by the principle that what T supports is closed under entailment, T supports $\Box_N\alpha$. Even the qualified way in which scientific theories arguably support subjunctive conditionals requires them to support claims of nomic necessity too.

In brief, if we want to apply a scientific theory freely in the scope of subjunctive reasoning about nomic possibilities, the theory had better be at least nomically necessary, even if the content of the theory itself is purely non-modal. Furthermore, if such modal applications have a track record of success, it provides abductive confirmation for the relevant claims of nomic necessity. It is a serious mistake to picture those objective modal claims as supported by nothing more than folk habits of thought and metaphysical speculation. Nevertheless, that point does not eliminate the suspicion that conceptualizing matters in such supposedly objective modal terms somehow misleads us about the underlying fundamental joints in nature. Thinking in terms of natural language subjunctive conditionals may look prescientific. These concerns are fuelled by the absence of objective modal constructions from the language L, in which by hypothesis the scientific theory T was formulated. Such suspicions can best be answered by considering cases where the content of the scientific theory itself is objectively modal. That is the task for the rest of the paper.

4. Objective probabilities

Talk of probabilities is, of course, widespread in the natural sciences. Whilst casual uses of the word 'probably' may merely express caution, explicit quantification of probabilities — for instance, in the interpretation of statistics — presupposes some form of modality, for any probability distribution is defined over a probability space of mutually exclusive, jointly exhaustive 'possibilities': in any given circumstances, all but one of them is counterfactual. Probabilities are assigned to all members of a field of 'events', that is, subsets of the set of all those possibilities. The possibilities behave like possible worlds, and the events behave like coarse-grained propositions, sets of possible worlds. The probability of a proposition is a measure of its closeness to necessity. If the number of events is finite, all nonempty propositions may be assigned nonzero probability, in which case probability 1 corresponds to necessity, because it is equivalent to 'truth everywhere' in the space, and nonzero probability corresponds to possibility, because it is equivalent to 'truth somewhere' in the space. If the number of events is infinite, there are technical obstacles to assigning all nonempty propositions nonzero probability; probability 1 is equivalent only to 'truth almost everywhere' in the space.[10] But even in the latter case, probability 1 and nonzero probability still behave logically like dual modal operators in a finitary modal language.[11] Moreover, necessity and possibility can still be defined in a natural way directly over the probability space itself as 'truth everywhere' and 'truth somewhere' respectively. A one-way connection still holds between probability and possibility: whatever has nonzero probability is possible in the corresponding sense, even though the converse fails. Moreover, probabilistic distinctions resemble modal distinctions in being coarse-grained: just as truth-functionally equivalent formulas are necessarily equivalent, they also have the same probability, as a consequence of the standard axioms for probability.

For present purposes, however, not any old probabilities will do. Only *objective* probabilities are appropriately related to objective possibilities. Often the probabilities discussed in science are epistemic, dependent on an evidence base, and so not suitable here. Subjective probabilities (credences, degrees of rational belief) help still less. Indeed, not even all objective probabilities are interesting for our purposes, since some of them are in effect distributions only over sets of actual cases. In particular, we are not concerned with probabilities understood as actual frequencies, even though they are objective at least in being agent-independent. But frequentist interpretations of probability are in any case unpromising, because actual frequencies may happen to be utterly wayward, in principle even over a very long run. A fair coin can come up heads any number of times in succession. Frequencies are better understood as good *evidence* for underlying probabilities that explain, and so should not be identified with, the frequencies.

The most familiar genuine objective probabilities are *chances*. Consider some physical system of scientific interest, perhaps the whole universe. We can ask: given that the system is in a maximally specific state s_0 at a time t_0, what is the *chance* that it will be in a maximally specific state s_1 at a later time t_1? If the system is deterministic, the answer will be 1 or 0. But if the system is indeterministic, the answer may, more interestingly, be some intermediate real number. The most celebrated example of indeterminism and intermediate chances in science is of course in quantum mechanics, under some interpretations. It is widely accepted that the probabilities in the formulation of quantum mechanics are not merely epistemic or subjective. However, given the notorious difficulties of interpreting quantum mechanics, we shall leave discussion of its probabilities to the experts.

Significantly, the very distinction between deterministic and indeterministic systems itself involves objective modality. Suppose that you are given the entire history of a system, past, present, and future, all described in purely non-modal terms, and that the history contains no recurrences: the system is in each maximally specific state at most once. There is no way of reading off from the history whether the system is deterministic or indeterministic. That depends on whether there are two *possible* total histories of the system, h and h*, maximally specific states s_0 and s_1, and times t_0 and t_1, such that in h the system is in s_0 at t_0 and in s_1 at t_1, while in h* the system is in s_0 at t_0 but is not in s_1 at t_1. If so, the system is indeterministic (its state at one time does not determine its state at another time); if not, the system is deterministic. The type of possibility at issue is nomic and objective. Chance is a measure of closeness to a timebound sort of objective necessity, such as nomic necessity conditioned on the circumstances at the time.

Non-trivial objective probabilities may also arise for deterministic systems. For scientists may explain some general features of the system's actual total history by showing them to be *typical* of its possible histories. That is, it is highly probable that the system will have a total history with those features. The relevant probabilities here are not chances given the state of the system at a time but rather something like probabilities over initial conditions. For the explanation to work properly, those probabilities should be objective. If we were merely told that it would be rational for someone in a particular evidential situation to be confident that the system would have a total history with the features at issue, we should be unsatisfied, because such a hypothetical agent is quite extraneous to what was to be explained. A better explanation would strip out the irrelevant material about the agent, and isolate the relevant facts about the system itself that the non-objective 'explanation' was clumsily attempting to communicate. An example of such an explanation of the general behaviour of a deterministic system in terms of objective probabilities over its initial conditions is the derivation of standard thermodynamic principles from classical statistical mechanics (see Loewer 2001 and Maudlin 2007 for discussion).

For illustrative purposes, a toy example will suffice instead. Suppose that a coin was tossed 1000 times. It came up heads approximately 500 times (the explanandum); why? A potential explanation is that the coin was fair and the tosses mutually independent (the explanans). Once the relevant calculations are made, the explanans gives a reasonable explanation of the explanandum. It is a piece of proto-science. Probability enters the explanation in at least two ways. First, the explanans itself is implicitly probabilistic: the coin is said to be fair in the sense that the probability of heads on a given toss is ½, and the tosses are said to be mutually independent in the sense that the unconditional probability of an outcome of a given toss equals its probability conditional on given outcomes of other tosses. Second, the connection between the explanans and the explanandum is also probabilistic, since the explanans does not entail the explanandum — the explanans is consistent with the coin's coming up heads every one of the 1000 times — but instead only makes the explanandum probable (in the same sense of 'probable'). The relevant probabilities are not subjective or epistemic, since the degrees of belief or evidential situation of an actual or ideal agent played no relevant role in the event to be explained. They are quite extraneous to the explanandum and should not figure in the explanans.[12] Nor are the relevant probabilities frequentist. For consider any given toss in the long run over which such frequencies would have to be calculated. If the toss is one of the 1000 in the explanandum, that would make for circularity in the explanation, but if the toss is not one of the 1000, then it played no role in bringing about the explanandum. Either way, it should be excluded. The example is best understood as involving a reasonable proto-scientific explanation in terms of objective probabilities.

The example does not require the physics underlying coin-tossing to be indeterministic. Instead, each cell of the macroscopic probability space may correspond to one equivalence class of a coarse-grained macroscopic partition of possible microscopic deterministic histories that differ from each other on the past and present as well as on the future; microscopically different ways of tossing the two coins lead deterministically to macroscopically different outcomes. Such possibilities are just as objective as indeterministic chances; no 'initial conditions' were nomically necessary. We should not suppose that an explanation in terms of the detailed microscopic histories of the actual tosses would in principle be better. For an explanation of the latter sort involves a drastic loss of generality: its microscopic explanans would obtain in only a tiny fraction of the cases in which the explanandum (as characterized above) would obtain. To capture the generality of the explanandum, we need the generality of the macroscopic objectively probabilistic explanans. A proper microscopic explanation would involve objective probabilities over different microscopic possibilities that realize the initial conditions of the coin-tossing.

5. State spaces

Probability is far from the only form in which objective modalities become the object of natural scientific inquiry. It is standard practice to study a physical system by analysing its state space or phase space, the abstract space of its *possible* states. The system may be as large as the universe or as small as a few interacting fundamental particles. The type of possibility is objective, more or less nomic. The states are maximally specific. This way of thinking is widespread in science. As a recent historian of the idea of phase space puts it, 'Listen to a gathering of scientists in a hallway or a coffee house, and you are certain to hear someone mention phase space' (Nolte 2010, p. 33).

State spaces have played a philosophically significant role in various connections. For example, in his critique of Hartry Field's nominalizing programme (1980), David Malament objects that if Field's method of nominalization is applied to various theories of mechanics, its effect is to replace quantification over abstract objects by quantification over the '"possible dynamical states" (of particular physical systems)', to which, he argues, a nominalist is not entitled (1982, p. 533). Aidan Lyon and Mark Colyvan have taken the latter objection further, arguing that attempts to nominalize standard phase-space theories in physics would result in a loss of explanatory power; as they explain, 'phase spaces are spaces of possible, but mostly non-actual, initial conditions' (2008, p. 227). In none of these cases are the possibilities subjective or epistemic; they are aspects of the physical domain under study, not of any real or ideal physicist's state of knowledge or belief. Rather, they are in some sense objective possibilities. For present purposes, our concern is not with the prospects for nominalisation.[13] Rather, it is with the objectively modal dimension of the physics.

To develop the point, we may consider for a case study the theory of dynamical systems (Strogatz 2001). In itself it is a mathematical theory, but it has intended applications in physics, chemistry, biology, and engineering, for instance to a pendulum, the solar system, the population growth or decline of predator and prey species, the weather, and so on. As the last case suggests, it is a standard framework for the study of chaotic systems.

Mathematically, a dynamical system consists of a set S on which some geometrical or topological structure is defined, a set T (usually either the set of real numbers or the set of integers) with an additive structure, and a family of functions $\{f_t\}_{t \in T}$ indexed by T, obeying the following constraints for all $s \in S$ and $t, t^* \in T$:

(i) $f_0(s) = s$

(ii) $f_t(f_{t^*}(s)) = f_{t+t^*}(s)$

Informally, we understand the formalism thus. S is the set of *instantaneous states* of the target system; they are maximally specific in relevant respects, mutually exclusive, and jointly exhaustive. T is the set of *directed lengths of time*; thus +1 and −1 may represent one second into the future and one second into the past respectively, distinct directed lengths of time whose sum is 0 (seconds). The system is assumed to be deterministic in both past and future directions; thus given its state at any one time, the dynamics fixes its state at any directed length of time from then (T includes negative as well as positive lengths of time).[14] Thus it is legitimate to understand $f_t(s)$ as the state of the system a length of time t after an instant when its state was s. For this interpretation to make sense, conditions (i) and (ii) must hold: (i) because zero time after an instant it is still that instant, and (ii) because the instant a length of time t after the instant a length of time t^* after a given instant is just the instant a length of time $t + t^*$ after the given instant.

Dynamical systems may be either *continuous* or *discrete*, depending on the structure of T. For a continuous dynamical system, T is the set of positive and negative real numbers, and the functions f_t are typically continuous with respect to the selected topology on S and implicitly defined by some differential equations. They form a *flow*. For a discrete dynamical system, T is the set of positive and negative integers, and the functions f_t are typically implicitly defined by some difference equations, though they may still be continuous with respect to the designated topology on S.

The mathematical theory of dynamical systems is just a branch of regular, non-modal mathematics. However, most *intended applications* of that mathematical theory are modal, in the sense that S is interpreted as the set of *possible* states of the target dynamical system — not, for instance, just the set of actual past, present, and future states of the system. To be more precise, given a dynamical system, let an *orbit* be any set of the form $\{f_t(s): t \in T\}$ for some state $s \in S$, in other words, the set of states which the system goes through at some time or other if at some time it is in s. It is easy to show that if any orbit exhausts S (so the system sooner or later goes through every state in S), then every orbit exhausts S. But, typically, no orbit exhausts *S*. Thus some of the possible states in *S* are mutually incompossible, given the dynamics, in the strong sense that if the system is *ever* in one of them, then it is *never* in the others. The states in *S* are *possible states*, not all of which are ever actualized. It would be foolish to try to eliminate all the counterfactual states in the system by cutting it down to just its actual orbit, because that would typically destroy the geometrical or topological structure defined over *S*: that structure is crucial to the explanatory power of the theory of dynamical systems. The point is to study the dynamical system of possible states as a whole, exploiting that mathematical structure.

As before, the sort of possibility at issue is not subjective or epistemic. It depends on the nature of the physical system under study, not on the psychological or epistemic states of the theorist who studies it, or of anyone else, real or ideal. It is some sort of objective possibility, usually nomic rather than metaphysical, perhaps even more restricted (it is not nomically necessary for there to be a pendulum). Of course, dynamical systems are mathematical models of complex and often messy natural structures, and as such are likely to involve some degree of simplification, idealization, and approximation. But that is just the normal case with natural science. It does not mean that dynamical systems have nothing to tell us about reality.

The possible states in *S* are clearly quite like *possible worlds*. However, since states are instantaneous, they are even more like ordered pairs of a world and a time, such as one evaluates formulas at in some formal theories of semantics for languages with both modal and temporal operators. But not even that comparison is perfectly apt, for nothing in a state specifies *when* the system is in it: indeed, unlike world-time points, states may be *repeatable*: when a dynamical system exhibits cyclic behaviour, it will be infinitely many times in each state that it is ever in (Nietzsche's eternal recurrence). One might try saying that the states in *S* are qualitative in a way that world-time points are not, but even that claim may be misleading, since the mathematical structure on states may require treating the result of spatially rotating, reflecting, or translating a given state as a different state, even though they are qualitatively indistinguishable. Still, the states correspond to equivalence classes of some world-time pairs under some relevant equivalence relation. Thus, although states cannot be straightforwardly assimilated into the framework of possible world semantics, they still quite clearly have an objective modal aspect, as well as a qualitative-temporal one.

We can make the modal aspect of dynamical systems explicit by treating them as models over which we evaluate formulas of a propositional modal language. This can be done in a very smooth and natural way, without applying any Procrustean methods. By their structure, dynamical systems ask to be so treated. More specifically, with respect to a given dynamical system, we will evaluate a formula as true or false at a state relative to an assignment of values to variables, just as a formula of an ordinary modal language is evaluated as true or false at a world in Kripke models for modal logic. We write s, $\underline{a} \models \alpha$ to mean that the formula α is true at the state *s* relative to the assignment $\underline{a}$. The variables of the formal language are ‘propositional’: they take sentence position. There are infinitely many such variables U, V, …. Formally, an assignment assigns each of them a set of states, a subset of *S*. Such sets play the role of propositions in the model, just as sets of worlds (subsets of *W*) play the role of propositions in Kripke models. The language has the usual truth-functors ¬, v, &, →, ↔, which behave as expected. It has dual modal operators ◊ and □, treated like quantifiers ranging over all states of the system (in this language, they need not express *metaphysical* modalities).[15] It also has the ‘tense’ operators F (‘at some future state’), G (‘at every future state’), P (‘at some past state’), H (‘at every past state’), and G_t, (‘at the state *t* after the current state’), for each directed length of time $t \in T$. To govern the propositional variables, there are propositional’ quantifiers $\forall$ and $\exists$, ranging over all subsets of *S*. Further sentence operators are needed to express relevant mathematical aspects of the dynamical system, such as a monadic operator ‘Open’ to express the openness of a subset of *S* in the underlying topology of the model.

Here is an explicit definition of truth in a given model:

[atom]	$s, \underline{a} \models V$	iff	$s \in \underline{a}(V)$
[¬]	$s, \underline{a} \models \neg\alpha$	iff	not $s, \underline{a} \models \alpha$
[v]	$s, \underline{a} \models \alpha \vee \beta$	iff	$s, \underline{a} \models \alpha$ or $s, \underline{a} \models \beta$
[&]	$s, \underline{a} \models \alpha \,\&\, \beta$	iff	$s, \underline{a} \models \alpha$ and $s, \underline{a} \models \beta$
[◊]	$s, \underline{a} \models \Diamond\alpha$	iff	for some $s^* \in S$: $s^*, \underline{a} \models \alpha$
[□]	$s, \underline{a} \models \Box\alpha$	iff	for all $s^* \in S$: $s^*, \underline{a} \models \alpha$
[F]	$s, \underline{a} \models F\alpha$	iff	for some $t \in T$, $t > 0$: $f_t(s), \underline{a} \models \alpha$
[G]	$s, \underline{a} \models G\alpha$	iff	for all $t \in T$, $t > 0$: $f_t(s), \underline{a} \models \alpha$
[P]	$s, \underline{a} \models P\alpha$	iff	for some $t \in T$, $t < 0$: $f_t(s), \underline{a} \models \alpha$
[H]	$s, \underline{a} \models H\alpha$	iff	for all $t \in T$, $t < 0$: $f_t(s), \underline{a} \models \alpha$
[G_t]	$s, \underline{a} \models G_t\alpha$	iff	$f_t(s), \underline{a} \models \alpha$
[Op]	$s, \underline{a} \models \mathrm{Open}(\alpha)$	iff	$\{s^*: s^*, \underline{a} \models \alpha\}$ is open
[$\exists$]	$s, a \models \exists V \alpha$	iff	for some $X \subseteq S$: $s, \underline{a}[V/X] \models \alpha$
[$\forall$]	$s, a \models \forall V \alpha$	iff	for all $X \subseteq S$: $s, \underline{a}[V/X] \models \alpha$

Such is the naturalness of the interpretation, these clauses require very little commentary: they are just what one would expect. The dynamics of the system is built into the clauses for the temporal operators, which are therefore not *purely* temporal. We continue to call them 'tense operators' rather than 'dynamical operators' only for the sake of familiarity. The underlying topology is used in the clause for 'Open'. In the quantifier clauses, $\underline{a}$[V/*X*] is the assignment like $\underline{a}$ except for assigning the set of states *X* to the variable V.

A formula is *valid* on a model if and only if it is true at every state on every assignment with respect to the model. It would be mathematically pointless to equip the models with a designated actual state, so we avoid doing so, and therefore could not define validity in terms of truth at the actual state of the model. A formula α is *valid* without qualification if and only if it is valid on every dynamical system model.

We briefly note some valid formulas, most of them standard. The underlying non-modal propositional logic is classical: all truth-functional tautologies are valid, and modus ponens preserves validity. The unrestricted modal operators □ and ◊ obey all the principles of the modal system S5.[16] The unrestricted necessity operator □ entails all the tense operators F, G, P, H, and G_t for all $t \in T$. Consequently, if something is ever possible, it is always possible:[17]

$$(P\Diamond V \vee \Diamond V \vee F\Diamond V) \rightarrow (H\Diamond V \;\&\; \Diamond V \;\&\; G\Diamond V)$$

We also have standard principles of tense logic for linear time with no first or last moment, which derive from the additive structure of the real numbers or the integers and conditions (i) and (ii) above.[18] In particular, conditions (i) and (ii) themselves correspond to the respective validity of these two axioms of metric tense logic (compare Prior 1967):

$$G_0V \leftrightarrow V$$

$$G_tG_{t^*}V \leftrightarrow G_{t+t^*}V$$

Other axioms, such as those corresponding to the density or discreteness of the time order, are validated if we restrict validity to continuous or to discrete models. The quantifiers obey the standard principles for propositional quantifiers in a modal setting, including for each state the existence of a proposition true at exactly that state:

$$\exists U\, [U \;\&\; \forall V\, [V \rightarrow \Box[U \rightarrow V]]]$$

Of course, the state may be a recurrent one: in the spirit of dynamical systems theory, the semantic theory does not distinguish between distinct times when the system is in the same state, although we can say that the system will again be in the same state:

$$\forall U\, [U \rightarrow FU]$$

This formula has the right effect because it entails that the atomic proposition true now will be true again.

We can see how the formal language can express characteristic ideas of the theory of dynamical systems by means of an example. The idea of an *attractor* plays an important role in the theory, helping us understand the long-term qualitative behaviour of dynamical systems. Very roughly, an attractor is a region of state space that the system gets pulled toward and stuck close to or in, once it has entered a surrounding region. Not all authors define the term in exactly equivalent ways, but the following definition is fairly standard (see Strogatz 2001, p. 324). As is typical, it assumes a topological structure on S. An *attractor* is a closed set of states $A \subseteq S$ such that:

(a) for all $s \in A$, $f_t(s) \in A$ whenever $t \geq 0$;
(b) for some open $U \supseteq A$: for all $s \in U$ and open $V \supseteq A$, for some t, $f_{t^*}(s) \in V$ for all $t^* \geq t$;
(c) if $A^* \subseteq A$ and A^* satisfies (a) and (b), then $A^* = A$.

Informally, (a) means that once the system is in A, it stays in A; (b) means that A draws and keeps all trajectories that ever come sufficiently close to it arbitrarily close to it (A attracts such trajectories); (c) means that A is minimal in these respects (it excludes redundant members). The *basin of attraction* of A is the largest set U satisfying the condition in (b); if A is an attractor, there is bound to be such a set. A *strange attractor* is an attractor that exhibits sensitive dependence on initial conditions. Such attractors are often fractal sets. They are important for the theory of chaos.

We can formalize (a)-(c) as a(A)-c(A) respectively (replacing italicized set variables by roman propositional variables):

a(A) $\quad \Box[\mathrm{A} \rightarrow \mathrm{GA}]$

b(A) $\quad \exists \mathrm{U}\ [\mathrm{Open(U)}\ \&\ \Box[\mathrm{A} \rightarrow \mathrm{U}]\ \&\ \forall \mathrm{V}\ [[\mathrm{Open(V)}\ \&\ \Box[\mathrm{A} \rightarrow \mathrm{V}]] \rightarrow \Box[\mathrm{U} \rightarrow \mathrm{FGV}]]]$

c(A) $\quad \forall \mathrm{A}^*\ [[\Box[\mathrm{A}^* \rightarrow \mathrm{A}]\ \&\ \mathrm{a(A^*)}\ \&\ \mathrm{b(A^*)}] \rightarrow \Box[\mathrm{A}^* \leftrightarrow \mathrm{A}]]$

For c(A), note that $\Box[\mathrm{A}^* \leftrightarrow \mathrm{A}]]$ behaves like $\mathrm{A}^* = \mathrm{A}$, because it requires the regions of state space A* and A to contain exactly the same states. We can now formalize '*A* is an attractor' thus (since a closed set is the complement of an open set):

Attractor(A) $\quad \mathrm{Open}(\neg\mathrm{A})\ \&\ \mathrm{a(A)}\ \&\ \mathrm{b(A)}\ \&\ \mathrm{c(A)}$

To handle 'basin of attraction', we formalize '*A* is an attractor and *B* is its basin of attraction':

Basin-of-attraction(A, B) $\quad \mathrm{Attractor(A)}\ \&\ \Box[\mathrm{B} \leftrightarrow \exists \mathrm{U}\ [\mathrm{U}\ \&\ \mathrm{b(U)}]]$

Both Attractor(A) and Basin-of-attraction(A, B) are *state-independent* formulas, in the sense that each is true either at every state or at no state (on a given assignment of values to variables): this reflects the lack of reference to the current state in what they formalize. Whether a given region is an attractor and whether one given region is the basin of attraction of another does not depend on where we are in state space. Clearly, any formula of the form □α or Open(α) is state-independent, for the variable '*s*' for the state of evaluation does not appear on the right-hand sides of the semantic clauses for □ and Open. Clearly too, the truth-functors and quantifiers preserve state-independence, because in their semantic clauses '*s*' is used only to express the truth-conditions of their inputs, and so makes no difference when the inputs are state-independent. Since the displayed formulas result from applying truth-functors and quantifiers purely to formulas of the form □α and Open(α), they are state-independent. By contrast, the subformulas A and B themselves are typically state-dependent; whether the system is in an attractor, or its basin of attraction, depends on which state it is in. As such examples suggest, the 'tensed' modal language has considerable power to express key ideas in dynamical systems theory.

The language with modal and temporal operators has some advantages over the set-theoretic notation. It renders some ideas more simply and perspicuously: for instance, a(A) is simpler than (a). It also avoids making some distinctions that lack physical significance. For instance, in the set-theoretic framework we must distinguish between members and subsets of *S*; the state *s* is distinct from the singleton region {*s*} of state space. By contrast, no such distinction arises in the modal-temporal language. In that respect, the latter stays closer to what is physically significant.

The definitions of scientifically significant ideas in the modal-temporal language make uninhibited use of its modal resources. For instance, the natural way to express the condition that every state is in a basin of attraction is with this formula:

□∃A∃B [B & Basin-of-attraction(A, B)]

Since b(U) in Basin-of-attraction(A, B) already contains □, this is an example of how modal operators naturally occur in the scope of further occurrences of those operators in such applications.[19]

It is also notable that the definition of 'Attractor' involves quantification into the scope of a modal operator, in clauses b(A) and c(A). That is the equivalent for quantification into sentence position of *de re* modality, Quine's third and most reprehensible grade of modal involvement (Quine 1966). Since Quine's official methodology involves taking our metaphysics from our best theory of the world, which is supposed to include physics, it is unfortunate for him that our best theory employs something like his bugbear, *de re* modality. Indeed, any dynamical system validates *de re* formulas that attribute incompatible necessary features to different 'propositions', for instance:

∃U □U

∃U □¬U

Any dynamical system with more than one state also validates this formula:

∃U [◊U & ◊¬U]

Thus Quine's naturalistic deference to natural science is in tension with his rejection of quantified modal logic. Although the readings of the operator □ associated with dynamical systems theory are weaker than metaphysical necessity, Quine's objections of principle to quantifying into the scope of a modal operator apply here just as much as they do elsewhere; they generalize to any non-trivial objective modal operator. If his arguments fail here, they fail generally.

From the present perspective, the non-modal nature of the mathematics that constitutes dynamical systems theory looks no more metaphysically significant than the non-modal nature of the mathematics that constitutes possible worlds model theory. In both cases, the modal connection is made by the intended applications of the mathematics. It is very convenient to reason in the non-modal language of mathematics, but in many applications we implicitly or explicitly characterize the entities we are reasoning about in modal terms. In metaphysics we may reason about modality by quantifying over possible worlds; in natural science we reason about modality by quantifying over possible states of a physical system. In the former case, the relevant modality is metaphysical; in the latter, it is more like nomic, but in both those cases it is objective. Natural science studies the structure of spaces of objective possibilities just as much as metaphysics does.

6. *Necessitism and contingentism in dynamical systems theory*

Sections 4 and 5 explained a general connection between the study of objective modalities by modal metaphysics and their study by the natural sciences. The present section will explore a much more specific connection between some contested issues in modal metaphysics and the modal logic underlying intended applications of dynamical systems theory as sketched in section 5.

In the model theory of first-order modal logic, one key choice-point is between *constant domain semantics*, which interprets the first-order quantifiers as ranging over a fixed domain of individuals, irrespective of the world of evaluation, and *variable domain semantics*, which interprets them as ranging over a domain that depends on the world of evaluation; informally, it is conceived as containing just the individuals that exist in that world. Every model in the constant domain semantics is equivalent to a model in the variable domain semantics, but not *vice versa*. Various formulas are valid (true in all models) on the constant domain semantics but invalid (false in some models) on the variable domain semantics, famously including the controversial first-order Barcan schema and its converse.[20] More simply, this formula is valid on the constant domain semantics, invalid on the variable domain semantics:

NNE $\Box\forall x\,\Box\exists y\; x=y$

Informally, NNE says that necessarily everything is necessarily something. We read $\Box$ in NNE as expressing some sort of objective necessity. Without relying on the model theory, we can see that NNE raises a significant metaphysical issue. *Necessitists* assert NNE; *contingentists* deny NNE (for metaphysical modality). For necessitists, it is necessary which individuals there are. For contingentists, it is contingent which individuals there are. Normally, a necessitist and a contingentist agree that it is contingent which things are concrete (or are in space-time, or have causes and effects, …). But the necessitist then adds that, in addition to the concrete things, there are also non-concrete things that merely could have been concrete, so that contingency in what is concrete does not generate contingency in what there is. The contingentist rejects any such way of saving NNE, insisting that a concrete thing is contingent in the strictest sense that there could have been no such thing as it at all. Each side has an internally coherent view; the issue is by no means easy to decide, although I have made a provisional case for necessitism (Williamson 2013a).

The issue between necessitism and contingentism might look like a paradigm of the sort of scholastic metaphysical dispute that utterly fails to engage with anything in natural science. But appearances can mislead. There are in fact quite specific connections.

The 'tensed' modal language for dynamical systems theory in section 5 lacks individual quantifiers and so does not contain NNE (for whatever objective modality is associated with the interpretation at hand). However, the language does have propositional quantifiers, and so contains the propositional analogue of NNE:

$\mathrm{NNE_P}$ $\Box\forall \mathrm{U}\,\Box\exists \mathrm{V}\,\Box[\mathrm{U} \leftrightarrow \mathrm{V}]$

Whereas NNE concerns the necessary being of *individuals*, $\mathrm{NNE_P}$ concerns the necessary being of *propositions* (represented mathematically by sets of states).[21] Informally, $\mathrm{NNE_P}$ says that necessarily every proposition is necessarily some proposition (as already noted, the formula $\Box[\mathrm{U} \leftrightarrow \mathrm{V}]$ is tantamount to $\mathrm{U} = \mathrm{V}$): for propositions, being is state-independent. *Propositional necessitists* assert $\mathrm{NNE_P}$; *propositional contingentists* deny $\mathrm{NNE_P}$ (for the given sort of objective modality). Necessitists tend to be propositional necessitists too (Williamson 2013a defends both views). Many contingentists are also propositional contingentists, holding that what propositions there are depends on what individuals there are (Stalnaker 2012). However, the two views do not always go together. Plantinga 1983 seems to defend a combination of contingentism about individuals with necessitism about propositions, on which it is (metaphysically) contingent that there is Socrates, but necessary that there is the proposition that there is not Socrates. Whether such a combination is well-motivated is another question (see Williamson 2013a, pp. 267-77). Henceforth we will ignore such hybrid positions, and concentrate on those which are either necessitist about all orders or contingentist about all orders.

Notably, NNE_P is valid on the semantics in section 5 for the modal language of dynamical systems. Thus the modal logic of dynamical systems embodies a necessitist metaphysics of propositions (for the given sort of objective modality). The logic also validates several other characteristically necessitist principles related to NNE_P, including an unrestricted comprehension schema for propositions, which guarantees that there is (necessarily) a proposition for each formula α of the language:

$COMP_p$ $\quad\quad \exists V \, \Box[V \leftrightarrow \alpha]$

Here α is any formula in which the variable V does not occur free, though other variables may. Strengthening $COMP_p$ by prefixing it with any sequence of universal quantifiers and □ operators in any order preserves validity (see Williamson 2013, p. 290 for the same schema, interpreted with respect to metaphysical modality). Versions of the Barcan schema and its converse are also valid, with propositional quantifiers in place of individual ones:

BF_P $\quad\quad \Diamond \exists V \, \alpha \rightarrow \exists V \, \Diamond\alpha$

CBF_P $\quad\quad \exists V \, \Diamond\alpha \rightarrow \Diamond \exists V \, \alpha$

By BF_p, if there could have been a proposition that met a given condition, then there is a proposition that could have met that condition. By CBF_p, if there is a proposition that could have met a given condition, then there could have been a proposition that met that condition. The validity of all these principles is not an artefact of a gerrymandered semantics. Quite the opposite: without extreme gerrymandering, there is no way of making explicit the modal content of intended applications of the mathematics of dynamical systems without validating these necessitist principles. In effect, intended applications of the mathematics of dynamical systems theory take necessitism about propositions for granted (for the relevant sort of objective modality). The modal logic for dynamical systems theory in section 5 just makes that metaphysical commitment explicit. That is not surprising; logic is not metaphysically neutral in any deep sense (see Williamson 2013a).

What about necessitism and contingentism for individuals? In dynamical systems, individuals are typically not represented as such. To find them, one must look into the internal structure (if any) of the states of the system. Typically, the states are treated as assignments of values to one or more independent variables. The number of variables is the dimension of the state space. For mathematical purposes, each state in an *n*-dimensional state space may be treated as just an *n*-tuple of numbers, if *n* is finite. A state space may also be of infinite dimension. For instance, if one is interested in the dynamics of temperature, it might be convenient to have a temperature variable for each point in a continuous physical space. The variables need not represent individuals: instead, they may represent global features of the target physical system, such as the number of predators and the number of prey. For many purposes, representing individuals one by one would involve needless, perhaps intractable, complexity. However, in some applications of dynamical systems theory, the variables *are* in principle associated with individuals. For a system of *n* particles, we may need to keep track of, say, three independent features (such as position or energy) per particle, so we need 3*n* variables altogether, and so a 3*n*-dimensional state space. In such cases, some distinctions between states correspond to distinctions between individuals: for instance, one state may differ from another only in that the values of the variables associated with a given particle in the former are those of the corresponding variables associated with another particle in the latter, and *vice versa*. Dynamical systems whose variables are associated with distinct individuals are sometimes called *agent-based*.

In many of the systems studied in natural science, individuals are sometimes created or destroyed. Such individuals may be particles, cells, organisms, whatever.[22] It is a temporary matter what individuals there are. It may also be a contingent matter what individuals there *ever* are; a given individual may be created on one trajectory but not on another. This is handled by having variables for each individual that *can* occur in some state or other (it would be horribly messy to handle it any other way). Thus, on the given application, each state *s* in effect encodes the identities of all *possible* individuals, whether or not they are present in *s* or in any other state on the same orbit as *s*. In making generalizations about states and sets of states, as is continually done in applications of dynamical systems theory, one is in effect quantifying over merely *possibly* present individuals, as well as actual ones: possible particles, possible cells, possible organisms, whatever they happen to be. On the face of it, this assumes a necessitist conception of what there is to quantify over rather than a contingentist one.

Of course, in applying the mathematics of state spaces, natural scientists typically have in mind a *type* of physical system, rather than a single token of that type. Thus the dimensions of the state space are not really associated with particular individuals. But the underlying point remains. For although the statements made about the application implicitly or explicitly have that extra level of generality, they are still generalizations about merely possibly present individuals as if they were all there to be generalized about, contrary to the contingentist view. For a necessitist, by contrast, such quantification is unproblematic.

A natural strategy for contingentists is to try to *simulate* in their own terms the necessitist effect of quantification, using crafty combinations of modal operators and quantifiers understood in contingentist terms. That strategy can be taken quite far. More specifically, the contingentist can simulate the necessitist effect of ordinary first-order quantification over possible individuals. Since each possible state is in effect an assignment of values to variables associated with possible individuals, the contingentist may well be able to simulate the necessitist effect of first-order quantification over possible states too. However, the contingentist simulation strategy arguably fails for second-order quantification over properties or sets of possible individuals (Williamson 2013a, pp. 305-

375; see also Fritz 2013). The necessitist effect of such second-order quantification cannot always be simulated in contingentist terms. Corresponding problems may therefore arise for a contingentist's attempt to simulate quantification over sets of possible states. Such quantification is ubiquitous in dynamical systems theory. For instance, as seen above, it is used to define basic terminology such as 'attractor' and 'basin of attraction'. The crucial feature of the definitions is that they quantify not only over possible states (with the variable '*s*') but also over sets of possible states (with the variables '*U*' and '*V*'). Without such quantification the definitions make no sense. But the intended effect of such quantification over sets of possible states characterized in terms of possibly concrete individuals is just the sort of necessitist move that contingentists cannot always simulate. They certainly cannot just help themselves to it without explanation.

In some special cases, contingentists can simulate the necessitist effects of higher-order quantification, for instance when all the possibly present individuals are compossible (they can all be present together in the same state). If the variables associated with distinct individuals are wholly independent of each other, then some state will assign them all non-zero values, which presumably means that all those individuals are present at that state. However, that argument does not work if some combinations of assignments are excluded (as they are by the Pauli exclusion principle, for instance). A contingentist simulation also works when the total number of possibly present individuals for the system is finite. But if there are infinitely many possible individuals for the system, while only finitely many of them can be present together, then neither of those special cases applies.

Contingentists could undertake the strategy of trying to show that, in practice, all the cases that arise in real-life natural science admit contingentist simulation, not only in dynamical systems theory but in scientific applications of the state space approach more generally. But that would be to give a very significant hostage to fortune. Pending the successful execution of the strategy, why should we assume without evidence that it will succeed? What is striking is that natural scientists themselves seem to feel no need of such precautions. In applying mathematics to state spaces, they make free use of quantification over possible individuals, possible states, sets of possible states, and so on, with no checks on whether they are straying beyond the limits of contingentist simulation. They do not treat the legitimacy of their practice as dependent on the availability of such simulation. In effect, they unreflectively rely on an invisible framework of necessitist modal logic. In that way, necessitism is more hospitable than contingentism to the normal practice of natural science.[23]

The absence of modal expressions from the core language of mathematics does not mean that its applications in natural science are non-modal. Rather, it manifests the necessitist presuppositions on which those scientific applications rely. They are no more non-modal than are the investigations of a metaphysician who reasons freely in a language without modal operators about possible worlds and their inhabitants. Contingentists who want to reconcile their modal metaphysics with scientific practice face a major reconstructive challenge: to vindicate within their own framework the free scientific use of quantification over possible states, sets of possible states, and so on. We currently have no good reason to expect that they will be able to meet the challenge.

7. Necessitism and contingentism in probability spaces

Similar metaphysical issues arise for the application of standard probability theory to implicitly modal matters. Suppose that we are reasoning about a counterfactual circumstance *C*, specified just as one in which there would have been exactly two fair coins tossed independently of each other at time *t*; *C* itself is not maximally specific, and in particular does not specify the microscopic details of the coins or the outcomes of the tosses. Our interest is in objective probabilities, such as chances (*t* is in the distant future), not in subjective or epistemic ones. What is the actual probability, conditional on *C*, that the result is one heads, one tails? The standard, correct answer is: ½. For there are four equiprobable possible outcomes: (HH) both coins come up heads; (HT) the first coin comes up heads, the second tails; (TH) the first coin comes up tails, the second heads; (TT) both coins come up tails. Since 'one heads, one tails' results in two of the four equiprobable out comes, the probability is 2/4 = ½. But familiarity should not make us regard the correctness of that argument as immediate. For an intelligent person without a suitable background in probability could instead have argued thus. There are three equiprobable possible outcomes: (HH) two heads; (H+T) one heads, one tails; (TT) two tails. Since 'one heads, one tails' results in just one of the three equiprobable outcomes, the probability is 1/3. One very intelligent person who made just such a mistake was Leibniz, who claimed that on a throw of two dice, 11 and 12 are equally likely outcomes, because each can be obtained in only one way (a five and a six; two sixes).[24] Kripke uses this very example of the two ways of getting 11 to explain his conception of possible worlds and trans-world identity (Kripke 1980, pp. 16-18).

What did we just mean by 'the first coin' and the 'the second coin'? Obviously, we did nothing to pick out one coin from the other. In effect, we used variables: 'Let *x* be one of the two, and *y* the other'. Reasoning that way is fine when given two objects, no further distinguished from each other. But that is not exactly what we were told to suppose. The supposition was that we are reasoning in an actual circumstance @ about a counterfactual circumstance *C*. The circumstance postulated to contain two coins was *C*, not @. In @, we are not given two coins; we are merely given that in *C* there would be two coins. For necessitists, that difference does not matter. The two coins in *C* are also in @, even if there they are not coins but merely possible coins. They are there in @ to be reasoned about. But for contingentists, the difference is crucial. They cannot assume that in @ there are two possible coins for them to reason about. Consequently, they cannot assume that there are two possible outcomes such as (HT) and (TH) above for them to reason about, because (HT) and (TH) were described in terms of the supposed difference between the two possible coins. Robert Stalnaker (2012), one of the most thoughtful defenders of a contingentist position, is quite clear that in a case like @ there is only a single qualitative possibility, which can be characterized in quantificational terms. That corresponds to (H+T) above. Thus contingentism undermines the standard probability calculation for those cases, because it provides only three possibilities, not four.

The point is easy to miss. For if we imagine reasoning in *C* itself, there are two coins, and the standard calculation is unproblematic. What that shows is that if *C* had obtained, the probability of 'one heads, one tails' *would have been* ½. But that was not the question. What was in question was the *actual* probability of 'one heads, one tails' conditional on *C*. For a contingentist such as Stalnaker, probabilities in @ are distributed over the actual space of possibilities, which may differ from the space of possibilities over which probabilities would have been distributed in *C*, because it is contingent what possibilities there are.

None of this is yet to say that contingentists are *forced* to assign probability 1/3 to 'one heads, one tails'. They might try to argue that some actual way of differentiating between the coins will always be available, for instance in terms of spatial location, though it is doubtful that such tactics will succeed with sufficient generality. Alternatively, in the style of Plantinga rather than Stalnaker, they might insist that there are actually two possibilities such as (HT) and (TH) even though there are not actually two possible coins to distinguish them, and somehow explain why they take such different attitudes to possible states and possible coins. If all else fails, they might say without further explanation that although there are actually only three possibilities, (HH), (H+T), and (TT), they are not equiprobable; (H+T) must have twice the probability that either (HH) or (TT) has. But what contingentists cannot do is simply endorse the standard calculation. It is not available to them just as it stands. At best, they will have to work hard to recover the standard calculation, and it is not clear that they will succeed.

The bearing of the necessitism-contingentism issue on calculations of probability is not confined to toy examples like those above. The choice between treating (HH), (HT), (TH), and (TT) as the four equiprobable possibilities and treating (HH), (H+T), and (TT) as the three equiprobable possibilities corresponds to the choice between Fermi-Dirac statistics and Bose-Einstein statistics in particle physics. For some applications, Bose-Einstein statistics do better; for others, Fermi-Dirac statistics do better. The danger for contingentism is that it may pressure us towards the Bose-Einstein statistics on general metaphysical grounds when the physics requires the Fermi-Dirac statistics. In general, different views in modal metaphysics mandate different ways of individuating objective possibilities, which in turn motivate different assignments of objective probability, and thereby have knock-on effects in natural science. The two enterprises are nothing like as disconnected as many philosophers and many scientists assume.[25]

8. *Metaphysical versus other objective modalities*

Appeals to objective modal aspects of natural science seem to have this limitation: they concern at best some form of nomic modality, but not metaphysical modality. How much light do the arguments of sections 3-7 cast on metaphysical modality?

The gap between nomic and metaphysical may be narrower than is usually thought. Following Saul Kripke (1980), Alexander Bird (2007) has argued in detail that laws of nature may be metaphysically necessary. If what it is to be an F involves being a G, then it is metaphysically, not just nomically, necessary that all Fs are Gs. It is a good question how far such arguments can be taken: could not motion have obeyed different laws? In any case, the total assimilation of nomic modality to metaphysical modality is not only rather implausible: it is not even relevant to all the cases discussed above. For the possibilities in a probability space or state space may not even exhaust all nomic possibilities, let alone all metaphysical ones. They may cover just the possible states of a highly contingent system, such as the tossing of coins or the weather on earth. Still, we may assume that in a typical case they *are* nomic possibilities, even if they are not all of the nomic possibilities.

A simple point is that if nomic modality is an objective modality, nomic possibility entails metaphysical possibility, the most general type of objective probability. Thus if science shows something to be nomically possible, it thereby shows it to be metaphysically possible too. In particular, if science shows something *nonactual* to be nomically possible, it thereby shows it to be metaphysically possible. The nomic possibility of various states is built into applications of probability spaces and phase spaces. Curtailing the phase space typically disrupts its mathematical structure and thereby reduces the explanatory power of the theory. Of course, someone might challenge the entailment from nomic possibility to metaphysical possibility. We saw in section 2 that if nomic possibility is just logical consistency with the laws of nature, it does not entail metaphysical possibility. However, that account of nomic possibility fared very badly. If instead nomic possibility is metaphysical compossibility with the laws of nature, then it trivially entails metaphysical possibility, for a metaphysical impossibility is not metaphysically compossible with anything. Another argument is that if α is nomically possible, then the subjunctive conditional $\alpha > \neg\alpha$ is false, but if α were metaphysically impossible that conditional would arguably be vacuously true (Williamson 2007). There is no good reason to deny the entailment from nomic possibility to metaphysical possibility.

Although no attempt will be made here to argue in general from nomic necessity to metaphysical necessity, often the main challenge to a claim that something is metaphysical necessary is also by implication a challenge even to the claim that it is nomically necessary. Consider, for instance, the necessitist thesis NNE. Perhaps the strongest contingentist objection to it is of this sort: if my parents had never met, there would have been no such thing as me; therefore, I am a counterexample to the claim that (necessarily) everything is necessarily something (and so is everyone else). Presumably, it is nomically as well as metaphysically possible for my parents never to have met. Thus the putative counterexample tells just as much against NNE on a nomic reading of the modal operators as against NNE on the intended metaphysical reading of them. But scientific evidence *for* the nomic version of NNE (as suggested in sections 6-7) is also evidence that such putative counterexamples do not work against the nomic version, and if they fail against the nomic version they fail against the metaphysical version too. It is evidence for nomically possible people who are not actually people, and for nomically possible particles which are not actually particles. Once such things are accepted, there is little to be gained by holding out against the metaphysical version of NNE; its theoretical virtues triumph in the absence of a compelling counterexample. In cases like these, the main dialectical action is within the realm of nomic possibility. The distinction between nomic and metaphysical necessity, though granted, sometimes makes less difference than might have been expected to the modal upshot of natural science.

Those considerations combine naturally with the more general observations in sections 1-2 of how the objective modalities tend to stand or fall together. It is implausible to treat the objective modal dimension of natural science as merely an artefact of folk cognitive architecture or overheated metaphysical speculation. We have no good evidence that it is a proxy for something else.

We can also draw a more general moral in epistemology. The epistemology of modality cannot be treated in isolation; it is not an autonomous branch of epistemology. Our natural scientific knowledge of objective modality is too tightly integrated with the rest of our scientific knowledge to permit such a division. Not very surprisingly, the abductive methodology of natural science plays a major role in the epistemology of modality.[26]

Notes

1 See Kratzer 2012, pp. 49-62, Portner 2009, pp. 144-84, and Vetter 201X, for instance. It is not denied that the same word can express an objective modality in one context and an epistemic modality in another: compare 'She could run a marathon in three hours' (objective) with 'Goldbach's Conjecture could be true' (epistemic).

2 There is a plausible argument that, in propositional modal logic, if metaphysical modality obeys *at least* the principles of S5, then it obeys *at most* the principles of S5 (Williamson 2013a, p. 111).

3 Contingentists may wish to insert a qualification 'if *x* exists' within the scope of the necessity operator to handle the possible non-existence of the objects. What matters is that $x = y$ licenses the inter-substitution of the free variables '*x*' and '*y*' in objective modal contexts; the same object is at issue under different guises.

4 See Strohminger 2015 for a detailed development of the case for perceptual knowledge of nonactual possibilities. This strikes at the Humean assumption that impressions are non-modal in content. Roca-Royes 201X makes a more empiricist argument for inductive knowledge of nonactual possibilities via their similarity to perceived actualized possibilities: if you have seen cups break, and thereby know that they *can* break, you may infer that a similar unbroken cup can break (though that does less to confront empiricist worries about how we come to understand 'can' in the first place).

5 It is worth noting that some philosophers of mathematics interpret the language of mathematics itself as implicitly modal: mathematics becomes a science of possible structures (Putnam 1967, Hellman 1989). Despite taking such views seriously, for present purposes I prefer not to rely on philosophical interpretations so distant from the way mathematicians explicitly talk and think. More recent modal interpretations of the language of set theory, such as Linnebo 2013 and Studd 2013 (see also Parsons 1983, pp. 298-341 and Fine 2006), have been motivated by a (laudable) desire to avoid Russell's paradox for sets without *ad hoc* restrictions; that too is quite far from the concerns of most working

mathematicians. Moreover, the latter motivation requires a (so far somewhat obscure) non-objective reading of the modal operators, because even the hierarchy of pure sets must involve such a modal aspect (since Russell's paradox arises even for pure sets, and it is generally agreed that the existence of pure sets is *metaphysically* non-contingent). Non-objective modal interpretations of the language of mathematics are not strictly relevant to the concerns of this paper.

6 The principle in the text is a form of *single-premise closure*. We do *not* assume the multi-premise closure principle that if a formula α of L supports the formulas $\beta_1, \ldots, \beta_n$ of L^+, and $\beta_1, \ldots, \beta_n$ jointly entail γ, then α also supports γ. Unlike multi-premise closure, single-premise closure is consistent with an interpretation of 'support β' as 'confer a probability above the threshold *c* on β', where $0 < c < 1$.

7 See Williamson 2007, pp. 293-9, for relevant background on the logical relations between counterfactuals and metaphysical modality.

8 As already hinted, it is controversial how much weaker than metaphysical necessity nomic necessity really is (see also section 8). It is also controversial how much of natural science really aims at nomic necessity. The present remarks about nomic modality should be taken in the spirit of a first approximation.

9 Some Lewisians may object to the assumption because it clashes with the 'small miracle' conception of subjunctive conditionals, but the assumption is nonetheless very plausible.

10 Allowing infinitesimal probabilities does not solve the problem (Williamson 2007b).

11 Let Ω be the set of all possibilities. If we define $\Box E = \Omega$ if E has probability 1 and $\Box E = \{\}$ otherwise, and $\Diamond E = \Omega$ if E has nonzero probability and $\Diamond E = \{\}$ otherwise, then in the finite case in the text □ and ◊ satisfy the principles of the propositional modal system S5 (= KT45); in the infinite case they satisfy only the principles of the weaker modal system KD45, where the T principle ($\Box E \subseteq E$) is weakened to the D principle ($\Box E \subseteq \Diamond E$).

12 If we used evidential or subjective probabilities, the putative explanation would at best show that the explanandum 'was to be expected'. But to show that an outcome was to be expected is not to explain why it occurred, in the relevant broadly causal sense. To revert to the example at the beginning of section 1, it

was trivially to be expected that *n* would number the inhabited planets, since '*n*' was defined to name their number. For instance, given that $n = 29$, to explain causally why there are exactly *n* inhabited planets is to explain causally why there are exactly 29 inhabited planets, but the trivial 'was to be expected' explanation does not advance the latter project. The difference between the two non-obviously co-referential names '*n*' and '29' is epistemically relevant but causally irrelevant.

13 One may question the assumption that merely possible states of a physical system are abstract objects (Malament 1982, p. 533; Lyon and Colyvan 2008, p. 233). On the approach of Williamson 2013a, p. 7, their non-concreteness does not make them abstract. But that point is irrelevant to the present argument.

14 For some purposes we might require directions to be non-negative, which would require only forwards determinism.

15 Of course, this is not the only way of interpreting modal operators over a dynamical system. If a topology is defined over the states, one can interpret □ as the topological interior operator, which is a much more 'local' form of necessity (though it does not involve an accessibility relation between worlds). It yields an S4 modal logic, whereas the present 'global' interpretation of □ yields an S5 modal logic. The two interpretations are not rivals; they simply pick out different aspects of the system for study. The global interpretation is more general, because it does not depend on what kind of mathematical structure is defined over the states. For more on topological interpretations of modal logics on dynamical systems see Artemov, Davoren, and Nerode 1997 and Davoren and Goré 2002.

16 They are unrestricted in the sense of ranging over all states of the system; as already noted, they are typically not equivalent to metaphysical necessity and metaphysical possibility.

17 This 'diamonds are forever' principle is reminiscent of, but not equivalent to, the principle defended by Dorr and Goodman 201X; the latter concerns metaphysical possibility and a more standard reading of the tense operators.

18 For an introduction to tense logic that explains the relevant background see, for instance, Müller 2011.

19 In unquantified S5, every formula is equivalent to one without such embedded occurrences of modal operators. That is not in general so for quantified S5, even with constant domains; see Fine 1978, pp. 146-51, for the case of first-order S5.

20 See Williamson 2013a for a discussion in more depth. I am assuming that the rest of the model theory is more or less standard.

21 For reasons explained in Williamson 2013a, pp. 254-61, reading higher-order quantifiers such as those in NNE_P as first-order quantifiers restricted to objects of a special sort (propositions) is ultimately inappropriate: semantically, the difference between name position and sentence position runs deeper than a difference between objects of one kind and objects of another, or between objects in general and objects of a special kind (where every object can in principle be named). Nevertheless, for present purposes the talk of propositions as objects is a harmless over-simplification.

22 For discussion of the biological case see Gunawardena 2009.

23 The complaint on pp. 286-8 of Williamson 2013a that various contingentist comprehension principles for second-order modal logic are too weak to serve the purposes of 'modal mathematics' relates to just this point. Consider the free application of non-modal mathematics to an implicitly modal subject matter, as in reasoning about dynamical systems or Kripke models intended for some objective modality. When the intended modal content of the application is made explicit model-theoretically in the manner of section 5 (which can be extended to other forms of quantification), unrestricted modal comprehension principles such as $Comp_P$ can be proved valid in the model theory by non-modal mathematics (for instance, set theory). Since the free application of non-modal mathematics to the modal subject matter is committed to all such valid modal formulas, in particular it is committed to those unrestricted modal comprehension principles. That is fine for necessitists but not for full-blooded contingentists such as Stalnaker (2012).

24 See Cussens 201X for a recent discussion of Leibniz's failure to contribute to the mathematics of probability.

25 For a related application of probability to an issue in modal metaphysics see Kment 2012, although I doubt that Kment would endorse the conclusions of this paper.

26 The material in this paper has evolved over several years. Various parts of it were presented as the Ruth Manor Lecture at Tel Aviv University, the Saul Kripke Lecture at City University New York, and the Wade Lecture at St Louis University. Earlier versions of the material were presented as talks at conferences on the epistemology of modality at Belgrade University, Aarhus University (where Daniel Dohrn provided a detailed response), and Stirling University, a conference on logic and metaphysics at the University of Southern California, a workshop on modal metaphysics in Montreal, and the Universities of Athens, Connecticut (Storrs), Michigan (Ann Arbor), and Oxford. Embryonic predecessors were presented to workshops at the Centre for the Study of Mind in Nature in Oslo and the Institute of Philosophy in London. I am grateful to all the participants at those events who helped me develop the material with their questions and comments, and for discussion or correspondence on the issues to Kit Fine, Peter Fritz, Peter Godfrey-Smith, Jeremy Goodman, Lloyd Humberstone, Matthias Jenny, Øystein Linnebo, Maurico Suárez, and Trevor Teitel. I believe that Saul Kripke envisaged an analogy with states in phase space early in his thinking about possible worlds.

Bibliography

Artemov, Sergei, Davore, Jennifer, and Nerode, Anil 1997: ‘Modal logics and topological semantics for hybrid systems’. Technical report MSI 97-05, Cornell University.

Bird, Alexander 2007: *Nature’s Metaphysics: Laws and Properties*. Oxford: Clarendon Press.

Chisholm, Roderick 1973: ‘Parts as essential to their wholes’, *Review of Metaphysics*, 26, pp. 581-603.

Cussens, James 201X: ‘Probability and statistics’, in M.R. Antognazza (ed.), *The Oxford Handbook of Leibniz*. Oxford: Oxford University Press.

Davidson, Donald 1970: ‘Mental events’, in L. Foster and J.W. Swanson (eds.), *Experience and Theory*. London: Duckworth.

Davoren, Jen, and Goré, Rajeev 2002: ‘Bimodal logics for reasoning about continuous dynamics’, in F. Wolter, H. Wansing, M. de Rijke, and M. Zakharyaschev (eds.), *Advances in Modal Logic*, vol. 3. London: World Scientific, pp. 91-111.

Dorr, Cian, and Goodman, Jeremy 201X: ‘Diamonds are forever’, *Noûs*, forthcoming.

Field, Hartry 1980: *Science without Numbers: A Defence of Nominalism*. Oxford: Blackwell.

Fine, Kit 1978: ‘Model theory for modal logic: Part I — the *de re/de dicto* distinction’, *Journal of Philosophical Logic*, 7, pp. 125-156.

Fine, Kit 1994: ‘Essence and modality’, *Philosophical Perspectives*, 8, pp. 1-16.

Fine, Kit 2006; ‘Relatively unrestricted quantification’, in A. Rayo and G. Uzquiano (eds.), *Absolute Generality*. Oxford: Clarendon Press.

Fritz, Peter 2013: ‘Modal ontology and generalized quantifiers’, *Journal of Philosophical Logic*, 42, pp. 643-678.

Gibson, James 1979: *The Ecological Approach to Visual Perception*. Boston: Houghton Mifflin.

Gunawardena, Jeremy 2009: ‘Models in systems biology: the parameter problem and the meanings of robustness’, in H. Lodhi and S. Muggleton (eds.), *Elements of Computational Systems Biology*. Hoboken: Wiley.

Hellman, Geoffrey 1989: *Mathematics without Numbers: Towards a Modal-Structural Interpretation*. Oxford: Clarendon Press.

Kim, Jaegwon 1993: *Supervenience and Mind: Selected Philosophical Essays*. Cambridge: Cambridge University Press.

Kment, Boris 2012: 'Haecceitism, chance, and counterfactuals', *Philosophical Review*, 121, pp. 573-609.

Kment, Boris 2014: *Modality and Explanatory Reasoning*, Oxford: Oxford University Press.

Kratzer, Angelika 1977: 'What "must" and "can" must and can mean', *Linguistics and Philosophy*, 1, pp. 337-355.

Kratzer, Angelika 2012: *Modals and Conditionals*. Oxford: Oxford University Press.

Kripke, Saul 1980: *Naming and Necessity*. Oxford: Blackwell.

Lewis, David 1968: 'Counterpart theory and quantified modal logic', *Journal of Philosophy*, 65, pp. 113-126.

Lewis, David 1986: *On the Plurality of Worlds*. Oxford: Blackwell.

Linnebo, Øystein 2013: 'The potential hierarchy of sets', *Review of Symbolic Logic*, 6, pp. 205-228.

Loewer, Barry 2001: 'Determinism and chance', *Studies in History and Philosophy of Modern Physics*, 32, pp. 609-620.

Lyon, Aidan, and Colyvan, Mark 2008: 'The explanatory power of phase spaces', *Philosophia Mathematica*, 16, pp. 227-243.

Mackie, Penelope 2006: *How Things Might Have Been: Individuals, Kinds, and Essential Properties*. Oxford: Clarendon Press.

Malament, David 1982: 'Review of *Science without Numbers: A Defense of Nominalism* by Hartry Field', *Journal of Philosophy*, 79, pp. 523-534.

Maudlin, Tim 2007: 'What could be objective about probabilities?', *Studies in History and Philosophy of Modern Physics*, 38, pp. 275-291.

Müller, Thomas 2011: 'Tense or temporal logic', in L. Horsten and R. Pettigrew (eds.), *The Continuum Companion to Philosophical Logic*. London: Continuum.

Nolte, David 2010: 'The tangled tale of phase space', *Physics Today*, 63 (4), pp. 33-38.

Parsons, Charles 1983: *Mathematics in Philosophy: Selected Essays*. Ithaca, NY: Cornell University Press.

Plantinga, Alvin 1983: 'On existentialism', *Philosophical Studies*, 44, pp. 1-20.

Portner, Paul 2009: *Modality*. Oxford: Oxford University Press.

Prior, Arthur 1956: 'Modality and quantification in S5', *Journal of Symbolic Logic*, 21, pp. 60-62.

Prior, Arthur 1967: *Past, Present and Future*. Oxford: Clarendon Press.

Putnam, Hilary 1967: 'Mathematics without foundations', *Journal of Philosophy*, 64, pp. 5-22.

Quine, Willard van Orman 1953: 'Reference and modality', in *From a Logical Point of View*. Cambridge, Mass.: Harvard University Press.

Quine, Willard van Orman 1966: 'Three grades of modal involvement', in *The Ways of Paradox and Other Essays*, pp. 156-174. New York: Random House.

Roca-Royes, Sonia 201X: 'Similarity and possibility: an epistemology of *de re* possibility for *concreta*', in B. Fischer and F. Leon (eds.), *Modal Epistemology after Rationalism*, forthcoming.

Salmon, Nathan 1982: *Reference and Essence*. Oxford: Blackwell.

Salmon, Nathan 1989: 'The logic of what might have been', *Philosophical Review*, 98, pp. 3-34.

Salmon, Nathan 1993: 'This side of paradox', *Philosophical Topics*, 21, pp. 187-197.

Sider, Ted 2011: *Writing the Book of the World*. Oxford: Clarendon Press.

Stalnaker, Robert 2012: *Mere Possibilities: Metaphysical Foundations of Modal Semantics*. Princeton: Princeton University Press.

Strogatz, Steven 2001: *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Boulder: Westview Press.

Strohminger, Margot 2015: 'Perceptual knowledge of nonactual possibilities', *Philosophical Perspectives*, 29, forthcoming.

Studd, James 2013: 'The iterative conception of set: a (bi-)modal axiomatization', *Journal of Philosophical Logic*, 42, pp. 697-725.

Vetter, Barbara 201X: 'Williamsonian modal epistemology, possibility-based', in M. McCullagh and J. Yli-Vakkuri (eds.), *Williamson on Modality*, London: Routledge.

Williamson, Timothy 2006: 'Indicative versus subjunctive conditionals, congruential versus non- hyperintensional contexts', in E. Sosa and E. Villanueva, eds., *Philosophical Issues*, *Volume 16: Philosophy of Language*, Oxford: Blackwell, pp. 310-333.

Williamson, Timothy 2007a: *The Philosophy of Philosophy*. Oxford: Wiley Blackwell.

Williamson, Timothy 2007b: 'How probable is an infinite sequence of heads?', *Analysis*, 67, pp. 173-180.

Williamson, Timothy 2013a: *Modal Logic as Metaphysics*. Oxford: Oxford University Press.

Williamson, Timothy 2013b: *Identity and Discrimination*, 2nd ed. Oxford: Blackwell.

Woodward, James 2003: *Making Things Happen: A Theory of Causal Explanation*. New York: Oxford University Press.