Dummett on the Relation between Logics and Metalogics

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Abstract

The paper takes issue with a claim by Dummett that, in order to aid understanding between proponents and opponents of logical principles, a semantic theory should make the logic of the object-language maximally insensitive to the logic of the metalanguage. The general advantages of something closer to a homophonic semantic theory are sketched. A case study is then made of modal logic, with special reference to disputes over the Brouwerian formula (B) in propositional modal logic and the Barcan formula in quantified modal logic. Semantic theories for modal logic within a possible worlds framework satisfy Dummett's desideratum, since the non-modal nature of the semantics makes the modal logic of the object-language trivially insensitive to the modal logic of the metalanguage. However, that does not help proponents and opponents of the modal principles at issue understand each other. Rather, it makes the semantic theory virtually irrelevant to the dispute, which is best conducted mainly in the object-language; this applies even to Dummett's own objection to the B principle. Other forms of semantics for modal languages are shown not to alter the picture radically. It is argued that the semantic and more generally metalinguistic aspect of disputes in logic is much less significant than Dummett takes it to be. The role of (non-causal) abductive considerations in logic and philosophy is emphasized, contrary to Dummett's view that inference to the best explanation is not a legitimate method of argument in these areas.

1. Introduction

Philosophically, I grew up in the Oxford of the period 1973-80, where Michael Dummett's thought was under more or less continuous debate, day and night. Familiarity with his work made us underestimate the difficulty of reading it cold: I remember a now-distinguished contemporary taken aback at the suggestion that some Americans found Dummett's writing obscure. There was no presumption that Dummett was actually *right*: most Oxford philosophers hoped to avoid his anti-realism. Indeed, many of them spent their time struggling desperately to do so. Yet they felt it always there, ready to engulf them if they made one false move. They tried to beat Dummett at his own game, perhaps with one or two changes in the rules. At a broader methodological level, Dummett represented the generation that had rejected ordinary language philosophy in favour of a more abstract and systematic mode of theorizing, still concerned to understand the use of natural languages but willing to model them by first-order formal languages and insistent on an explicit compositional theory of meaning. Less congenial were the behaviourist tendencies we suspected to be lurking in his arguments. But his originality, fertility, versatility, philosophical resourcefulness, technical power, and sheer idiosyncrasy defied such labels.

Dummett was my supervisor for the year 1979-80, in which I completed my doctoral dissertation on the concept of approximation to the truth. He remains for me a paradigm of intellectual seriousness. He was also remarkably tolerant of my aggressively realist arguments, ready to discuss them on their own terms rather than seeking to impose his own starting-point. At least my work was different from the relentless concentration on his own writings in much of what he had to supervise: in that respect it may have come as something of a relief to him, despite its blithe tacit assumption of the futility of his life's work. One incident may be of interest. We were discussing an argument I had given for regarding a four-termed relation of comparative similarity as more basic than a three-termed one, the only idea in my dissertation I subsequently developed for publication (Williamson 1988). Dummett thought it utterly fallacious. He thought for a while, then said: 'The difference between us in philosophical method is that you think that inference to the best explanation is a legitimate method of argument in philosophy, and I don't'. On his view, the deep philosophical action came in determining whether a supposed explanation was so much as intelligible. After that, little was left for inference to the best explanation to do. On my view, philosophy does not fall into unintelligibility as easily as Dummett thought — plain falsity is a far more urgent danger — and in any case the way to test an explanation for intelligibility is by trying to work within its framework, not by seeking a guarantee of intelligibility in advance. I have recently defended such an abductive methodology for logic and metaphysics (Williamson 2013, pp. 423-429). How else Dummett expected to argue for a theory of meaning I do not fully understand. But, holding the Wykeham Chair of Logic that Dummett held when he taught me, I wonder whether I am as tolerant of disagreement in my students as he was of mine.

Despite my socialization into a philosophical culture steeped in Dummett's work, I find much of it deeply puzzling to read. I try to put my finger on a crucial premise of his arguments that I reject, but the difference between us is more global than that. I have to imagine my way into an alien but intriguing and powerful mindset, and find the experience surprisingly reminiscent of reading a philosophical masterpiece from a distant era. Although, sentence by sentence, the hermeneutic effort is less, putting the pieces together in one's own terms can be almost equally hard. Latching onto a clear point of disagreement is therefore not simply a matter of philosophical point-scoring, even if it is that too. It is a clue to deeper differences of theoretical outlook.

One such clear point of disagreement emerges in *The Logical Basis of Metaphysics*, when Dummett says 'a difference over fundamental laws of logic must reflect a difference over the meanings of the logical constants' (1991, p. 54). Elsewhere I have argued at length for the opposite view (2007, pp. 85-133). Rather than confront that issue again head-on, I will explore some characteristically thought-provoking remarks that Dummett then makes about the best way for proponents and opponents of a fundamental law of logic to make progress in their dispute by means of semantic theory. For Dummett, a semantic theory states in outline how the semantic values of whole sentences are determined by the semantic values of their constituents, explicitly articulating the contributions of the logical constants in a manner that would enable a soundness or completeness theorem to be stated and proved (if true) for a corresponding deductive system. The semantic theory does *not* attempt to show how its principles are correct in virtue of the use of the object-language by the community of its speakers. For Dummett, that essential task is to be carried out by a deeper meaning-theory for the language. Nevertheless, he envisages the semantic theory playing a role in enabling the parties to a dispute over a fundamental law of logic to understand their disagreement.[1]

2. Dummett on the virtue of insensitivity

According to Dummett (1991, pp. 54-5):

> A thoroughly pernicious principle has gained considerable popularity in recent years. It is that in formulating a semantic theory the metalanguage must have the same underlying logic as the object-language. When this principle is followed, the proponent of a non-classical logic has a perfect counter to an argument in favour of a classical law that he rejects, namely, that the argument assumes the validity of the law in the metalanguage.

Dummett illustrates such a stalemate with the dispute over the distributive law between classical and so-called quantum logic, as to whether A & $(B \vee C)$ entails $(A$ & $B) \vee (A$ & $C)$. He then continues in general terms:

> What is needed, if the two participants to the discussion are to achieve an understanding of each other, is a semantic theory as insensitive as possible to the logic of the metalanguage. Some forms of inference must be agreed to hold in the metalanguage, or no form of inference can be shown to be valid or to be invalid in the object-language; but they had better be ones that both disputants recognise as valid. Furthermore, the admission or rejection in the metalanguage of the laws in dispute between them ought, if possible, to make no difference to which laws come out valid and which invalid in the object-language. […] If both disputants propose semantic theories of this kind, there will be some hope that each can come to understand the other; there is even a possibility that they may find a common basis on which to conduct a discussion of which of them is right.

As an illustration of such convenient insensitivity, he gives the semantics of intuitionistic sentential logic based on Beth or Kripke trees. Whether the metalogic is classical or intuitionistic, that semantics can be shown to validate exactly intuitionistic logic for the object-language.

The aim of the rest of this paper is to assess Dummett's claims in the quoted passages.

3. Some advantages of sensitivity

Say that a semantic theory S *projects* a metalogic ML onto a logic L when L is the strongest logic whose validity is derivable from S, *qua* semantics for the language of L (the object-language), in ML, *qua* logic for the language of S (the metalanguage). In the first displayed passage, Dummett denies that a semantic theory should project each metalogic onto itself. The logic need not preserve the metalogic. In the second passage, he asserts that a semantic theory should project different metalogics onto the same logic (over as wide as possible a range of metalogics). The logic must be robust with respect to the metalogic. Dummett's second claim implies the first, for trivially if the logic always preserves the metalogic, it is maximally sensitive to the metalogic. The converse does not hold, for a semantic theory might in principle constitute a one-one projection with no fixed points: then the logic would be maximally sensitive to the metalogic without ever preserving it.

One obvious target for Dummett's argument is the preference, often associated with Davidsonian philosophy of language, for homophonic semantics. The association was especially salient in Oxford when Dummett wrote. At least to a first approximation, a homophonic semantic theory projects each metalogic onto itself. By contrast, the Beth or Kripke semantics for intuitionistic logic is non-homophonic in structure, and projects even a classical metalogic onto intuitionistic logic for the object-language.

Dummett's argument applies to all semantic theories translatable into a homophonic theory, not just to the homophonic theory itself, since they will all project intertranslatable metalogics onto the same logic for the given object-language. Yet elementary considerations suggest that a semantic theory *should* be translatable into a homophonic theory. The key fact about the sentence "Snow is white" which a semantic theory for English in any metalanguage should capture is that it means that snow is white. More generally, for any sentence *s* of the object-language, a semantic theory for that language should entail some truth like this:

(M) *s* means that *p*

In this schema, "*s*" gets replaced by a name of an object-language sentence and "*p*" by a sentence of the metalanguage. For the instance of (M) to be true, the former object-language sentence must indeed mean what the latter metalinguistic sentence means, or express the same proposition. In general, therefore, the object-language must be translatable into the metalanguage. Davidsonians try to achieve the same effect, despite substituting the extensional construction "is true in the object-language if and only if" for the non-extensional "means that", by imposing complex constraints on the semantic theory.

That elementary argument must be qualified in various ways.

First, purely homophonic semantics is inadequate for languages with context-dependence. For example, although the sentence "I was born in Sweden" as uttered by me means that I was born in Sweden, as uttered by you it does not mean that *I* was born in Sweden; it means that *you* were born in Sweden. For present purposes we can ignore the complexities of context-dependence.

Second, a more pertinent qualification is that a semantic theory, as Dummett envisages it, is primarily a theory of logical consequence, and so typically abstracts from the meanings of non-logical expressions, by generalizing over all interpretations or models of the object-language which fix the intended interpretations of the purely logical expressions. Thus not all sentences of the object-language need be translatable into a metalanguage adequate to express the metalogic of the object-language. For example, the metalanguage may lack synonyms of "snow" and "white", even though the object-language has them. Nevertheless, since the model theory is supposed to abstract away only the logically irrelevant aspects of the semantics of the object-language, we may expect its semantic clauses for the logical constants to translate more or less homophonic semantic clauses for them, relativized to a model and usually other parameters too, such as an assignment of values to variables. That is just what we find in standard first-order model theory. It has clauses like the following, where M, $\underline{a} \models A$ means that the formula

A is true in the model M under the assignment $\underline{a}$ of values to variables, dom(M) is the domain of quantification for M, and $\underline{a}(x/d)$ is the assignment like $\underline{a}$ except that $\underline{a}(x/d)(x) = d$:

$$M, \underline{a} \models \neg A \text{ if and only if not } M, \underline{a} \models A$$

$$M, \underline{a} \models A \,\&\, B \text{ if and only if } M, \underline{a} \models A \text{ and } M, \underline{a} \models B$$

$$M, \underline{a} \models \exists x\, A \text{ if and only if for some } d \in \mathrm{dom}(M)\text{: } M, \underline{a}(x/d) \models A$$

These are not homophonic clauses, since what correspond to the object-language symbols ¬, & and ∃ in English as the informal metalanguage are “not”, “and” and “some”. Even if the clauses were formalized in an extension of the object-language, some non-homophonic features would remain, such as the reference to the assignment $\underline{a}(x/d)$ and, more significantly, to the domain of the model (Williamson 2003). Nevertheless, since the main connective in the right-hand side of the biconditional for each logical constant is a rough translation of the logical constant into the metalanguage, and little more than the technically necessary minimum of extra structure has been introduced, we may still loosely call such model-theoretic semantics *quasi-homophonic*.

Dummett’s main objection to quasi-homophonic semantics is that it blocks rational debate about questions of validity, because each side uses its preferred logic as a metalogic to vindicate that very logic for the object-language, and accuses the other side of begging the question when it does the same thing. More generally, on his view, quasi-homophonic semantics lacks explanatory power.

Imagine a dispute over the validity of a fundamental logical law [!], which is in fact valid, although some philosophers argue otherwise. Friends of [!] correctly explain why it is valid. Their explanation invokes both a semantic theory and a metalogic. May it invoke [!] itself in the metalanguage? If it does, it will not persuade critics of [!]. But the purpose of explaining *why* something obtains is often not to persuade anyone that it *does* obtain. In explaining *why* there is life on Earth, a scientist is not trying to persuade anyone that there *is* life on Earth.[2] It may be quite unreasonable to demand that friends of [!] explain why it is valid in the object-language without invoking [!] in the meta-language. If [!] is a fundamental logical law, a metalogic without [!] cannot be expected to project onto a logic with [!]. On this view, the metalogic *should* contain all the fundamental logical laws of the object-language. If the fundamental laws generate the non-fundamental ones, it follows that the metalogic should be at least as strong as the logic. It may need to be stronger: for instance, the metalanguage for a quantifier-free object-language must itself contain quantifiers, to express metalinguistic generalizations (such as soundness and completeness), so the metalogic must include quantificational logic.

In quasi-homophonic semantics, the metalanguage may contain metalinguistic vocabulary beyond the expressive resources of the object-language. For example, the theory of logical consequence for a first-order language with unrestricted quantification may require a second-order metalanguage (Williamson 2003). This leaves the logic some room for insensitivity to the metalogic, since metalogics which coincide over the fragment of the metalanguage translatable into the object-language may differ elsewhere. But that is not the room Dummett wanted, since the metalogics of the parties to a dispute over validity for the object-language will thereby differ over the fragment of the metalanguage translatable into the object-language, not just elsewhere.

Dummett may still complain that quasi-homophonic semantics lacks explanatory power. But the problem is not that its explanations of validity are strictly circular. For example, if we assume $\forall x\, x=x$ in explaining why that very formula is valid, the explanans is distinct from the explanandum; to suppose otherwise is to confuse use and mention. Nevertheless, the recipient may be left with an uneasy feeling that the explanation was too cheap: it did not cost enough in hard labour. If we assume $\forall x\, x\neq x$, we can "explain why" $\forall x\, x\neq x$ is valid in a formally parallel way.

However, the task was not to explain why $\forall x\, x=x$, that is, why everything is self-identical. No explanation of that non-metalinguistic fact is on offer. We may well doubt that any is possible for something so simple and fundamental. The task was to explain why the *formula* $\forall x\, x=x$ is valid. The quasi-homophonic semantics enables us to explain that semantic fact, taking the non-semantic facts for granted. To expect more than that from a semantic theory is to look in the wrong place. In general, the task for a semantic theory is to explain the semantic facts, given the non-semantic facts. It is not to explain the non-semantic facts, even when they are logical facts.

The same point applies to explaining why a rule of inference is valid, but is easier to miss there because the use-mention distinction is harder to apply to such rules than to single sentences. But the distinction still does apply, for simply to employ a logical rule such as modus ponens is not yet to think metalinguistically.

None of this yields a methodology for resolving disputes about fundamental logical laws. Why should it? We reasonably expect such disputes to be hard to resolve. But they have not been put beyond the reach of reason. Once we explore and compare in detail the consequences of adopting different systems of logic, we have plenty of evidence on which to base a reasoned choice. In Dummett's example, if rejecting one distributive law was all it took to resolve the puzzles of quantum mechanics in all other respects, with no other resolution in sight, we might indeed have bluntly rejected that law for both object-language and metalanguage, without constructing any elaborate semantic theory to project a classical metalogic onto a quantum logic. The real trouble for quantum logic may be just that rejecting the distributive law does much less than advertised towards resolving the physical puzzles.

The discussion so far has proceeded in highly schematic terms. We can test it by considering a case study. In this paper, the case is modal logic.

4. Case study I: propositional modal logic

The standard metatheory of modal logic seems to meet Dummett's constraint that the logic should be insensitive with respect to the metalogic. The semantic framework is possible worlds model theory, which is used to characterize the relevant consequence relations. The metalanguage is simply an extensional language with enough non-logical primitives to express set theory and the syntax of the modal object-language. It contains no modal operators. It does not even contain modal predicates such as "is possible" or "is a possible world". In short, it is the language of mathematics and syntax. The logic onto which such a non-modal semantic theory projects the metalogic depends on no specifically modal principle of the metalogic.

Let us check the point in detail for propositional modal logic. Syntactically, the object-language is standard: it contains countably many atomic formulas *p*, *q*, *r*, …, the one-place sentential operators ¬ and □, and the two-place sentential operator &. Other symbols are introduced as metalinguistic abbreviations in the usual way. For example, ◊ is ¬□¬. A *model* is any triple <W, R, V>, where W is any nonempty set, R is any set of ordered pairs of members of W (a binary relation on W), and V is any function from atomic formulas to subsets of W. We define a three-place |= between a model M = <W, R, V>, an element w ∈W and a formula *A*, by recursion on the complexity of *A*, thus (where "wRx" abbreviates "<w, x> ∈R"):

If *A* is atomic, M, w |= *A* if and only if w ∈V(*A*)

M, w |= ¬*A* if and only if not M, w |= *A*

M, w |= *A* & *B* if and only if M, w |= *A* and M, w |= *B*

M, w |= □*A* if and only if M, x |= *A* for every x ∈W such that wRx

Those definitions are stated purely in terms of mathematics and syntax. Although the clauses for ¬ and & are quasi-homophonic, the clause for □ is not, since the modal operator in the object-language is handled by non-modal quantification in the metalanguage.

The model theory of propositional modal logic is developed simply as a piece of mathematics. For example, we can prove mathematically that for any binary relation R on a set W: M, w |= *A* → □◊*A* (the 'Browerian' schema) for every model M = <W, R, V>, w ∈W and formula *A* if and only if R is symmetric. Similarly, we can prove that M, w |= □*A* → □□*A* (the 4 schema) for every model M = <W, R, V>, w ∈W and formula *A* if and only if R is transitive. The proofs involve no modal considerations whatsoever: they do not mention possibility or necessity.

To work through an even simpler example, let us prove that M, w |= □*A* → *A* for every model M = <W, R, V>, w ∈W and formula *A* if and only if R is reflexive. First, suppose that R is reflexive on W. Consider any model M = <W, R, V>, w ∈W and formula *A*. If M, w |= □*A*, then, by the clause for □, M, x |= A for every x ∈W such that wRx; but wRw because R is reflexive, so M, w |= A. Thus M, w |= $\Box A \rightarrow A$ by the clause for →, as required. For the converse, suppose that R is not reflexive on W. Hence for some w ∈W, not wRw. Consider a model M = <W, R, V> such that V(p) = W-{w}. Thus by the relevant clauses M, w |= $\Box p$ but not M, w |= p, so not M, w |= $\Box p \rightarrow p$, which completes the proof.

In motivating the semantics informally, we may describe W as a set of worlds, and R as a relation of relative possibility between worlds, where x is possible relative to w if and only if x would be possible if w obtained. We may call a formula A true at a world w in a model M if and only if M, w |= A. However, such ideas play no official role in the formal definitions or proofs. For example, in providing a model M = <W, R, V> and w ∈W such that not M, w |= $\Box p \rightarrow p$, we made no attempt to provide a possible case in which something obtains necessarily without obtaining; there is no such possible case. The set W may simply be {0}, and R simply {}.

In effect, the technical study of modal logic has made such dramatic progress over the past fifty years by eliminating all modal considerations from its reasoning. Questions in the model theory of modal logic are answered by purely mathematical proofs. Philosophical disputes about possibility or necessity are irrelevant to this process. For that very reason, however, the model theory does not resolve those disputes.

Let us explore the relation between the model theory of modal logic and philosophical questions about modality in more detail. For definiteness, we fix on an informal interpretation of the modal operators □ and ◊: as symbolizing metaphysical necessity and possibility respectively, say, rather than physical or epistemic modalities. Call a formula A *metaphysically universal* if and only if A is true on every interpretation of the atomic formulas with the intended interpretation of the operators. For present purposes we may assume that it is uncontroversial for the given object-language that (i) every truth-functional tautology is metaphysically universal; (ii) whenever A and $A \rightarrow B$ are metaphysically universal, so is B; (iii) every instance of the schema $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$ is metaphysically universal; (iv) whenever A is metaphysically universal, so is $\Box A$.

Since the analogues of (i)-(iv) for provability axiomatize the weakest "normal" modal logic K, (i)-(iv) make every theorem of K metaphysically universal. This implies that there is a class of models (in the above sense) such that any formula is metaphysically universal if and only if it is true at every world in every model in the class. For let C be the class of all models M such that every metaphysically universal formula is true at every world in M. We need to show that every formula true at every world in every model in C is metaphysically universal. Suppose that A is not metaphysically universal. Thus $\neg A$ is true on some interpretation Int of the atomic formulas and the intended interpretation of the operators (we assume bivalence: if A is not true on Int then $\neg A$ is true on Int). The set of all formulas which are true on Int is K-consistent, in the sense that for no such formulas $B_1, \ldots, B_n$ is $\neg(B_1 \,\&\, \ldots \,\&\, B_n)$ a theorem of K: for if $B_1, \ldots, B_n$ are all true on Int, so is $B_1 \,\&\, \ldots \,\&\, B_n$, so $\neg(B_1 \,\&\, \ldots \,\&\, B_n)$ is not true on Int, so $\neg(B_1 \,\&\, \ldots \,\&\, B_n)$ is not metaphysically universal, so by the above $\neg(B_1 \,\&\, \ldots \,\&\, B_n)$ is not a theorem of K. It follows that the canonical model M = <W, R, V> of K contains a world w at which all formulas true on Int are true. Let M* = <W*, R*, V*> be the submodel of <W, R, V> generated by w; thus W* is the set of all x ∈W to which w has the reflexive ancestral of R, $R^* = R \cap W^{*2}$ and $V^*(p) = V(p) \cap W^*$ for every atomic formula p. Generated submodels preserve the truth-values of formulas at any point: for every x ∈W* and formula B, M*, x |= B if and only if M, x |= B.[3] Hence if B is true on Int then M*, w |= B because M, w |= B. In particular, since $\neg A$ is true on Int, M*, w |= $\neg A$, so not M*, w |= A. But M* ∈C: for if x ∈W then x is n steps of the relation R* from w; thus if a formula B is metaphysically universal, so is $\Box^n B$ by (iv) above (where $\Box^n$ is a sequence of n occurrences of □), so in particular $\Box^n B$ is true on Int, so M*, w |= $\Box^n B$, so M*, x |= B by the clause for □. Thus A is not true at every world in every model in C. This proves that a formula is metaphysically universal if and only if it is true at every world in every model in C. We seem to have reduced questions of metaphysical universality to questions of model theory, given the rather modest assumptions (i)-(iv) about metaphysical universality.

The catch is that the class C of models was itself defined in terms of metaphysical universality. Suppose, for example, that two philosophers disagree over the principle that what obtains necessarily possibly obtains. They both understand the modal operators as symbolizing metaphysical modalities; they are not talking past one another. In effect, they disagree on whether the B axiom $p \rightarrow \Box\Diamond p$ is metaphysically universal. They both accept constraints (i)-(iv) on metaphysical universality, and so agree that the axiom is metaphysically universal if and only if it is true at every world in every model in the class C. By the fact noted above, if C contains only models <W, R, V> for which R is symmetric, then the B axiom is true at every world in every model in C, so the axiom is metaphysically universal; in the other direction, if for some non-symmetric relation R on a set W, C contains all models of the form <W, R, V>, then the B axiom is false at some world in some model in C, so the axiom is informally invalid. But to ask whether C contains a model <W, R, V> with a non-symmetric relation R boils down to asking whether every metaphysically universal formula is true at every $w \in W$ in <W, R, V>. If $p \rightarrow \Box\Diamond p$ is false at some $w \in W$ in <W, R, V> then the question arises whether $p \rightarrow \Box\Diamond p$ *is* metaphysically universal. We are back where we started.

We may therefore expect arguments for or against the B principle to bypass the mathematical model theory, and address the modal issues directly. That is just what we find in practice, in particular when we look at Dummett's own critique of the principle. He argues, against Kripke (1980, p. 157), that there could have been unicorns (Dummett 1993a, p. 346):

> They might, for instance, be of the order Artiodactyla, like deer, or of the order Perissodactyla, like horses. In the language of possible worlds, there are no unicorns in the actual world *w*, but there is a possible world *u* in which there are unicorns, which belong to the order Artiodactyla, and another possible world *v* in which there are also unicorns, which in that world belong to the order Perissodactyla. […] In world *u*, any animal, to be a unicorn, must have the same anatomical structure as the unicorns in *u*, and hence, in particular must belong to the order Artiodactyla. It follows that the world *v* is not possible relatively to *u*, and, conversely, that *u* is not possible relatively to *v*. How about the actual world *w*—is that possible relatively to either *u* or *v*? It would at first seem so, since the principal difference we have stipulated is that there are no unicorns at all in *w*. But *u* is a world in which it holds good that unicorns are necessarily of the order Artiodactyla, whereas in *w* it is possible for unicorns to be of the order Perissodactyla. Since a proposition necessarily true in *u* is possibly false in *w*, *w* cannot be possible relatively to *u*, although *u* is possible relatively to *w*. The relation of relative possibility (accessibility) is therefore not symmetrical.

In effect, Dummett is arguing that the principle $p \rightarrow \Box\Diamond p$ is invalid when p is interpreted as expressing the proposition that it is possible for unicorns to be of the order Perissodactyla.[4]

Although Dummett puts his argument in terms of accessibility relations between worlds, he is not attempting to use model theory to avoid metaphysical controversy. Rather, he is adverting to an *intended* model. In speaking of possibility and necessity, he means the genuine articles, not whatever happens to play similar structural roles in an arbitrary model. The talk of worlds and their relative possibility serves to present the argument more perspicuously, not to replace metaphysics by semantics. One could articulate an equivalent argument using modal operators, without reference to worlds. Indeed, the crucial first sentence of the quoted passage is already so phrased: in effect, "There could have been unicorns of the order Artiodactyla and there could have been unicorns of the order Perissodactyla". The switch to talk of possible worlds is presented as terminological: "in the language of possible worlds".

Critics of Dummett's argument have accused it of confusing what could have been truly said with the word "unicorn" in counterfactual circumstances with what can be truly said with it in the actual circumstances about the counterfactual circumstances.[5] If they are right, as they surely are, the argument rests on a semantic confusion, but that does not make it a semantic argument. It does not make the argument's modal dimension redundant.

Similar considerations apply to controversy over the 4 axiom $\Box p \rightarrow \Box\Box p$. Suppose that two philosophers disagree over the principle that what necessarily obtains necessarily necessarily obtains. They both understand the modal operators as symbolizing metaphysical modalities; they are not talking past one another. In effect, they disagree on whether the 4 axiom is metaphysically universal. They both accept constraints (i)-(iv) on metaphysical universality, and so agree that the axiom is metaphysically universal if and only if it is true at every world in every model in the class C. If C contains only models $\langle W, R, V\rangle$ for which R is transitive, then the 4 axiom is true at every world in every model in C, so the axiom is metaphysically universal; in the other direction, if for some non-transitive relation R on a set W, C contains all models of the form $\langle W, R, V\rangle$, then the 4 axiom is false at some world in some model in C, so the axiom is informally invalid. But to ask whether C contains a model $\langle W, R, V\rangle$ with a non-transitive relation R boils down to asking whether every metaphysically universal formula is true at every $w \in W$ in $\langle W, R, V\rangle$. If $\Box p \rightarrow \Box\Box p$ is false at some $w \in W$ in $\langle W, R, V\rangle$ then the question arises whether the model $\langle W, R, V\rangle$ is really in C, which in turns partly depends on whether $\Box p \rightarrow \Box\Box p$ *is* metaphysically universal. Again, we are back where we started.

Whether what necessarily obtains necessarily necessarily obtains is a metaphysical question, not a semantic one. It cannot be settled by purely model-theoretic means. Not surprisingly, the most salient criticism of the principle has been overtly metaphysical, and couched in explicitly modal terms. Friends of the principle have responded at the same level. Both sides use semantic considerations in an auxiliary capacity, not as the core of their argument.[6, 7]

5. Case study II: the Barcan formula

The case of quantified modal logic is similar. The central controversy concerns the Barcan formula (Barcan 1946):

BF $\Diamond \exists x A \rightarrow \exists x \Diamond A$

Informally, BF says that if there could have been something which met a certain condition, then there is something which could have met that condition. Many metaphysicians hold that there are actual counterexamples to BF. For instance, Queen Elizabeth I never had a child, but she could have done. By BF, it follows that there is something which could have been a child of Elizabeth I. But what is it? Given the essentiality of one's actual origins, no actual person could have had Elizabeth I as a parent (Kripke 1980). Although some actual collection of atoms could have *constituted* a child of Elizabeth I, the collection would not have been *identical* with the child. According to those metaphysicians, there is nothing which could have been a child of Elizabeth I. Thus BF is false. Again, given the necessity of identity, BF implies that there could not have been more things than there actually are; but many metaphysicians regard the numerosity of the universe as contingent.

There are two natural ways of extending models of propositional modal logic to interpret quantified modal logic: constant domain semantics and variable domain semantics. For both, the function V now maps each n-place atomic predicate F to an intension V(F), which maps each w $\in$W to an extension V(F)(w) for F.

On constant domain semantics, each model has a single component set D to serve as the domain of the quantifiers. The semantic clause for a quantifier takes a form like this:

$$\mathrm{M, w, \underline{a}} \models \exists x\, A \text{ if and only if for some } \mathrm{d} \in \mathrm{D}\text{: } \mathrm{M, w, \underline{a}}(x/\mathrm{d}) \models A$$

As an easily proved non-modal mathematical fact, on constant domain semantics every instance of BF is true at every world in every model under every assignment.

On variable domain semantics (Kripke 1963), each model has a component function which maps each w $\in$W to a set D(w) to serve as the domain of the quantifiers when evaluated at w. The semantic clause for a quantifier takes a form like this:

$$\mathrm{M, w, \underline{a}} \models \exists x\, A \text{ if and only if for some } \mathrm{d} \in \mathrm{D(w)}\text{: } \mathrm{M, w, \underline{a}}(x/\mathrm{d}) \models A$$

As an easily proved non-modal mathematical fact, on variable domain semantics some instance of BF is false at some w $\in$W in some model under every assignment. For consider any model with w, x $\in$W such that wRx but not D(x) $\subseteq$ D(w). In some such model M, V(F)(y) = D(y)-D(w) for all y. Thus M, x, $\underline{\mathrm{a}}$ $\models \exists x\, Fx$, so M, w, $\underline{\mathrm{a}}$ $\models \Diamond\exists x\, Fx$. But if M, w, $\underline{\mathrm{a}}(x/\mathrm{d}) \models \Diamond Fx$ then d $\in$V(F)(y) for some y $\in$W, so not d $\in$D(w); hence not M, w, $\underline{\mathrm{a}}$ $\models \exists x\, \Diamond Fx$. Thus for A = Fx, BF is false at w in M.

Informally, the set D in a constant domain model is imagined to contain whatever there is, with the background assumption that being (unlike being concrete) is non-contingent. Similarly, the set D(w) in a variable domain model is imagined to contain whatever there is in the world w, with the background assumption that being (like being concrete) is contingent. But these informal understandings play no role whatsoever in the model theory itself.

As characterizations of metaphysical universality, constant domain semantics and variable domain semantics give incompatible results. We cannot accept both. The choice between them returns us to the metaphysical question of the contingency or otherwise of being. We have been given no way to bypass the modal controversy and settle the metaphysical universality of BF on non-modal grounds.

Although variable domain models falsify BF, opponents of BF have a specific reason not to regard them as more than convenient representational devices. They typically hold that BF has actually false instances (read *A* as "*x* was a child of Elizabeth I"). Moreover, they typically hold that some such instances in no way depend on any tacit contextual restriction of the quantifiers. For example, although there could have been something which was a child of Elizabeth I, there is absolutely nothing, however widely the quantifier ranges, which could have been a child of Elizabeth I. Try capturing this idea in an intended variable domain model M. M should contain the actual world @ as one of its worlds. If BF has false instances at @ in M, for some world w D(@) does not include D(w), so some d in D(w) is not in D(@), so D(@) does not contain everything there is. Thus D(@) does not contain everything over which the quantifier ranges on its intended interpretation. Consequently, such opponents of BF should deny that there is an intended model.[8] They should regard models as more like mere representational devices, which may picture a failure of BF but cannot instantiate one. Such a model may contain a world at which the true closed formulas coincide with those true on a given intended interpretation, but the explanation of their truth at the world in the model will be quite different from the explanation of their truth on the intended interpretation. We can explain on purely mathematical grounds why there are counterexamples in the variable domain semantics to BF. Indeed, in some of those counter-models the domains of all worlds are numbers, whose being is presumably non-contingent. The explanation ignores modal matters. Any argument for the coincidence of the sentences true at the world in the model with the sentences true on the intended interpretation would itself have to proceed partly in modal terms.

6. Other semantic theories for modal languages

Given the results of section 5, opponents of BF may therefore seek a semantic theory more faithful to the intended meanings of the modal operators. The natural idea is to use a quasi-homophonic semantics in a modal metalanguage. A little work has indeed been done along such lines. It is a laborious business, by comparison with possible worlds semantics in a non-modal metalanguage. Even very simple results are very hard to prove; various obstacles remain to be overcome. Moreover, if we use such a metatheory, we cannot expect to achieve very much by semantic ascent. Our assessments of modal principles in the object-language simply reflect our assessments of the same principles in the metalanguage.[9]

In general, we cannot expect to settle modal questions by non-modal reasoning. Of course, some philosophers try to reduce the modal to the non-modal. Perhaps the best example is David Lewis (1986). On his modal realism, quantification over possible worlds in a non-modal language represents the underlying metaphysical reality more perspicuously than does the use of modal operators. The possible worlds themselves are explained in non-modal terms, as mutually isolated spatiotemporal systems. The actual world is merely one such system amongst many, just as here is merely one place amongst many, privileged only from its own perspective. Claims about how given objects could have been different describe or misdescribe how counterparts of those objects in other systems are different. Philosophers who agree on modal realism can use it to settle some modal questions by non-modal reasoning. But most philosophers reject modal realism as hopelessly implausible. They insist that this actual world contingently but objectively has a privileged metaphysical position. For them, the use of modal operators gives a more perspicuous representation of the underlying metaphysical reality in that respect than does quantification over worlds in a non-modal language.

If modal realism is false, of what philosophical use is possible worlds model theory for modal logic? It is a powerful instrument for consistency proofs. We can show a formula to be underivable from a set of axioms and rules of inference by constructing a model with a world at which all the axioms are true and all the rules preserve truth but the formula is false. Moreover, if we have proved an axiomatic system sound and complete for a given class of models, we can sometimes derive consequences from it more efficiently by reasoning about the models than by reasoning within the axiomatic system itself. Alternatively, one might avoid the process of axiomatization altogether by specifying a modal theory as comprising just those formulas true at all worlds in all models in a formally specified class, and deriving its consequences by reasoning about the models. But in such applications the model theory plays no more than a useful auxiliary role. It does not enable us to bypass modal reasoning. For example, it provides no way of settling the disputes over the B, 4 and BF schemas by semantic means.

Might some other form of semantics for modal languages do better? Possible worlds semantics is by far the best developed approach, and so constitutes the most authoritative test in this area of Dummett's argument quoted at the start of this paper. It turned out to provide no independent criterion for determining the metaphysical universality of philosophically contentious modal schemas. *A fortiori*, nor does quasi-homophonic semantics. In the spirit of Dummett (1991), we might also consider more proof-theoretic semantic theories for modal languages. So far, proof theory has contributed remarkably little to the development of modal logic. Robert Brandom has recently attempted to apply his inferentialist approach to the semantics of modal logic. However, since his strategy is to show how to construct an inferentialist semantics for any normal modal logic, it is of no great help in deciding *between* normal modal logics (Brandom 2008, pp. 170-1).

On current evidence, semantics is no royal road to the resolution of disputes in modal logic. It can clarify and discipline them, but it cannot transmute questions of logic into questions of semantics. Of course, confusion in semantics can cause error in logic. For unnoticed ambiguities can cause error in any inquiry, and false assumptions of synonymy may have the same effect. Confusion as to whether names are rigid designators can cause error in the modal logic of identity. Sorting out the confusion may be a precondition for correcting the error. But if getting the semantics wrong usually results in getting the logic wrong, it does not follow that getting the semantics right usually results in getting the logic right. In this respect, logic may differ less from the natural sciences than is often supposed. A bad semantic theory for the language of physics may mislead us into accepting false conclusions in physics. For example, on the basis of crude verificationism in semantics, a philosopher may assert that all past events have left causal traces in the present, which may turn out to be physically false. But that is no reason for physicists to expect much help from a *good* semantic theory for the language of physics. We should not be too quick to assume that the case of logic is radically different.[10]

7. Understanding and disagreement in logic

In a passage from Dummett quoted at the beginning of the paper, he emphasizes the need for the two parties in a dispute over a logical principle to understand each other. He envisages this mutual understanding as produced by the insensitivity of the logic to the metalogic, given the sort of semantic theory he recommends, because the two sides can agree on what logic a given semantic theory of that sort would generate, assuming only uncontentious metalogical principles. For Dummett, the dispute may thereby evolve into one over the choice of semantic theory for the object-language.

"Understanding someone's utterance" can mean different things. If someone says "The Moon is larger than the Earth", we may understand his utterance in one sense, because we know on the basis of our competence with the English language and awareness of the conversational context what proposition he strictly and literally expressed, but not understand his utterance in another sense, because we have no idea why he felt warranted in making it, thereby asserting what is generally known to be a falsehood. In a fundamental dispute over principles of logic, the initial understanding we lack of our opponent's utterances is typically of the latter rather than the former kind. Whatever its formal ramifications, the dispute applies at least to principles formulated in a natural language, whose expressions both sides use with their standard public meanings, not with artificially concocted ones.[11] For the natural language as object-language, the role of the rival semantic theories is descriptive, not stipulative. But they are supposed to give each side understanding in the latter sense of the other's utterances, by explaining the considerations on which they were based.

In the case studies of modal logic above, neither side has much difficulty in understanding what the other is strictly and literally saying. The challenge is to understand why they are saying it. In that task, however, the semantic theories are virtually useless. Those who accept symmetry as a constraint on the accessibility relation of a model must accept the B principle for the object-language, but that only raises the further question: why do they accept the symmetry constraint? An analogous point applies to those who reject both the symmetry constraint and the B principle. If you are puzzled by your opponents' stance on the B principle, you will be equally puzzled by their stance on the symmetry constraint. If they explain their putative counterexample to symmetry, you may start to understand why they say what they do. But the example could just as well have been presented as a direct counterexample to the B principle itself, without the digression through model theory. One can often come to understand why people take the stance they do in a logical dispute when they articulate the considerations which move them. That does not depend on whether the considerations are semantic, logical or metaphysical.

When we are challenged to explain why we accept what we take as our most fundamental logical principles, we may find ourselves with embarrassingly little to say. We can try to rebut objections, provide corroboration in the form of our logic's strong track record, simplicity, elegance and integration with mathematics and science, and emphasize the problematic features of rival logics, while still feeling those considerations to be secondary by comparison with the utter obviousness of its principles. However, there are several reasons for not reworking the semantic theory with a view to deriving the contested principles of the logic from uncontested principles of the metalogic.

First, the strategy may be impossible to implement. If the principle is fundamental enough, it may be derivable from less contested principles of the metalogic only given a contested semantic theory. The extra semantic structure invoked to avoid appeal to the contested principle in the metalanguage may simply support doubts about the faithfulness of the semantics. For instance, it is uncontentious that constant domain semantics validates BF; sceptics about BF simply focus their doubts on whether constant domain semantics is faithful to the intended meanings of the formulas.

Second, contentiousness depends on one's opponents. Virtually every putative principle of logic has been contested by some philosopher or other. The reworking of semantic theory needed to validate a principle of logic from principles of metalogic uncontested by one opponent may be quite different from the reworking needed to validate that principle from principles of metalogic uncontested by a quite different opponent. Semantics is a theoretical inquiry into meaning, not a debating technique. We should not tailor our semantic theories to suit the accident of which opponent currently exercises us the most, thereby distorting our semantic theories to gain a short-term dialectical advantage in logic.

Of course, we cannot assume without further investigation that the case of modal logic is typical of the interrelations between logic, metalogic and semantics. In any difficult choice between logics, we must reflect explicitly on whether the argument forms they endorse coincide with the valid ones, which we can do systematically only if we consider semantic theories for the logical constants and syntactic constructions which characterize those forms. But it does not follow that the initiative lies with semantics in such an inquiry; as we have seen, its role may still be a subsidiary, clarificatory one.

One simple line of thought, adumbrated earlier, suggests that the role of semantics will indeed be secondary. The starting point is that logical theorems are normally not metalinguistic in content. To return to the case of modal logic, against the background of a quantified modal system as strong as S5, the dispute over BF can resolve itself into a dispute over the single formula $\Box\forall x \Box\exists y\ x=y$ (NNE), which says that necessarily everything is necessarily something, or equivalently: there could not have been something which could have been nothing. NNE and ¬NNE are not metalinguistic claims in any interesting sense. To try to explain why there could not have been something which could have been nothing, or why there could have been something which could have been nothing, by invoking semantic considerations would be to go off on a digression through the irrelevant. The same goes for any other theorem of any standard logical system. But if individual axioms and theorems are not metalinguistic in content, axiom and theorem schemas too do not introduce metalinguistic content, for a schema is only a convenient way of collecting its instances. Furthermore, if axiom and theorem schemas do not introduce metalinguistic content, nor do derived and underived inference rules. For an axiom or theorem schema is just the special case of a derived or underived inference rule with no premises, and leaving room for premises as well as a conclusion does not by itself introduce metalinguistic content where none was before. Thus logical principles in general are not metalinguistic in content, and so should not be explained (in the relevant sense) in semantic terms. Although we may need semantics to clear away confusions which block us from accepting valid logical principles, it is not semantics which explains the principles themselves. The same applies to rejecting invalid logical principles, once they are put in the form of universal generalizations, perhaps in higher-order logic. We should not expect semantics to exceed its proper task.

This paper has concentrated on the relative roles of metalogic and of semantic theory in logical disputes, the topic of the passages originally quoted from *The Logical Basis of Metaphysics*. In the case studies, the metalogic and the semantic theory conforms to Dummett's requirements, since which modal logic for the object-language the semantic theory projects the metalogic onto is trivially independent of any specifically modal feature of the metalanguage. Nevertheless, the semantic theory does not help the parties to the disputes in modal logic to understand each other, let alone to resolve their disputes. Indeed, we found no evidence for Dummett's view that such disputes are fundamentally metalinguistic. Of course, according to Dummett, the ultimately decisive considerations should come from a deeper level of the theory of meaning, at which competing claims are supposed to be cashed out in terms of the hard currency of speakers' observable use of the language. But we have virtually no idea what such a process might look like for modal logic, or why we should expect the semantic theorizing it would involve to be less contentious than the non-metalinguistic logico-metaphysical theorizing involved in a more direct approach to the modal issues. My own view is that in such matters we have no promising alternative to abductive theorizing in a modal object-language.[12] Thus my disagreement with Dummett can be traced back to the methodological difference he diagnosed between us more than a third of a century ago.

Notes

1 With permission, the remaining sections of this paper use material, significantly revised, from Williamson 2011. That paper was presented, in different forms, to both the 2010 Lauener Symposium in honour of Michael Dummett and a 2010 conference on Logic and Knowledge at the Sapienza University of Rome, whose proceedings form the kernel of Cellucci, Grosholz, and Ippoliti 2011. Cesare Cozzo gave a helpful commentary on the latter occasion (Cozzo 2011). I thank audiences at both events for discussion.

2 Dummett 1975 emphasizes the distinction between explaining validity and persuading someone of it.

3 See Hughes and Cresswell (1984, pp. 22-5 and 78-81 respectively) for more on the relevant properties of canonical models and of generated submodels.

4 Dummett's argument requires the transitivity of the accessibility relation to argue from the accessibility of *v* from *w* and the inaccessibility of *v* from *u* to the inaccessibility of *w* from *u*, or alternatively the 4 schema (amongst other principles) to derive $\neg\Box\Diamond\Diamond q$ from $\neg\Box\Diamond q$, where *q* expresses the proposition that unicorns are of the order Perissodactyla (so p corresponds to $\Diamond q$). Dummett endorses the 4 schema in the next paragraph (1993, p. 347).

5 See Reimer 1997 and Rumfitt 2010. For a related dispute over B, involving a proper name rather than a natural kind term, see Stephanou 2000 and Gregory 2001.

6 See Salmon 1989 and 1993 and Williamson 1990, pp. 126-43.

7 For a more extensive discussion of metaphysical universality in propositional modal logic see Williamson 2013, pp. 92-118.

8 See Williamson 1998 for this point and a defence of BF. See also Linsky and Zalta 1994 and 1998 and Parsons 1995 for similar defences of BF. The point is argued in more depth in Williamson 2013, pp. 130-139. For the failure of the criterion of representational significance in Stalnaker 2010 to handle the example of BF see Williamson 2013, pp. 188-194.

9 For modal truth theory see Fine 1977, Davies 1978, Peacocke 1978, Gupta 1978 and 1980, and Rumfitt 2001; for modal model theory, see Humberstone 1996.

10 For related discussion see Williamson 2007, pp. 10-47.

11 For more on logical disagreement without diversity of meaning see Williamson 2007, pp. 73-133.

12 Williamson 2013 undertakes such an approach.

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