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Leibniz's argument for the Identity of Indiscernibles in *Primary Truths*[*]

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1. Leibniz stated his *Principle of Identity of Indiscernibles* (PII) in several places, but only in few of them he gave arguments for it. One such place is his correspondence with Clarke. Another such place is his paper entitled *Primary Truths*. In both places Leibniz derives (PII) from the *Principle of Sufficient Reason* (PSR). But the two derivations, or arguments, are very different from each other. The argument in the correspondence with Clarke applies a theological version of (PSR). That argument presupposes that the actual world was created by God, and concerns the reasons God would have had to create a world with indiscernibles. The argument in *Primary Truths*, on the contrary, does not presuppose God (in fact, in *Primary Truths* God is mentioned only after Leibniz has argued for (PII)).

The purpose of this essay is to reconstruct Leibniz's argument for (PII) in *Primary Truths*[1]. Leibniz is very succinct in the relevant passage and the argument Leibniz explicitly advances is, as we shall see, problematic. The structure of this paper is as follows. In section 2 I shall present Leibniz's argument as it appears in the text of *Primary Truths*. In section 3 I shall mention two objections, advanced by Jan Cover and John Hawthorne, to this argument. In section 4 I shall present my own reconstruction of Leibniz's case for (PII) in *Primary Truths*, and I shall briefly explain why this recostruction of Leibniz's is not subject to Cover and Hawthorne's objections. Finally, in section 5, I shall present and discuss Cover and Hawthorne's own reconstruction of Leibniz's argument for (PII). I shall argue that there are both systematic, or philosophical, and hermeneutical reasons to prefer my reconstruction.

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[1] I have reconstructed Leibniz's argument for (PII) in the letters to Clarke in my "Leibniz's Argument for the Identity of Indiscernibles in his Correspondence with Clarke", *Australasian Journal of Philosophy* 77/4 (1999), pp. 429-438.

2. What is Leibniz's argument for (PII) in *Primary Truths*? To answer this question I need first to briefly summarise the parts of *Primary Truths* that precede Leibniz's argument for (PII) there.

Leibniz begins his essay by asserting that primary truths, or identities, are those that assert the same thing of itself or deny the opposite of its opposite (C 518). Examples are: 'A is A', 'A is not not A' and 'every thing is as it is'. Why are identities *primary* truths? Because, according to Leibniz, all other truths reduce to identities with the help of definitions. From this Leibniz extracts a general theory of truth:

> The predicate or consequent, therefore, is always in the subject or antecedent, and this constitutes the nature of truth in general, or, the connexion between the terms of a proposition, as Aristotle also has observed. In identities this connexion and inclusion of the predicate in the subject is express, whereas in all other truths it is implicit and must be shown through the analysis of notions, in which *a priori* demonstration consists (…) But this is true in the case of every affirmative truth, universal or particular, necessary or contingent, and in the case of both an intrinsic and an extrinsic denomination (C 518-19)[2].

From other texts, including his references to Peter and Judas later in *Primary Truths* (C 520), it is clear that Leibniz intends his theory to apply also to singular truths, that is, truths like 'Peter sins' or 'Alexander is a king'. This theory is known as the *concept containment theory of truth*. It is called the *concept* containment theory because normally Leibniz states it by saying that the *concept* of the predicate is contained in the *concept* of the subject (GP II, 56). The text quoted above might be thought to be ambiguous between requiring for truth a connection between the concepts or a connection between the predicate and that to which the concept of the subject corresponds. But if there is any ambiguity, I don't think we need to resolve it, for I agree with Robert Adams that Leibniz believed both that the predicate was contained in the concept of the subject and in the substance to which the subject corresponds.[3]

Leibniz goes on to derive (PSR) from the above considerations:

[2] "Primae Veritates"; C 518-19; quoted from *Leibniz. Philosophical Writings*, trans. by M. Morris and G. H. R. Parkinson, London, Dent (Everyman's Library), 1995, pp 87-8.

[3] R. M. Adams: *Leibniz. Determinist, Theist, Idealist*, New York, Oxford University Press, 1994, pp. 71, 79.

> From these facts, which have not yet been sufficiently considered because of their excessive easiness, there follow many things of great importance. For from this there at once arises the accepted axiom, ‘There is nothing without a reason’, or, ‘There is no effect without a cause’. For otherwise there would be a truth which could not be proved *a priori*, i.e. which is not analysed into identities; and this is contrary to the nature of truth, which is always, either expressly or implicitly, identical (C 519, Morris and Parkinson 88).

Leibniz then goes on to derive some minor consequences, after which he states and argues for (PII):

> From this it follows also that *there cannot be in nature two individual things which differ in number alone*. For it must be possible to give a reason why they are diverse, which must be sought from some difference in them (C 519, Morris and Parkinson 88).

This is a simple application of *modus tollens* based on (PSR). Ignoring the modalities involved (‘there *cannot* be…’, ‘it *must* be *possible*…’, ‘which *must* be sought...’), this argument can be represented as follows:

1. Everything must have a reason (PSR).
2. For every x and every y, if x and y are indiscernible, there is no reason why they are numerically different.
3. Therefore, for every x and every y, if x and y are indiscernible, they are numerically the same (PII).

Leibniz’s passage also contains an explanation of why premise (2) holds. There is no reason why indiscernible individuals are numerically different because such a reason must refer to some difference the indiscernibles contain. But being indiscernible they differ in no way, and so there is no reason why they are numerically different. (Of course, even being indiscernible, numerically different individuals differ in one way: numerically. But their numerical difference would hardly be a reason for their being numerically different).

Thus, on the face of it, Leibniz's argument here for (PII) is simply that everything must have a reason but there is no reason why numerically different but indiscernible individuals would be numerically different. This is why Cover and Hawthorne dub this argument the *No Reason* argument for (PII)[4].

3. No doubt the No Reason argument is a faithful representation of the relevant passages in Leibniz's text. The problem with this argument is that it is objectionable, as Cover and Hawthorne have shown. They put forward two different but related objections to the No Reason argument. Their first objection is that the No Reason argument assumes the onus of proof must be on those who deny (PII). But it is not obvious that this must be so: in a case of indiscernibles *a* and *b*, 'there would seem to be no less a reason for denying that they are two than for denying that they are one' (Cover and O'Leary-Hawthorne 190).

They illustrate the point with a mathematical analogy. Suppose it is established there is no direct proof for *p* and no direct proof for *not–p*. Shall we conclude that we have indirectly proved *not–p*, or shall we conclude that we have indirectly proved *p*? Cover and Hawthorne rightly say one is hard pressed to see why one should prefer one conclusion over the other unless some further data can be brought to bear. All one should conclude in such a case, they suggest, is either that one doesn't know which of *p* or *not–p* has been proved, or else that there is no fact of the matter as to whether *p* or *not–p* (Cover and O'Leary-Hawthorne 190).

And this leads to their second objection to the No Reason argument. This is that Leibniz assumed that there is a fact of the matter, in cases of indiscernibility, as to whether the indiscernible entities are one or two, numerically identical or numerically different. If there is no fact of the matter concerning numerical identity and difference in cases of indiscernibility, then lack of reason for indiscernibles *a* and *b* being two does not license the conclusion that *a* and *b* are one.

One might think that Leibniz can respond to this second objection by bringing in considerations about bivalence. Thus, Cover and Hawthorne note, Leibniz might have replied as follows: 'Bivalence tells me that *p* is either true or else it is false; (PSR) tells me that if *p* is true, then *p* has a sufficient reason; and so in the absence of reasons for *p* I can deduce *not–p*'. But Cover and Hawthorne point out that even if bivalence and (PSR) are true, it might still be the case that '*p*' fails to

[4] J. A. Cover and J. O'Leary-Hawthorne: *Substance and Individuation in Leibniz*, Cambridge, Cambridge University Press, 1999, p. 189.

express a state of affairs or proposition, in which case the absence of reasons for the truth of '*p*' does not entitle one to conclude that *not–p*. That is, if Leibniz is going to argue that indiscernibles *a* and *b* are one from lack of reasons for their being two, he must first make sure that 'a and *b* are two' expresses a genuine proposition or state of affairs (Cover and O'Leary-Hawthorne 191-2).

What shall we make of this? It seems clear to me that the objections show Leibniz needs to give positive reasons, or a direct argument, for (PII). For if Leibniz had a direct argument for (PII), then there would be no question about the onus of proof: he would have shown there is no reason for the non-identity of indiscernibles *and* that there is a reason for their identity. Similarly, once a direct argument has been given for the identity of indiscernibles, it has been shown there is a fact of the matter, in cases of indiscernibility, as to whether the indiscernible entities are numerically identical or numerically different.

Cover and Hawthorne think Leibniz could meet their objections. But before discussing their reconstruction of Leibniz in section 5, I shall in the next section propose my own, and show that Leibniz had available in *Primary Truths* a positive reason, or direct argument, for (PII).

4. Leibniz did have a direct argument for (PII) in *Primary Truths*. It is an argument based on what Leibniz says at the beginning of his article. For him all truths are reduced, through definitions of terms and analysis of notions, to primary truths or identities, that is, truths where 'the connexion and inclusion of the predicate in the subject is express'. But although such connection and inclusion are only explicit in primary truths, they are implicit in all other propositions (C 519). This means that all truths, or at least all affirmative truths, including singular ones, have the form $X = X$. In some cases, when the truth in question is contingent, as he says elsewhere and suggests in *Primary Truths* (C 519), the analysis of notions and substitution of definitions has no end. Nevertheless, even in those cases where an infinite analysis is required, all truths have the form $X = X$. Now if all truths have that form, Leibniz could have argued for (PII) in the following way, where '*P*' stands short for the conjunction of all simple properties or predicates of *a* and *b*:

(1) *a* is *P*; *b* is *P*.

(2) All affirmative truths are either implicit or explicit identities.

(3) $a = P$ (from premises (1) and (2))

(4) $b = P$ (from premises (1) and (2))

(5) Therefore, $a = b$ (from premises (3) and (4) by Leibniz's Law)

Since 'P' stands short for the conjunction of all properties or predicates of a and b, what premise (1) states is that a and b are indiscernible. So the argument proves that if a and b are indiscernible, they are identical, i.e. (PII) is true. Let us call this argument the *Identity argument*.

Note that here Leibniz's Law cannot be understood, as sometimes it is, as the conjunction of (PII) and the *Indiscernibility of Identicals*. For that would make the argument question begging. But this is not a problem, for all we need to employ here is the so-called *Principle of Substitutivity of Identicals*, according to which the names of one and the same thing can be substituted for each other, *salva veritate*, in those sentences where they occur purely referentially. This is what I take Leibniz's Law to be in this argument, and this licenses the inference of (5) from (3) and (4)[5].

It is important not to confound the Identity argument with another argument for (PII) in which identity plays a crucial role. This is the argument that there cannot be indiscernible but numerically different things a and b because in that case a would have the property of *being identical to a*, which b would lack. This argument, which admittedly makes (PII) trivial, depends neither on the concept-containment theory of truth, nor on the claim that all truths are identities. Furthermore, this argument trivisalises (PII) because it does not need, and typically its proponents do not, make *being identical to a* a property reducible to other more basic, purely qualitative properties of a. This is why this argument tends to deliver a very weak version of (PII). The Identity argument, on the other hand, does require that a property like *being identical to a* be reducible to a conjunction of purely qualitative properties. In particular, the property of *being identical to a* is simply the property of *being P*, where *being P* is a complex property consisting of all the qualitative properties that a has. For instance, the property of *being Julius Caesar* is simply the property of *being a Roman general, who was assassinated, etc*[6]. This

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[5] No doubt the way I have stated Leibniz's Law is more Quinean than Leibnizian. But Leibniz did maintain what I have here called *Leibniz's Law*. He held that identicals are those that (or, better, whose names or denominations) can be substituted for each other *salva veritate* in any proposition; see C 362. Yet he was aware that this principle failed in oblique contexts, which is why formulating the law in terms of the Quinean notion of purely referential occurrence is not inappropriate. For Leibniz on the failure of substitutivity in oblique contexts see B. Mates: *The Philosophy of Leibniz. Metaphysics and Language*, New York, Oxford University Press, 1986, pp. 130-32.

[6] Admittedly, the picture is more complicated, since some of these properties are had only at certain times. For instance, Julius Caesar does not have the property of *being assassinated* at all times, but only in 44 B.C. I am ignoring this complication here, but for ways of dealing with it see Mates 87-94.

is why Leibniz's version of (PII) is so controversial and not at all trivial: it requires every two things to differ in some qualitative properties.

Yet to the Identity argument it may be objected that premises (1) and (2) do not entail premises (3) and (4), for surely not all truths are reduced to identities like '$a = P$'. For even if such reduction is suggested in the opening lines of *Primary Truths*, Leibniz also says, in the first passage quoted above, that the connection and inclusion of the predicate in the subject is explicit in identities. So identities are, for Leibniz, truths where the predicate is explicitly *contained in* the subject, not necessarily truths where the predicate is explicitly *identical with* the subject (the latter are only a proper subclass of the former). Leibniz was of course well aware of this, and there are passages where he gives truths like 'homo albus est albus' as examples of identities (C 11). So it may be that P is contained in the concept of a but without being identical with the concept of a. This is the case, for example, with the concepts of Alexander and of king: the former includes, but is not identical with, the latter. But if P need not be identical to the concept of a then it may be contained both in the concept of a and the concept of b, without being identical to either. So a and b need not be identical.

The objection is thus based simply on the idea that the concept of a may contain P but not be identical with P. Now if P is not identical with the concept of a, then there is more to the concept of a than simply P. That is, $a = P+Q$, where $P \neq Q$. Of course, it may be that $b = P+R$, where $Q \neq R$, in which case $a \neq b$.

But if $a = P+Q$, then P does not contain all properties or predicates of a, for Q can be predicated of a and is not contained in P. So if P contains all properties or predicates of a, $a = P$. Similarly, if P contains all properties or predicates of b, $b = P$. And in such a case, by Leibniz's law, $a = b$.

No doubt premise (2) of the Identity argument is extremely contentious. My claim is not that this argument is compelling; my claim is that the argument is a valid argument that would have seemed compelling to Leibniz himself. It is likely that Leibniz thought of something like the Identity argument, but did not state it because he thought it was too obvious. But whether or not Leibniz thought of this argument when writing *Primary Truths*, my claim is that it was available to Leibniz in *Primary Truths*, and captures the spirit, and makes sense of the letter, of the text.

My proposal is thus that we see the Identity argument as complementing Leibniz's claim in *Primary Truths* that there is no reason why indiscernibles a and b are numerically different. So, according to this reconstruction, Leibniz's case for (PII) is a conjunction of two different arguments, the No Reason and the Identity argument. Thus Leibniz is not only saying that there is no reason why indiscernibles *a* and *b* would be two, he is also adding to this claim reasons why indiscernibles *a* and *b* must be one. The reason why they must be one is articulated in the Identity argument: if *a* and *b* are indiscernible, then $a = P$ and $b = P$ and so, by Leibniz's law, $a = b$. No doubt Leibniz is able in this way to meet the two objections by Cover and Hawthorne: he is not open to the charge of assuming the onus of proof is on those who deny (PII), for he has directly argued for (PII). And he is not open to the charge of having assumed there is a fact of the matter as to whether indiscernible entities are one or two, for having argued for (PII) and given reasons for it Leibniz has thereby shown there is such a fact of the matter.

5. Cover and Hawthorne also defend Leibniz from their two objections to the No Reason argument. Based on what Leibniz says in 1704 in a letter to De Volder, they present Leibniz as presupposing that there is a fact of the matter with respect to the identity and difference of laws-of-the-series. Given these facts of identity and diversity about laws-of-the-series, to be entitled to say that *a* and *b* are one in a case where *a* and *b* have different substantial forms (or laws-of-the-series, since for Cover and Hawthorne laws-of-the-series are identical to substantial forms), one needs a principle of unity/identity that underlies difference with regard to substantial form. Similarly, if one is to say that *a* and *b* are two in a case where the substantial form is the same, then one needs '*a principle of plurality/difference that underlies unity* which will entitle one to posit many [entities], despite a single substantial form/law' (Cover and O'Leary-Hawthorne 196; italics in the original).

Thus facts of identity and diversity of substantial form are the only basis in reality for talk of identity and diversity of entities. If this is so, then, presumably, the two specific objections Cover and Hawthorne devised against the No Reason argument evaporate. And Leibniz's argument against indiscernibles *a* and *b* will be good if there is no such principle of plurality/difference that can ground the numerical difference between *a* and *b* despite their sharing a substantial form. And there is no such principle available to Leibniz, Cover and Hawthorne claim: (difference of) accidents, relations, matter and haecceities are all options not available to Leibniz; so they think the true form of the No Reason argument is the following (Cover and O'Leary-Hawthorne 197):

(1) There is at least one instance of a substantial form.
(2) Nothing in the world can ground more than one instance of that substantial form.
(3) But there is more than one instance only if something in the world grounds more than one instance.
(4) So there is one and only one instance of that substantial form.

This is an alternative reconstruction of Leibniz's argument for (PII). But I have both philosophical and hermeneutic reasons for preferring my reconstruction of Leibniz's case for (PII).

First, the argument as reconstructed by Cover and Hawthorne is still subject to their own objections. The conclusion says that there is one and no more than one instance of the substantial form in question. Let us grant there is a reason why there is at least one instance of that substantial form. Yet why think there is no more than one? Because nothing in the world can ground more than one? That is, because there is no reason why there is more than one? This is what the argument suggests. But why assume that the onus of proof is on those who deny there is only one instance of the substantial form? After all it is not obvious that there is more reason for denying than for asserting that there is more than one instance of the substantial form in question. Cover and Hawthorne have not shown why we should prefer their version of the No Reason argument to the following argument, that concludes there is more than one instance of the relevant substantial form:

(1) There is at least one instance of a substantial form.
(2) Nothing in the world can ground exactly one instance of that substantial form.
(3) But there is exactly one instance only if something in the world grounds exactly one instance.
(4) So there is more than one instance of that substantial form.

It might be argued that premise (2) of this argument is false. If so, something in the world grounds exactly one instance of the substantial form. Yet what is it? All Cover and Hawthorne have provided is evidence that there is nothing that grounds more than one instance. They are explicit on this, as they say: '[t]hat *there is not more than one* [instance of the substantial form] is guaranteed by the lack of any principle for diversification of that substantial form' (Cover and O'Leary-Hawthorne 197-8; italics in the original). But what if nothing can ground exactly one instance either? In that case we should conclude that there is no fact of the matter as to whether there is exactly one or more than one such instance.

So it looks as if Cover and Hawthorne have assumed that there is a fact of the matter as to whether there is exactly one or more than one instance of a substantial form when there is at least one such instance. Yet why assume this? Why rule out the conventionalist option according to which the fact is simply that there is at least one instance, and whether there is exactly one, or more than one, is just talk?

Thus Cover and Hawthorne's objections to the No Reason argument are also objections to their reconstructed argument. To meet these objections Leibniz would need to say what would ground that there is exactly one instance of the substantial form in question. Yet at one single point Cover and Hawthorne assert that there is something that would ground there being exactly one instance of the substantial form. They say:

> In the absence of any grounds in reality for finding unity underlying different laws or diversity underlying the same law, one is left only with the facts of diversity and unity of law/form. Insofar as one's methods of counting reflect such facts, they will have a grounding in the world. Insofar as one's method of counting does not reflect such facts, the "diversity" will have solely a nominal basis. It is, as it were, mere talk (Cover and O'Leary-Hawthorne 199)

The problem with this is that it does not explain why what is said to be a 'grounding', or a reason, is such a 'grounding' or reason. All it says is that if there is only one substantial form then that is a reason for there being *only* one instance of the form. Yet while it may be clear that if there is only one substantial form there is *at least* one instance, it still remains to be shown why there must be *only* one instance in that case as well. To say that that must be so because it 'reflects the fact of the unity of the form' is simply to insist that when we count only one form, we must count only one instance. But it does not tell us why there being only one form *grounds*, or *constitutes a reason for*, there being only one instance.

Thus I conclude that the No Reason argument as reconstructed by Cover and Hawthorne does not give Leibniz what he needs. Leibniz is still open to the charges of assuming the onus of proof is on those who deny (PII) and of assuming there must be a fact of the matter as to whether there is exactly one or more than one entity in a case where *a* and *b* are indiscernible.

Thus there are systematic or philosophical reasons to reject Cover and Hawthorne's reconstruction of Leibniz's argument. But there are hermeneutic reasons as well. For Cover and Hawthorne's reconstructed No Reason argument fails to make sense of the context of Leibniz's argument. What they say about accidents, relations, matter and haecceities, namely that they are not principles of difference available to Leibniz, may be right, but Leibniz never suggests, in *Primary Truths*, that they play any role in the derivation of (PII). Leibniz says that (PII) follows 'from these considerations', but surely the considerations Leibniz was referring to were either (PSR) or the idea that all truths reduce to identities, or both – but not that accidents, relations, matter and haecceities cannot serve as a principle of difference for entities sharing substantial forms. Thus Cover and Hawthorne's reconstruction breaks the connection of Leibniz's argument for (PII) with its immediate context.

Things are otherwise with my reconstruction of Leibniz's argument. First, my reconstruction is based only on what Leibniz says in *Primary Truths*. For the Identity argument is based on Leibniz's idea that all truths reduce to identities. Since this idea is a consequence of the concept containment theory of truth, my reconstruction links (PII) with Leibniz's theory of truth. And the concept containment theory of truth figures prominently in *Primary Truths*[7].

Second, my reconstruction of Leibniz's is able to represent an important feature of the connection between (PII) and the concept containment theory of truth. No doubt any reconstruction of Leibniz's argument that derives (PII) from (PSR) represents a connection between (PII) and the theory of truth, for (PSR) itself was derived from the theory of truth. But there is more than this to the connection between (PII) and the concept containment theory of truth. For Leibniz says that (PSR) is true because 'otherwise there would be a truth which could not be proved *a priori*, i.e. which is not resolved into identities' (C 519). And Leibniz then implies that if (PII) were not true then (PSR) would

[7] It might be objected that my reconstruction is not based *only* on what Leibniz says in *Primary Truths*, for the Identity argument rests on an application of Leibniz's law, and Leibniz does not mention this law in *Primary Truths*. But the Identity argument rests on Leibniz's law as a rule of inference, not as a premise. So Leibniz's law is not properly stated in the Identity argument. So the fact that Leibniz did not state Leibniz's law in *Primary Truths* should be no objection. The premises of the Identity argument are based on, and presuppose, only what Leibniz explicitly says in *Primary Truths*. No considerations alien to *Primary Truths* are required to support it.

not be true. But then if (PII) were not true, there would be a truth which could not be resolved into identities.

What is this truth? Presumably, there are two such truths: the truth that *a* is *P* and the truth that *b* is *P*. For if they were reduced to explicit identities, they would be reduced to $P = P$. But then, of course, $a = b$. So if *a* and *b* are two indiscernible things, the truths that *a* is *P* and *b* is *P* do not reduce to identities. That those truths would not reduce to an explicit identity were (PII) false is a feature of the connection between (PII) and the theory of truth that is lost in Cover and Hawthorne's reconstruction[8].

Now, it may be objected that although my reconstruction is faithful to one element present in Leibniz's text, namely the connection between (PII) and the theory of truth, it does not represent another element also present in the text, namely the connection between (PII) and (PSR). But this charge is not good. I see Leibniz's case for (PII) as being a conjunction of two arguments: the No Reason argument and the Identity argument. Leibniz shows there is no reason for indiscernibles *a* and *b* being numerically different (the No Reason argument), and he gives a reason why they must be numerically identical (the Identity argument). The connection between (PII) and (PSR) is thus strengthened by my reconstruction.

Thus there are hermeneutic reasons to recommend my reconstruction of Leibniz's argument. But, as we saw in section 4, there are not only hermeneutic reasons. For my proposed reconstruction of Leibniz's argument nicely meets Cover and Hawthorne's objections to the No Reason argument. Leibniz did not assume the onus of proof was on those who deny (PII). For, apart from arguing there is no reason why indiscernibles *a* and *b* are numerically different, he gives a reason, articulated in the Identity argument, why they must be numerically identical. And he does not assume without argument that there must be a fact of the matter as to whether indiscernible entities are one or two: having given a reason why indiscernibles *a* and *b* must be one, he has shown why there is a fact of the matter as to whether they are one or two.

[8] When I say that truths like *a* is *P* and *b* is *P* reduce to identities I do not mean to say that they can be reduced to explicit identities by means of a finite process of analysis. All I mean is that there is a process of analysis, either finite or infinite, that reduces them to explicit identities.