Lecture 2. What is the Normative Role of Logic?

What is the connection between (deductive) logic and rationality?

One extreme: Frege. A law of logic is a law of rational thought.

Seems problematic, if rational change of logic is possible.

> It seems to say that in a debate over logic, the party who advocates the incorrect logic is automatically irrational.

It seems that the connection between logic and rationality must be subtler.

The other extreme: Harman. Logic has no more of a connection to rationality than any other important discipline does.

> On this view, logic is a science on par with all others: its goal is to discover a certain kind of truth, viz., truths about what forms of argument must preserve truth.
>
> Rational people will try to have the right views about this, but they'll try to have the right views about physics and sociology too.
>
> So the tie between logic and rationality is no closer than the tie between physics or sociology and rationality.

This does have the advantage of not ruling out rational change in logic.

> > We can have a rational change of logic whenever we have a rational change in our beliefs about what forms of argument must preserve truth;
> >
> > No obvious reason to doubt that these beliefs can rationally change, any more than that our beliefs about physics can rationally change.

But aside from that, is there reason to believe the Harman view?

Harman's argument is that there are a large number of obstacles to finding a believable connection between logic and rational belief.

In the first part of today's lecture, I'll argue that these obstacles can be overcome.

In the second part of the lecture, I'll argue that logic is *not* the science of what forms of inference necessarily preserve truth: or even, the science of what forms of inference preserve truth *by logical necessity*.

This will make it hard to see what the subject of logic could possibly be, if it isn't somehow connected to laws of rational thought.

So we'll be driven back to something like the Fregean view.

But I'll suggest a slight modification that does not obviously preclude rational debate about logic.

PART ONE

Harman cites four problems for a connection between logic and rational belief. (I'm not sure how seriously he takes them all.)

1. Reasoning doesn't follow the pattern of logical inference. When one has beliefs $A_1, ..., A_n$, and realizes that they together entail B, sometimes the best thing to do isn't to believe B but to drop one of the beliefs $A_1, ..., A_n$.
2. We shouldn't clutter up our minds with irrelevancies. But we'd have to if whenever we believed A we believed all its consequences.
3. It's sometimes rational to have beliefs even while knowing they are inconsistent, if one doesn't know how the inconsistency should be avoided.
4. No one can recognize all the consequences of his or her beliefs.

    Because of this, it is absurd to demand that one's beliefs be closed under consequence.

    And for similar reasons, it is absurd to demand that one's beliefs be consistent.

I take 3. and 4. to be of most interest, but the solutions to 1. and 2. will affect them.

Problem 1: "When one has beliefs A_1, ..., A_n, and realizes that they together entail B, sometimes the best thing to do isn't to believe B but to drop one of the beliefs A_1, ..., A_n."

This shows that the following is not a correct principle:

> If one realizes that A_1, ..., A_n together entail B, then if one believes A_1, ..., A_n, one ought to believe B.

But the obvious solution is to give the 'ought' wider scope:

(*) If one realizes that A_1, ..., A_n together entail B, then one ought to see to it that if one believes A_1, ..., A_n one believes B.

Problem 4a: should one strengthen this, by weakening the antecedent from

> "If *one realizes that* A_1, ..., A_n together entail B"

to just

> "If A_1, ..., A_n together entail B"?

(*) If [one realizes that?] $A_1, ..., A_n$ together entail B, then one ought to see to it that if one believes $A_1, ..., A_n$ then one believes B.

This would show a lot more of a bearing of logic on rational belief in the strengthened form in which the 'one realizes that' is dropped.

John MacFarlane:

> [if the only normative claims that logic imposes are from *known* implications, then] the more ignorant we are of what follows logically from what, the freer we are to believe whatever we please—however logically incoherent it is.
>
> But this looks backward. We seek logical knowledge so that we know how we ought to revise our beliefs: not just how we *will* be obligated to revise them when we acquire this logical knowledge, but how we are obligated to revise them even now, in our state of ignorance.

So the strengthened form is desirable, but can we have it?

The obvious problem: Believing all the logical consequences of ones beliefs is simply not humanly possible, so failure to do so can hardly be declared irrational.

For similar reasons, the idea that it is always irrational to be inconsistent seems absurd.

> I think *any* rational person would have believed it impossible to construct a continuous function mapping the unit interval onto the unit square, until Peano came up with a remarkable demonstration of how to do it.
>
> The belief that no such function could exist (combined with certain set-theoretic beliefs) was eminently rational, but inconsistent.

Interim solution: Take the principle to be

> If $A_1, ..., A_n$ together *obviously* entail B, then one ought to see to it that if one believes $A_1, ..., A_n$ then one believes B.

Problem 3: "It's sometimes rational to have beliefs even while knowing they are inconsistent, if one doesn't know how the inconsistency should be avoided."

A famous example: the Paradox of the Preface.

> One says in the preface that probably one has made an error somewhere in the book, even though this amounts to the disjunction of negations of claims in the book.

More interesting examples: sometimes good physical theories (classical electrodynamics) have absurd consequences, and one doesn't know how to fix them.

For each claim in them, one thinks its probably right.

This seems a rational attitude, and is licensed by Bayesian views: one can have a high degree of belief in each of A_1 through A_n, but not in their conjunction or in some other claims entailed by their conjunction.

> (Take belief to be just degree of belief over some high contextually determined threshold.)

So while examples like this do create a problem for (*), it seems at first blush obvious how to fix it:

Replace ‘if one believes A_1, ..., A_n’ by ‘if one believes $A_1 \wedge ... \wedge A_n$’:

(W) If A_1, ..., A_n together obviously entail B, then one ought to see to it that if one believes $A_1 \wedge ... \wedge A_n$ then one believes B.

Or slightly more generally:

(W^+) If A_1, ..., A_n together obviously entail B, then one’s degree of belief in B should be at least as high as one’s degree of belief in $A_1 \wedge ... \wedge A_n$.

But even in the stronger form (W^+), this is excessively weak, for two reasons.

First: The force of $\wedge$-Introduction on degrees of belief is completely lost.

$\wedge$-Introduction should be a substantive constraint on our degrees of belief:

> if one believes A_1 to degree 1 and A_2 to degree 1, one ought to believe $A_1 \wedge A_2$ to degree 1;
>
> and if one believes A_1 to degree 0.95 and A_2 to degree 0.95, one ought to believe $A_1 \wedge A_2$ to degree at least 0.9.

But (W^+) tells us only that the degree of belief in $A_1 \wedge A_2$ should be at least as high as itself!

Second problem: People don't have degrees of belief for everything. A principle governing a person's degrees of belief ought to be understood as having the tacit assumption that the person has all the degrees of belief in question.

For instance, a person can have high degrees of belief in A and in $A \supset B$, but *have no degree of belief at all* in their conjunction.

But in that case, (W) and (W^+) allow very high degrees of belief in A and $A \supset B$ while at the same time having extremely low degree of belief in B.

We can handle both problems simultaneously as follows:

(D) If $A_1, ..., A_n$ together obviously entail B, then one ought to see to it that P(B) (one's degree of belief in B) is at least $P(A_1) + ... + P(A_n) - (n-1)$.

● The n=1 case just says that if A obviously entails B, one's degree of belief in B should be at least that of A.

● The n=0 case just says that if B is an obvious logical truth, P(B) should be 1.

(D) seems the proper generalization. Gives e.g. that if $A_1, A_2 \vDash B$, $P(B) \geq P(A_1) + P(A_2) - 1$.

> (Shouldn't we get $P(B) \geq P(A_1 \wedge A_2)$? No problem: if the logic includes $\wedge$-elim, we'll also have $A_1 \wedge A_2 \vDash B$, so we do get this when the agent has the degree of belief.)

Note: This principle

(D) If $A_1, ..., A_n$ together obviously entail B, then one ought to see to it that P(B) (one's degree of belief in B) is at least $P(A_1) + ... + P(A_n) - (n-1)$.

is quite neutral to the underlying logic (and thus to the full principles of Bayesianism, which require that the underlying logic be classical).

It says that whatever logic is assumed correct,

(i) if B is (known to be) entailed by A in that logic, a proponent of that logic should believe B to at least as high degree as A;

(ii) if B is (known to be) a theorem of the logic, it should be believed to degree 1;

and so forth.

Some other features of degrees of belief in Bayesian theories fall out of Principle (D) *together with the assumption of classical logic*.

E.g. in classical logic or any other logic in which contradictions entail everything, (D) yields that $P(A) + P(\neg A)$ can never be more than 1.

(Principle (D) still applies to probability theories that allow $P(A) + P(\neg A)$ to be less than 1, or greater than 1 when the logic is paraconsistent.)

A stronger generalization than (D) is available in standard Bayesian theories. In the n=2 case it is:

(D^+) If A_1 and A_2 together obviously entail B, then one ought to see to it that P(B) is at least $P(A_1) + P(A_2) - P(A_1 \vee A_2)$;

a tighter bound than the $P(A_1) + P(A_2) - 1$ that is delivered by (D).

But this tighter bound is a special feature of Bayesian theories. (For instance, it doesn't hold in the Dempster-Shafer theory.) It isn't simply due to the impact of logical implication on our degrees of belief.

Moreover, the point about people not having degrees of belief in every proposition shows that (D) gives information in cases where (D^+) doesn't:

> The n=2 case of (D) has the tacit condition that the person has degrees of belief in A_1, A_2 and B.
>
> But (D^+) has the tacit condition that the person has degrees of belief not only in these but also in $A_1 \vee A_2$.
>
> Since one can have degrees of belief in the former without having them in the latter, (D^+) doesn't give the info of (D).

Problem 2: “Even though ‘The earth is round’ entails ‘Either the earth is round or there are now Martian elephants in the Oxford St. tube station’, it would be a bad thing to clutter up one’s brain with such irrelevancies.”

The obvious solution, as Harman himself notes, is to distinguish *explicit belief* from *implicit belief*.

Explicit beliefs are ones that are directly stored; one implicitly believes something when one is disposed to explicitly believe it should the question arise.

So we should change (*) to something like

(**) If A_1, ..., A_n together obviously entail B, then one ought to see to it that if one *explicitly* believes A_1, ..., A_n then one *at least implicitly* believes B.

But how does one fit this with a degree of belief model, used in (D)?

I've mentioned the idea of generalizing standard Bayesian theories, so that an agent needn't have a degree of belief in every sentence of her language.

An obvious addition is to make an explicit/implicit distinction among one's actual degrees of belief: explicit degrees of belief are ones represented explicitly in the agent; an implicit degree of belief is a disposition to have that degree of belief explicitly.

It turns out (for reasons that will become apparent) that the notion of implicit degree of belief is not general enough for our needs. Our principle

(D) If $A_1, ..., A_n$ together obviously entail B, then one ought to see to it that P(B) (one's degree of belief in B) is at least $P(A_1) + ... + P(A_n) - (n-1)$.

cannot be suitably generalized using it alone.

But whatever problem there is here will go away, once I've addressed another aspect of Problem 4.

Problem 4b: While the most overt problems of excessive computational demands have been avoided by the use of '*obvious* entailment' in our principles, some have been re-introduced by talk of degree of belief.

Standard discussions of degrees of belief totally ignore computational limitations.

A minimal computational limitation is Turing-computability: but no Bayesian probability function on a rich language is computable, at least if it satisfies a very minimal condition of adequacy.

> Every Bayesian probability function must assign the value 1 to every logical truth. By Church's theorem (undecidability of classical logic), this means any computable probability function would have to assign value 1 to things other than logical truths as well.
>
> One could live with that. But it is easy to extend the proof of Church's theorem, to show that any computable function on an arithmetic language that assigns value 1 to all logical truths must also assign value 1 to something inconsistent with Robinson arithmetic (a *very* weak arithmetic theory).
>
> So any computable probability function would have to assign probability 0 to a very weak fragment of arithmetic!

Rather than giving up on computability, I think the proper conclusion is that (except in highly idealized contexts) *(even if we are Bayesians) we shouldn't focus on probability functions*.

Instead we should focus on certain *probabilistic constraints*: constraints such as that the conditional degree of belief in A given B is no greater than that of C given D.

It is constraints such as these that we explicitly represent.

These constraints evolve, both by something akin to the Bayesian process of conditionalization, and also *by thinking*.

The process of thinking can impose new explicit constraints, e.g. a new theorem will henceforth be explicitly constrained to get value 1.

Before, it may have been constrained by logic to get value 1, but only by a very unobvious proof. So the agent may not even implicitly believe the theorem to high degree. In fact, he may implicitly or explicitly believe the negation to a high degree (Peano curve).

There will also be cases where obvious constraints of logic give implicit constraints on degrees of belief, but no explicit constraints. These constraints needn't determine an implicit degree of belief, they can be much weaker.

So the key distinction is explicit *v.* implicit *constraints on* degrees of belief.

The natural idea for handling the computational aspect of Problem 4, while keeping the solutions to the others:

(D*) If A_1, ..., A_n together obviously entail B, then one ought to see to it that if one's explicit constraints obviously entail lower bounds of at least p_1, ..., p_n on A_1, ... A_n respectively, then one will impose a lower bound of at least $\Sigma p_i - (n-1)$ on B should the question arise.

Much more would need to be done to turn my remarks on probabilistic constraints into a serious theory; what I've said doesn't go much beyond common sense.

But my goal wasn't to deliver a theory, but to say why I think there's no problem in supposing that logic imposes a rationality constraint on our degrees of belief.

The story I've told avoids the excessive demands of logical closure.

It also avoids excessive demands of logical consistency.

The constraints may be probabilistically inconsistent; an account of the updating procedure should be such that when inconsistency is discovered, adjustments are made to try to eliminate it.

Moreover, the story avoids these excessive demands without confining the normative requirements to cases where the logical relations are *known by the agent*.

The requirements are there whenever the entailments are obvious, even if the agent doesn't know them.

What counts as obvious? In my view, there's no general answer: it depends on both who is being assessed and who is doing the assessing. But this is no problem for using the notion in describing normative requirements, for normative requirements are relative in both these ways. (More on this in Lecture 5.)

Instead of using the notion of obviousness, we could list specific rules that we count as obvious and insist that they impose obligations. E.g.:

(D^*_{alt}) If B follows from $A_1, ..., A_n$ *by such and such a simple rule*, then one ought to see to it that:

> if one's explicit constraints directly entail (by such and such other rules) lower bounds of at least $p_1, ..., p_n$ on $A_1, ... A_n$ respectively, then one will impose a lower bound of at least $\Sigma p_i - (n-1)$ on B should the question arise.

This would give the veneer of objectivity (for better or worse).

This imposes obligations only for simple inferences.

Even if complicated inferences can be obtained by putting together simple inferences, there's no obligation to have ones beliefs accord with the complex inference. There's only

> an obligation to take the first step,
>
> a potential obligation to take the step after that once one has fulfilled that obligation,
>
> a still more potential obligation to take the third step,

and so forth.

For long complicated proofs, we have *at most* a long chain of potential obligations; this is far short of an obligation to believe the conclusion if one believes the premises.

MOREOVER: For typical proofs, we don't even have a long chain of potential obligations.

For there is a distinction to be made between two kinds of obvious inference.

(i) In some, like the inference from $\forall xA(x)$ to $A(t)$ for specific A and t,

> the inference is totally obvious, but nonetheless
>
> *ex*plicit belief in the conclusion based on explicit belief in the premise is atypical because one needs the specific t to be brought to one's attention.

Famous proofs like Russell's disproof of naive comprehension remain unobvious for so long, even though the derivation involved there is so quick, because of this.

So if one is going to maintain a believable form of (D^*_{alt}), one must exclude such rules from its antecedent (or restrict the antecedent to certain instances of the rule, intuitively the obvious ones).

(ii) In the case of other inferences like that from $A \wedge B$ to A, it's hard not to explicitly think of the conclusion when one thinks of the antecedent.

In these cases, when one has an explicit (constraint on) degree of belief in the premise *and also attends to it*, it's very likely that one will have an *explicit* constraint on ones degree of belief in the conclusion. (The distinction here is probably one of degree.)

Perhaps there are hard-to-see proofs that use only premises and rules of inference of the second sort, where the conclusion suggests itself to us.

If so, then in *those* cases it's mere length of proof that makes the proofs unobvious.

But that is certainly atypical of hard proofs. In the case of hard proofs, then, there doesn't seem to be even the long chain of potential obligations contemplated before.

This seems to me to fully handle the problem of computational limitations.

Problem 4c:

Should the facts of logical implication impose an obligation on those who don't accept the logic? Especially those who have serious (even though not ultimately correct) reasons for not accepting it?

On a natural interpretation of (D^*_{alt}), the obligations come from the simple rules *of the correct logic*. That dictates the answer 'yes'. But there is a case to be made against this:

> Suppose that classical logic is in fact correct, but that X has made a very substantial case for weakening it. (Indeed, suppose that no advocate of classical logic has yet given an adequately answer.)
>
> Suppose that usually X reasons in accordance with the non-classical logic he advocates, but that occasionally he slips into classical reasoning that is not licensed by his own theory.
>
> Isn't it *when he slips and reasons classically* that he is violating rational norms?

But (D^*_{alt}) (on the interpretation suggested) gives the opposite verdict.

One response: Switch to another interpretation of (D^*_{alt}), according to which it's the simple rules of the agent's logic (or alternatively, of the logic that the agent has most reason to accept), that provides the obligations.

This relativist reponse removes the normative pull of reasoning in accord with *the correct* logic, when that logic is at odds with the logic that one accepts or that one has most reason to accept.

A good response? The MacFarlane quote suggests a discomfort. Paraphrasing slightly:

> "This looks backward. We seek logical knowledge so that we know how we ought to revise our beliefs: not just how we *will* be obligated to revise them when we have the correct logical theory, but how we are obligated to revise them even now, in our state of logical error."

Second response: (MacFarlane). There is an obligation to reason in accordance with the correct logic, but there can also be competing obligations.

In the case of those with serious reasons for doubting what is in fact the correct logic, these competing obligations are quite strong. So

> There is simply no way to satisfy all of one's obligations until one corrects one's mistaken views about the logic.

This idea of considering competing obligations rather than an undifferentiated notion of *overall obligation to believe* or of *rational belief* seems to allow a basically Fregean view that closely connects *one facet of* rationality to logic.

> (The view says little about the facet of rationality that gives the contrary obligations.)

My view: each of these two quite different responses contains a considerable element of truth.

This is, to some extent, predicted by my preferred version of the normative constraint, which began

(D*) If A_1, ..., A_n together *obviously* entail B, then one ought to see to it that ....

On one reading of 'obviously' (obviously *to the agent*) we would get a response like the first, and on another (obviously *to someone with the correct logic*) we would get a response like the second.

Really it isn't an ambiguity in 'obvious'. Rather, 'obvious' is normative. We evaluate primarily via the evaluator's norms, but sometimes via the agent's. So (D*) yields both interpretations, without an ambiguity.

My discussion may seem to have presupposed normative realism. ("What are the objective normative constraints that logic imposes?")

But there's an alternative that I prefer. Its core is

(1) The way to characterize *what it is for a person to employ a logic* is in terms of *norms the person follows*: norms that govern the person's degrees of belief by directing that those degrees belief accord with the rules licensed by that logic.

More specifically, we recast (D^*_{alt}) into something much less normative, as follows:

(E) Adhering to a logic L involves trying to bring it about, for the simple inferences $A_1, ..., A_n \vdash B$ licensed by the logic, that if one's explicit constraints entail lower bounds of at least $p_1, ..., p_n$ on $A_1, ... A_n$ respectively, then one will impose a lower bound of at least $\Sigma p_i - (n-1)$ on B should the question arise.

We get a certain kind of normativity derivatively, by the following obvious principle:

(2) In externally evaluating someone's beliefs and inferences, we go not just by what norms the person follows, but also by what norms *we take to be good ones*: we will use *our* logic in one facet of the evaluation, though we may use the agent's logic in another.

(2) doesn't connect up *actual oughts* with *the actually correct logic*, but connects *ought judgements* with *what we take to be good logic*.

But my suggestion would be that there are no "actual oughts" that this leaves out: normative language is to be construed expressivistically. (More on this in Lec. 5.)

So construed, a normative principle like (D*) will turn out to be correct, but will be seen as something like an epiphenomenon of (E) together with the evaluative practices alluded to in (2).

These evaluative practices allow the consideration of both our own logic and the other person's in evaluating the other person's beliefs—the best resolution of Problem 4c.

PART TWO

Harman proposed an alternative to the idea that logic has a normative role:

> Logic is the science of what forms of argument necessarily preserve truth.

I'll argue against any such alternative characterization.

This should substantially increase the plausibility of the idea that logic is to be characterized in part by its normative role. (What else would be left?)

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My claim: **We must reject the claim that all logically valid inferences preserve truth.** (To be *slightly* qualified.)

Motivation: Gödel's second incompleteness theorem.

Says that no remotely adequate mathematical theory can prove its own consistency [or even, its own non-triviality].

This can seem puzzling: why can't we prove the consistency [non-triviality] of a mathematical theory *T* within *T* by

(A) inductively proving within *T* that *T* is sound, i.e. that all its theorems are true,

and

(B) arguing from the soundness of *T* to the claim that *T* is consistent [non-trivial]?

The problem will lie in (A) (except in the case of quite uninteresting theories of truth).

But why can't we argue

(Ai) that all the axioms are true;

(Aii) that all the rules of inference preserve truth;

and conclude by induction that all the theorems are true?

[Or a variant, using satisfaction.]

In standard mathematical theories, with only defined truth predicates, the resolution of this is clear: you can't define a general truth predicate. So you can't even *formulate* (Ai) and (Aii), let alone *prove* them.

But this precludes identifying the valid inferences with the necessarily truth-preserving ones: that would require such a general notion of truth.

Tarski overcame this by identifying the valid inferences with those that preserve truth *in all classical models.*

Truth in a classical model *is* definable, since it is very different from truth; but *those very differences* mean that his account of validity doesn't have the philosophical punch that necessary truth-preservation would have.

(i) Non-classical logicians agree that classical inferences preserve *truth in classical models*; but they don't agree that they preserve *truth*. They think that classical models misrepresent reality.

(ii) Even classical logicians think that classical models misrepresent reality: classical models have domains restricted in size, whereas set-theoretic reality doesn't. (That's why truth-in-a-model can be defined when truth can't be.)

The problem of having no general truth predicate comes from insisting on *defining* truth.

But what if we introduce a general truth predicate as a primitive?

In this case, the paradoxes mean we have a choice:

(i) a classical-logic theory in which truth obeys unusual laws,

(ii) a theory with a non-classical logic that keeps the usual laws of truth (e.g. True(<A>) equivalent to A).

But in every such theory of any interest, it is either

impossible to argue that all the axioms are true

or **impossible to argue that all the rules preserve truth**.

[And the impossibility isn't due to the generality.]

Example: classical "truth value gap" theories. Typically they

Include every sentence of form "True(<A>) → A" as an axiom;

Include some sentences of form "¬True[<True(<A>) → A>]" as a theorem.

Belief in the axiom is licensed—the axiom is taken to be valid, in the normative sense—but it is declared untrue!

One could simply *define* ‘valid’ to mean ‘necessarily truth-preserving’ (or in the case of axioms, ‘necessarily true’). Doesn’t get around main point: these theories give a special positive status to all claims of form “True(<A>) → A”, but declares some of them untrue.

I regard this as a serious defect: such theories seem somehow “self-undermining”.

Most other theories with a general truth predicate

imply the truth of all their own axioms,

but *not* the truth-preservingness of their own rules.

Indeed, they employ certain rules (say modus ponens, or the inference from True(<A>) to A) while *rejecting* the claim that those rules generally preserve truth.

Is this as counterintuitive as rejecting the truth of some of one's axioms? I don't think so. Reason: With most such theories, there's no reason to doubt that all the rules preserve truth *when it matters*.

The rejection of the claim that (say) modus ponens *preserves truth generally* arises because of a rejection of the claim that it *preserves truth when applied to certain pathological premises*.

But these premises are ones that the theory won't accept, if it is consistent. So the rejection of the claim that the rule preserves truth *generally* doesn't seem to undermine the use of the rule.

Still: **legitimately using the rule is compatible with *rejecting* the claim that the rule generally preserves truth.**

Again, issue isn't how to define 'valid': one must *either* say that **validity doesn't require truth-preservation** *or* that **it's legitimate to employ deductive rules that one doesn't think are valid**.

Perhaps we should redefine validity, not as (necessarily) preserving truth in general but as (necessarily) doing so when it matters?

> This would require careful definition, and there are different ways in which this might be done.
>
> But however it is done, it won't help: basically, a theory is still not going to be able to *prove* that it's own rules preserve truth when it matters, because if it could, it could prove its own soundness.

If validity isn't defined in terms of necessary truth-preservation (whether general or restricted), how is it to be understood?

I propose that we take it as a primitive notion that governs our inferential or epistemic practices. (That was the suggestion earlier in the lecture: e.g., when we discover that the inference from *p* and *q* to *r* is valid, then we should ensure that our degree of belief in *r* is no lower than the sum of our degrees of belief in *p* and in *q*, minus 1.)

From this viewpoint, we can easily explain why it's *natural* to think that validity coincides with necessary truth preservation.

Consider four claims:

(1) The inference from $p_1,...,p_n$ to q is valid

(2) The inference from $True(<p_1>)$, $True(<p_n>)$ to $True(<q>)$ is valid.

(3) The inference from $True(<p_1>)$ *and ... and* $True(<p_n>)$ to $True(<q>)$ is valid.

(4) The sentence *If* $True(<p_1>)$ *and ... and* $True(<p_n>)$, *then* $True(<q>)$ is valid.

These seem equivalent: (1) to (2) by the usual truth rules, (2) to (3) by the usual rules for conjunction, (3) to (4) by the usual rules for the conditional. But the validity of a sentence is necessary truth (by virtue of form), so (4) says that the inference necessarily preserves truth (by virtue of form).

This argument looks very persuasive. But it turns on principles that can't be jointly accepted! (The Curry paradox, which I may discuss next time.)

In particular, the paradox shows that we can't subscribe *both* to the truth rules employed in rewriting (1) as (2) *and* to the rules for the conditional employed in rewriting (3) as (4).

There are different views on how the Curry paradox is to be resolved. But every one of them undermines this *argument* that validity is to be identified with necessary truth preservation.

And in a sense, they undermine the identification itself.

One can stipulate that ‘valid’ is to mean ‘necessarily preserves truth’.

But *that* notion of validity isn’t what underwrites our notion of goodness in deductive argument—validity in that sense isn’t even extensionally equivalent to goodness of deductive argument.

Our notion of good argument is an essentially normative notion, not capturable even extensionally in terms of truth preservation. In this sense, logic is essentially normative.