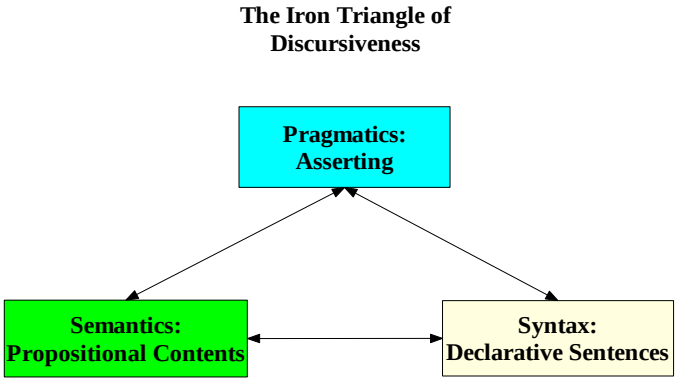
# *Between Saying and Doing: Towards an Analytic Pragmatism*

## Lecture Five: Incompatibility, Modal Semantics, and Intrinsic Logic

### Section 1: Introduction




### Section 2: Incompatibility

Pragmatic definitions of material incompatibility and two kinds of consequence:

- Incompatibility of *p* and *q*: If *S* is committed to *p*, then *S* is not entitled to q.
- Committive consequence: If *S* is committed to *p*, then *S* is committed to q.
- Permissive consequence: If *S* is committed and entitled to *p*, then *S* is (*prima facie*) entitled to q.

"And those who introduce the notion of connexion say that a conditional is sound when the contradictory of its consequent is incompatible with its antecedent." (Sextus Empiricus)

Definition of a third kind of consequence:
*p incompatibility-entails q* just in case everything incompatible with *q* is incompatible with *p*.

### Section 3: Incompatibility Semantics

Semantic suggestion: Represent the propositional content expressed by a sentence by the set of sentences that express propositions incompatible with it. More generally, associate with each *set* of sentences, as its semantic interpretant, the set of *sets* of sentences that are incompatible with it.

The semantically primitive incompatibility relations are *standard* if and only if they satisfy:
1. Symmetry: If S is incompatible with S', then S' is incompatible with S.
2. Persistence: If one set of claims is incompatible with another, so is any larger set containing it. That is, one cannot remove or repair an incompatibility by throwing in some further claims.

Incompatibility semantics is *directly* modal. That is, it does not proceed by defining (non-modal) truth at an index, and then generalizing over indices.

## Section 4: Introducing Logical Operators

Incompatible sentences are Aristotelian contraries. A sentence and its negation are contradictories. The contradictory is the *minimal* contrary: the one that is entailed by all the rest. We already have an entailment relation, defined wholly in terms of incompatibility. So we can introduce a sentence *q* to be the negation of *p* by the condition that *q* is the *minimal* incompatible of *p*: the one entailed by everything else incompatible with it.

Negation so defined has, for all standard incompatibility relations, all the familiar, desirable properties we expect. Because *p* and *Np* are guaranteed to be incompatible, every set of sentences that contains or entails both—what we are now in a position to characterize as the *inconsistent* sets of sentences—is guaranteed to be incoherent. Negation contraposes appropriately with incompatibility entailment. That is, p|=q if and only if Nq|=Np. And every sentence is incompatibility-equivalent to its double negation: p|=NNp and NNp|=p.

A corresponding procedure permits the introduction of *conjunction*. Here the most important fact to acknowledge is that something can be incompatible with a conjunction even though it is not incompatible with either conjunct. What is incompatible with the conjunction *Kpq* should just be whatever is incompatible with the *set* {p,q}.

Conjunction so defined acts like conjunction. It is obvious from the semantic definition that {p,q}|={Kpq}. It follows immediately from persistence of incompatibility that {Kpq}|={p} and {Kpq}|={q}. It is less obvious, but true, that this definition also validates the principle that If p|=q and p|=r, then p|=Kqr: that if p entails q and p entails r, then p entails their conjunction.

Fact: The incompatibility semantics over standard incompatibility relations with these semantic definitions of connectives validates classical propositional logic.

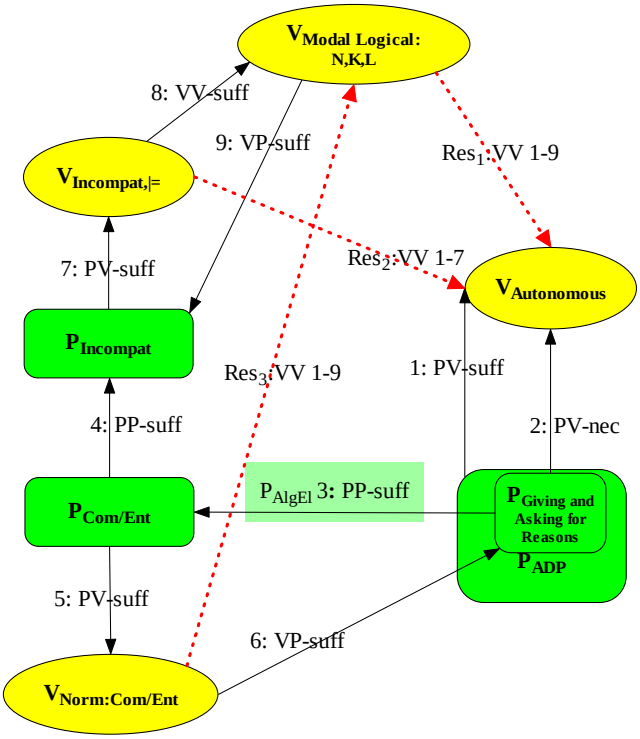
Modality:
On the incompatibility-semantic approach, to introduce a connective, one specifies how to compute its incompatibilities. So the question is: what intuitively should be taken to be *incompatible* with *necessarily p* (Lp)—that is, with the *necessity of p*? Put otherwise, what claims rule out the *necessity* of *p*? Clearly anything incompatible with *p* is incompatible with *necessarily-p* (ensuring that Lp|=p). But what else is incompatible with the *necessity* of *p*, besides the things that are incompatible with *p*? Here is the basic thought:
**To be incompatible with *necessarily-p* is to be** (self-incompatible or) **compatible with something that *does not entail p***.
For anything compatible with something that does not entail *p* is compatible with something that does not necessitate *p*, and so leaves open the possibility that not *p* is not necessary.

A similar line of thought applies to *possibility* in relation to incompatibility, permitting us to introduce *possibly-p* (Mp) as well as *necessarily-p* (Lp). Whatever is incompatible with *possibly-p* should be incompatible with *p* (ensuring that *p* entails *possibly-p*, p|=Mp). But only some things that rule out *p* also rule out the *possibility* of *p*. Which are those? Here is an idea:
**To be incompatible with *possibly-p* is to be incompatible with *everything* that is *compatible* with something *compatible* with *p***.
For anything compatible with something compatible with *p* is compatible with something that leaves the *possibility* of *p* open.

Fact: The incompatibility semantics over standard incompatibility relations and these semantic definitions of connectives validates the modal logic Lewis called ‘S5’.

## Section 5: Meaning-Use Analysis

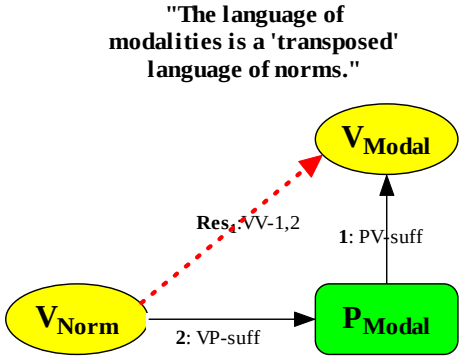
**Incompatibility Semantics for Modal Logic**




Here is some help in reading this meaning-use diagram:

- Basic MURs (1)-(3) are by now familiar. Every autonomous discursive practice must include practices of giving and asking for reasons—as part of the iron triangle of discursiveness, and that that involves distinguishing in practice between the deontic statuses of commitment and entitlement.
- We saw last time that that is sufficient to introduce normative vocabulary, specifically the deontic modal vocabulary of ‘commitment’ and ‘entitlement’, which is VP-sufficient to specify the triadic inferential substructure of practices of giving and asking for reasons. Those facts are represented by MURs (5) and (6).
- We also saw how practically distinguishing commitments and entitlements underwrites a notion of practical incompatibility of commitments, where commitment to one claim is taken or treated as sufficient to rule out entitlement to another. That is MUR (4), which permits the introduction of semantic metavocabulary letting one say that two claims are incompatible, and that claims stand in the relation of incompatibility-entailment, which is MUR (7).
- We have now seen how that semantic metavocabulary allows one to extend the original vocabulary by introducing modal-logical vocabulary (MUR (8)), which has the expressive power to define a connective that says in that object-vocabulary *that* two claims are incompatible: LNKpq. Basic MUR (9) accordingly exhibits modal-logical vocabulary as a kind of *semantic* metavocabulary for incompatibility.

- Complex resultant MUR $Res_1$ analyzes the sense in which the vocabulary of modal logic S5 is *implicit* in the use of any autonomous vocabulary. This analysis is a further cashing-out of what last time I called "The modal Kant-Sellars thesis."
- Complex resultant MUR $Res_2$ codifies an analysis of the possibility of using incompatibility and incompatibility-entailment as a semantic metavocabulary for any autonomous vocabulary. Finally, complex resultant MUR $Res_3$ represents a new relation between the *normative* vocabulary of commitment and entitlement and the *modal* vocabulary of necessity and possibility. It represents a detailed analysis of a sense in which we could understand Sellars's dictum that "the language of modality is a 'transposed' language of norms."




We are now in a position to unpack what in that diagram are represented as two basic MURs. The PV-sufficiency of a set of modal practices for the deployment of a modal vocabulary in this simple diagram corresponds to the complex MUR that is the resultant of basic MURs (7) and (8) in the MUD for modal logic above. The VP-sufficiency of a normative vocabulary to specify those implicitly modal vocabulary-deploying practices now shows up as the complex MUR that is the resultant of relations (3), (4), (5), and (6). In fact we ought to include the other basic MURs that occur in our diagram, and identify the resultant representing the fact that normative vocabulary can serve as a pragmatic metavocabulary for modal vocabulary in the simple MUD with the complex resultant relation (3) in the more complex MUD.

## Section 6: Semantic Holism: Recursive Projectibility without Compositionality

The metavocabulary in which incompatibility semantics is conducted is entirely *extensional*. The semantic interpretants of sentences (and theories) are just sets (of sets of sentences), and the semantic interpretants of logically compound sentences are computed by purely set-theoretic operations on those sets.

The operators defined by the extensional incompatibility semantics are strongly *intensional*, however:

- Something can be incompatible with a conjunction without being incompatible with either of its conjuncts.
- In each incompatibility-interpretation, the semantic value of *not-p* is not determined by the semantic value of *p* alone, but only by it together with the semantic values of a *lot* of other sentences not mentioned in the formula—namely those incompatible with those incompatible with *p*.
- What is incompatible with *possibly-p* is what is *in*compatible with everything compatible with something compatible with *p*. Once again, we can fix the semantic interpretant of *p*, its incompatibility set, and still vary the semantic interpretant of *possibly-p*, by varying the semantic

interpretants of things compatible with what is compatible with *p*. The same phenomenon is exhibited by the incompatibility definition of *necessarily-p*.

Incompatibility semantics is *holistic* in the following precise sense: The classical and modal-logical connectives, as semantically defined by incompatibilities, do not have the *semantic sub-formula property*. That is, it is *not* the case that the semantic interpretants of logical compounds formed by applying those connectives is a function of the semantic interpretants of their components.

Surprising fact: Although incompatibility semantics is *not compositional* it *is* fully *recursive*. The semantic values of logically compound expressions are wholly determined by the semantic values of logically simpler ones. It is holistic, that is, noncompositional, in that the semantic value of a compound is not computable from the semantic values of its components. But this holism *within* each level of constructional complexity is entirely compatible with recursiveness *between* levels. The semantic values of all the logically compound sentences are computable entirely from the semantic values of *less complex* sentences. It is just that one may need to look at the values of *many*—in the limit *all*—the less complex sentences, not just the ones that appear as sub-formulae of the compound whose semantic value is being computed. The semantics is projectible and systematic, in that semantic values are determined for all syntactically admissible compounds, of arbitrary degrees of complexity.

So: What semantic projectibility, systematicity, and learnability-in-principle require is not semantic atomism and compositionality, but semantic recursiveness with respect to complexity.

## Section 7: Consequence-Intrinsic Logic

The order of explanation I have been pursuing up to this point

- starts with practices of giving and asking for reasons,
- argues that they are PP-sufficient for practices of deploying basic normative vocabulary—in particular the deontic modal vocabulary of 'commitment' and 'entitlement',
- uses that as a pragmatic metavocabulary that specifies how to deploy a modal concept of incompatibility,
- uses that as the basic semantic metavocabulary in which to define a *consequence* relation of incompatibility-entailment,
- and on that basis offers semantic definitions of logical vocabulary, including modal operators.

It is possible to exploit the pragmatic and semantic relations appealed to in this approach in service of a different order of explanation, however. In particular, instead of defining a semantic consequence relation in terms of a prior notion of *incompatibility*, we can start with a consequence relation—either a logical consequence relation or a material one that depends on the contents of the non-logical vocabulary articulating its premises and conclusions—and *impute* an incompatibility relation on that basis that will semantically generate just that consequence relation.

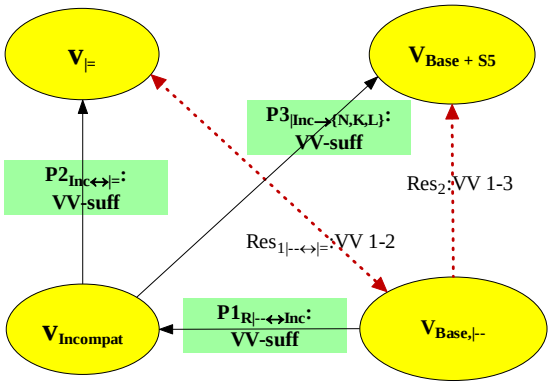
To make this work, we have to ask what conditions a consequence relation defined on an arbitrary set of sentences must meet in order to make it possible to define from it an incompatibility relation such that sets of sentences X and Y stand in the consequence relation (which I'll write 'X|—Y') just in case everything incompatible with Y is incompatible with X (which I will continue to write 'X|=Y'). It turns out that two conditions suffice:

General Transitivity: $\forall X,Y,Z,W \subseteq L[ (X|\!\!-Y \ \& \ Y \cup W|\!\!-Z) \rightarrow X \cup W|\!\!-Z)]$.
Defeasibility: $\forall X,Y \subseteq L[\sim(X|\!\!-Y) \rightarrow \exists Z \subseteq L[[\forall W \subseteq L[Y \cup Z|\!\!-W] \& \exists W \subseteq L[\sim(X \cup Z|\!\!-W)] ]$.

A representation theorem shows that any consequence relation that meets these two conditions—whether it be a material or a logical consequence relation—can be codified by a standard incompatibility relation definable in a natural way from that consequence relation. And we have seen that any standard incompatibility relation has a logic whose non-modal vocabulary behaves classically and whose modal vocabulary is S5, in the sense that the natural semantic definitions of such vocabulary in terms of incompatibility yields that logic. Putting these results together, we can say that in this precise sense, S5 (whose non-modal fragment is just classical logic) is the logic *intrinsic to* standard incompatibility relations, and hence standard consequence relations. But since not only classical logic, but *all* the usual modal logics—not only S5, but K, T, S3, S4, and B, have standard consequence relations, classical logic and S5 are the intrinsic logic of, for instance, S4, as well as the others. The logic that is in this sense *intrinsic* to the consequence relations of *most* other familiar logics is classical S5. S5 accordingly has some claim to being *the* modal logic of consequence relations, whether material or logical.

The concept of the logic that is *intrinsic* to the consequence relation characteristic of some vocabulary presents a new kind of pragmatically mediated semantic relation between vocabularies. It could not have been discovered apart from our considerations concerning normative and modal vocabularies, nor understood apart from the apparatus of meaning-use analysis.

**Consequence-Intrinsic Logic**




I have been concerned to fill in the three sets of practices that implement the basic VV-sufficiency relations of which the relation of *intrinsicness* of a logic to a vocabulary is the resultant:

- imputing a standard incompatibility relation from a standard consequence relation,
- defining incompatibility-entailment in terms of that incompatibility relation, and
- semantically introducing logical vocabulary, including modal vocabulary, in terms of incompatibility.

Having put the technical material behind us, next time I will shift focus by turning attention to what is expressed by *intentional* vocabulary and take up once again the issue of the relations between normative and modal vocabularies—that is, between deontic and alethic modalities—as it bears on the nature of intentionality.