3. Extrapolation and its Limits (5/16/12)

Why should the unobserved part of reality resemble the observed part? Unobserved emeralds could just as easily be blue, as green. This is a puzzle about type 1 or "inductive" extrapolation.

How is the unobserved part of reality even *supposed* to resemble the observed part? Resemblance could just as easily be on the score of grue as green. This is a puzzle about type 2 or "projective" extrapolation.

What does it even *mean* to call unobserved emeralds green? The samples guiding my use were just as much grue, as green. The word could just as easily be true of unexamined *grue* emeralds. This is a puzzle about type 3 or "alethic" extrapolation.

Extrapolation of the 4th kind. You know what *5 o'clock on the sun* means! "Without the intervention of any arcane philosophical scepticism about rule-following, there really is a difficulty about extending the old concept—certain presuppositions of our application of this concept are lacking." Defying presuppositions/implications does seem difficult! And yet.....

"[Suppose] I were to say: "You surely know what 'It is 5 o'clock here' means; so you also know what 'It is 5 o'clock on the sun' means." (PI, 350)

A gratin is a quiche, except it might not be baked in a shell.
Similar figures are congruent, except perhaps not the same size.
Necessity is like duress, absent the requirement of coercive pressure.
They did equally well, apart from some minor addition errors.

Fuhrmann [1996]

How to extend a content to a region of logical space where its presuppositions/implications are false? This is a puzzle about type 4 or "content" extrapolation.

'Wittgenstein seems to mean that, waiving his basic and general sceptical problem, there is a special intuitive problem...illustrated by the 5 o'clock on the sun example" (Kripke [1982])

what	in	from	to	result
regularities	actual emeralds	examined	unexamined	**e** *will* be green, since past emeralds were
similarities	possible emeralds	examined	unexamined	**f** *should* be green, to resemble past emeralds
applicability	possible things	assessed	unassessed	**g** *counts* as green, to go by actual assessments
contents	possible worlds	p-satisfying	p-violating	**w** *satisfies* Grass is green–waiving the grass requirement–to go by macro-worlds

Type-4 extrapolation is in one way easier: The traditional problems are sceptical problems. No one doubts inductive, projective, and alethic extrapolation "work." The question is how. Content extrapolation (bracketing, ...) may or may not work. The problem of saying when and how is substantive, not skeptical.

Type-4 extrapolation is also harder: *Green* does not imply *examined*. No logical obstacle, then, to extending *green* into the unexamined region. *Grass is green* does imply *Grass is colored*, and it assumes *There is grass*. This creates all kinds of obstacles to extending *Grass is green* to worlds where the grass is uncolored, or the green stuff that constitutes grass in macro-worlds does not sum to anything.

SEMANTIC ARITHMETIC "Why should we need a theory of how and when subtraction "works"? It's enough if we can tell in particular cases." We can't. Our judgments stem as much from temperament as features of the case.

Optimists plunge ahead mindless of the difficulties. They act like success is guaranteed. A story they might like.

Wittgenstein making fun of this attitude: "It means simply that it is the same time [on the sun] as it is here when it is 5 o'clock" (PI 350) "The same is the same — how identity is established is a psychological question. (High is high — it is a matter of psychology that one sometimes sees, and sometimes hears it)" (PI 377)

Q to Reagan: [something about de Gaulle]
Reagan: I don't believe I've heard the name.
Q to Mondale: Does it concern you what your opponent doesn't know?
Mondale: It's not that. It's what he does know, that just isn't true.

Mysterians appreciate that success is not guaranteed. Some think failure is guaranteed. Content extrapolation is a leap into the dark with no preordained outcome. A story they will like.

Wittgenstein as mysterian: "If one has to imagine someone else's pain on the model of one's own, this is none too easy a thing to do: I have to imagine pain which I do *not* feel on the model of the pain which I *do* feel" (PI 300).

A: How does the telegraph work? I don't get how words go down wires.
B: A dog stretches from Minsk to Pinsk. Pull the tail, the head barks.
A: OK, but what about the *wireless* telegraph? How does *that* work?
B: The same way, but without the dog.

Trying to raise my arm = raising it, except it might not go up.
A lawlike statement = a law, except it may or may not be true.
Having a tomato-experience = seeing one, except it may not be there.
Prehension is comprehension, except maybe not intellectual.
Quasi-remembering = remembering, but it might not have been me.
A theory is empirically adequate if it is true, ignoring what it says about theoretical entities.
A theory is nominalistically adequate if it is true, ignoring what it says about abstract entities
Solipsistic jealousy is jealousy, minus any implication the target exists (Putnam)
Warrant is whatever "makes the difference between knowledge and mere true belief" (Plantinga)
"A judgment = what is left of a belief after any ... phenomenal quality is subtracted" (Chalmers)
I am responsible for φing if I am to blame for it, bracketing any suggestion that φing is wrong.
To be fragile is to exemplify what breaking adds to being dropped.
A thing is green if it has what looking green adds to being under observation
An act is "courageous" iff it is courageous, minus any suggestion that it is thereby admirable.

"When I raise my arm, my arm goes up. Now the problem arises: what is left over if I subtract the fact that my arm goes up from the fact that I raise my arm?" (Wittgenstein [1953], 621)

"We can investigate the world, and man as a part of it, and find out what cues he could have of what goes on around him. Subtracting his cues from his world view, we get man's net contribution as the difference" (Quine [1960])

"Someone's claim to remember a past event implies that he himself was aware of the event at the time of its occurrence, but the claim to quasi-remember [it] implies only that someone or other was aware of it" (Shoemaker [1970])

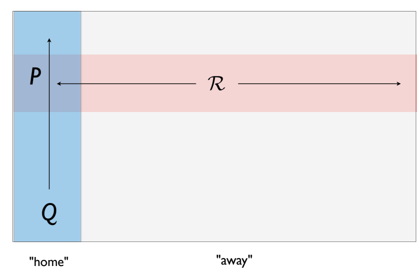
Jaeger: "The question 'What is left over?' presupposes that there is exactly one statement with certain logical properties" "[But] whereas there is exactly one number r such that $r+2=5$, it is not the case that there is exactly one statement R such that 'R & my arm goes up' is logically equivalent to 'I raise my arm'."

Hudson: "[One] candidate for the role of P-Q is the material conditional $Q \supset P$...if there are several propositions whose conjunction with Q is P, then the weakest of these shall be considered the difference between P and Q."

Jaeger [1973], Hudson [1975]

THESIS One feels there *must* be a remainder, but there is really no must about it. The notion of a logical remainder is irremediably unclear.

ANTITHESIS A remainder always exists. It's the weakest R such that $Q\&R$ is equivalent to P—that is, the material conditional $Q \supset P$.



SYNTHESIS $R = P$-Q only if (I) *within* the Q-region, R agrees with P, while (II) *outside* the Q-region, R's behavior is modelled somehow on its behavior within Q.

The truth in THESIS. Doubts about P-Q are doubts about what? The existence of an R that

(i) behaves like P in the Q-region, and
(ii) has its truth controlled by the same factors outside the Q-region as within.

If $R = Q \supset P$ is our best option, we're sunk. $Q \supset P$ does agree with P on Q-worlds. But it does not follow the track laid down by $Q\&P$ vs $Q\&\overline{P}$. Leaving Q, it does not keep to its modest rightward path, but balloons to fill the entire $\neg Q$-region.

$Q \supset P$ is true in too many worlds. It holds in *all* $\neg Q$-worlds, not the smallish strip P draws through the Q-region. Also it's true in too many ways. $Q \supset P$ is true in the Q-region for Q-compatible reasons only. Outside, no. Some ways for $P \supset Q$ to hold involve the failure of Q; these aren't Q-compatible.

The truth in ANTITHESIS: P-Q is to be an R that agrees with P within Q (P iff $Q\&R$), and holds/fails elsewhere for the same reasons it holds/fails within. The claim is that *an R satisfying these conditions is always available.*

Wait....haven't we agreed that subtraction is not always well defined? Two questions are getting confused. Subtraction is well-defined *as a logical operation* if there is always such a proposition as P-Q. It is well-defined if, go to any world you like, the proposition is true or false there. When Q is intuitively unsubtractible from P, P-Q still *exists*,......just don't try evaluating it at (too many) worlds outside the Q-region. Examples later.

REMAINDER-MAKING How to construct a proposition R (strictly, $\mathcal{R}$) for P-Q to express? Let's think first about the *kinds* of condition we'd like R to meet. These divide up as follows, thinking of the Q-region as "home" (because it's the region we're extrapolating from) and the $\overline{Q}$ region as "away."

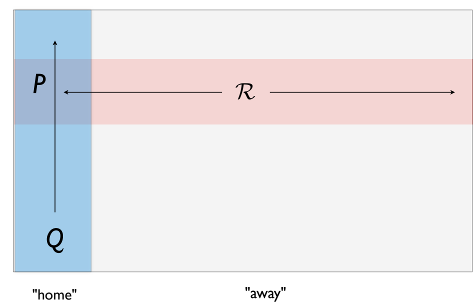
"Home" conditions speak to R's behavior at home, that is, within the Q-region. If R is to count as continuing P beyond that region, then within it, R should follow P's lead. "Away" conditions speak to R's behavior *outside* the Q-region. If R is to divide up away-worlds on the same principle as at home, its away behavior should agree in some still undetermined way with its behavior at home.

"Classifying" conditions have to do with *whether* R is true in a given world. "Rationalizing" conditions are to do with *how* R is true in a world. This gives us four types of extrapolation condition overall:

Why not treat P-Q as the conjunction of those of P's consequences that are free of any taint of Q? P-Q would be false if it had a false Q-free consequence..... This is intuitive but hard to implement; for one thing, "free of any taint of Q" is not obviously closed under conjunction. Better is the *pointwise* approach that tries to determine of each world whether P-Q ought to be true in it, or false, or neither; the proposition is the function mapping worlds to appropriate truth-values.

(HC) home-classifying;
(HR) home-rationalizing;
(AC) away-classifying; and
(AR) away-rationalizing.

The following approach seems logical: (HC), (HR), (AR), then (AC).
First, R should have the same truth-value as P in the Q-region.



Agreement (HC)

R is $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$ at home in the same worlds as P is $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$.

R's reason for being true (false) in a home-world w is whatever makes w a $Q\supset P$-world ($Q\supset\neg P$-world)....whatever makes P true (false) in w, *given that Q is true*.

Another way to think of it. Take a P-world w. It will also be Q, since P implies Q. Ask yourself, why is w on the $Q\&P$ side of the line rather than the $Q\&\overline{P}$ side? *That's* why R is true in w. Why is w a *The # of witches = 0* world rather than one where *The # of witches ≠ 0*? It doesn't have any witches.

Reasons (HR)

R is $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$ at home for the same reasons as P is $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$ *given Q*.

Next we have to specify R's ways of being true/false in away worlds. When does a hypothesis "go on in the same way" from the home-region? When it is true/false in the same ways. R should not acquire new truthmakers (falsemakers) upon leaving home.

Integrity (AR)

R is $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$ for the same reasons away as it was $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$ at home.

Now we have to determine R's truth-value in away-worlds. This is a function, presumably, of the available *reasons* for R to be true/false in such worlds. One can't *quite* say that R is true in an away-world w if a home-style truthmaker for R obtains there. For there could be a home-style falsemaker too (example in margin). But one can say something close. R has "reason just to be true" in w $=_{df}$ home-grown truthmakers obtain in w and home-grown falsemakers do not.

Let P and Q be *Both my children are girls* and *I have exactly two children*. Consider away-world w: I have three children, of which two are girls. R has a truthmaker in w, since *I have exactly two children⊃Both are girls* is true by virtue of my two girls, a fact compatible with my having exactly two children. R also has a falsemaker in w, since *I have exactly two children ⊃ They are not both girls* is true by virtue of my one boy, a fact also compatible with my having exactly two children. Overall: P-Q is true in worlds where I have two or more girls and no boys; gappy if I have two or more girls and some boys; and false if I have one girl and any number of boys.

Projection (AC)

R is $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$ away in the worlds where it has reason just to be $\left\{\begin{matrix}\text{true}\\ \text{false}\end{matrix}\right\}$.

Putting it all together, writing home-grown for Q-compatible,

> P-Q is true (false) in w iff $Q \supset P$ has, and $Q \supset \neg P$ lacks, a home-grown truthmaker in w (vice versa).

Suppose we say that P *adds truth* to Q where $Q \supset P$ has a Q-compatible truthmaker, and that P *adds falsity* where $Q \supset \neg P$ has such a truthmaker. Then it all boils down to this memorable biconditional:

> P-Q is true (false) in w iff P adds just truth (falsity) to Q in w.

Snow is rare and white adds just truth to *Snow is rare*. *Snow is rare and expensive* adds just falsity. Imagine I say, *All five planets are uninhabited*. You deny it. Your statement adds truth to *There are five planets*, because of Earth. It adds falsity, because of any five of the others. The incremental content—what P adds to Q—is thus unevaluable in our world.

Where is it true? in worlds with five or more uninhabited planets and no inhabited ones.

DEGREES OF EXTRICABILITY The truth-value of P-Q is fixed by its components, except where both components are false—the last line of the would-be truth-table. How extricable Q is from P is played out entirely on that last line. Whenever P adds just truth, or just falsity, the higher Q's degree of extricatibility.

Table 1: "TruthTable" for Subtraction

P	Q	P adds truth?	P adds falsity?	P-Q
t	**t**	$Q \supset P$ is true for a Q-friendly reason	$Q \supset \neg P$ is not even true	**t**
t	**f**	IMPOSSIBLE............	P IMPLIES Q.	N/A
f	**t**	$Q \supset P$ is not even true.	$Q \supset \neg P$ is true for a Q-friendly reason.	**f**
f	**f**	....IF P adds just truth, P-Q is		**t**
		....IF P adds just falsity, P-Q is		**f**
		....IF P adds both, or neither, P-Q is		**n**

(1) *Tom is red* is highly inextricable from *Tom is crimson.*
(2) *Numbers exist* is highly extricable from $\#$(*Red stars in the* n^{th} *GY*) $= 2^n$
(3a) *My arm goes up* is partly extricable from *I raised my arm*
(3b) The truth of what is known is partly extricable from *S knows it*

Try to evaluate *Tom is crimson - Tom is red* in a w where Tom is green. *Tom is crimson-if-red* and *Tom is non-crimson-if-red* do have truthmakers in w—the fact that Tom is green—but Tom's greenness is not red-friendly. Is there something *else* about green-Tom—something Tom can keep when he's red—in virtue of which he is crimson-if-red, or non-crimson-if-red? What would this red-friendly feature be? Looks like a case of perfect inextricability.

Wittgenstein in *Remarks on Color* says: *There can be transparent red, but not transparent white*, *A luminous grey is impossible*. Imagine he has discovered that crimson is, of its nature, vibrant and glorious. Perhaps *Tom is crimson* is false due to the lackluster quality of Toms color *whatever it is*. I don't know that it's impossible to develop a system along these lines. But it seems silly. One seeks in vain for a property of green-Tom that red things can possess, but not unless they are crimson.

How far can *#s exist* be extricated from the conjecture that $\#(n)$—the number of red stars n galactic years after the big bang—is 2^n? For perfect extricability, $\#(n) = 2^n$ should add just truth to *#s exist*, or just falsity, in every numberless world. Does it? Either we have one red star to begin with and doubling every galactic year, or we don't. The first is a number-friendly truthmaker for *#s exist*$\supset \forall n(\#(n) = 2^n)$. Other worlds have number-friendly truthmakers for *#s exist*$\supset \neg\forall n(\#(n) = 2^n)$, like the stars tripling each galactic year.

Between lies a vast region of imperfect extricabiliity. *I raise my arm* adds just falsity in worlds where I'm unconscious, or committed at every level to keeping it down. For it to add just truth, $Up \supset Raised$ would need an *Up*-friendly truthmaker in some arm-down world. What would it be? *Trying* to raise my arm is up-friendly, but not sufficient for raising it if it goes up. Trying *effectively* is sufficient but does not obtain in home-worlds = worlds where the arm stays down.

The kind of knowledge attributed to Reagan: *S knows that P - P*. A knowledge claim adds just falsity to its complement, maybe, if *S* is too young, or confused, or strongly opinionated the other way. For truth, we'd need that the *one and only obstacle to S's knowing something* is that the something is false.

Extricability issues arise already in propositional logic, with minimal models (countermodels) playing the role of truthmakers (falsemakers). Notation: $X \wedge \partial Y$ is undefined if Y is false, and is otherwise equivalent to X.

X	Y	X-Y
$p\&q$	q	p
p	$p \vee q$	$p \ \& \ \partial(p \vee q)$
$p \leftrightarrow q$	$p \supset q$	$q \supset p$
$p \dot{\vee} q$	$p \vee q$	$\neg(p\&q)$
$p\&q$	$p \leftrightarrow q$	$(p \vee q) \ \& \ \partial(p \leftrightarrow q)$
$(p\&q) \vee r$	$p \vee r$	$q \vee r$

This is hard to arrange! Imagine we get fabulous, overwhelming evidence of an afterlife, and believe accordingly. Somehow, flukily, incomprehensibly, it's not true. *I know I'll survive* might then *appear* to add truth to *I'll survive*. Had there been life beyond death, I could have honestly said I knew in advance.

SUBTRACTION AS A PHILOSOPHICAL TOOL In philosophical analysis, we tend to approach the target from below, by conjoining weaker conditions. Why not "analysis from above," in which we overshoot the target and then backtrack? There have been some real failures here: narrow content, e.g. But how else are we to understand lawlikeness, quasi-memory, necessity as opposed to duress, or scare quotes uses of moral terms? (Or prehension?)

Some existence questions are hard, or harder, to take seriously. Skeptics will say: one can "divide through" by the objects and be left with essentially the same claim. This dividing-through metaphor is not easy to cash out. Logicall subtraction is perhaps better conceived as division; it undoes conjunction which is akin multiplication. One can divide through by the Xs, if the hypothesis of their existence is perfectly extricable from the hypotheses that rely on it. Plausible or not I don't know, but this is a reading that's at least *available*.

"An obstacle to the development of an adequate philosophy of mathematics is our limited understanding of... how the truth of a mathematical statement may be ensured by the concepts employed rather than by the objects described. Whether with a more developed understanding of these notions we can, to use a Wittgensteinian phrase, "divide through" by the objects of mathematics, be they abstract objects or mental constructs, is a question that remains open" (Lear [1977]).

"When you have taken full account [of our notion of Napoleon, still] you have not touched the actual man; but in the case of Hamlet, you have come to the end of him. If no one thought about Hamlet, there would be nothing left of him; if no one had thought about Napoleon, he would have soon seen to it that some one did" (Russell [1919])

Subtraction has been presented as a way of *cancelling* the subtrahend's content, as opposed to *negating* its content. But negation is itself sometimes seen as a cancellation device.

> A man who contradicts himself may have succeeded in exercising his vocal chords. But from the point of view of imparting information, or communicating facts (or falsehoods), it is as if he had never opened his mouth Contradiction is like writing something down and erasing it, or putting a line through it. A contradiction cancels itself and leaves nothing. (Strawson [1952])

He would have us think of "p as running up a flag, $\neg p$ as running it down again; p as writing something on a board, $\neg p$ as rubbing it out again, or putting a line through it, cancelling it out; p as recording a message, $\neg p$ as erasing it; p as stating something, $\neg p$ as withdrawing it."

Is it just me, or is Strawson wrong about this? Even if we grant that $\neg A$ erases the earlier assertion of A, why think that A returns the favor, erasing the later assertion of $\neg A$? This raises an interesting question. What *does* one say to wipe the slate clean, after making an assertion one then thinks better of? What goes in for X in the update rule

(i) $A + X =$ nothing asserted?

$\neg A$ is too strong; it leaves us with something still asserted, viz. that it is not the case that A. To cancel A cleanly, one says, *hold on, it might be that* $\neg A$. Putting $\Diamond \neg A$ in for X in (i) gives us

(ii) $A + \Diamond \neg A =$ nothing asserted.

We know of one *other* operation that returns us from A to the nothing-asserted state, viz. the operation of *subtracting* A.

(iii) A minus $A =$ nothing asserted.

(ii) and (iii) suggest a hypothesis about what is accomplished by adding a might-statement to the conversational record:

(iv) adding $\Diamond\neg A$ = subtracting A; adding $\Diamond A$ = subtracting $\neg A$.

This is just the shell of a theory, but one worth exploring, because of the help it gives with two puzzles.

As epistemic modals are usually understood, "Bob might be in his office" says that my information (or certain information) is consistent with his being there. I said this gets the subject matter wrong. But how are we going to get the claim to be about Bob and his office? Negating A doesn't change its subject matter; and disavowing something, as opposed to asserting it, doesn't either. Attaching "might" is running those two changes in sequence; it's disavowing $\neg A$.

Now a puzzle due to Seth Yalcin (Yalcin [2007]), though something in the same ballpark is noted by Hempel. The following argument is invalid: $\Diamond\neg A$, so $\neg A$. An argument is invalid only if the conclusion fails in a scenario where the premise holds—a scenario, in this case, where A holds and $\Diamond\neg A$ also holds. A scenario like that makes no sense, one may feel, not even hypothetically.

"[It's been suggested that] the statement 'it is not raining' implies 'it is not the case that it is probably raining.' But then, by contraposition 'it is probably raining' would imply 'it is raining'. And while this construal would give a strong empirical content to sentences of the form 'probably, p', it is of course quite unacceptable" (Hempel [1960]).

Suppose might is a cancellation device. Why is $\Diamond\neg A$, *therefore* $\neg A$ invalid? It's not that the truth of $\Diamond\neg A$ falls short of establishing $\neg A$. $\Diamond\neg A$ disavows A; disavowing A doesn't commit one to $\neg A$. Similarly if A & $\Diamond\neg A$ is used as a supposition. The instructions it gives to would-be supposers are self-contradictory: we are to suppose that A, while at the same time not supposing that A.

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