KANT'S CONSTRUCTION OF NATURE

Kant's *Metaphysical Foundations of Natural Science* is one of the most difficult but also most important of Kant's works. Published in 1786 between the first (1781) and second (1787) editions of the *Critique of Pure Reason*, the *Metaphysical Foundations* occupies a central place in the development of Kant's philosophy, but has so far attracted relatively little attention compared with other works of Kant's critical period. This book develops a new and complete reading of the work, and reconstructs Kant's main argument clearly and in great detail, explaining its relationship to both Newton's *Principia* and eighteenth-century scientific thinkers such as Euler and Lambert. By situating Kant's text relative to his pre-critical writings on metaphysics and natural philosophy, and, in particular, to the changes Kant made in the second edition of the *Critique*, Michael Friedman articulates a radically new perspective on the meaning and development of the critical philosophy as a whole.

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A Reading of the *Metaphysical Foundations of Natural Science*

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521198394

First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Friedman, Michael, 1947–
Kant's construction of nature : a reading of the *Metaphysical foundations of natural science* / Michael Friedman.
p. cm.
Includes bibliographical references (p. m) and index.
ISBN 978-0-521-19839-4 (hardback)
1. Kant, Immanuel, 1724–1804. Metaphysische Anfangsgründe der Naturwissenschaft. 2. Physical sciences–Philosophy. I. Title.
B2786.Z7F75 2013
113–dc23
2012026434

ISBN 978-0-521-19839-4 Hardback

To the memory of
Robert E. Butts (1928–1997)
in gratitude for his support, encouragement, and friendship

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Preface and acknowledgments

This book represents the culmination of an intellectual journey of more than thirty years, beginning in 1980 with my reading of Gerd Buchdahl's *Metaphysics and the Philosophy of Science*, published in 1969. To be sure, I had harbored a long-standing serious interest in Kant's philosophy since my days as an undergraduate. I had been gripped by the *Critique of Pure Reason* and excited by the rebirth of interest in Kant within the Anglo-American tradition sparked by the publication of P. F. Strawson's *The Bounds of Sense* in 1966. This rebirth, however, did not include a corresponding serious interest in Kant's philosophy of science. On the contrary, in Strawson, as in much of traditional Kant scholarship, Kant's engagement with the largely Newtonian science of his time tended to be downplayed or dismissed as involving an unjustified a priori commitment to principles that we now know to have been superseded by the later progress of science, and the hope was to preserve what was still viable in Kant's philosophy independently of this commitment. My own interest in Kant, before reading Buchdahl, ran squarely along such traditional lines.

Meanwhile, however, also during my years as an undergraduate, I began working in contemporary philosophy of science, especially the philosophy of physics. I continued this work as a graduate student, resulting in a dissertation (which later appeared in print, much expanded and revised, in 1983) on space-time physics in both its Newtonian and Einsteinian versions. Reading Buchdahl's book against this background appeared to me as a revelation, for I then saw a way to combine my long-standing interest in Kant with my newer interest in the philosophy of space-time physics from Newton to Einstein. Kant – *of course!* – was centrally concerned with our representations of space and time, which serve for him (together with the categories) as an a priori framework underlying what virtually everyone during the eighteenth-century Enlightenment took to be our best example of rational and objective knowledge of the natural world, namely Newton's *Principia*. More generally, as Buchdahl himself had argued, the *Critique of Pure Reason* could be read in the context of the development of the modern philosophical and scientific tradition from Galileo and Descartes through the eighteenth-century Enlightenment, as philosophers and natural scientists together (often combined in the same person) struggled to adjust our conceptions of both nature and humanity to the profound intellectual and spiritual upheavals of the scientific revolution and its aftermath.

Nevertheless, although Buchdahl, unlike Strawson, gives a very significant role to Kant's philosophy of science, he still agrees with Strawson (and most traditional interpreters) in seeking sharply to separate the *Critique of Pure Reason* in particular from the Newtonian science of Kant's time. Buchdahl insists, more specifically, on a sharp distinction between ordinary and scientific experience, and, accordingly, he conceives the nature in general considered in the transcendental analytic of the *Critique* as a world of common-sense particulars constituted independently of scientific laws. He then conceives the world as it is described by modern scientific theories such as Newton's as a product of the regulative use of reason discussed in the transcendental dialectic, and it is only here, for Buchdahl, that properly scientific laws of nature come into play. Thus, while he of course acknowledges the importance of the *Metaphysical Foundations of Natural Science*, Buchdahl sees a significant "looseness of fit" between its project and that of the *Critique*. The specific scientific laws figuring prominently in the former work (such as Kant's three mechanical laws of motion) are merely *modeled* on the corresponding pure principles of the understanding articulated in the transcendental analytic (in this case the three analogies of experience), and such properly scientific laws of nature are in no sense intended to *follow* from the transcendental principles.

Buchdahl's sharp separation between ordinary and scientific experience is motivated, among other things, by a desire to make room for the later development of twentieth-century physics – and therefore for Kuhnian scientific revolutions – within the more general framework of the first *Critique*. The approach I began to develop after reading Buchdahl, by contrast, aimed to turn this perspective on its head. Against the background of my earlier work on the foundations of space-time physics from Newton to Einstein, I was forcibly struck, above all, by how deep Kant's insights into the presuppositions of Newtonian mathematical physics really were. It appeared to me that such depth of insight into the conceptual structure of the best available science of the time was an astonishing philosophical achievement all by itself, entirely independent of its relationship to the more modern scientific developments that were yet to come. Moreover, if one does want seriously to inquire into this relationship, I believe that the best way to proceed consists in carefully tracing out the way in which our modern (Einsteinian) conception of space, time, and motion (along with its corresponding philosophical motivations) is the result of a deep conceptual transformation that began with Kant's scientific situation at the end of the eighteenth century and concluded with the revolutionary new (Einsteinian) space-time theories characteristic of the early years of the twentieth.

My project here, however, concerns the interpretation of Kant in the intellectual context of his own time. And what is most distinctive of my approach is the central place I give to the *Metaphysical Foundations of Natural Science* within the philosophy of Kant's mature or "critical" period. I am convinced, in particular, that it is not possible adequately to comprehend this critical philosophy without paying very detailed and intensive attention to Kant's engagement with Newtonian science. Nevertheless, I do not wish to claim that the standpoint of the *Critique* is simply identical with that of the *Metaphysical Foundations*. On the contrary, the relationship between the former and the latter is mediated by what Kant himself calls the *empirical* concept of matter – a concept which, as such, belongs among neither the categories or pure concepts of the understanding nor the pure sensible concepts (e.g., geometrical concepts) employed in mathematics. Consequently, the propositions of what he calls pure natural science that Kant attempts to "prove" in the *Metaphysical Foundations* – on the basis of transcendental principles of the understanding, to which, in some cases, he explicitly appeals as premises – require for their derivation an additional specifically empirical element not found in the first *Critique*. The standpoint of the first *Critique*, on Kant's own account, is therefore significantly more abstract and general than that of the *Metaphysical Foundations*.

The precise relationship between the first *Critique* and the *Metaphysical Foundations*, and the precise sense of Kant's assertion that the concept of matter he develops in the latter work is an empirical concept, involve complex and subtle issues that can only be explored in detail in what follows. For now, however, I simply note that my approach to finding a central place for the *Metaphysical Foundations* within the critical philosophy proceeds by "triangulating" this work within a threefold philosophical and scientific context: (i) that created by the great turn of the century debate with Newton recorded in Leibniz's correspondence with Clarke, whose aftermath, from Kant's point of view, centrally involved the work of both

Leonhard Euler and Johann Heinrich Lambert; (ii) Kant's own intellectual development from the earlier (and more Leibnizean) metaphysics and natural philosophy of his "pre-critical" period; (iii) Kant's further development in the critical period from the first edition of the *Critique*, through both the *Prolegomena* and the *Metaphysical Foundations*, to the second edition. We thereby see, in much more detail and with much more precision than is possible otherwise, exactly how Kant's life-long struggle delicately to situate himself at the intersection of Leibnizean metaphysics and Newtonian physics is finally brought to a successful – and deeply revolutionary – conclusion in the critical period.

I proceed by what I have called a *reading* of Kant's text, which, as I understand it, is distinct from both a fully contextualized intellectual history and a more traditional line-by-line commentary. Thus, for example, while the first element of my triangulation begins with the stage-setting debate between Newton and Leibniz at the turn of the eighteenth century, I do not proceed by analyzing this debate in its own right and then tracing its influence throughout the century up to Kant. Nor do I comment upon each "explication" and "proposition" of the *Metaphysical Foundations* in turn, providing intellectual context and analysis as needed. Instead, I attempt to reconstruct what I understand to be Kant's main argument as it develops through all of its manifold twists and turns, where the evidence for my reconstruction is provided primarily by Kant's words themselves. I then introduce the first element of intellectual context in my triangulation (beginning with Newton and Leibniz, and continuing with such later figures as Euler and Lambert) when, in the course of my reconstruction, I find good reason to take Kant to be responding to or engaged with the work of one or another of these authors at some particular point in the argument – to have such works open on his desk, as it were, or at least in his mind. Indeed, the works that I list in Part II of my Bibliography (as primary sources other than Kant's own works) are limited to precisely these.

Similarly, while one might well take it to be the task of a traditional commentary to situate the author's analyses against the background of as much relevant secondary literature as possible, I have by no means attempted to do so here. Instead, I engage with secondary literature only to the extent that I have found it necessary and fruitful in order to clarify and develop various specific points in my reconstruction of Kant's argument. Thus, while I have learned much about Kant's philosophy of natural science from many more authors than those cited here, most of whom are cited in my earlier book on *Kant and the Exact Sciences*, published in

1992, the secondary literature with which I now explicitly engage comprises precisely those works (in addition to the primary sources) that I have had open on my desk (or at least in my mind) while writing this book. Accordingly, the works that I list in Part III of my Bibliography (as secondary sources) are limited to these.

My reconstruction of Kant's argument has resulted in a long and in some respects rather complicated book. This reflects the fact that Kant's treatise is extremely compressed, and my attempt to comprehend it involves the extended procedure of triangulation described above. I hope that reading the *Metaphysical Foundations* together with those elements of the surrounding context, which, in my reconstruction, I find beneath the surface, will greatly enhance our understanding of this text. Its structure and organization add a further layer of complexity. For, although Kant does present a continuous linear argument, earlier parts of the argument typically point towards later parts for their completion and full articulation. In this sense, the text is more "dialectical" than linear, in that the meaning and point of what Kant is saying at any given stage only becomes fully articulated at a later stage. My reading, which also unfolds in a linear sequence following the four principal chapters of Kant's text, thereby inherits this dialectical character; and I find it necessary, accordingly, to go back and forth repeatedly (mostly in footnotes and cross references) in considering earlier and later passages together.

However, there is one especially important part of the *Metaphysical Foundations* that I do not subject to a more or less linear reconstruction: the Preface or *Vorrede*. Here Kant discusses what he is doing from a much more general point of view, and he explicitly considers the relationship between the project of the *Metaphysical Foundations* (which he here calls the special metaphysics of corporeal nature) and that of the first *Critique* (which he here calls general metaphysics or transcendental philosophy). So the Preface, in this sense, stands outside of the main line of argument developed in the four succeeding chapters. Because of this, and because of the great importance of the question of the precise character of the relationship between Kant's special metaphysical foundations of natural science and the general metaphysics of the *Critique*, I consider central themes and passages from the Preface in two distinct steps that frame my reconstruction of the main argument – prospectively in my Introduction and retrospectively in the Conclusion.

The organization of my book into four main chapters preceded by an introduction and followed by a conclusion follows the structure of my reading. The four main chapters correspond to the four principal chapters of Kant's text – which themselves correspond, in turn, to the four main headings of the table of categories in the *Critique*. The consecutively numbered sections, however, reflect the structure of the continuous linear argument (cumulatively extending from chapter to chapter) that I find in the text. I thereby develop my reconstruction of this argument within the framework of Kant's architectonic – where, in particular, the final section in each of my four chapters concerns the relationship between this part of the argument and the corresponding categories.

My reading of Kant's treatise is Newtonian, in so far as I place Newton's *Principia* at the very center of Kant's argument. This much is signaled in the text of the *Metaphysical Foundations* by the circumstance that the name of Newton occurs far more often than that of any other author – and most of these references, in fact, are to the *Principia*. For this reason, among others, the idea that Newton's *Principia* is paradigmatic of the natural science for which Kant attempts to provide a metaphysical foundation has often been simply taken for granted – by both Buchdahl and myself, for example. More recent authors, however, have begun to challenge this idea and, in particular, have brought to light previously underemphasized connections between Kant's argument in the *Metaphysical Foundations* and the Leibnizean tradition in which he received his philosophical education. This development, I believe, has been a healthy one, and there is one important issue on which I have accordingly changed my views significantly. Whereas I (along with many others) had assumed that the three mechanical laws of motion Kant articulates in his third chapter or Mechanics correspond closely to Newton's three Laws of Motion, I have now been convinced by the work of Erik Watkins and Marius Stan that this was a mistake. I shall discuss the issue substantively in what follows, but here I want to insist that this recent work has not compromised my overriding emphasis on Newton's *Principia* in the slightest. On the contrary, the very close and detailed reading I now give of Kant's fourth chapter or Phenomenology is intended, among other things, to establish the depth and centrality of Kant's engagement with Book 3 of Newton's masterpiece beyond any reasonable doubt.

The first fruits of my study of the *Metaphysical Foundations of Natural Science* were presented in a series of seminars at the University of Western Ontario in the Spring of 1984, when I held a Canada Council Visiting Foreign Scholars Fellowship. I am grateful to William Demopoulos for nominating me for this Fellowship, and to the participants in these

seminars – which were initiated by Demopoulos and Robert E. Butts – for very valuable discussions and feedback. The result was my first publication on this topic, entitled "The Metaphysical Foundations of Newtonian Science," in a volume edited by Butts appearing in 1986 commemorating the bicentennial anniversary of the publication of the *Metaphysical Foundations*. I am very much indebted to both Butts and Demopoulos for providing me with this first opportunity to develop my ideas and for encouraging me in their further development thereafter. I continued to return to Western Ontario in subsequent years and to receive significant encouragement and feedback from a number of others there as well, including, especially, Richard Arthur, Robert DiSalle, William Harper, and Itamar Pitowsky.

In coming to terms with Kant's relation to Newton's *Principia* I have had the great good fortune of receiving invaluable help from perhaps the two leading philosophical Newton scholars of our time: Howard Stein and George E. Smith. Indeed, I moved to Chicago in 1982 largely to take advantage of Stein's deep knowledge of Newton, and I was not disappointed. Building on multiple readings of Stein's classic discussion of "Newtonian space-time," I was then able to interact with Stein himself and, for example, to attend his year-long course on the conceptual development of physics from Ancient astronomy through Einstein. Stein's work, more generally, provided the basis for my understanding of the conceptual framework for describing space, time, and motion that Newton had created – and, therefore, the basis for my understanding of Kant's treatment of space, time, and motion in the *Metaphysical Foundations*. I would not have been able even to get started in developing my reading of Kant's treatise without Stein's help and example.

If I could not have gotten started without Stein, I could not have finished without Smith. Smith's course on the *Principia* had become legendary, and I was therefore extremely pleased when we were able to bring him to Stanford as a Distinguished Visiting Professor in the Winter and Spring quarters of 2009. I learned more about the detailed internal workings of the *Principia* then than I could have possibly imagined. Moreover, at the end of his visit Smith did me the inestimable service of reading the then current draft of my manuscript with extraordinary patience and care, and of discussing my treatment of the relationship between Kant and Newton with me over a period of several weeks. These discussions provided the indispensable basis, in connection with this issue, for my rewriting of the manuscript in the following years – as will be readily apparent to any attentive reader of the final result.

I first became acquainted with Robert DiSalle during Stein's course on the development of physics mentioned above. I then got to know him much better at a conference on "Philosophical Perspectives on Newtonian Science" in 1987, where DiSalle commented on a paper of mine. This paper went on to become my second publication on the *Metaphysical Foundations* when it appeared, together with DiSalle's incisive comments, in a volume bearing the title of the conference in 1990. His comments, and our subsequent interactions, have been invaluable to me, as has the lasting philosophical friendship that we have enjoyed ever since. I was able to see much more of DiSalle after he subsequently moved to the University of Western Ontario, where, together with Demopoulos (one of my oldest philosophical friends), the three of us shared numerous profitable exchanges on the meaning and significance of conceptual foundations (and conceptual transformations) in the exact sciences from Newton and Kant to the present.

I have also enjoyed, for many years, lasting philosophical friendships with two leading scholars of Kant's scientific thought: Gordon Brittan and Erik Watkins. I have learned much from both of them and was particularly inspired at the beginning of my intellectual journey by Brittan's pioneering "analytical" approach to the subject in his 1978 book on *Kant's Philosophy of Science*. Watkins's later work on the Leibnizean background to the *Metaphysical Foundations* has left a significant imprint on the present book. In addition, both Brittan and Watkins read the penultimate version of my manuscript, and I am grateful to both for their supportive and helpful comments – which decisively influenced its final structure and content.

I was a Visiting Professor at the University of Konstanz in the Spring-Summer term of 1994, where I taught a course on *Kant's Philosophie der Physik* together with Martin Carrier. I had already become acquainted with Carrier through his own work on Kant's philosophy of physics, and I was then able to learn much more from him at Konstanz. His philosophical friendship has also been invaluable to me, and, more specifically, an exchange between the two of us concerning Kant's "mechanical estimation" of quantity of matter in the *Metaphysical Foundations* occupies a pivotal position in the reading that I develop here.

In the Fall of this same year I moved to the Department of History and Philosophy of Science at Indiana University, Bloomington, where I remained until finally moving to Stanford in 2002. During my years at Indiana I presented a seminar on the *Metaphysical Foundations* on a number of occasions, and to a number of gifted students with backgrounds in both philosophy and the history of science. Scott Tanona contributed an outstanding paper on Kant and Newton that has since appeared in *Philosophy of Science* and has significantly helped me in my further thinking. Mary Domski and Andrew Janiak are now well-known scholars of Newton and modern philosophy (including Kant) in their own right.

A particularly memorable year was 1998, when both Konstantin Pollok and Daniel Sutherland came to Bloomington as Visiting Scholars. Pollok was then in the process of completing his dissertation (later published as his *kritischer Kommentar* on the *Metaphysical Foundations*) at the University of Marburg, Sutherland in the process of completing his dissertation (on the role of Kant's concept of magnitude) at the University of California at Los Angeles. I formed lasting philosophical friendships with both of them, and the work of both figures prominently in my book.

It was during these last years at Bloomington that I began the serious writing of what eventually became this book. One of the first new steps I took was to engage in detail with the second or Dynamics chapter of Kant's treatise, and I was immediately struck by the stark contrast between the atomism of discrete point-centers developed in his pre-critical version of a dynamical theory of matter and the new view of matter as a true continuum (substantially present in each part of the space that it occupies) developed in the *Metaphysical Foundations*. I had fruitful discussions about this with my old friend Mark Wilson, who, although no Kantian, is a devoted student of continuum mechanics. His help and advice on this topic was invaluable, and it led to a more extensive (and fruitful) engagement with Euler's early work on the subject than I had previously envisioned.

Since arriving at Stanford I have presented my seminar on the *Metaphysical Foundations* on several more occasions, while I continued to work on my manuscript. I have been fortunate to have been involved with the dissertations of a number of outstanding students with serious interests in Kant (and the *Metaphysical Foundations*) here as well, including Ludmilla Guenova, Teru Miyake, Samuel Kahn, and Tal Glezer. Special thanks are due to Dustin King, who took my seminar while still an undergraduate and contributed an extraordinary paper on Kant's use of the mathematical method in the *Metaphysical Foundations* that is still influencing my thinking. Moreover, the seminar completed in the Winter quarter of 2011–2012 was particularly important, since I there distributed (almost) final versions of my chapters from week to week and thereby received valuable feedback. I am grateful, in this connection, to all those who participated; and I owe special thanks to two students – Greg Taylor and Paul Tulipana – who prepared the index.

The most important and substantial revisions of my manuscript were accomplished in the academic year 2010–11 at the Max Planck Institute for the History of Science in Berlin. I am indebted to the many scholars I encountered there, and particularly to the three Directors at the time, Hans-Jörg Rheinberger, Lorraine Daston, and Jürgen Renn, who kindly provided me with successive positions in their three Departments as a Visiting Scholar. I am especially indebted, however, to Vincenzo De Risi, who was leading a research group at the Institute and whom I had earlier met as an outside examiner of his dissertation at the University of Pisa. This dissertation, a deeply original study of Leibniz's geometry and monadology, was published (in English) in 2007 and has since (and justly) attracted considerable attention. I am indebted to it personally for a significantly improved understanding of the relationship between the Leibnizean–Wolffian philosophy of Kant's time and Leibniz himself. De Risi is now embarked on a study of later developments in the philosophy of space and geometry, including Kant's, and we had many extremely fruitful conversations during my year in Berlin. De Risi also provided me, during this same year, with very helpful comments on my manuscript.

Towards the end of my year in Berlin I met Marius Stan, who spent time in De Risi's research group on the history and philosophy of geometry and the concept of space. It was during this time that I read (and discussed with him) Stan's important paper on Kant's Third Law of Mechanics, which exerted a significant influence (as already suggested) on my current understanding of Kant's three laws. I was also stimulated by discussions with Stan concerning his work in progress on Kant's treatment of rotation in the Phenomenology to clarify my own views on this central but difficult topic.

The last – but by no means least – important experience during my year in Berlin involved becoming reacquainted with Daniel Warren. I had first become acquainted with him many years ago, when I taught at the University of Pennsylvania in the late 1970s and he was a medical student there. He attended my course on the first *Critique*, and I was immediately struck by his philosophical talent. He later obtained a doctorate in philosophy from Harvard, and I had the privilege of serving on his committee. Warren's dissertation, completed in 1994, has since appeared in the Routledge Outstanding Dissertations Series and has now become a classic study of Kant's Dynamics – its influence is also very clear and present in my book. But what was most inspirational for me during my year in Berlin were a number of intensive conversations I had with Warren concerning the crucial question of the precise relationship between the *Metaphysical Foundations* and the first *Critique*. The conception that I ultimately arrived at and developed (in my Conclusion) of the different yet complementary perspectives of the two works was stimulated by these memorable conversations.

My year in Berlin was supported by a Research Award from the Alexander von Humboldt Foundation, for which I was nominated (and later sponsored) by two of my closest German philosophical friends: Carrier (now at Bielefeld) and Gereon Wolters (at Konstanz). The previous research leave during which I made significant progress on the manuscript, in the academic year 2006–7, was supported by a Fellowship at the Center for Advanced Study in the Behavioral Sciences. I am grateful to both the Humboldt Foundation and the Center for Advanced Study, and also for the additional support during these two years provided by Stanford University, for making it possible for me to bring this long intellectual journey to a conclusion.

My final and most important debt of gratitude, however, is to Graciela De Pierris, with whom I enjoy much more than a lasting philosophical friendship. We first met, appropriately, at a conference on the history and philosophy of mathematics at Indiana University in the Fall of 1984, when she was a member of the faculty in the Department of Philosophy there, and we have been philosophical partners and life partners ever since. We both are devoted students and admirers of Kant's philosophy, although, when we first met, our approaches diverged significantly. Our paths have since begun to converge (although by no means monotonically), as she has come increasingly to appreciate the scientific dimension of Kant's thought and I have come increasingly to appreciate its transcendental dimension. I hope that I have achieved a satisfactory balance between the two in this book, and, if I have, it is due primarily to her philosophical penetration, wise advice, and unstinting support and encouragement throughout these years.

Introduction: The place of the Metaphysical Foundations *in the critical system*

The *Metaphysical Foundations of Natural Science* appeared in 1786, at the height of the most active decade of Kant's so-called "critical" period – which began with the first edition of the *Critique of Pure Reason* (1781) and included the *Prolegomena to any Future Metaphysics* (1783), the *Groundwork of the Metaphysics of Morals* (1785), the second edition of the *Critique of Pure Reason* (1787), the *Critique of Practical Reason* (1788), and finally the *Critique of the Power of Judgement* (1790). The *Metaphysical Foundations*, however, is by far the least well known of Kant's critical writings. Although it has received some significant scholarly attention during the more than 200 years that have elapsed since its first appearance, it has received far less than most other Kantian works, including some of his earlier writings on natural science and metaphysics from the "pre-critical" period. The reasons for this are not far to seek. The *Metaphysical Foundations* is a particularly dense and difficult work, even by Kantian standards. It is engaged with relatively technical problems in the foundations of the physical science of Kant's time, and, what is worse, it is structured in a forbidding quasi-mathematical style via "definitions" ("explications"), "propositions," "proofs," "remarks," and so on. I believe, nonetheless, that this little treatise of 1786 is one of the most important works in Kant's large corpus, and, in particular, that it is impossible fully to understand the theoretical philosophy of the critical period without coming to terms with it. Kant himself addresses the question of the place of the *Metaphysical Foundations* within the critical system most explicitly in his Preface, and so my discussion of this issue will also serve as my initial reading of the Preface.[1]

[1] As explained in my Preface, I shall discuss the Preface to the *Metaphysical Foundations* in two stages: here in the Introduction and then (retrospectively) in the Conclusion.

THE *METAPHYSICAL FOUNDATIONS* AND THE *CRITIQUE OF PURE REASON*

The *Metaphysical Foundations* is centrally implicated in the important changes Kant made between the first and second editions of the *Critique*. In January of 1782, soon after the appearance of the first edition, a highly critical review contributed by Christian Garve and revised by the editor J. G. Feder was published in the *Göttinger Anzeigen von gelehrten Sachen*.[2] This review, as is well known, maintained that what Kant had produced is simply a new version of an old doctrine – a version of psychological or subjective Berkeleyean idealism. Kant, not surprisingly, was displeased, and his very next statement of the critical philosophy, in the *Prolegomena*, was clearly intended, at least in part, to answer this charge of subjective idealism. Indeed, the appendix to the *Prolegomena*, "On what can be done to make metaphysics as a science actual," is almost exclusively devoted to a reply to the Garve–Feder review.

Kant attempts, in particular, conclusively to differentiate his view from Berkeley's by focussing on the critical doctrine of space (together with that of time):

> I show, by contrast [with Berkeley], that, in the first place, space (and also time, which Berkeley did not consider) together with all of its determinations can be cognized by us a priori, because it, as well as time, inheres in us prior to all perception, or experience, as pure form of our sensibility, and makes possible all sensible intuitions and therefore all appearances. It follows [in the second place] that, since truth rests on universal and necessary laws, as its criterion, experience for Berkeley can have no criterion of truth – for the appearances (for him) had nothing a priori at their basis, from which it then followed that they are nothing but mere semblance [*Schein*]. By contrast, for us space and time (in combination with the pure concepts of the understanding) prescribe their law a priori to all possible experience, which, at the same time, yields the secure criterion of truth for distinguishing, within experience, truth from semblance. (4, 375)

Kant continues by asserting that his "so-called (properly critical) idealism is thus of an entirely peculiar kind, in such a way, namely, that it overturns the customary [idealism], [so] that through it all a priori cognition, even that of geometry, first acquires objective reality," and he therefore begs permission to call his philosophy "formal, or better critical idealism, in order to distinguish it from the dogmatic [idealism] of Berkeley and the skeptical [idealism] of Descartes" (4, 375).

[2] A translation can be found in Kant (2004a).

The second edition of the *Critique* appeared four years later. Here Kant extensively revised some of the most important (and most difficult) chapters of the book: the transcendental deduction of the categories, the paralogisms of pure reason, and the system of the principles of pure understanding. The former two chapters are completely rewritten. In the case of the principles chapter the revisions are not as extensive, but Kant did add two entirely new sections: the famous refutation of idealism and a general remark to the system of principles, which, among other things, is intended "to confirm our previous refutation of idealism" (B293).[3] Moreover, Kant also substantively revised the structure of the transcendental aesthetic by separating two distinct lines of argument with respect to both space and time: a "metaphysical exposition," which articulates the synthetic a priori character of the representation in question (space or time) by elucidating "what belongs to it" in so far as it is "*given a priori*" (B37–38), and a "transcendental exposition," which demonstrates the synthetic a priori character of the representation in question by showing that only on this assumption is a certain body of assumed synthetic a priori knowledge possible (B40).

In the case of the transcendental exposition of the concept of space, of course, the synthetic a priori science in question is geometry (B40): "a science that determines the properties of space synthetically and yet a priori." It is important to note, however, that the science of geometry thereby enables us (synthetically and yet a priori) to determine the objects of outer intuition – the appearances of outer sense – as well (B41):

> Now how can an outer intuition dwell in the mind that precedes the objects themselves and in which the concept of the latter can be a priori determined? Obviously not otherwise except in so far as it has its seat merely in the subject, as its formal constitution to be affected by objects, and thereby to acquire an *immediate representation*, i.e., *intuition*, of them, and thus only as the form of outer *sense* in general.

Thus Kant's argument for transcendental or "formal" idealism here depends, just as much as in the *Prolegomena*, on the idea that the synthetic

[3] The text of the refutation of idealism begins by echoing the remarks on "formal" idealism (in connection with both Berkeley and Descartes) from the appendix to the *Prolegomena* (B274):

> Idealism (I mean *material* [idealism]) is the theory that declares the existence of objects in space outside us to be either merely doubtful and *indemonstrable* or false and *impossible*. The *first* is the *problematic* [idealism] of *Descartes*, which declares only a single empirical assertion – namely, *I think* – to be indubitable; the *second* is the *dogmatic* [idealism] of *Berkeley*, which declares space, and with it all things to which it attaches as an inseparable condition, to be something impossible in itself, and therefore also [declares] the things in space to be mere figments of the imagination.

a priori representation of space, along with the synthetic a priori science of geometry, plays a crucial role in making experience or *empirical* knowledge first possible.[4]

In the general remarks to the transcendental aesthetic (§8) Kant clinches his argument for transcendental idealism – so as to make its certainty "completely convincing" – by choosing "a case whose validity can become obvious" (A46/B63–64). This, once again, is the case of space and geometry, which (Kant adds in the second edition) "can serve to clarify what has been adduced in §3 [namely, the transcendental exposition]" (B64). And the point, in harmony with §3, is that only on the assumption of transcendental idealism is synthetic a priori geometrical knowledge of the objects of outer intuition possible (A48/B66): "If space (and thus also time) were not a mere form of your intuition, which contains a priori conditions under which alone things can be outer objects for you, without which subjective conditions they are nothing in themselves; then you could a priori constitute nothing at all about outer objects synthetically." Geometry is a synthetic a priori science, in other words, precisely because our pure intuition of space is a subjectively given a priori condition for all appearances or objects of experience.[5]

The changes introduced in the second edition of the *Critique*, following the *Prolegomena*, are intended further to delimit Kant's view from subjective idealism. They do this, in particular, by emphasizing the importance of the representation of space (and thus geometry) in Kant's system, together with the circumstance that what Kant means by

[4] This point illuminates, and is illuminated by, what Kant says in the aesthetic prior to the transcendental exposition. The introduction to the metaphysical exposition states (A22/B37): "By means of outer sense (a property of our mind) we represent objects to ourselves as outside of us, and all of these *in space*. Therein is their figure, magnitude, and relation to one another determined, or determinable." The conclusion of the first argument then asserts (A23/B38): "Therefore, the representation of space cannot be obtained from the relations of outer appearance through experience; rather, this outer experience is itself only possible in the first place by means of the representation in question." The conclusion of the second argument similarly asserts (A24/B38): "Space is a necessary a priori representation, which lies at the basis of all outer intuition … It must therefore be viewed as the condition of the possibility of appearances, not as a determination depending on them, and is an a priori representation, which necessarily lies at the basis of outer appearances."

[5] Kant makes this explicit in the immediately following (and concluding) sentence (A48–49/B66):

> It is thus indubitably certain, and not merely possible, or even probable, that space and time, as the necessary conditions of all (outer and inner) experience, are merely subjective conditions of all our intuition, in relation to which therefore all objects are mere appearances and not things in themselves given in this manner, about which much a priori can also be said in reference to what pertains to their form, but never the least about the things in themselves that may lie at the basis of these appearances.

"appearances" includes – indeed centrally includes – material physical bodies located outside me in space. This is especially true of the refutation of idealism, of course, which argues that even my knowledge of my own mental states in inner sense is only possible on the basis of my perception (my *immediate* perception) of external material bodies located outside my mind in outer sense. And the more general point, as we have seen, is that space and geometry play a privileged constitutive role in making experience or empirical knowledge first possible. In terms of the constitution of *experience*, therefore, outer sense is prior to inner sense. As Kant explains in the preamble to the refutation of idealism, his proof aims to show "that even our *inner* experience (which was not doubted by Descartes) is only possible under the presupposition of *outer* experience" (B275).[6]

Kant explains in the Preface to the *Metaphysical Foundations* that the task of this work is to delineate the a priori principles governing what Kant calls the *doctrine of body* – which depends, more generally, on "the form and the principles of outer intuition" (4, 478). Kant begins the Preface by asserting (467) that nature, in its "*material* meaning," "has two principal parts, in accordance with the principal division of our senses, where the one contains the objects of the *outer* senses, the other the object of *inner* sense." "In this meaning," Kant continues, "a twofold doctrine of nature is possible, the *doctrine of body* and the *doctrine of the soul*, where the first considers *extended* nature, the second *thinking* nature" (467). (Note here the clear echo of Descartes.) At least in principle, therefore, two different branches of the metaphysics of nature are possible – two different species (470) of "*special* metaphysical natural science (physics or psychology), in which the above transcendental principles [of the first *Critique*] are applied to the two species of objects of our senses."[7] It turns out, however,

[6] As Kant explains in the aesthetic, there is another sense in which time, as the form of inner sense, is prior to space (A34/B50):

> *Time* is the formal a priori condition of all appearances in general. Space, as the pure form of all outer intuition is, as a priori condition, limited merely to outer appearances. By contrast, because all representations, whether they have outer things as object or not, nevertheless belong in themselves, as determinations of the mind, to our inner state; and because this inner state belongs under the formal condition of inner intuition, and thus to time; [it follows that] time is an a priori condition of all appearances in general – the immediate condition of inner appearances (of our soul) and precisely for this reason the mediate condition of outer appearances as well.

This, however, is a priority with respect to intuition or appearance, not a priority with respect to experience. Thus Kant also asserts, in the second edition aesthetic, that "within [inner intuition] the representations of the *outer senses* constitute the proper material with which we occupy our mind" (B67).

[7] More precisely, according to the paragraph of which this sentence is the conclusion, there is a more general species of the metaphysics of nature (general metaphysics or transcendental

that only the special metaphysics of *corporeal* nature can serve to ground a genuine science.

Kant articulates the reason for this in the following paragraph, which begins with the statement (470) that "in any special doctrine of nature there can be only as much *proper* science as there is *mathematics* to be found therein" and concludes with the claim that "since in any doctrine of nature there is only as much proper science as there is a priori knowledge therein, a [special] doctrine of nature will contain only as much proper science as there is mathematics capable of application there" (470).[8] Kant goes on to argue that chemistry (unlike pure physics or the mathematical theory of motion) will "only with great difficulty" ever become a proper science (470–71) and that the situation is even worse in psychology:

> Yet the empirical doctrine of the soul must remain even further from the rank of a properly so-called natural science than chemistry. In the first place, because mathematics is not applicable to the phenomena of inner sense and their laws, the only option one would have would be to take the *law of continuity* in the flux of inner changes into account – which, however, would be an extension of cognition standing to that which mathematics provides for the doctrine of body approximately as the doctrine of the properties of the straight line stands to the whole of geometry. For the pure intuition in which the appearances of the soul are supposed to be constructed is *time*, which has only one dimension … Therefore, the empirical doctrine of the soul can never become anything more than an historical doctrine of nature, and, as such, a natural doctrine of inner sense which is as systematic as possible, that is a natural description of the soul … This is also the reason for our having used, in accordance with common custom, the general title of natural science for this work, which actually contains the principles of the doctrine of body, for only to it does this title belong in the proper sense, and so no ambiguity is thereby produced. (471)

In other words, since geometry cannot apply in any substantive way to the object of inner sense, there can be no proper science of this object (the soul). Consequently, there can be no metaphysical foundations of natural science applying specifically to the soul – no Kantian explanation of how our supposed knowledge of the soul is grounded in a priori principles governing both concepts and intuitions. Our empirical knowledge of the contents of inner sense, to the extent that we have such knowledge, rather presupposes (like all empirical knowledge or experience in general) "the form and the principles" of *outer* intuition.

philosophy) of which the two species of special metaphysics are *subspecies*. I shall return to this paragraph below.

[8] I shall return below to the difficult argument on behalf of these claims presented in the body of the paragraph.

To see the connection between this argument and the refutation of idealism articulated in the second edition of the *Critique*, it is helpful to look at the argument Kant provides in the general remark to the system of principles as confirmation of the refutation of idealism. In this later argument Kant has already stated (B288) that there is something "remarkable" in the circumstance "that we cannot comprehend the possibility of things in accordance with *the mere category*, but must rather always have an intuition at hand in order to establish the objective reality of the pure concept of the understanding." He now goes further by emphasizing the need for specifically *spatial* intuitions:

> It is even more remarkable, however, that, in order to understand the possibility of things in accordance with the categories, and thus to verify the *objective reality* of the latter, we require not merely intuitions, but always even *outer intuitions*. If, for example, we take the pure concepts of *relation*, we find, first, that in order to supply something *permanent* in intuition corresponding to the concept of *substance* (and thereby to verify the objective reality of this concept), we require an intuition *in space* (of matter), because space alone is determined as permanent, but time, and thus everything in inner sense, continually flows. (B291)

This argument is clearly reminiscent of that of the refutation of idealism.[9]

Unlike in the refutation, however, Kant now elaborates the argument in terms of all three categories of relation: substance, causality, and community. After his discussion of the category of causality (to which I shall return below), Kant describes (B292) the "proper ground" for the necessity of specifically outer intuition in this case as the circumstance "that all alteration presupposes something permanent in intuition, in order even to be perceived as alteration itself, but in inner sense no permanent intuition at all is to be found." The point of the remark is then summed up as follows (B293–94): "*This entire remark is of great importance*, not only in order to confirm our previous refutation of idealism, but even more so [later], when we will speak of *self-knowledge from mere inner consciousness* and the determination of our nature without the assistance of outer empirical intuitions, in order to indicate the limits of the possibility of such knowledge." Kant thereby points forward to the discussion of the

[9] This becomes especially clear in the second remark to the refutation of idealism, where Kant asserts (B278) that "we have absolutely *nothing permanent*, which could underlie the concept of *a substance*, as intuition, *except merely matter*, and even this permanent is not derived from outer experience but is rather presupposed a priori as a necessary condition of all *determination of time*, and thus as a determination of *inner sense* through *the existence of outer things* with respect to *our own existence*."

(second edition) paralogisms, as well as backward to the refutation of idealism.

But there is an important passage towards the end of the Preface to the *Metaphysical Foundations* of which this discussion in the general remark is a clear echo. After carefully distinguishing between general metaphysics or transcendental philosophy and the special metaphysics of (corporeal) nature, Kant continues:

> It is also indeed very remarkable (but cannot be expounded in detail here) that general metaphysics, in all instances where it requires examples (intuitions) in order to provide meaning for its pure concepts of the understanding, must always take them from the general doctrine of body, and thus from the form and the principles of outer intuition; and, if these are not exhibited completely, it gropes uncertainly and unsteadily among mere meaningless concepts ... [here] the understanding is taught only by examples from corporeal nature what the conditions are under which such concepts can alone have objective reality, that is, meaning and truth. And so a separated metaphysics of corporeal nature does excellent and indispensable service for *general* metaphysics, in that the former furnishes examples (instances *in concreto*) in which to realize the concepts and propositions of the latter (properly speaking, transcendental philosophy), that is, to give a mere form of thought sense and meaning. (478)

Where this matter *is* "expounded in detail," it appears, is precisely the general remark added to the second edition of the *Critique*. And there is thus a significant connection indeed, I believe, between the argument concerning the priority of outer sense for experience developed in the refutation of idealism and the argument we have been considering from the Preface to the *Metaphysical Foundations* – according to which only the metaphysics of *corporeal* nature is capable of grounding a genuine natural science.[10]

In order properly to appreciate this point, however, we need also to observe that the a priori experience-constituting principles derived from "the form and the principles of outer intuition" include not only spatial geometry but also what Kant variously calls the "pure" or "general" or "mathematical" *doctrine of motion* [*Bewegungslehre*]. Indeed, when Kant, in the Preface, comes to describe how the *Metaphysical Foundations* will actually carry out the program of a special metaphysics of corporeal

[10] Although there is such a connection, I believe, between the argument of the Preface to the *Metaphysical Foundations* and the refutation of idealism, there remain crucially important differences between the two. I shall return to a consideration of these important differences (which are closely related, in turn, to the more general differences, for Kant, between the perspectives of the *Metaphysical Foundations* and the first *Critique*) in the Conclusion. I am especially indebted to illuminating conversations with Daniel Warren for a better appreciation of these differences.

nature, he begins by placing this doctrine of motion at the basis (476–77): "The basic determination of something that is to be an object of the outer senses had to be motion, because only thereby can these senses be affected. The understanding traces back all other predicates of matter belonging to its nature to this one, and so natural science is either a pure or applied *doctrine of motion* throughout."[11] Kant continues by stating (477) that "[t]he *metaphysical* foundations of natural science are therefore to be brought under *four* chapters" – arranged in accordance with the table of categories – where each chapter adds a new aspect or "determination" to the concept of motion. In particular, the first chapter or Phoronomy begins by defining or explicating the concept of matter – that which is to be "an object of the outer senses" – as *the* ***movable*** *in space* [*das Bewegliche im Raume*]. In addition, Kant makes it clear at the end of the Preface that the doctrine of motion he has in mind here is a "*mathematical* doctrine of motion [*mathematische Bewegungslehre*]" (478, emphasis added).

According to the transcendental exposition of the concept of *time* added to the second edition of the *Critique* (§5) it is precisely this mathematical theory that stands to the concept of time as geometry stands to the concept of space (B48–49):

> Here I may add that the concept of alteration and, along with it, the concept of motion (as alteration of place) is possible only in and through the representation of time: so that, if this representation were not an a priori (inner) intuition, no concept, whatever it might be, could make an alteration – i.e., the combination of contradictorily opposed predicates (e.g., the being and not-being of one and the same thing at one and the same place) – conceivable. Only in time can two contradictorily opposed determinations in one thing be met with, namely, *successively*. Therefore, our concept of time explains as much synthetic a priori knowledge as is set forth in the general doctrine of motion [*allgemeine Bewegungslehre*], which is by no means unfruitful.[12]

As we shall see, the *general doctrine of motion* to which Kant refers here (and which, he laconically remarks, "is by no means unfruitful") is the mathematical theory of motion Newton develops in the *Principia*.

[11] It is presumably the *pure* doctrine of motion that figures in what Kant calls *pure* natural science. I shall discuss Kant's cryptic argument for the priority of motion here ("because only thereby can these senses be affected") in my chapter on the Phoronomy below.

[12] The passage corresponding to the transcendental expositions of space and time in §10 of the *Prolegomena* reads (4, 283): "Geometry takes as its basis the pure intuition of space. Arithmetic produces even its concepts of number by the successive addition of units in time – above all, however, pure mechanics can produce its concepts of motion only by means of the representation of time." Here "pure mechanics [*reine Mechanik*]" appears to correspond to the "general doctrine of motion [*allgemeine Bewegungslehre*]" in the second edition of the *Critique*.

Kant's emphasis on the concept of *succession* in the transcendental exposition suggests that the general doctrine of motion is important not only in the transcendental aesthetic but also in the transcendental analytic as well – and, in particular, that it is intimately connected with the concept of causality.[13] This suggestion is confirmed in the discussion of causality in the general remark to the system of principles that immediately follows the treatment of the concept of substance (the passage at B291 quoted above):

> Second, in order to exhibit *alteration*, as the intuition corresponding to the concept of *causality*, we must take motion, as alteration in space, for the example. Indeed, it is even the case that we can make alteration intuitive to ourselves solely in this way, as no pure understanding can conceive its possibility. Alteration is the combination of contradictorily opposed determinations in the existence of one and the same thing. How it may now be possible that an opposed state follows from a given state of the same thing is not only inconceivable to any reason without example, but is not even understandable without intuition – and this intuition is the motion of a point in space, whose existence in different places (as a sequence of opposed determinations) alone makes alteration intuitive to us in the first place. For, in order that we may afterwards make even inner alterations intuitive, we must make time, as the form of inner sense, intelligible figuratively as a line – and inner alteration by the drawing of this line (motion), and thus the successive existence of our self in different states by outer intuition. (B291–92)

Kant here, once again, suggests a connection between the concept of alteration (and the concept of causality) and the refutation of idealism. Now, however, it appears that a deeper ground for the priority of space in the constitution even of inner experience is that an a priori basis for specifically *temporal* experience depends on the general doctrine of motion – where the concept of motion "unites" time with space (A41/B58).[14]

[13] The *schema* of the concept of causality, for Kant, consists in "the succession of the manifold, in so far as it is subject to a rule" (A144/B183).

[14] I shall return to the full passage below. The idea that time might require an outer (spatial) representation is introduced as early as the first edition transcendental aesthetic (A33/B50):

> [P]recisely because this inner intuition [that is, time] provides no figure, we seek to make up for this lack by analogies, and we represent the temporal sequence by a line progressing to infinity, in which the manifold constitutes a series that is of only one dimension, and we infer from the properties of this line to all the properties of time – except in the case of the single [property] that all parts of the former are simultaneous, but those of the latter are always successive.

In §24 of the second edition transcendental deduction Kant introduces the idea of an "outer *figurative* representation of time" explicitly (B154, emphasis added), states the need for this representation in much stronger terms, and is also explicit that the representation in question essentially involves motion (and therefore succession) (B156, emphases added): "[W]e can make time representable to ourselves *in no other way* than under the image of a line, in so far as we *draw* it, without which mode of presentation we could in no way cognize the unity of its measure or dimension [*Einheit ihrer Abmessung*]." Kant immediately goes on to indicate a connection between this point and the argument of the refutation of idealism that is yet to come.

That the pure (or general or mathematical) doctrine of motion is thereby connected with the category of causality, and thus with the analogies of experience, clarifies the sense in which this doctrine figures crucially in the a priori grounding of *experience* in Kant's technical sense. According to the general principle of the [three] analogies in the second edition (B217): "*Experience is only possible through the representation of a necessary connection of perceptions.*" Thus experience of appearances adds the representation of necessary connection to the mere intuition or perception of appearances (in accordance with the axioms of intuition and the anticipations of perception), and it thereby subjects the appearances to necessary laws of nature – including, first and foremost, the analogies of experience themselves (compare A216/B263). This is why, in the passage from the *Prolegomena* with which we began (4, 375), Kant says that "since truth rests on universal and necessary laws, as its criterion, experience for Berkeley can have no criterion of truth," whereas, for Kant himself, "space and time (in combination with the pure concepts of the understanding) prescribe their law a priori to all possible experience." That space and time can prescribe their law a priori to all possible experience only in combination with the pure concepts of the understanding (especially the relational categories issuing in the analogies of experience) is the central reason, as we shall see, that pure natural science requires not solely a mathematical but also a *metaphysical* a priori grounding.

KANT, NEWTON, AND LEIBNIZ

Kant's argument for "formal" or transcendental idealism in the transcendental aesthetic is framed by the conflict between the Newtonian and Leibnizean conceptions of space and time – a conflict that was very well known at the time through the *Leibniz–Clarke Correspondence* (1715–17). The aesthetic begins (in the discussion of space) by indicating three possible alternatives (A23/B37–38):

> What, now, are space and time? Are they actual beings? Are they only determinations, or even relations of things, but in such a way that they would pertain to them also in themselves, even if they were not intuited? Or are they such that they only attach to the form of intuition alone, and thus to the subjective constitution of our mind, without which these predicates cannot be attributed to any thing at all?

In the remainder of the aesthetic the last alternative – Kant's alternative – emerges as the only possible conception not subject to the overwhelming difficulties afflicting the other two. The crucial passage begins (A39–40/B56–57) by describing the conflict between "the mathematical investigators of nature [*mathematische Naturforscher*]" and "some metaphysical students of nature." The former "assume two eternal and infinite non-things [*Undinge*] subsisting in themselves, which are there (without there being anything actual), only in order to contain all actuality within themselves." For the latter, by contrast, "space and time are taken to be relations between appearances (next to or after one another), abstracted from experience, although in the abstraction represented confusedly."

Kant's argument for his own position – transcendental idealism – then follows:

> The [mathematical investigators] gain this much, that they make the field of appearances free for mathematical assertions. On the other hand, they confuse themselves very much by precisely these conditions when the understanding pretends to extend beyond this field. The [metaphysical students] gain much in the latter respect, namely, the representations of space and time do not get in the way when they wish to judge of objects not as appearances but merely in relation to the understanding; however, they can neither give an account of the possibility of a priori mathematical cognitions (in so far as they lack a true and objectively valid a priori intuition) nor bring empirical propositions into necessary agreement with these [mathematical] assertions. In our theory of the true constitution of these two original forms of sensibility both difficulties are remedied. (A40–41/B57–58)

Only Kant's transcendental idealism – his conception of "a true and objectively valid a priori intuition" serving as the form of our sensibility – can give a proper foundation for the necessary application of mathematics to nature while simultaneously avoiding the error of projecting space and time into the realm of the supersensible (God and the soul). This last error, as we shall see, prominently involves the Newtonian conception of divine omnipresence.

In the *Inaugural Dissertation* (1770), where he first introduces his doctrine of sensibility and transcendental idealism, Kant is more explicit about both the identity of his targets and the nature of his criticism. This is especially clear, for example, in the corresponding argument concerning space (§15, D):

> *Space is not something objective and real*, nor is it a substance, nor an accident, nor a relation; it is, rather, subjective and ideal; it issues from the nature of the mind in accordance with a stable law as a schema, so to speak, for coordinating everything that is sensed externally. Those who defend the reality of space either conceive of it as an *absolute* and boundless *receptacle* of possible things – an opinion

> which finds favor among the English – or they contend that it is the relation *itself* which obtains between existing things, and which vanishes entirely when the things are taken away, and which can only be thought as existing between actual things – an opinion which most of our own people, following Leibniz, maintain. As for the first empty invention of reason: since it invents an infinite number of true relations without any existing beings that are related to one another, it belongs to the world of fable. But the error into which those who adopt the second opinion fall is much more serious. Specifically, the proponents of the first view only put a slight impediment in the way of certain concepts of reason, or concepts relating to noumena, and which are in any case particularly inaccessible to the understanding, for example, questions about the spiritual world, omnipresence, etc. The proponents of the second view, however, are in headlong conflict with the phenomena themselves, and with the most faithful interpreter of all phenomena, geometry. (2, 403–4)[15]

Kant thus asserts much more explicitly, in particular, that one of his main objections to the Newtonian view concerns the concept of divine omnipresence.[16]

Kant's philosophical ambition to find a middle way between Newton and Leibniz, between the representatives of geometry and those of metaphysics, extends all the way back into the pre-critical period, before he had

[15] In the corresponding argument concerning time (§14, 5) Kant's description of his two opponents is also revealing (2, 400):

> Those who assert the objective reality of time either conceive of it as some continuous flux within existence, and yet independently of any existing thing (a most absurd invention) – a view maintained, in particular, by the English philosophers; or else they conceive of it as something real that has been abstracted from the succession of internal states – the view maintained by Leibniz and his followers.

There follows a very interesting and important argument concerning *simultaneity*, to which I shall return below.

[16] In the second edition version of the transcendental aesthetic Kant adds a remark that makes this point more explicit as well (B71–72):

> In natural theology, where one thinks an object that is not only no object of sensible intuition for us, but cannot even be an object of sensible intuition for itself, one takes care to remove the conditions of space and time from all of its intuition (for all of its cognition must be intuition and not *thought*, which is always a manifestation of limitations). But with what right can one do this, if one has previously made both into forms of things in themselves – and, indeed, into forms which, as a priori conditions of the existence of things, even remain when one has annihilated the things themselves? (For, as conditions of all existence in general, they must also be conditions for the existence of God.) There is therefore no alternative, if one does not pretend to make them into objective forms of all things, except to make them into subjective forms of our outer and inner mode of intuition. [This kind of intuition] is called sensible, because it is *not original* – i.e., it is not such that the existence of objects of intuition is itself given through it (which, as far as we can comprehend, can only pertain to the primordial being), but it depends on the existence of the objects, and is thus only possible in so far as the representative faculty of the subject is affected by them.

ever conceived the doctrine of transcendental idealism. The most important of Kant's pre-critical works in this respect (and the one most closely resembling the *Metaphysical Foundations*) is the *Physical Monadology* of 1756, which is intended to demonstrate *The Use in Natural Philosophy of Metaphysics Combined with Geometry*. The main problem considered is how to reconcile the existence of absolutely simple material substances – physical monads – with the infinite divisibility of space, and Kant articulates the problem very sharply in the Preface:

> But how, in this business, can metaphysics be married to geometry, when it seems easier to mate griffins with horses than to unite transcendental philosophy with geometry? For the former peremptorily denies that space is infinitely divisible, while the latter, with its usual certainty, asserts that it is infinitely divisible. Geometry contends that empty space is necessary for free motion, while metaphysics hisses the idea off the stage. Geometry holds universal attraction or gravitation to be hardly explicable by mechanical causes but shows that it derives from forces that are inherent in bodies at rest and act at a distance, whereas metaphysics dismisses the notion as an empty delusion of the imagination. (1, 475–76)

The Leibnizean–Newtonian background stands out particularly clearly. For, on the one hand, the whole problem is to reconcile the substantial simplicity of physical monads with geometrical infinite divisibility, and, on the other, the opposing view of "geometry" is committed to universal gravitation acting immediately at a distance.[17]

In the Preface to the *Metaphysical Foundations* Kant does not mention Leibniz, but the name of Newton figures prominently. Indeed, in the final paragraph of the Preface Kant makes it explicit that the physics of the *Principia* is paradigmatic of the "mathematical doctrine of motion" he has just mentioned in the immediately preceding sentence (478; see the paragraph to which note 11 above is appended). He also asserts that precisely this physics still requires a "small amount" of metaphysics:

[17] Of course Leibniz himself never doubted that mathematical or geometrical space is infinitely divisible, but he did assert (partly because of this infinite divisibility) that neither space nor material bodies in space are in any way substantial: both are rather ideal "well-founded phenomena" arising out of the pre-established harmony between ultimate non-spatial simple substances. It was the later Leibnizean–Wolffian tradition that went further by insisting that physical space cannot be infinitely divisible but rather consists of a finite number of "physical points" or "physical monads." (I shall return to this issue in my chapter on the Dynamics below.) For that matter, Newton himself never endorsed genuine gravitational action at a distance – a point, as we shall see, of which Kant was well aware and indeed explicitly addressed. Only the second generation of Newtonians (including Kant) unreservedly endorsed such action at a distance. This is the context for Kant's talk of "the English" or "mathematical investigators of nature," on the one side, and "metaphysical students of nature" or "or own people, following Leibniz," on the other.

Newton, in the preface to his *Mathematical First Principles of Natural Science*, says (after he had remarked that geometry requires only two of the mechanical operations that it postulates, namely, to describe a straight line and a circle): *Geometry is proud of the fact that with so little derived from without it is able to produce so much.** By contrast, one can say of metaphysics: *it is dismayed that with so much offered to it by pure mathematics it can still accomplish so little.* Nevertheless, this small amount is still something that even mathematics unavoidably requires in its application to natural science; and thus, since it must here necessarily borrow from metaphysics, it need also not be ashamed to let itself be seen in community with the latter. (478–79)[18]

Moreover, it is by no means surprising that Kant specifically cites Newton and the *Principia* here. For the *Metaphysical Foundations* (as I pointed out in my Preface) cites Newton far more than any other author, and most of these references, in fact, are to the *Principia*.[19]

Kant claims in this final paragraph that metaphysics is "unavoidably require[d]" to explain the application of mathematics in natural science – a claim already developed at greater length earlier in the Preface. Kant there argues (472) that explaining "the application of mathematics to the doctrine of body … is a task for pure philosophy," and that, for this purpose, the latter "makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept [of matter] itself, in relation to the pure intuitions in space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*." He then concludes:

[18] In the footnote Kant quotes (in Latin) from the Author's Preface to the Reader of Newton's *Mathematical Principles of Natural Philosophy* (1687), which reads (in the translation of I. B. Cohen and A. Whitman (1999, p. 382)): "Geometry can boast that with so few principles obtained from other fields, it can do so much." It is generally agreed that Newton adopted the title "*mathematical* principles of natural philosophy" in deliberate contrast to Descartes's largely non-mathematical *Principles of Philosophy* (1644) – by which, nonetheless, Newton had been significantly stimulated in his youth (see §3.1 of Cohen's "A Guide to Newton's *Principia*" in Newton 1999). From now on I cite the Cohen-Whitman volume in the form 'Pnn', where 'nn' denotes page numbers. It is striking that Kant here gives Newton's title as "mathematical first principles of *natural science* [*mathem. Grundlehren der Nat.-Wiss.*]." This is a reflection, perhaps, of Kant's own view that mathematical principles are quite distinct from philosophical (or metaphysical) principles – a point to which I shall return below.

[19] In the immediately preceding paragraph Kant has expressed the hope (478) that "stimulated by this sketch, mathematical investigators of nature should find it not unimportant to treat the metaphysical part, which they cannot leave out in any case, as a special fundamental part in their general physics, and to bring it into union with the mathematical doctrine of motion." The "mathematical doctrine of motion [*mathematische Bewegungslehre*]," then, is that which is pursued by "mathematical investigators of nature [*mathematische Naturforscher*]" – and, once again, it is Newton, above all, who is paradigmatic of such an investigator.

Hence all natural philosophers who have wished to proceed mathematically in their occupation have always, and must have always, made use of metaphysical principles (albeit unconsciously), even if they themselves solemnly guarded against all claims of metaphysics upon their science. Undoubtedly they have understood by the latter the folly of contriving possibilities at will and playing with concepts, which can perhaps not be presented in intuition at all, and have no other certification of their objective reality than that they merely do not contradict themselves. All true metaphysics is drawn from the essence of the faculty of thinking itself, and it is in no way feigned [*erdichtet*] on account of not being borrowed from experience. Rather, it contains the pure actions of thought, and thus a priori concepts and principles, which first bring the manifold of *empirical representations* into the law-governed connection through which it can become *empirical* **cognition**, that is, experience. Thus these mathematical physicists could in no way avoid metaphysical principles, and, among them, also not those that make the concept of their proper object, namely, matter, a priori suitable for application to outer experience, such as the concept of motion, the filling of space, inertia, and so on. But they rightly held that to let merely empirical principles govern these concepts would in no way be appropriate to the apodictic certainty they wished their laws of nature to possess, so they preferred to postulate such [principles], without investigating them with regard to their a priori sources. (472)

Kant portrays the "mathematical physicists [*mathematische Physiker*]" in question as seeking to avoid "metaphysics" understood as "the folly of contriving possibilities at will and playing with concepts" – possibilities (and concepts) which, in this sense, are merely "feigned." And he recommends his own peculiar understanding of "metaphysics" (a "metaphysics of experience") instead. Given that Kant is of course familiar with Newton's well-known protestations against "feigning" hypotheses – whether "metaphysical or physical" – in the General Scholium added to the second (1713) edition of the *Principia*, there can be very little doubt (especially in light of the final paragraph of the Preface) that the "mathematical physicist" Kant has foremost in mind here is Newton.[20]

[20] Newton's famous discussion of hypotheses in the General Scholium reads as follows (P943): "I have not as yet been able to deduce from phenomena the reason for these properties of gravity, and I do not feign [*fingo*] hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy." Here, in particular, Newton is replying to the objections of contemporary mechanical philosophers, principally Leibniz and Huygens, that his theory of universal gravitation involves an unintelligible or "occult" action at a distance by insisting, on the contrary, that he leaves the question of the true cause of gravitational attraction entirely open. As suggested in note 17 above, it was important to Kant to call into question this Newtonian agnosticism – for reasons that will become clear in what follows.

If Newton is the main representative of "mathematical physics," then who (aside from Kant himself) is the representative of "metaphysics?" In light of all we have seen the most natural choice is Leibniz. Indeed, the very beginning of the *Physical Monadology* – right before the passage quoted above on the opposition between "metaphysics" and "geometry" (1, 475–76) – advances quite similar considerations in support of the need for metaphysics in natural science:

> Clear-headed philosophers who seriously engage in the investigations of nature indeed unanimously agree that punctilious care must be taken lest anything concocted with rashness or a certain arbitrariness of conjecture should insinuate itself into natural science, or lest anything be vainly undertaken in it without the support of experience and the mediation of geometry. Certainly, nothing can be thought more useful to philosophy, or more beneficial to it, than this counsel. However, hardly any mortal can advance with a firm step along the straight line of truth without here and there turning aside in one direction or another. For this reason, there have been some who have observed this law to such a degree that, in searching for the truth, they have not ventured to commit themselves to the deep sea but have considered it better to hug the coast, only admitting what is immediately revealed by the testimony of the senses. And, certainly, if we follow this sound path, we can exhibit the laws of nature but not the origin and causes of these laws. For those who only hunt out the phenomena of nature are always that far removed from a deeper understanding of the first causes … Metaphysics, therefore, which many say may be properly absent from physics is, in fact, its only support; it alone provides illumination. (1, 475)

In the *Physical Monadology* Kant's project is to show how metaphysics in the Leibnizean tradition (as appropriately modified by Kant) can nonetheless lie at the foundation of a genuinely Newtonian mathematical physics. In the *Metaphysical Foundations*, I submit, Kant's project is analogous: to show how metaphysics in this same tradition (now even more fundamentally modified by Kant) can provide a metaphysical foundation (in a new sense) for this same physics.

KANT ON MATHEMATICS AND METAPHYSICS

In the paragraph from the Preface to the *Metaphysical Foundations* concerning the relationship between metaphysics and mathematical natural philosophers (or "mathematical physicists") quoted above (472) Kant asserts that "[a]ll true metaphysics is drawn from the essence of the faculty of thinking itself," in so far as "it contains the pure actions of thought, and thus a priori concepts and principles, which first bring the manifold of *empirical representations* into the law-governed connection through which it can become *empirical* **cognition**, that is, experience." These "pure actions of thought" or "a priori concepts" are the categories or pure concepts of the understanding – which, in the metaphysical deduction, are derived from the logical forms of judgement.[21] In the second paragraph following this one Kant explains that his project in the *Metaphysical Foundations*, as thus guided by the table of categories, can therefore be brought to completion:

In everything that is called metaphysics one can hope for the *absolute completeness* of the sciences, of such a kind as one may expect in no other type of cognition. Therefore, just as in the metaphysics of nature in general, here also the completeness of the metaphysics of corporeal nature can confidently be expected. The reason is that in metaphysics the object is only considered in accordance with the general laws of thought, whereas in other sciences it must be represented in accordance with data of intuition (pure as well as empirical), where the former, because here the object has to be compared always with *all* the necessary laws of thought, must yield a determinate number of cognitions that may be completely exhausted, but the latter, because they offer an infinite manifold of intuitions (pure or empirical), and thus an infinite manifold of objects of thought, never attain absolute completeness, but can always be extended to infinity, as in pure mathematics and empirical science of nature. I also take myself to have completely exhausted the metaphysical doctrine of body, so far as it may extend, but not to have thereby accomplished any great work. (473)[22]

At the beginning of the immediately following paragraph Kant adds (473–74): "But the schema for completeness of a metaphysical system, whether it be of nature in general or of corporeal nature in particular, is the table of categories.* For there are no more pure concepts of the understanding which can be concerned with the nature of things."[23]

This paragraph continues:

[21] At the end of the metaphysical deduction, immediately before the table of categories, Kant says (A79/B105): "In this way there arise just as many pure concepts of the understanding, which apply a priori to objects of intuition in general, as there were logical functions in all possible judgements in the previous table: for the understanding is completely exhausted by the functions in question, and its capacity is thereby completely measured."

[22] Kant here expresses the same kind of modesty concerning the achievements of his new kind of metaphysics – especially in comparison with mathematics and natural science – as he does in the last paragraph of the Preface (478–79; see the passage to which note 18 above is appended). His talk of how his metaphysical project may thereby be "completely exhausted" echoes the passage from the metaphysical deduction quoted in note 21 above.

[23] It is no wonder, then, that Kant's single explicit reference to the *Metaphysical Foundations* in the second edition of the *Critique* concerns precisely the table of categories (B109–10):

[T]hat this table is uncommonly useful – indeed indispensable – in theoretical philosophy, in order completely to outline, and mathematically *to divide in accordance with determinate principles the plan of the whole of a science*, in so far as it rests on a priori concepts, is already self-evident from the fact that the table in question completely contains all elementary concepts of

All determinations of the general concept of a matter in general must be able to be brought under the four classes of [pure concepts of the understanding], those of *quantity*, of *quality*, of *relation*, and finally of *modality* – and so, too, [must] all that may either be thought a priori in this concept, or presented in mathematical construction, or given as a determinate object of experience. There is no more to be done, to be discovered, or to be added here except, if need be, to improve it where it may lack in clarity or exactitude [*Gründlichkeit*]. (474–76)[24]

After the passage on the priority of the concept of motion in his enterprise that I have already quoted (476–77; see again the paragraph to which note 11 above is appended) Kant explains how this concept, in particular, is to be brought under the four classes of categories:

The *metaphysical* foundations of natural science are therefore to be brought under *four* chapters. The *first* considers *motion* as a pure *quantum* in accordance with its composition, without any quality of the movable, and may be called **phoronomy**. The *second* takes into consideration motion as belonging to the *quality* of matter, under the name of an original moving force, and is therefore called **dynamics.** The *third* considers matter with this quality as in *relation* to another through its own inherent motion, and therefore appears under the name of **mechanics**. The *fourth* chapter, however, determines matter's motion or rest merely in relation to the mode of representation or *modality*, and thus as appearance of the outer senses, and is called **phenomenology**. (477)

Thus the way in which the motion of matter is investigated in the four chapters that make up the body of the *Metaphysical Foundations* (including the order of this investigation) is, as promised, entirely determined by the table of the categories. It is precisely this circumstance that makes the investigation *metaphysical* in Kant's new sense.

But why does the Newtonian mathematical theory of motion require a metaphysical foundation in this sense? Why can it not simply stand on its own? The answer, from Kant's point of view, is that we need an explanation for how the motion of matter described by Newton's theory becomes

the understanding, and even the form of a system of these in the human understanding, and it consequently gives an indication of all the *moments* of a projected speculative science, and even their *order*, as I have made an attempt to show elsewhere.[1]"

The footnote reads: "*Metaphys. Anfangsgr. der Naturwissensch.*" (It has been suggested – and this suggestion has been adopted in the Akademie edition – that 'mathematically' in this passage should be replaced by 'systematically'.)

[24] The reason so many pages in the Akademie edition are taken up by these few sentences is that the note attached to 'the table of categories' in the penultimate sentence of the preceding passage is extremely long – for Kant here reconsiders the relationship between the metaphysical and transcendental deductions in response to criticism. For an illuminating discussion of this note, which is important for understanding the revisions to the transcendental deduction made in the second edition, see Pollok (2008).

a genuine object of experience – and we need to do this, moreover, without invoking what Kant takes to be the metaphysically objectionable notion of Newtonian absolute space. In particular, we need to explain how the distinction between "true" and merely "apparent" motion is made within experience without anywhere invoking the objectionable idea of motion relative to space itself – that is, relative to *empty* space.[25] The crucial point is that the motion of matter can become a genuine object of experience only in virtue of the three relational categories (substance, causality, community) and their corresponding principles (the analogies of experience). These categories and principles are realized or instantiated in the *Metaphysical Foundations* by Kant's version of the mechanical laws of motion, which are derived from the empirical concept of matter and the analogies of experience in the third chapter or Mechanics.[26] Moreover, in the fourth chapter or Phenomenology Kant invokes the modal categories and the corresponding postulates of empirical thought to explain how the motion of matter can be determined either as (merely) possible, as actual ("true"), or as necessary.[27]

More specifically, the Phenomenology describes a procedure for reducing all motion and rest to "absolute space" that is intended to generate a determinate distinction between true and merely apparent motion despite the acknowledged relativity of all motion as such to some given empirically specified relative space or reference frame. The procedure begins by considering our position on the earth, indicates how the earth's state of true (axial) rotation can nonetheless be empirically determined, and concludes by considering the cosmos as a whole – together with the "common center of gravity of all matter" (563) – as the ultimate relative space (or reference frame) for correctly determining all true motion and

[25] There is no doubt that Kant, in line with the Leibnizean tradition, takes the Newtonian tradition to be committed to just such an objectionable conception of absolute space – as (2, 403) an "*absolute* and boundless *receptacle* of possible things" or (along with time) as (A39/B56) one of the "two eternal and infinite non-things subsisting in themselves, which are there (without there being anything actual), only in order to contain all actuality within themselves": see the paragraph to which notes 15 and 16 above are appended, together with the preceding two paragraphs.

[26] I use the phrase "mechanical laws of motion" here to refer to both Kant's three Laws of Mechanics and Newton's three Laws of Motion. I shall indicate the ways in which Kant's Laws diverge from Newton's in note 37 below.

[27] As Kant explains at the beginning of the Phenomenology, this determination of the motion of matter in relation to the three modal categories centrally involves the distinction between "semblance" and "truth" (555), and it thereby makes contact with the discussion in the *Prolegomena* with which we began of Kant's view of the difference between his "formal" idealism and Berkeley's "material" idealism (4, 375). I shall return to the connection with the *Prolegomena* on this point in my chapter on the Phenomenology.

rest. Moreover, Kant makes it explicit, as we shall see, that Newton's famous treatment of "absolute" space, time, and motion in the *Principia* constitutes the essential background for this discussion. The upshot, from a modern point of view, is that Kant takes the mechanical laws of motion (implicitly) to define a privileged frame of reference in which they are satisfied: they do not state mere empirical facts, as it were, about a notion of (true or absolute) motion that is already well defined independently of these laws. The mechanical laws of motion are thereby revealed to be (synthetic) a priori principles governing the (true or actual) motion of matter, and Kant has thus explained their "a priori sources" in the sense of the crucial paragraph on the relationship between metaphysics and mathematical natural philosophers (472) already discussed several times above.[28]

But what is the relationship between metaphysics in Kant's new (critical) sense and the metaphysics of the Leibnizean tradition? Kant's pre-critical *Physical Monadology* – which, as we know, also attempted to give Newtonian mathematical physics a metaphysical foundation – appeals to Leibnizean metaphysics in a more standard sense. There are ultimate, non-spatial simple substances (monads) underlying the phenomena of matter and motion, but these Kantian simple substances, unlike properly Leibnizean ones, genuinely interact with one another by forces of nature modeled on those of Newtonian physics. More precisely, while there is still a Leibnizean distinction between the (non-spatial) noumenal realm of ultimate simple substances and the phenomenal realm of matter in motion, the real relations (external determinations) connecting these ultimate substances with one another appear to us phenomenally (in space) as forces of attraction and repulsion acting at a distance.[29]

[28] The modern perspective on Newtonian absolute, space, time and motion, which crystallized in the late nineteenth century, takes the mechanical laws of motion implicitly to define the concept of an *inertial frame of reference*: an inertial frame, that is, is simply any frame of reference in which these laws are satisfied. Kant's strategy is similar to this, except that he does not appeal to what we now call Galilean relativity: he does not have the concept of a *class* of privileged (inertial) reference frames, all of which are moving inertially (rectilinearly and with constant speed) relative to one another. He seeks to construct, rather, a *single* privileged frame – defined in the limit as the "common center of gravity of all matter" – which can then serve as a surrogate for Newtonian absolute space. I invoke this modern perspective here in order to give the reader a preliminary sense of Kant's motivations for taking the mechanical laws of motion to be synthetic a priori principles, which therefore require a "metaphysical" explanation. Kant's own procedure for reducing all motion and rest to "absolute space" – which constructs (in the limit) a single privileged frame rather than a class of such frames – can only be properly understood by following the twists and turns of his quite complicated text in considerable detail. When we do this, moreover, we shall also better understand why and how metaphysics in Kant's sense plays an essential role in this procedure.

[29] As we shall see in my chapter on the Dynamics, it is in precisely this way that the *Physical Monadology* resolves the tension between absolutely simple physical monads corresponding to

In the critical period, by contrast, the only kind of substance of which we can have (theoretical) cognition is phenomenal substance (*substantia phaenomenon*), and this kind of substance, as we shall see, must always be spatially extended and thus infinitely composite.[30] Indeed, in the critical period there is no possibility of (theoretical) knowledge extending beyond the phenomenal realm at all, and the categories or pure concepts of the understanding only have objective (theoretical) meaning for us when applied to or schematized in terms of our pure forms of sensibility.

Nevertheless, Kant agrees with Leibniz that certain fundamental intellectual concepts (such as substance, causality, community, possibility, actuality, necessity, and so on) underlie the science of metaphysics, and Kant takes these concepts to derive from the logical structure of our pure understanding independently of sensibility. He also agrees that these metaphysical concepts are indispensably required properly to ground the mathematical science of nature governing spatio-temporal phenomena. Yet they can no longer perform this function in an ontological sense – as describing a deeper (theoretical) reality lying behind the phenomenal realm. They only perform it in the context of Kantian transcendental idealism, as formal a priori rules for constituting spatio-temporal phenomena within experience: for constituting *phenomenal* substances, *phenomenal* causal relations, and *phenomenal* relations of mutual interaction or community.

This last case, corresponding to the category of community, helps us to see why Kant takes specifically Newtonian physics – the Newtonian mathematical theory of motion – as the only proper natural science for which

the smallest constituents of matter and the infinite divisibility of space. Such monads are to be conceived as point-like centers of attractive and repulsive forces, where the repulsive force, in particular, generates a region of solidity or impenetrability in the form of a tiny "sphere of activity" emanating from a central point. Geometrically dividing this region of impenetrability in no way divides the actual substance of the monad but merely the "sphere of activity" in which the point-like central source manifests its repulsive capacity to exclude other monads from the region in question. The Leibnizean commitment to ultimate simple substances or monads is therefore perfectly consistent with the infinite divisibility of space after all – but (and here is Kant's characteristic twist) it can only be maintained by explicitly adopting the Newtonian conception of "impressed forces" acting at a distance, in particular, that of a short range repulsive force acting at a very small distance given by the radius of the relevant "sphere of activity."

30 Accordingly, as we shall also see in my chapter on the Dynamics, the problem posed by the infinite divisibility of space that the *Physical Monadology* had attempted to solve by invoking finite "spheres of activity" is now solved, in the *Metaphysical Foundations*, by invoking the transcendental idealism articulated in the antinomy of pure reason of the first *Critique* – more specifically, the argument of the second antinomy resolving the apparent incompatibility between the infinite divisibility of space and the *presumed* absolute simplicity of the material or phenomenal substances found in space. It is in precisely this context that the name of Leibniz finally appears in the text.

we can provide such a metaphysical foundation. The law of the equality of action and reaction corresponds in the *Metaphysical Foundations* to the third analogy of experience in the first *Critique*. And the primary application of this law in Kant's procedure for reducing all motion and rest to absolute space involves the force of universal gravitation, now conceived as a genuine action at a distance through empty space. Indeed, it is precisely the universal gravitational interaction at a distance between all bodies in the universe, for Kant, that constitutes their (phenomenal) coexistence or simultaneity in space and time.[31] This, in fact, is why Kant insists on genuine gravitational action at a distance and, more generally, why he decisively rejects the Leibnizean criticisms of such action on behalf of Newtonian physics.[32]

This (critical) perspective on the category of community is clearly reflected in Kant's discussion in the general remark to the principles (added to the second edition) from which I have already quoted above, where, as explained, Kant consistently emphasizes the priority of space:

> Finally, the category of *community*, with respect to its possibility, cannot at all be conceived through mere reason, and thus it is not possible to comprehend the objective reality of this category without intuition, and indeed outer [intuition] in space … *Leibniz*, therefore, in so far as he attributed a community to the substances of the world only as the understanding thinks it alone, needed a divinity for mediation; for from their existence alone it seemed to him, correctly, to be inconceivable. But we can make the possibility of community (of substances as appearances) conceivable very well, if we represent them to ourselves in space, and therefore in outer intuition. For the latter already contains within itself a priori formal outer relations as conditions of the possibility of real [relations] (in action and reaction, and thus community). (B292–93)

Formal outer relations in space are the conditions of the possibility of *real* relations between bodies effected by forces, and the reference to "action and reaction" here appears clearly to invoke the argument of the *Metaphysical Foundations*. Newtonian universal gravitation is thereby suggested as well – as the one and only interaction (at a distance) among

[31] In the text of the third analogy there are two specific relations of causal interaction instantiating the category of community among the heavenly bodies: the universal gravitational interaction between them, which establishes an *immediate* or *direct* relationship of community, and "the light, which plays between our eyes and the heavenly bodies" and thereby "effects a *mediate* [or *indirect*] community between us and them" (A213/B260, emphasis added). I shall explain this in detail below.

[32] In this context, as we shall also see, Kant rejects Newton's own hesitations about action at a distance as "set[ting] him at variance with himself" (515; compare note 20 above, together with the paragraph to which it is appended).

all the bodies in the universe that can (immediately) constitute their thoroughgoing coexistence or simultaneity.[33]

The point of the discussion of the categories of relation in the general remark is to emphasize the importance of space in all three cases. At the end of the discussion of the three relational principles in the main body of the text (in both editions) there is a parallel passage emphasizing time:

These, then, are the three analogies of experience. They are nothing else but the principles for the determination of the existence of appearances in time with respect to all of its three modes, the relation to time itself as a magnitude (the magnitude of existence, i.e., duration), the relation in time as a series (successively), and finally [the relation] in time as a totality of all existence (simultaneously). This unity of time determination is thoroughly dynamical; that is, time is not viewed as that in which experience immediately determines the place of an existent, which is impossible, because absolute time is no object of perception by means of which appearances could be bound together; rather, the rule of the understanding, by means of which alone the existence of the appearances can acquire synthetic unity with respect to temporal relations, determines for each [appearance] its position in time, and thus [determines this] a priori and valid for each and every time. (A215/B262)

This passage is remarkable, among other reasons, because it suggests that the procedure Kant calls dynamical time determination is his substitute for Newtonian absolute time – just as the procedure of reducing all motion and rest to absolute space will, in the *Metaphysical Foundations*, become his substitute for Newtonian absolute space.

[33] I observed in note 15 above that Kant, in the *Inaugural Dissertation*, had already argued against the Leibnizean conception of time by appealing to the concept of simultaneity. In particular, Kant there accuses the Leibnizean view of "completely neglecting *simultaneity**, the most important consequence of time" (2, 401), and he continues as follows in the note:

Simultaneous [things] are not so because they do not succeed one another. For if succession is removed, then some conjunction which was there because of the series of time is indeed abolished, but *another* true relation, such as the conjunction of all at the same moment, does not thereby immediately arise. For simultaneous [things] are joined together at the same moment of time in just the same way that successive [things] are at different moments. Therefore, although time has only one dimension, still the *ubiquity* of time (to speak with Newton), whereby *all* [things] sensitively thinkable are at *some time*, adds another dimension to the magnitude of actual [things], in so far as they hang, as it were, from the same point of time. For if one designates time by a straight line proceeding to infinity, and that which is simultaneous at any point of time by ordinates to this line, then the surface thereby generated represents the *phenomenal world*, with respect to both substance and accidents. (2, 401)

The kinship between this argument and that of the general remark to the system of principles is striking, but further discussion will have to wait until I come back to the question in more detail in my chapter on the Mechanics.

Here is not the place to discuss this analogy further, and much of the body of what follows is devoted to its clarification.[34] What I want to focus on now is Kant's assertion in the first sentence of his summary explanation of the three analogies of experience quoted above, which characterizes them as "nothing else but the principles for the determination of the *existence* of appearances in time" (A215/B261, emphasis added). For this claim illuminates Kant's argument in the Preface to the *Metaphysical Foundations* that proper natural science, in the first instance, requires metaphysics of nature. Kant has just argued (469) that "[a]ll proper natural science therefore requires a *pure* part, on which the apodictic certainty that reason seeks therein can be based." But, he points out, there are two different sources of pure rational cognition: "Pure rational cognition from mere *concepts* is called pure philosophy or metaphysics; by contrast, that which grounds its cognition on the *construction* of concepts, by means of the presentation of the object in an a priori intuition, is called mathematics" (469). The argument for the priority of metaphysics then follows: "*Properly* so-called natural science presupposes, in the first instance, metaphysics of nature. For laws, that is, principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be presented a priori in any intuition. Thus proper natural science presupposes metaphysics of nature" (469).[35]

The laws in question – the "principles of the necessity of that which belongs to the *existence* of a thing" – are, first and foremost, the analogies of experience. Yet, Kant continues, the metaphysics of nature in question can be either *general* or *special* metaphysics:

> [Metaphysics of nature] must always contain solely principles that are not empirical (for precisely this reason it bears the name of a metaphysics), but it can still either: *first*, treat the laws that make possible the concept of a nature in general,

[34] The connection between the two sides of the analogy – absolute time and absolute space – is mediated by the general remark, which emphasizes the importance of matter, motion, and the interaction of moving forces in space for verifying the objective reality of the relational categories. The relationships between motion and succession, on the one hand, and interaction and simultaneity, one the other, are relatively straightforward. But the relationship between matter and its quantitative conservation, which instantiates the first analogy of experience as formulated in the second edition (B224), and the quantitative determination of time ("the magnitude of existence, i.e., duration") is more obscure – and will take very many pages fully to elaborate.

[35] In the first sentence of the Preface (467) Kant characterizes *nature*, "in its *formal* meaning," as "the first inner principle of all that belongs to the existence of a thing*." And, in the note, Kant sharply distinguishes on this basis between *essence* and *nature*: "Essence is the first inner principle of all that belongs to the possibility of a thing. Therefore, one can attribute only an essence to geometrical figures, but not a nature (since in their concept nothing is thought that would express an existence)" (467).

even without relation to any determinate object of experience, and thus undetermined with respect to the nature of this or that thing in the sensible world, in which case it is the *transcendental* part of the metaphysics of nature; or *second*, concern itself with a particular nature of this or that kind of thing, for which an empirical concept is given, but still in such a manner that, outside of what lies in this concept, no other empirical principle is used for its cognition (for example, it takes the empirical concept of matter or of a thinking being as its basis, and it seeks that sphere of cognition of which reason is capable a priori concerning these objects), and here such a science must still always be called a metaphysics of nature, namely, of corporeal or of thinking nature. However, it is then not a general, but a *special* metaphysical natural science (physics or psychology), in which the above transcendental principles are applied to the two species of objects of our senses. (469–70)[36]

In the case of the special metaphysics of *corporeal* nature, therefore, Kant's version of the mechanical laws of motion (which realize or instantiate the analogies of experience in the case of the empirical concept of matter) are also "principles of the necessity of that which belongs to the *existence* of a thing" – the existence, namely, of material things or bodies.[37]

We already know, however, according to the argument of the immediately following paragraph, that the special metaphysics of corporeal nature is the only special metaphysics of nature that can actually ground a proper natural science. The reason is that, in the case of any special metaphysics of nature, a *mathematical* a priori foundation is indispensably required along with the *metaphysical* foundation:

I assert, however, that in any special doctrine of nature there can be only as much *proper* science as there is *mathematics* to be found therein. For,

[36] This is the paragraph referred to in note 7 above.

[37] Compare note 26 above. Kant's three Laws of Mechanics, which are derived in the Mechanics chapter from the three analogies of experience, comprise (i) a principle of the conservation of the total quantity of matter; (ii) the law of inertia; (iii) the law of the equality of action and reaction. These same three laws are invoked in §VI of the second edition Introduction to the *Critique* as an indisputable proof that "pure natural science" is actual (B20–21):

[O]ne need only consider the various propositions that occur at the beginning of proper (empirical) physics, such as those of the permanence of the same quantity of matter, of inertia, of the equality of action and reaction, etc., and one will be soon convinced that they constitute a pure (or rational) physics, which well deserves to be separately established, as a science of its own, in its entire extent, whether narrow or wide.

Newton's three Laws of Motion, by contrast, are the law of inertia, the Second Law equating force with change of momentum, and the Third Law equating action and reaction. Thus, Newton's Second Law does not occur among Kant's three Laws, and Kant's principle of the conservation of matter does not occur among Newton's. The significance of this divergence will be explored in detail below, especially in my chapter on the Mechanics, but it is not unconnected with the issue concerning the relationship between the conservation of material substance and the quantitative determination of time remarked upon in note 34 above.

according to the preceding, proper science, and above all proper natural science, requires a pure part lying at the basis of the empirical part, and resting on a priori cognition of natural things. Now to cognize something a priori means to cognize it from its mere possibility. But the possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). Hence, in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the *intuition* corresponding to the concept be given a priori, that is, that the concept be constructed. Now rational cognition through the construction of concepts is mathematical. Hence, although a pure philosophy of nature in general, that is, that which investigates only what constitutes the concept of a nature in general, may indeed be possible even without mathematics, a pure doctrine of nature concerning *determinate* natural things (doctrine of body or doctrine of soul) is only possible by means of mathematics. And, since in any doctrine of nature there is only as much proper science as there is a priori knowledge therein, a [special] doctrine of nature will contain only as much proper science as there is mathematics capable of application there. (470)[38]

The difference between a metaphysical and a mathematical foundation for a proper natural science therefore depends on the difference between actuality (existence) and possibility.[39] A metaphysical foundation provides a priori principles governing the existence or actuality of things, while a mathematical foundation provides a priori principles governing their real (as opposed to merely logical) possibility.[40] Whereas the real possibility (objective reality) of things standing under the pure concepts of the understanding is secured by the transcendental deduction of the categories independently of mathematical construction, the real possibility of more specific or determinate kinds of things falling under an *empirical* concept – such as the empirical concept of matter – cannot be a priori established in this way. The only remaining alternative, therefore, is

[38] This is the argument referred to in note 8 above. For the argument that only the special metaphysics of corporeal nature (as opposed to a purported special metaphysics of thinking nature) can ground a proper natural science see the paragraph to which note 7 above is appended, together with the following paragraph.

[39] The three moments under the categories of modality (A80/B106) are "*possibility* – impossibility," "*existence* [*Dasein*] – non-existence," and "*necessity* – contingency."

[40] This fundamental difference between metaphysics and mathematics is obviously closely connected with the distinction between *nature* and *essence* emphasized in the first sentence of the Preface to the *Metaphysical Foundations*: see note 35 above, and recall that the (real) possibility of geometrical figures, for Kant, can only be secured by the construction in pure intuition of the corresponding concepts.

mathematical construction, which exhibits intuitions corresponding to the concept a priori.

Does this mean that it is the task of the *Metaphysical Foundations* to show how the empirical concept of matter can be constructed a priori in pure intuition? However natural such a reading may at first sight appear, this cannot be Kant's intention. For, in the first place, the general remark to dynamics in the Dynamics chapter sharply distinguishes between the *mechanical* and *dynamical* natural philosophies – or, equivalently, between the mathematical-mechanical and metaphysical-dynamical modes of explanation. The former, which corresponds to the mechanical or corpuscular philosophy pursued by Descartes and others, seeks to explain all the properties and actions of matter by its purely geometrical properties (figure, size, and motion) without appealing to fundamental forces of attraction and repulsion. The latter, by contrast, which corresponds to the concept of matter Kant develops in the *Metaphysical Foundations*, explicitly appeals to such forces, and, indeed, its "concept of matter is reduced to nothing but moving forces" (524).[41] The most important difference, however, is that the possibility of the mechanical concept of matter, as purely geometrical, "can be verified with mathematical evidence" (525), while this is emphatically not the case for the dynamical concept (525): "By contrast, if the material itself is transformed into fundamental forces … we lack all means for *constructing* this concept of matter, and presenting what we thought universally as possible in intuition." Kant explicitly and emphatically asserts, therefore, that his own preferred concept of matter *cannot* be constructed in pure intuition.[42]

More generally, and in the second place, the first *Critique* extensively discusses the (real as opposed to merely logical) possibility of concepts in the postulates of empirical thought. The first postulate, governing the

[41] In the body of the Dynamics chapter (591–92) Kant has already sharply distinguished between the *absolute* or *mechanical* concept of impenetrability or the filling of space ("which presupposes no moving force as originally belonging to matter") and the *relative* or *dynamical* concept (which "rests on a physical basis"). Kant concludes that, in accordance with his preferred concept, "*the filling of space must be viewed only as relative impenetrability*." Kant's conception here is thus similar to the dynamical theory of matter developed earlier in the (pre-critical) *Physical Monadology*, according to which solidity or impenetrability is not taken to be a primitive absolute quality of bodies but is rather constituted by the "sphere of activity" of repulsive force surrounding a point-like central monad (see note 29 above, together with the paragraph to which it is appended). In the *Metaphysical Foundations*, however, this idea is combined with the (critical) conception of matter as a genuine space-filling continuum, according to which *every* point within a region of impenetrability equally exerts forces of attraction and repulsion.

[42] Nevertheless, the notes and remarks to Kant's eighth proposition of the Dynamics appear to suggest such a construction. I shall discuss this issue in detail in my chapter on the Dynamics.

(real) possibility of things, requires (A218–20/B265–67) that their concept "agree with the formal conditions of experience in general [in accordance with intuitions and concepts]." "A concept that comprises a synthesis," Kant continues (A220/B267), "is to be taken as empty, and related to no object, if this synthesis does not belong to experience – either as borrowed [*erborgt*] from it, in which case it is an *empirical concept*; or as one that rests, as an a priori condition, on experience in general (the form of experience), in which case it is a *pure concept*, which nevertheless belongs to experience, because its object can only be found there."[43]

Kant considers the pure concepts of substance, causality, and community in the following discussion (A221–22/B268–69), and he then takes up more specific (empirical) instantiations of these concepts:

> However, if one wanted to make entirely new concepts of substances, of forces, and of interactions out of the material offered to us by perception, without borrowing [*entlehnen*] the example of its connection from experience itself, then one would fall into mere phantoms of the brain, whose possibility would have no indications at all – since one does not accept experience as instructress, and yet these concepts are borrowed [*entlehnt*] from it. (A222/B269)

Such (empirical) concepts (A222/B269–70) "cannot acquire the character of their possibility a priori, like the categories, as conditions on which all experience depends, but only a posteriori, as such that are given through experience itself – and their possibility must either be cognized a posteriori and empirically, or it cannot be cognized at all." But Kant's own empirical concept of matter involves a specific kind of *material substance*, specific (empirically given) fundamental *forces* (of attraction and repulsion), and a specific type of *interaction* (in accordance with the third law of motion). Unlike a pure sensible or mathematical concept (compare note 43), therefore, it cannot be constructed in pure intuition, and, in any case, its (real) possibility can by no means be established purely a priori.[44]

What then *is* Kant saying in the crucial – but, as we have now seen, very difficult – paragraph concerning possibility and mathematical

[43] Kant makes it clear in what follows that the pure concepts in question include both categories and pure sensible concepts – that is, mathematical concepts. That a triangle, for example, is really possible depends not only on its construction in pure intuition but also on the circumstance (A224/B271) that "space is a formal a priori condition of outer experiences" and that "precisely the same image-forming [*bildende*] synthesis by which we construct a triangle in the imagination is completely identical with that which we exercise in the apprehension of an appearance, in order to make for ourselves an empirical concept of it."

[44] In the general remark to dynamics, once again, Kant makes this point explicitly concerning the two fundamental forces of attraction and repulsion (524):

construction in the Preface to the *Metaphysical Foundations* (470)? Important clues are provided in a closely following paragraph (472):

> But in order to make possible the application of mathematics to the doctrine of body, which only through this can become a natural science, principles for the *construction* of the concepts that belong to the possibility of matter in general must be introduced first. Therefore, a complete analysis of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions of space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*.[45]

Two points are of special interest here: first, Kant speaks of "principles for the *construction* of the concepts" (in the plural) "that belong to the possibility of matter in general," and second, Kant infers that "a complete *analysis* of the concept of a matter in general" (emphasis added) is needed. Such an analysis will break up the concept of matter in general into its constituent or partial concepts [*Teilbegriffe*], and what Kant appears to be suggesting is that "principles for the *construction* of concepts" (in the plural) will apply to these (partial) concepts rather than to the concept of matter in general.[46]

> But who pretends to comprehend the possibility of the fundamental forces? They can only be assumed if they unavoidably belong to a concept that is demonstrably fundamental and not further derivable from any other (like that of the filling of space), and these, in general, are repulsive forces and the attractive forces that counteract them. We can indeed certainly judge a priori about the connection and consequences of these forces, whatever relation among them one can think without contradiction, but cannot yet presume to suppose one of them as actual. For to be authorized in erecting an hypothesis, it is unavoidably required that the *possibility* of what we suppose be completely *certain*, but with fundamental forces their possibility can never be comprehended.

[45] Compare the paragraph to which note 20 above is appended, where a few lines from this passage have already been quoted at the beginning.

[46] It is also important to observe that, according to precisely this passage, "principles for the *construction* of concepts" belong to the "*metaphysics of corporeal nature*" and therefore to "pure philosophy" or metaphysics – not (like the relevant constructions themselves) to mathematics. Compare Kant's important discussion of "mathematical" and "dynamical" principles of pure understanding in the first *Critique*, which are carefully distinguished from the principles of mathematics and physical dynamics respectively (A162/B201–2):

> One should well take note, however, that I here have as little to do with the principles of mathematics, in the one case, as I do with the principles of general (physical) dynamics, on the other. Rather, I have only in mind the principles of pure understanding in relation to inner sense (without distinction of the representations given therein), by which the former principles all acquire their possibility. I am therefore entitling them [mathematical and dynamical] more on account of their application than their content.

In the immediately following paragraph Kant further suggests that the principles of construction in question apply to the concepts into which the concept of matter can be analyzed rather than to this concept itself (472): "Thus these mathematical physicists could in no way avoid metaphysical principles, and, among them, also not those that make the concept of their proper object, namely, matter, a priori suitable for application to outer experience, such as the concept of motion, the filling of space, inertia, and so on."[47] It seems clear from what we have already seen, moreover, that not *all* of these (partial) concepts can themselves be mathematically constructed. This applies especially, of course, to the concept of the filling of space or impenetrability.[48] But the concept of motion, by contrast, appears eminently capable of mathematical construction. Indeed, as we shall see, the sole proposition in the Phoronomy is concerned with explaining how motion can be considered as a mathematical magnitude in terms of speed and direction, and it does so by providing what Kant calls a mathematical construction in pure intuition exhibiting how two speeds (in whatever direction) may be added or composed with one another. This, in particular, is why phoronomy, in Kant's sense, is nothing but "the pure doctrine of magnitude (*Mathesis*) of motions" (489), which aims simply to explain how the concepts of speed and velocity are possible as mathematical magnitudes.[49] It is only in this way, for Kant, that we can, in the first instance, secure the application of the mathematical theory of magnitude (the theory of proportion) to the concept of motion.

Thus, what Kant calls mathematical principles of pure understanding are principles that explain the possibility of mathematics, together with its characteristic procedure of the construction of concepts, but they do not themselves belong to mathematics.

[47] In §15 of the *Prolegomena* Kant gives the relevant list of (partial) concepts as (4, 295): "the concept of *motion*, of *impenetrability* (on which the empirical concept of matter rests), of *inertia*, and others."

[48] See again note 41 above, together with the paragraph to which it is appended; and observe that, according to the passage quoted in note 47 above, the "empirical" character of the concept of matter appears, in the first instance, to rest on the concept of impenetrability.

[49] Phoronomy, Kant says (495), should be considered "not as pure doctrine of motion, but merely as pure doctrine of the magnitude of motion, in which matter is thought with respect to no other property than its mere movability." In more detail (489):

> In phoronomy, since I am acquainted with matter through no other property but its movability, and may thus consider it only as a point, motion can only be considered as the *describing of a space* – in such a way, however, that I attend not merely, as in geometry, to the space described, but also to the time in which, and thus to the speed with which, a point describes the space. Phoronomy is thus the pure doctrine of magnitude (*Mathesis*) of motions.

It is because the motion in question can be considered as that of a mere mathematical point that the construction Kant describes can be exhibited in pure (rather than empirical) intuition.

Kant aims, more generally, to explain how the quantitative structure of each of the concepts required by the mathematical theory of motion becomes possible – and, in this way, to explain the application of mathematics in all of (Newtonian) mathematical physics. One of the most important goals of the Mechanics chapter, for example, is to explain how the concept of mass or quantity of matter becomes possible as a mathematical magnitude – to explain how, in Kant's terms, the quantity of matter of any body may be (mechanically) measured or "estimated" (537, Proposition 1). This quantity, in turn, then plays a crucial role later in the Mechanics (Proposition 4), where Kant explains, for the first time, how the "true" motions of matter can themselves be (quantitatively) determined. The idea is that, in a system of any two interacting bodies, their "true" motions are to be determined in such a way that their quantities of motion or momenta (momentum defined as mass times velocity) are equally apportioned between them (relative, therefore, to their common center of mass). This, in fact, is how Kant demonstrates the equality of action and reaction, by what he himself calls (546) a *construction* of "the action [and reaction] in the community of the two bodies." And it is in precisely this way, as Kant here puts it for the first time (545), that the respective motions of the two bodies are "reduced to absolute space."

Yet the construction presented in the fourth proposition of the Mechanics, unlike that presented in the sole proposition of the Phoronomy, cannot be carried out in pure intuition. In order completely to determine the relevant center of mass, for example, we need to measure or "estimate" the quantities of matter of the two bodies in question, and this, for Kant, can only be accomplished in empirical (rather than pure) intuition.[50] As we shall see, however, that empirical intuition is increasingly required in Kant's evolving explanation of the application of mathematics in pure natural science turns out to be a strength rather than a weakness of his position. For the aim of Kant's special metaphysics of corporeal nature is not to deduce a priori the quantitative structure of the (Newtonian) mathematical theory of motion from either metaphysics or pure mathematics. He aims, rather, to use all the resources of his revolutionary metaphysics of experience to explain, step by step, how the fundamental *empirical* concepts of this theory acquire their quantitative (measurable) structure and thereby become amenable

[50] As we shall see, this is connected with the circumstance that quantity of matter is defined in the first explication of the Mechanics (537) as "the aggregate of the movable in a determinate space" – and it therefore *cannot* pertain to a mere mathematical point: compare note 49 above.

to a mathematical (rather than merely metaphysical) a priori treatment.[51] It is in this way, and in this way alone, that we can finally explain how the modern mathematization of nature – the application of mathematics to the empirical or phenomenal world of our sensible experience – first becomes possible.[52]

[51] In these terms we can understand the argument of the difficult paragraph on possibility and mathematical construction (470) as follows. The general metaphysics of the first *Critique* only provides a priori principles governing all objects whatsoever of a nature in general. But any proper natural science restricted to a more specific or determinate subspecies of such objects must also rest, in addition, on more specific a priori principles appropriate to these objects. The principles of general metaphysics alone are clearly not sufficient for *this* purpose, and the only remaining a priori principles that could conceivably be of service here are those of mathematics. Therefore, a special metaphysics of any determinate subspecies of objects in nature must explain the possibility of applying mathematics to the specific empirical concepts involved in a proper natural science restricted to this subspecies.

[52] Kant emphasizes that his problem is to explain the *application* of mathematics to natural science or the doctrine of body at the end of the paragraph concerning possibility and mathematical construction (470), at the beginning of the paragraph on "principles for the *construction* of the concepts that belong to the possibility of matter in general" (472), and, once again, at the end of the final paragraph of the Preface concerning the "small amount" that can be contributed by metaphysics to natural science (479): "Nevertheless, this small amount is still something that even mathematics unavoidably requires in its application to natural science; and thus, since it must here necessarily borrow from metaphysics, it need also not be ashamed to let itself be seen in community with the latter."

CHAPTER ONE

Phoronomy

I MATTER AS THE MOVABLE IN SPACE

The first explication of the Phoronomy (480) characterizes matter as the movable in space. This characterization, in an important sense, is the linchpin of the entire treatise. For, on the one hand, the other three chapters all begin from the idea that matter is the movable and then proceed to add a further specification. In Dynamics matter is the movable in so far as it fills a space; in Mechanics matter is the movable in so far as it, as such a thing, has a moving force; in Phenomenology matter is the movable in so far as it, as such a thing, can be an object of experience. On the other hand, however, the initial explication of the Phoronomy also refers back to the Preface, where Kant first introduces the concept of motion (476–77) as "[t]he basic determination of something that is to be an object of the outer senses" and explains that "[t]he understanding traces back all other predicates of matter belonging to its nature to this one, and so natural science is either a pure or applied *doctrine of motion* throughout." In the same passage Kant explains that the metaphysical foundations of natural science considers the concept of motion in four chapters arranged in accordance with the four headings of the table of categories, where the first or Phoronomy chapter therefore considers motion under the heading of quantity. So here in the Preface Kant forges a connection between the concept of motion and his initial characterization of matter as simply an object of the *outer* senses (467).[1] I shall return to this centrally important connection below. But I first want to observe that what Kant adds in the Phoronomy to the concept of motion already introduced in the Preface is the explicit recognition that motion, in turn, always makes essential reference to space: matter is not simply the movable *simpliciter* but rather the movable *in space*. An explicit reference to

[1] See the paragraph to which note 7 of the Introduction is appended.

the relation of the movable (matter) to space is thus the counterpart in the Phoronomy of the further specifications of the movable added in the three remaining chapters.

This addition of an explicit reference to space is by no means trivial. For, as Kant points out in the first explication, we thereby raise the problem of absolute versus relative space (480): "*Matter* is the *movable* in space. That space which is itself movable is called material, or also *relative space*. That space in which all *motion* must finally be thought (and which is therefore itself absolutely immovable) is called pure, or also *absolute space*." Thus, Kant immediately introduces the distinction between absolute and relative space, which figures centrally in Newton's famous Scholium to the Definitions that initiate the *Principia*. In Newton's words (P408–9): "Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space."

Aside from these similarities in terminology and wording, why should we think that Kant has Newton's Scholium specifically in mind? First, like Newton, Kant goes on immediately to discuss the problem of absolute versus relative motion (481–82), and, in his further discussion of this problem (487–88) he invokes the example of relative motions of objects within a ship compared with relative motions of the ship itself to which Newton also appeals in the Scholium (P409).[2] Second, and more importantly, it turns out that Kant's own solution to the problem of absolute versus relative space, in *contrast* to his understanding of the Newtonian conception, is that "absolute space is *in itself* nothing and no object at all" (481–82) but rather, in the Kantian sense, some kind of idea of reason.[3] Nevertheless, when Kant elaborates on his solution in the Phenomenology chapter, where he states (560) that "absolute space is therefore not necessary as the concept of an actual object, but only as an idea, which is to serve as the rule for considering all motion and rest therein merely as relative," he explicitly refers several times to Newton's Scholium in the course of developing this thought (557–58, 562). Of course all these difficult issues have yet to be addressed. I now want simply to observe that, despite Kant's sharp metaphysical differences with Newton concerning

[2] Using motions on or within a ship to illustrate the relativity of motion was of course very familiar before Newton, extending back (at least) to Galileo. Nevertheless, there is some reason to think that Kant has Newton's discussion specifically in mind: see note 5 below.

[3] For Kant's understanding of the Newtonian conception see note 25 of the Introduction.

the concept of absolute space, it appears that Kant takes up this problem, first and foremost, from Newton's Scholium.[4]

Kant begins to address the problem of absolute versus relative space and absolute versus relative motion in the second remark to the first explication. He points out that any space in which we are to "arrange" or "set up" our experience of motion must be a movable, empirical, or relative space:

In all experience something must be sensed, and that is the real of sensible intuition. Therefore, the space in which we are to arrange [*anstellen*] our experience of motion must also be sensible, that is, it must be designated through what can be sensed. This space, as the totality of all objects of experience and itself an object of experience, is called *empirical space*. But this space, as material, is itself movable; and a movable space, however, if its motion is to be capable of being perceived, presupposes once again an expanded material space in which it is movable; this latter presupposes in precisely the same way yet another; and so on to infinity. (481)

Kant concludes that all motion must necessarily be relative and thus that absolute space, considered as some kind of object in itself, is impossible:

Thus all motion that is an object of experience is merely relative, and the space in which it is perceived is a relative space. This latter moves in turn in an expanded space – perhaps in the opposite direction, so that matter moved with respect to the first space can be called at rest in relation to the second space. And these variations in the concept of motions progress to infinity along with the change of relative space. To assume an absolute space – that is, one such that, because it is not material, it can also not be an object of experience – as *given in itself* is to assume something for the sake of the possibility of experience that can be perceived neither in itself nor in its consequences (motion in absolute space). Yet experience must always be arranged without it. Absolute space is thus *in itself* nothing and no object at all. (481)

As I have suggested, Kant's ultimate solution is that absolute space is not entirely to be banished, however, but is rather to be re-interpreted "as an idea, which is to serve as the rule for considering all motion and rest therein merely as relative" (560).

Once again, I am not yet in a position to examine this solution, and I shall not be able to do so adequately until I consider Kant's full elaboration of it in my chapter on the Phenomenology. But it is possible now to explore Kant's conception of the relativity of motion and of space more

[4] For these metaphysical difference see notes 15 and 16 of the Introduction, together with the paragraph to which they are appended and the preceding paragraph.

fully. As I have noted, Kant returns to the problem a little later in the Phoronomy in the course of articulating his own version of a principle of the relativity of motion:

> To make the motion of a body into an experience it is required that not only the body, but also the space in which it moves, be an object of outer experience and thus material. An absolute motion – that is, a motion in relation to a non-material space – is capable of no experience at all and hence is nothing for us (if one wanted to grant that absolute space were something in itself). But in all relative motion the space itself – since it is assumed to be material – is in turn represented as either at rest or as moved. The first case occurs when, beyond the space in which I viewed the body as moved, no further expanded space is given to me that includes this space (as when I see a ball moving on the table in the cabin of a ship). The second case occurs when, beyond the given space, another space that includes this one is given (in the example mentioned, the bank of the river), since I can then, in relation to the latter space, view the nearest space (the cabin) as moved and the body itself as possibly at rest. However, because it is completely impossible to determine for an empirically given space – no matter how expanded it may be – whether it may or may not be moved in turn in relation to an inclusive space of still greater extent, it must then be completely the same [*einerlei*] for all experience and every consequence of experience whether I wish to view a body as moved or at rest (but with the space moved in the contrary direction with the same speed). (487–88)

This passage is a clear echo of the passage from the second remark to the first explication presented immediately above. In both passages the point is that the concepts of motion and rest have no fixed application to any given empirical object. Rather, we can begin with a given empirical space and then "expand" this space indefinitely by considering wider and wider such spaces that include both the original space and (successively) one another. We thus obtain a nested sequence of relative spaces in which the originally given empirical object can be characterized (successively) by an indefinite variety of states of motion – relative to the indefinitely expanded empirical spaces in question. In such a sequence, Kant says (481), "these variations in the concept of motions progress to infinity along with the change of relative space."

This conception of the relativity of motion and of space is actually formulated quite early in Kant's intellectual career. It appears for the first time in a short pamphlet, "New System of Motion and Rest [*Neuer Lehrbegriff der Bewegung und Ruhe*]," published in 1758, which begins by articulating essentially the same view of relative motion and rest that we find in the Phoronomy. Since the explanation in 1758 is much more detailed and vivid (and is also considerably less well known, especially in the English-speaking world), I here present a lengthy extract:

In this position [of "mere sound reason"] I recognize that motion is change of place. I soon also grasp, however, that the place of a thing is recognized by means of the situation, the position, or the outer relation of this thing with respect to others that are around it. Now, I can consider a body in relation to certain outer objects that immediately surround it, and I will then say, if it does not change this relation, that it is at rest. But as soon as I view it in relation to a sphere of wider extent it is possible that precisely the body together with its neighboring objects changes its position in relation to this [sphere], and I will, from this point of view, impart [*mitteilen*] a motion to it. And I am completely free to expand my point of view as much as I wish, and to consider my body in relation to ever more distant surroundings. I [thereby] grasp that my judgement about the motion and rest of this body is never constant, but can always rather be changed by new points of view. Suppose, for example, that I find myself in a ship that lies at anchor on the river Pregel. I have a ball in front of me lying on the table: I consider it in relation to the table, the walls, and the other parts of the ship and say that is at rest. Soon thereafter I look out of the ship towards the riverbank, notice that the rope by which it was anchored has been cut and that the ship is slowly drifting down the stream, and immediately say that the ball is moving – and, indeed, from east to west in accordance with the direction of the flow. However, as soon as someone tells me that the earth is rotating daily with a much greater speed from west to east, I become of a different opinion, and I ascribe to the ball a completely different and contrary motion, with a speed that can easily be determined in astronomy. But now I am reminded that the whole sphere of the earth is in an even faster [state of] motion, from west to east, with respect to the planetary system. I am forced to ascribe this to my ball and to change the speed I had previously given it. Finally, *Bradley* teaches me that the entire planetary system together with the sun most likely undergoes a displacement with respect to the fixed stars. I ask: in what direction and with what speed? I receive no answer. And now I become dizzy: I no longer know whether my ball is at rest or in motion, in what direction and with what speed. At this point I begin to comprehend that there is something lacking in the expressions of motion and rest. I should never use them in an absolute sense but always respectively. I should never say that a body is at rest, without also specifying in relation to which things it is at rest; and I should never say that it is in motion, without at the same time naming the objects with respect to which it alters its relation. (2, 16–17)

The passage from Kant's articulation of a principle of relativity in the Phoronomy presented previously (487–88) is thus an abbreviation of the more vivid and expansive explanation Kant gives in 1758. The former presents the first two stages of an indefinite sequence of ever more comprehensive relative spaces and associated motions (ball relative to a ship, ship relative to a riverbank, riverbank relative to the rotating earth, the earth relative to the solar system, and so on) that is described much more fully in the latter.[5]

Kant's conception of the relativity of motion and of space is therefore quite concrete and, in particular, is organized around a rather specific sequence of relative spaces. We begin with the relative spaces determined by the positions of our own bodies, either with respect to the earth or with respect to some or another surrounding structure (such as a ship) that may itself be in motion relative to the earth; we next take account of the motions of the earth within the solar system – its rotation around its axis and its orbital motion with respect to the sun; we then take account of the motion of the sun, together with the entire planetary system, by means of a yet wider orbital motion of the solar system as a whole within the Milky Way galaxy (due to the rotation of the latter); and so on. Indeed, the reference to the astronomer James Bradley in the 1758 passage suggests that Kant has an orbital motion of the solar system within the Milky Way galaxy explicitly in mind and also how he intends the sequence to continue beyond this point. For the reference to Bradley here echoes a corresponding reference at the beginning of Kant's *Universal Natural History and Theory of the Heavens*, which appeared three years earlier. Kant there quotes a passage from Bradley (1748) indicating that a motion of the solar system relative to the fixed stars is possible, and Kant then interprets Bradley's results as supporting his own view that the Milky Way galaxy containing our solar system is rotating.[6] Kant proceeds

[5] This embedding of the example from the Phoronomy (487–88) into a larger sequence makes the parallel with Newton's discussion of a similar example in the Scholium more explicit (P409–10):

> But true rest is the continuance of a body in the same part of that unmoving space in which the ship itself, along with its interior and all its contents, is moving. Therefore, if the earth is truly at rest, a body that is relatively at rest on a ship will move truly and absolutely with the velocity with which the ship is moving on the earth. But if the earth is also moving, the true and absolute motion of the body will arise partly from the true motion of the earth in unmoving space and partly from the relative motion of the ship on the earth. Further, if the body is also moving relatively on the ship, its true motion will arise partly from the true motion of the earth in unmoving space and partly from the relative motions both of the ship and of the body on the ship, and from these relative motions the relative motion of the body on the earth will arise.

[6] Kant's (translated) quotation from Bradley (1748, pp. 39–40) occurs in the Preface to the *Theory of the Heavens* (1, 231–32). Bradley's original paper appeared in the *Philosophical Transactions of the Royal Society*, and Kant quotes from a (German) translation appearing in the *Hamburgisches Magazin* in the same year (pp. 616–17). Kant then cites Bradley (1, 252) in support of his own conception of a rotating Milky Way galaxy in the First Part of his treatise, entitled "Systematic Constitution among the Fixed Stars." Although it is true that Bradley (in the passage earlier quoted by Kant) does speculate that some observed motions of the fixed stars relative to the earth

to generalize this conception to infinity, finally, by viewing the Milky Way galaxy as part of a much larger rotating system of galaxies in turn, this last system as part of a much larger rotating system of such systems, and so on *ad infinitum*.[7]

A parallel conception of space and location in space is reflected, if only implicitly, in Kant's characteristic discussions of spatial orientation, as first presented in *The Ultimate Ground of the Differentiation of Regions in Space* (1768) and later in *What is Orientation in Thinking?* (1786). Kant begins from the idea, expressed in the first work (2, 378–79), that "we are acquainted with all things outside us by means of the senses only in so far as they stand in relation to ourselves," that is, in relation to our body. In particular, our body generates three perpendicular directions of spatial orientation, up and down, forward and backward, right and left, which then form the indispensable basis for orienting ourselves with respect to any given space – or, as Kant puts it in *Orientation in Thinking*, orienting ourselves mathematically (8, 135). Thus, for example, I can orient myself mathematically with respect to objects in a familiar room even in darkness by starting from a particular familiar object and then moving right and left, forward and back (8, 135). And, by the same token, I can orient myself geographically with respect to the surface of the earth by aligning myself relative to the points of the compass, north, south, east, and west. Finally, I can orient myself with respect to the heavens using these same geographical reference points by locating the earth, in turn, in relation to the heavenly bodies – so, for example, I can say that the earth itself orbits the sun from west to east.[8] All location of objects in a space, and, in particular, all location of objects in an oriented space, is therefore achieved by starting from our own body as given and then working outwards step by step – from everyday objects in our familiar surroundings, to our more

may be due to a motion of our solar system with respect to absolute space, the observations that Bradley tabulates are in fact due to other factors: the aberration of starlight arising from the combined effect of the (finite) velocity of light and the orbital motion of the earth, on the one side, and a small nodding or "nutation" of the earth's axis of rotation, on the other.

[7] See the striking passage towards the end of the First Part (1, 256):

> We see the first terms of a progressive relationship of worlds and systems, and the first part of this infinite progression already provides the knowledge of what we are to conjecture about the whole. There is here no end, but rather an abyss of true immeasurability, into which all the capability of human concepts must sink, even when it is elevated by the help of the science of number.

[8] For this notion of geographical orientation see *Orientation in Thinking* (8, 134–35), and compare the parallel discussion in *Regions in Space* (2, 379–80). It is noteworthy that the same notion introduces Kant's brief discussion of orientation in the third remark to the second explication of the Phoronomy, where Kant illustrates it by the motion of "a planet from west to east" (483).

general location with respect to the earth, and finally to the (changing) location of the earth itself with respect to the heavens.[9]

Kant's view of space, location and orientation in space, and motion and rest in space is therefore essentially connected from the very beginning with the new view of space, motion, and rest due to the Copernican revolution in astronomy. We initially take ourselves and our familiar surroundings to be at rest and thus to define a particular notion of space and location in space centered on ourselves. Everyday observation soon shows us that we ourselves may be in motion relative to the surface of the earth, however, so we then take the earth to define the relevant space and, accordingly, to be at rest (relative to this space) at the center of the universe. But this geocentric view of space, motion, and rest is then upset by the Copernican revolution, which, in the end, leaves us with an infinite and homogeneous space with no privileged center or any other distinguished point.[10] Choice of a privileged point – and thus of a privileged relative space or what we now call reference frame – can now only proceed from an entirely arbitrary initial space (defined by the position of our own body, for example) and then work its way outwards, as it were,

[9] In *Regions in Space* Kant follows his introductory discussion of spatial orientation in general with the example of incongruent counterparts, which is there taken (2, 381) to prove the reality of "universal absolute space, as conceived by geometers." I cannot examine this famous argument here. But it is important to note that the Kantian conception of spatial orientation in general is in no way incompatible with his rejection in the *Metaphysical Foundations* of what he understands as the *Newtonian* conception of absolute space. For the latter is essentially tied to the problem of absolute versus relative motion, which Kant explicitly excludes from consideration – citing Euler (1748) – in the 1768 essay (2, 378). Indeed, we find the same general conception of spatial orientation in 1786, in both *Orientation in Thinking* and the *Metaphysical Foundations*: see, in particular, the passage cited in note 8 above, where Kant again invokes incongruent counterparts, this time to suggest that the relevant difference between such counterparts (e.g., a left and right hand) is intuitive rather than conceptual.

[10] It is well known that Kant's Preface to the second edition of the *Critique of Pure Reason* compares his own revolution in philosophy to the Copernican revolution in astronomy (Bxv–xviii). Kant goes further several pages later by comparing the proof of his new "Copernican" hypothesis in metaphysics in the body of the *Critique* to Newton's proof of the original Copernican hypothesis in astronomy (note at Bxxii):

> Thus the central laws of motion of the heavenly bodies provided established certainty to that which *Copernicus* initially assumed merely as an hypothesis, and, at the same time, proved the invisible binding force of the universe (the *Newtonian* attraction), which would have always remained undiscovered if [Copernicus] had not dared, in a manner contrary to the senses and yet true, to seek the observed motions not in the objects of the heavens but in their observer. I here introduce in the Preface the transformation in thinking, analogous to [Copernicus's] hypothesis and explained in the *Critique*, also only as an hypothesis, in order to make clear that the first attempts at such a transformation are always hypothetical. In the treatise itself, however, it is proved not hypothetically but apodictally, from the constitution of our representations of space and time and the elementary concepts of the understanding.

through an indefinite sequence of ever more inclusive spaces. To say that an object is either in motion or at rest, on this view, only makes sense when we specify exactly where in the sequence of ever more inclusive relative spaces we are considering it: an object at rest in my room is not at rest relative to the earth's axis of rotation, this axis is moving relative to the solar system, the solar system is moving relative to the Milky Way galaxy, and so on.[11]

Kant's conception of space, location in space, and motion in space thus has a very specific structure, and it is only by keeping this structure firmly in mind that we can clarify otherwise puzzling features of Kant's argument in the *Metaphysical Foundations*. Thus, for example, it immediately becomes clear why Kant takes movability as an essential and defining feature of matter. This can appear counterintuitive initially, because one can naturally ask oneself why all matter – specified merely as the object of outer sense – must be movable: why can there not be material objects located in space that are nonetheless entirely immovable?[12] Suppose, for example, that I imagine looking out at the objects located in space around me and finding that none of them are moving; why, I might ask myself, must they *ever* move? The point, however, is that this whole way of framing the question presupposes a commonsensical (and thus fundamentally geocentric) conception of motion and rest. To see this, we have merely to raise the question whether I myself, and thus the entire relative space surrounding me, am not in motion in turn. If so, then it immediately follows that the objects previously viewed as unmoving are in fact in a state of motion after all – in relation, that is, to the new relative space in which my own motion (along with that of my relative space) is now considered.

[11] The essentially Copernican (and ultimately Newtonian in the sense of the previous note) character of Kant's conception of motion is also made clear in the second explication of the Phoronomy, according to which "motion of a thing" is to be defined as "change of its *external relations* to a given space" rather than as mere "change of place" (482). For the central example introduced in the first remark to this explication is the rotation of the earth (482): "Now a body can move without changing its place – as in the case of the earth rotating around its axis. But its relation to external space still changes thereby. It turns, for example, its different sides toward the moon in 24 hours – from which all kinds of varying effects then follow on the earth." The effects in question include, in particular, those involving tidal forces: both the tides of our seas as described by Newton in the *Principia* in terms of the varying gravitational pull exerted by the moon (together with the sun) and the corresponding effects on the earth's rate of rotation to which Kant first called attention (more than a century before it was verified) in his 1754 essay on *Whether the Earth has Undergone an Alteration of its Axial Rotation*.

[12] This question has been asked by many commentators on the *Metaphysical Foundations*. A particularly clear example is Walker (1974), which takes the supposed possibility of immovable matter to be explanatory of Kant's view that the concept of matter articulated in the *Metaphysical Foundations* is an *empirical* concept.

For Kant, as we know, space, motion, and rest are always relative concepts; and he holds, in particular, that any body initially taken to be at rest may equally well be taken to be in motion. For there always exists, at least in principle, an "expanded" relative space with respect to which the initial relative space is in motion. (Conversely, any body initially taken to be in motion may equally well be taken to be at rest.) As Kant himself explains (488): "[I]t must then be completely the same [*einerlei*] for all experience and every consequence of experience whether I wish to view a body as moved or at rest (but with the space moved in the contrary direction with the same speed)." It is in this precise sense that all matter without exception – all material bodies located outside me in space – must, in fact, be movable.

By the same token, finally, Kant's characteristic conception of space, location, and motion also illuminates the way in which he connects his original characterization of matter as simply an object of the *outer* senses with his more specific characterization in terms of motion and movability. This connection is first made, as I noted in the Introduction, in a crucial paragraph in the Preface – using an especially obscure and cryptic argument (476–77): "The basic determination of something that is to be an object of the outer senses had to be motion, because only thereby can these senses be affected. The understanding traces back all other predicates of matter belonging to its nature to this one, and so natural science is either a pure or applied *doctrine of motion* throughout." What does Kant mean by the claim that only *by motion* "can [the outer] senses be affected"? Is he making a physiological claim about our external sense organs – for example, that they can only be stimulated by moving matter impacting upon them? This, even if true, would be a merely empirical claim, with no special standing in a properly *metaphysical* treatment of matter.[13]

Let us begin by observing that, for Kant, to say that an object affects the outer senses is simply to say that the object is sensed [*empfunden*] by means of these senses.[14] Let us also recall that Kant begins his discussion

[13] This kind of puzzlement has of course been expressed by many commentators. A particularly clear discussion, once again, is Walker (1974, pp. 152–53). Note that Kant's next sentence emphasizes the *metaphysical* nature of his claim (477): "The *metaphysical* foundations of natural science are therefore to be brought under *four* chapters [arranged in accordance with the table of categories]."

[14] See the remarks at the very beginning of the transcendental aesthetic (A19–20/B34): "The action of an object on the faculty of representation, in so far as we are affected by it, is *sensation* [*Empfindung*]. That intuition which is related to an object through sensation is *empirical* [intuition]. The undetermined object of an empirical intuition is *appearance*."

of orientation in space with the assertion (2, 378) that "we are acquainted with all things outside us by means of the senses only in so far as they stand in relation to ourselves." He goes on to explain that this involves, more specifically, a relation of the object sensed outside us to our own body: we establish three perpendicular planes determined by the three principal directions of orientation of our body, and we then use these planes to specify corresponding "regions in space" wherein all outer objects of perception may be located.[15] But we have thereby established a particular relative space, reference frame, or coordinate system centered on our own body, in which the position of every object in space can be precisely specified by its corresponding perpendicular distances from the given planes. And we have thus introduced, at the same time, the first term of one of Kant's "infinite progressions" of ever more inclusive relative spaces, in relation to which the state of motion of any outer object can then be variously determined. Movability and relative motion are therefore built into Kant's conception of the objects of outer sensation from the very beginning. This is not a physiological claim concerning how our (external) sense organs are stimulated but a transcendental-philosophical analysis, based on Kant's characteristically Copernican conception of space, location, and motion, of what specifically *embodied* spatial perception involves.

When Kant first introduces the idea of a metaphysical doctrine of body in the Preface, it is characterized solely in terms of the structure of our forms of sensibility. After explaining (467) that nature (in its "material meaning") can be "understood as the whole of all appearances, that is, the sensible world, excluding all non-sensible objects," Kant (467) divides the contents of nature (in this meaning) into "the objects of the *outer* senses" and "the object of inner sense," respectively. Thus a twofold metaphysical foundation of natural science is possible in principle, where the first considers the objects of the outer senses and the second the (putative) object of inner sense. But since, as explained in the Introduction, it turns out that a rational science of the soul (the putative object of inner sense) is not in fact possible, the metaphysical foundations of natural science reduce to the metaphysical doctrine of body, matter, or specifically corporeal nature.

Kant explains how to move from this characterization of matter as mere object of the outer senses to matter as the movable in space in the second remark to the first explication of the Phoronomy. After officially

[15] See *Regions in Space* (2, 378–79). As I noted above, essentially the same view is presented in *Orientation in Thinking* in 1786.

characterizing matter by its most fundamental determination or predicate (motion or movability), Kant goes on to say that a second, merely "metaphysical" explication is also possible:

> If I am to explicate the concept of matter, not via a predicate that belongs to it itself as object, but only via relation to that cognitive faculty in which the representation can first of all be given to me, then every *object of the outer senses* is matter – and this would be the merely metaphysical explication of matter. Space would then be merely the form of all outer sensible intuition (we here leave completely aside the question whether just this form also belongs to the outer object we call matter *in itself* or remains only in the constitution of our sense). *Matter*, as opposed to *form*, would be that in the outer intuition which is an object of sensation [*Empfindung*]. (481)

This second explication is in essential agreement with the characterization of matter presented at the beginning of the Preface: here, too, we consider solely the relation of matter to the form of our outer sensible intuition. In such a "metaphysical" explication, accordingly, we consider space as the mere *form* of our outer intuition and matter as its mere *content* – whatever this content may be.

But in the following sentences (switching from the subjunctive to the indicative mood) Kant then makes the crucial transition to the more concrete and substantive notion of real, material, empirical, and *therefore movable* space:

> Matter would thus be the properly empirical element of sensible and outer intuition, since it can certainly not be given a priori. In all experience something must be sensed [*empfunden*], and that is the real of sensible intuition [*das Reale der sinnlichen Anschauung*]. Therefore, the space in which we are to arrange our experience of motion must also be sensible [*empfindbar*], that is, it must be designated through what can be sensed. This space, as the totality of all objects of experience and itself an object of experience, is called *empirical space*. But this space, as material, is itself moving. (481)[16]

In considering the actual sensation or perception of objects located outside us in space, we necessarily go beyond the consideration of space as a mere form of outer sensible intuition. We necessarily introduce the more concrete and substantive conception of space encapsulated in Kant's characteristically Copernican conception of spatial relativity, and on

[16] Kant's Copernican conception is first made explicit in the remainder of the final sentence quoted above: see the first quotation in the paragraph following the one to which note 4 above is appended.

this conception, as we have seen, matter must be characterized, first and foremost, as the movable.[17]

2 MOTION (AND REST) AS AN ENDURING STATE

The third explication and accompanying remark presents a particular conception of rest and, by implication, of motion as well.[18] The point, briefly, is that rest cannot be conceived merely negatively, as it were, as the lack or absence of motion; it must rather be understood as a positive state that a body may possess, that of what Kant calls "enduring presence [*beharrliche Gegenwart*]" in the same place. The explication reads (485): "*Rest* is enduring presence (*praesentia perdurabilis*) at the same place. What is *enduring* [*beharrlich*] is that which exists throughout a time, i.e., persists [*dauert*]." The following remark makes clear, however, that Kant does not mean that a body can be in a state of rest at a given place only if it is present at the place throughout some *interval* of time. On the contrary, he explicitly distinguishes the notion of an *instantaneous* yet still "enduring" state from that of a state actually persisting over time (486): "To be in an *enduring state* and to *endure in this state* (if nothing else displaces it) are two different (although not incompatible) concepts." Kant illustrates the notion of enduring presence or rest (486) by motion with an instantaneous "turn-around" point: that of a body rising and decelerating under the influence of gravity and then, after a single instant of rest, accelerating back down again in free fall. The state Kant is attempting to characterize, then, is that of an instantaneous *tendency* to remain in a given place – a tendency that may or may not issue in actually remaining at that place over a finite interval of time. The notions of "enduring"

[17] A parallel link between sense perception and the relativity of space is present in Newton's discussion in the Scholium. After observing that the "popular" view of space, position, and motion is "conceived solely with reference to the objects of sense perception," Newton famously states (P408–9):

> Relative space is any movable measure or dimension of … absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth. Absolute space and relative space are the same in species and in magnitude, but they do not always remain the same numerically. For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute sense.

[18] I touched on aspects of the second explication and accompanying remarks in note 11 above, and I shall touch on others in what follows.

and "persistence" here refer only to this mere potentiality or tendency to remain in a given place.

I have suggested that this conception of rest is, by implication, a conception of motion as well. In order to see this, it suffices to recall that, for Kant, motion and rest are relative terms, so that, in particular, a body at rest relative to one frame of reference will be in motion relative to a second frame of reference relative to which the original frame of reference is moving. In Kant's words (488): "[I]t must … be completely indifferent for all experience and every consequence of experience whether I wish to view a body as moved or at rest (but with the space moved in the contrary direction with the same speed)." Thus, in the example in question, suppose that we observe the motions of the rising and falling body, not from a relative space or reference frame fixed on the surface of the earth, but from one moving away from the earth in the line of upward projection of the body, at a constant speed equal to the speed of the initial upward projection of the body in the original frame. In this second frame of reference the body is initially in a (momentary) state of rest; it then accelerates away from the reference frame and towards the earth (which is itself moving away from the reference frame at a constant speed equal to the initial speed of projection) until it attains the speed of the initial projection (but towards the earth); finally, after "persisting" in this speed for one moment, it continues to accelerate towards the earth until it attains a speed double that of the initial projection, whereupon it crashes into the earth. In this frame of reference, therefore, there is no state of rest and thus no turn-around point: there is simply a continuous process of acceleration in which attaining the momentary speed of the initial projection (but in the contrary direction) corresponds to the original turn-around point in a state of rest. The moral, then, is that everything Kant here says about the state of rest also holds for an arbitrary state of motion. Any motion whatsoever (with speed equal to zero or any other value) must be conceived as a momentary state or instantaneous tendency – a tendency that may or may not (as in the present example) result in actually remaining in that state over a finite time.[19]

Nevertheless, examples of motion with a turn-around point are especially important to Kant. In the present remark, in particular, Kant emphasizes the difference between two such cases: that of a body rising and falling under the influence of gravity and that of uniform motion of a

[19] Such an instantaneous tendency was standardly characterized as a *conatus* by such writers as Huygens and Leibniz. As we shall see, Kant uses the corresponding German term "*Bestrebung* [striving]."

body from a point A to a point B, whereupon the body is instantaneously reflected back to point A with the same uniform speed in the contrary direction. In this second case, as Kant puts it (485), we are to assume a motion where:

> [T]he body travels along the line AB with uniform speed forwards and backwards from B to A, and that, since the moment when it is at B is common to both motions, the motion from A to B is traversed in ½ sec., the motion from B to A also in ½ sec., and both together in one whole second – so that not even the smallest part of the time pertains to the presence of the body at B.

This case, then, corresponds to the way in which perfectly elastic impact was conceived: an initial constant or uniform motion towards the point of impact is transformed instantaneously into a motion with the same constant speed in the opposite direction.[20]

Kant's argument in the present remark is that there is a crucial asymmetry between this case of perfectly reflected uniform motion and the case of rising and falling under the influence of gravity. In the case of perfect uniform reflection, according to Kant, no coherent state of motion or rest can be assigned at the turn-around point B. For, by assumption, the body travels with the same constant speed at every point along the line from A to B. Similarly, the body travels with the same constant speed, but in the opposite direction, on its return trip at every point along the line from B to A. However, if we assign either of these motions to the body at the instant at which it is at B, we misdescribe the situation: in the former case (since the motion is uniform) the body would continue beyond B, in the latter (for the same reason) the body would have arrived from beyond B (and thus not from A). The only state that we can coherently assign to the body at the point B, therefore, is what Kant calls a "complete lack of motion" (485) or "a lack of all motion" (486). But this, in turn, cannot be equated with a state of rest. For that would imply that, at B, "the motion

[20] This is the way in which the impact of "absolutely hard" bodies was standardly described at the time, for example, by Huygens. Another standard case of motion with a turn-around point, which Kant does not explicitly consider here, is the motion of a pendulum. This case, however, would be essentially the same as that of rising and falling under gravity. In the second remark to the second explication Kant distinguishes motions that "return on themselves [*in sich zurückkerhrende*]" from those that do not [*in sich nicht zurückkerhrende*] (483). The former are either "circulating" or "oscillating," such as circular or vibratory [*schwankende*] motions (483): "The former [circulating] always traverse precisely the same space in the same direction, the latter always alternately in contrary directions – as in the case of vibrating pendulums [*schwankende Penduln*]." (These two cases – circular and pendular motion – were also extensively investigated by Huygens.) In this terminology, then, both of Kant's examples in the remark to the third explication are oscillating motions that return on themselves: Kant describes the case of perfect reflection here as "*in sich selbst widerkehrende Bewegung*" (485).

AB had ceased and that from B to A was not yet there" (485) – which, once again, would be incompatible with the assumption that the motion (in either direction) is entirely uniform and therefore has the same constant speed at *every* point between A and B.[21]

Suppose, by contrast, that (485) one "imagines the line AB as erected above the point A, so that a body rising from A to B falls back again from B to A after it has lost its motion at B through gravity." In this case, as we know, Kant thinks that a state of rest – as enduring presence in the same place – can indeed be assigned to the body at B. So Kant asks himself, accordingly, why the concept of rest is appropriate in this case but not that of perfect uniform reflection:

> The reason for this lies in the circumstance that the [former] motion is not thought of as uniform at a given speed but rather first as uniformly slowed down [*verzögert*] and thereafter as uniformly accelerated. Thus, the speed at point B [is] not completely [diminished], but only to a degree that is smaller than any given speed. With this speed, therefore, the body would, if it were to be viewed always as still rising … uniformly traverse with a mere moment of speed (the resistance of gravity here being set aside) a space smaller that any given space in any given time no matter how large. And hence it would absolutely not change its place (for any possible experience) in all eternity. It is therefore put into a state of *enduring* presence at the same place – i.e., [a state] of rest – even though this is immediately annulled because of the continual influence of gravity (i.e., the change of this state). (486)[22]

The crucial asymmetry, then, is that in the motion under the influence of gravity the approach followed by withdrawal from the turn-around point is not constant or uniform but is rather uniformly *decelerated* and then uniformly *accelerated*. It is precisely this feature of the case, for Kant, that allows us to assign to the body at B, not "a complete lack of motion," but rather what Kant calls "a mere moment of speed" or a speed "smaller than any given speed."[23]

[21] This reasoning clearly involves some implicit *continuity* assumptions, which will be made explicit in what follows.

[22] Where I have inserted "is … diminished" in brackets, there is actually no verb at all in Kant's text. Hartenstein has made the plausible suggestion that "*verzögert werde*" should be inserted after "degree [*Grad*]" – but I have used "diminished" instead of "slowed down" to be a bit more neutral: see note 25 below.

[23] Lemma 1 of Section 1 of the *Principia* reads (P433): "*Quantities, and also ratios of quantities, which in any finite time constantly tend to equality, and which before the end of that time approach so close to one another that their difference is less than any given quantity, become ultimately equal.*" If Kant were following Newton here, then he would be saying that the speed of the decelerating body approaches a value of zero (i.e., rest) as its limit (at point B). By contrast, if Kant were speaking of an *infinitesimal* or literally *infinitely small* speed, then he would be saying that the speed of the decelerating body acquires (at point B) a non-zero value smaller than any finite value.

Kant's characterization of motion under the influence of gravity in terms of uniform acceleration and deceleration is an allusion to Galileo's law of fall – according to which all bodies near the surface of the earth experience the same uniform or constant acceleration downwards towards the center of the earth with the same value (g = 32 ft. per sec.^{2}). It follows (using modern notation for ease of comprehension) that all such bodies acquire a uniformly increasing velocity directed downwards ($v_{down} = gt$) and that the distances thereby traversed increase in proportion to the squares of the times ($s = \frac{1}{2}gt^2$). In Kant's example of upward projection followed by fall, therefore, we begin with an initial velocity of upward projection, v_{up}, to which the action of gravity continuously adds a uniformly increasing velocity directed downwards, $v_{down} = gt$. At any given time, then, the total velocity of the body is given by the sum of these two, $v_{up} - gt$, where the minus sign indicates the downward direction of the latter velocity. When gt = the initial speed of upward projection, we have a turn-around point of total velocity $v_{turn\text{-}around}$ = zero, and, at subsequent times, we then have simple uniform acceleration in fall in accordance with $v_{down} = gt$. The crucial point, for Kant, is that in this case the turn-around velocity is continuously approached (and then continuously receded from) by a decreasing (and then increasing) sequence of velocities. But this is emphatically not the case, of course, in the case of uniform perfect reflection: here the turn-around point stands out as a *discontinuity* between a perfectly constant sequence of incoming velocities and a second such sequence of outgoing velocities.

Suppose, in modern terms, that we represent the two trajectories in question not simply by their spatial paths but as trajectories in space and time (as spatial functions of the time). Then the case of uniform deceleration and acceleration under gravity appears as a continuous spatio-temporal curve (continuously rising and continuously falling) possessing a *tangent* (or derivative) at every point (including the turn-around point), whereas the case of perfect uniform reflection appears as two straight lines (incoming and outgoing) connected by a sharp cusp or corner at the turn-around point possessing *no* tangent (or derivative) at that point.[24] In these terms,

Newton explains in the Scholium to this Section (P440–43) that he is using his method of first and ultimate ratios precisely to *avoid* the hypothesis of infinitesimals ("indivisibles"), but Kant himself appears to have no hesitations concerning infinitesimals or infinitely small quantities (compare note 25 below).

[24] In modern terms, therefore, while the former trajectory is both continuous and smooth (everywhere differentiable), the latter is continuous but not smooth (not differentiable at the turn-around point).

what Kant is saying is that an instantaneous state of motion (speed or velocity) at a place and time is represented by a straight line or tangent to a continuous spatio-temporal curve or trajectory at that place and time. So, in particular, an instantaneous state of rest (with speed or velocity equal to zero) is represented by a straight line or tangent at the spatio-temporal point in question, where this tangent itself represents a *constant* state of rest in which the trajectory maintains its spatial position over time. If the body were to endure in a state of rest, it would follow this tangent. By contrast, if it instantaneously (but continuously) deviates from a state of rest (as in the case of fall), it then follows a spatio-temporal curve or trajectory whose succeeding tangents continuously deviate from the straight line representing a constant state of rest. Kant's point, therefore, is that instantaneous states of motion (whether with speed equal to zero or any other value) can be coherently assigned to a given trajectory at a time only if the trajectory in question possesses a definite spatio-temporal tangent (or derivative) at that time.

Kant's point, however, is not that the case of uniform perfect reflection in impact cannot be coherently described. It can certainly be coherently described, but this very description reveals a crucial discontinuity at the turn-around point – the lack of a well-defined spatio-temporal tangent (or derivative). The state of such a trajectory at the turn-around point can only be described, in Kant's words, as a "lack of motion," which, Kant explains, "can in no way be constructed" (486). By contrast, Kant's own characterization of rest, as enduring presence in the same place, *can* be constructed (486), and it can "therefore be used for the subsequent application of mathematics to natural science."[25] Kant will later argue, in the

[25] Kant's text reads more fully as follows (486):

> Thus rest cannot be explicated as lack of motion, which, as = 0, can in no way be constructed, but must rather be explicated as perduring presence in the same place, since this concept can also be constructed, through the representation of a motion with infinitely small speed throughout a finite time, and can therefore be used for the ensuing application of mathematics in natural science.

This certainly makes it look as if Kant is attributing a literally infinitesimal but non-zero speed to the decelerating body (at point B). Note, however, that Kant's notation "= 0" is used to denote a complete lack of motion (at B) rather than what we would call a well-defined state of rest (with speed equal to zero). We represent the latter, in particular, by a well-defined tangent to the motion (at B) with zero slope, and Kant (not surprisingly) is simply not clearly distinguishing this notion from a literally infinitesimal but non-zero state of motion. (Concerning the textual issue raised in note 22 above: if "completely diminished" – using my insertion – means something like "completely eliminated," then Kant would be again emphasizing the difference between rest and a complete lack of motion; if it means "completely slowed down," however, then Kant is invoking a deceleration towards an infinitely small but still non-zero speed.)

Mechanics, that the case of impact must also be described in terms of *continuous* deceleration and acceleration (552): "A moved body that impacts on a matter is thus set into a state of rest, not at once, but only by a continuous retardation; a body that was at rest is only set into motion by a continuous acceleration; and a body is changed from one degree of speed to another only in accordance with the same rule."[26] The case of impact is thereby assimilated, in this respect, to the case of continuous deceleration and acceleration under the influence of gravity described by Galileo.[27]

3 MOTION AS A MAGNITUDE

The remainder of the Phoronomy is devoted to the conceptualization of motion as a magnitude [*Größe*] or, as Kant sometimes puts it, as a quantum. Such a conceptualization, as Kant makes clear, essentially involves the application of mathematics to motion (and thus to the movable) and, in an important sense, is the central task of Phoronomy as a whole (489): "Phoronomy is thus the pure doctrine of magnitude [*Größenlehre*] (*Mathesis*) of motions." In this sense, all our considerations so far have been preliminary, in that the point of Kant's discussion of the relativity of motion – and, as we shall see, his discussion of motion as an enduring state as well – is to facilitate, and pave the way for, the present discussion of the "construction of motions in general as *magnitudes*" (487). It is precisely here, in the actual execution of the construction in question, that we find the single "proposition" of the Phoronomy, which is derived, in turn, from the single "principle" stated there (Kant's principle of the relativity of motion). Moreover, it is precisely this aspect of the Phoronomy that is then carried over into the following Dynamics chapter, whose first proposition depends on the single proposition of the Phoronomy as its premise (497). Finally, a consideration of motion as a mathematical magnitude or quantity is also where the Phoronomy is explicitly connected to the corresponding heading of the table of categories and principles:

[26] In the same passage, as we shall see, Kant therefore *denies* the existence of absolutely hard bodies in the sense of note 20 above.

[27] In the Scholium to the Laws of Motion in the *Principia* Newton illustrates and confirms his laws of motion first by Galileo's law of fall, then by the motion of oscillating pendulums, and then by the laws of impact – which (following Wren and Mariotte) he verifies (much more carefully and systematically) by experiments with pendulums. Interweaving and synthesizing these two already known cases of motion and interaction (gravity and impact) is a crucially important part of Newton's mathematical method. As we shall see, it is an equally important part of Kant's metaphysical foundations of natural science. Kant, on my reading, has already begun this process of synthesis here: see again note 20 above.

namely, the categories of quantity and their associated principle articulated in the axioms of intuition.

In the second edition version of the axioms of intuition Kant characterizes the concept of magnitude in general in terms (B202–3) of "the composition of the homogeneous [*Zusammensetzung des Gleichartigen*] and the consciousness of the synthetic unity of this (homogeneous) manifold."[28] Kant initiates his discussion of motion as a magnitude in the fourth explication of the Phoronomy (486): "*To construct* the concept of a *composite motion* [*zusammengesetzten Bewegung*] is to present a motion a priori in intuition, in so far as it arises from two or several given [motions] united in a movable." He continues, in the following remark, with the assertion that the sole object of Phoronomy (487) is "to determine these motions a priori as magnitudes, with respect to both their speed [*Geschwindigkeit*] and direction, and, indeed, with respect to their composition [*Zusammensetzung*]." The fifth explication then explains the notion of composition (489): "The *composition of motion* [*Zusammensetzung der Bewegung*] is the representation of the motion of a point as identical [*einerlei*] with two or several motions of this point combined together [*zusammen verbunden*]." So it follows, on the one hand, that the conceptualization of motion as a magnitude centrally involves its mathematical description in terms of what we now call the directed (vector) magnitude of *velocity*, and, on the other, that this conceptualization of motion as a (directed) magnitude has the task (as in all cases of magnitude in general) of exhibiting some sort of operation of *composition* [*Zusammensetzung*].

It is important to understand, however, that Kant is employing the traditional notion of (continuous) magnitude descended from ancient Greek mathematics, not our modern notion of physical magnitude. For us, a physical magnitude (such as mass) is a function from some set of physical objects (massive bodies) into the real number system, implemented by choosing some arbitrary unit object (a standard gram, for example) and then representing the ratios of this object to all other objects by real numbers. The output of our function is thus a dimensional real number encoding the system of units in question (grams). In the traditional theory, by contrast, there was not yet a single real number system, which was

[28] This characterization is followed by (B203):

> Now the consciousness of the manifold of the homogeneous in intuition in general, in so far as the representation of an object is first made possible thereby, is the concept of a magnitude (*quanti*). Therefore, even the perception of an object, as appearance, is only possible by means of the same synthetic unity of the manifold of a given sensible intuition, whereby the unity of the composition of the manifold of the homogeneous is thought in the concept of a *magnitude*.

only established in the late nineteenth century. Rather, each type or kind of magnitude (lengths, areas, volumes, times, weights, and so on) was thought to form a system on its own, characterized by its own particular operation of *addition*. On this conception, in particular, it makes sense to add two lengths together to obtain a third, longer or greater length, and similarly for areas and volumes, but it does not make sense to add a length to an area or an area to a volume. Magnitudes are said to be *homogeneous*, then, when they are of the same dimension and can therefore be added together, and it is only homogeneous magnitudes of the same dimension (belonging to the same type or kind of magnitude) that can meaningfully be said to have a *ratio* to one another.[29] In this tradition, moreover, since what we would now express in terms of equations between magnitudes represented by real numbers is rather expressed in terms of equations (or proportionalities) between ratios, there is no need for an arbitrary choice of unit: instead of operating with dimensional real numbers, as it were, the traditional theory operates directly with the dimensions (or magnitude-kinds) themselves.[30]

For Kant, in particular, to conceptualize something as a magnitude or quantity is to exhibit or construct an appropriate operation of addition (which he calls "composition [*Zusammensetzung*]"), whereby we obtain a homogeneous system of elements belonging to a single dimension or magnitude-kind. The main task of Phoronomy, accordingly, is to do precisely this for the case of the directed (vector) quantity of velocity:

Phoronomy is thus the pure doctrine of magnitude (*Mathesis*) of motions. The determinate concept of a magnitude is the concept of the generation of the representation of an object through the composition of the homogeneous [*Zusammensetzung des Gleichartigen*]. Now, since nothing is homogeneous with

[29] Definition 4 of Book v of Euclid (which presents the Eudoxean theory of ratio or proportion) states that two magnitudes *have* a ratio just in case for some (natural) number n, the first magnitude added to itself n times is greater than the second, and, similarly, for some number m, the second added to itself m times is greater than the first. It thus makes sense only where addition is always well defined, that is, within a single dimension or magnitude-kind. According to the crucial Definition 5 of Book v, however, *sameness* (equality) of ratios, or *proportionality* between the corresponding magnitudes, can hold even where the ratios in question are from two different magnitude-kinds. For this whole subject see especially Stein (1990). See also Sutherland (2004a, 2004b, and 2006) for a detailed interpretation of Kant's conception of magnitude in terms of the Euclidean-Eudoxean theory.

[30] In the *Meno* problem Socrates presents to the slave boy, for example, where we would now express the result by the (rather trivial) formula $\sqrt{2}$ ft. × $\sqrt{2}$ ft. = 2 ft.^{2}, the Greeks say that to double a given square one erects it on the diagonal of the given square, so that the square on the diagonal has the same ratio to the given square as the doubled side (the result of adding the original side to itself) does to the original side. Thus we here have an equality between two ratios of different dimensions in accordance with note 29 above.

motion except motion in turn, phoronomy is a doctrine of the composition of the motions of one and the same point with respect to its speed and direction, that is, the representation of a single motion as containing two or more motions at the same time, or two motions of precisely the same point at the same time, in so far as they constitute one motion *together* [*zusammen*]; that is, [they] are identical [*einerlei*] with the latter, and do not, for example, produce it, as causes produce their effect. (489)

Motion is conceptualized as a magnitude, then, when we exhibit or construct an appropriate addition operation on the system of (directed) velocities, so that precisely these velocities thereby constitute a single homogeneous magnitude-kind.

Kant's solution to this problem is presented in the single proposition of the Phoronomy, which is officially divided into three separate cases: one where the motions in question occur in the same line and the same direction, a second where they occur in the same line in opposite directions, and a third where they occur in two different lines meeting at a given point in a given angle. The third case, as Kant explains, is the most general one, and his solution here proceeds by a familiar parallelogram construction in which the composite motion in question appears as the diagonal of the parallelogram determined by the two initial motions (492).[31] In this sense, the operation of addition or composition Kant presents is given by a parallelogram of motions that initially appears to be very similar to those found in standard treatments in physics, for example in Newton's first Corollary to the Axioms, or Laws of Motion.[32] Kant emphasizes in

[31] Kant explains that the third case in a sense subsumes the other two; for when the angle in question is zero (Kant says "infinitely small") we have the first case, and when it is 180° (Kant says "differing from a straight line only infinitely little") we have the second (495). Only the third case need be considered in physics, then, although the separation into three different cases "has its uses in transcendental philosophy" (495): "But that one normally understood, by the term *composite motion* [*zusammengesetzten Bewegung*], only the single case where the directions comprise an angle, did no harm to physics, but rather to the principle of classification of a pure philosophical science in general." I shall return to the point of Kant's threefold classification in transcendental philosophy in section 7 below.

[32] Newton's derivation of his first corollary centrally involves the Second Law of Motion, and, in his comments on this Law, Newton in effect already presents the derivation – in the form, specifically, of the three cases separately distinguished by Kant (P416–17):

If some force generates any motion, twice the force will generate twice the motion, and three times the force will generate three times the motion, whether the force is impressed all at once or successively by degrees. And if the body was previously moving, the new motion (since motion is always in the same direction of the generative force) is added to the original motion if that motion was in the same direction or is subtracted from the original motion if it was in the opposite direction or, if it was in an oblique direction, is combined obliquely and composed [*componitur*] with it according to the directions of both motions.

the strongest possible terms, however, that his construction should not be confused with the standard (e.g., Newtonian) parallelogram construction in physics. For the latter, according to Kant, amounts to what he calls a mere "mechanical construction," "where one allows moving causes to produce a third motion by combining one given motion with another," and it is not, therefore, what Kant is aiming at: namely, a "mathematical" or "geometrical construction" in which we require that "one magnitude be *identical* [*einerlei*] with another or that two magnitudes in composition be *identical* with a third, not that they produce the third as causes" (493).

Kant emphasizes, accordingly, that (494) his "*mathematical construction*" should "only make intuitive what the object (as quantum) *is to be*, not how it may be *produced* by nature or art by means of certain instruments and forces." He also emphasizes (494) that his construction, unlike Newton's, is entirely independent of the mechanical laws of motion and, in particular, of the law of inertia – whereby "the body [in question] conserves itself in *free* motion with the first speed, while the second is added, which, however, is a law of nature of moving forces that can in no way be at issue here, where the question is solely how the concept of velocity as a magnitude is to be *constructed*."[33] Indeed, Kant presents these considerations considerably earlier in his remark to the fourth explication:

> It is required for the construction of concepts that the conditions of their presentation not be borrowed from experience, and thus not presuppose certain forces whose existence can only be derived from experience; or, in general, that the condition of the construction must not itself be a concept that can in no way be given a priori in intuition, such as, for example, the concept of cause and effect, action and resistance, etc. Now here it is above all to be noted that phoronomy has first to determine the construction of motions in general as *magnitudes*, and, since it has matter merely as *something movable* as its object, in which no attention at all is therefore paid to its quantity, [it has to determine] these motions a priori solely as magnitudes, with respect to both their speed and direction, and, indeed, with respect to their composition. For so much must be constituted wholly a priori, and indeed intuitively, on behalf of applied mathematics. For the rules for the connection of motions by means of physical causes, that is, forces, can never be rigorously expounded, until the principles of their composition in general have been previously laid down, purely mathematically, as basis. (486–87)[34]

[33] Newton's statement of the composition of motions in his first corollary (note 32 above) is given in terms of the composition of *forces*, and, of course, he also appeals explicitly to the law of inertia in arguing that the uniform motion impressed by the first force will continue undisturbed while the second force is impressed.

[34] In the sentence stressing that phoronomy "has matter merely as *something movable* as its object, in which no attention at all is therefore paid to its quantity," Kant means that matter is here

Moreover, that Kant's construction of the composition of motions is to proceed entirely independently of the mechanical laws of motion is also indicated by the fact that Kant himself does not officially present these laws until much later in his treatise, in the third chapter or Mechanics.[35]

Kant is not here attempting to prove purely mathematically, in pure intuition, what Newton proves only on the basis of his Laws of Motion. Kant is rather focussing on an entirely different question: how does motion acquire a mathematical structure and thus become a mathematical magnitude in the first place? Newton, in the second Definition of the *Principia*, takes velocity as an already well-understood primitive term – clearly conceived as a mathematical magnitude or quantity – and uses this term to define quantity of motion (mass times velocity). This latter quantity is then the subject of the Second Law of Motion, from which, in turn, the parallelogram of motions is derived (see note 32 above).[36] For Kant, by contrast, it is precisely Newton's starting point that is at issue. Kant is asking what he takes to be the prior question of how velocity comes to be conceived as a mathematical magnitude in the first place, and his answer, in accordance with the traditional theory of (continuous) magnitude, is given by exhibiting an appropriate addition relation. Kant's parallelogram

considered as merely a movable point, without any quantity of its own (such as mass or volume) *aside from motion*. See the first remark to the first explication (480):

> Since in phoronomy nothing is to be at issue except motion, no other property is here ascribed to the *subject* of motion, namely, matter, aside from *movability*. [Matter] can itself so far, therefore, also be considered as a point, and one abstracts in phoronomy from all inner constitution, and therefore also from the quantity of the movable, and concerns oneself only with motion and what can be considered as quantity in motion (speed and direction).

[35] In the Mechanics, in particular, Kant presents the law of inertia as the realization of the category of cause and effect, and the law of the equality of action and reaction as the realization of the category of community. That matter, in the Phoronomy, is considered entirely independently of *mass* (note 34 above) is another indication that the mechanical laws of motion are not yet under consideration. Indeed, the concept of mass or quantity of matter is only officially defined in the Mechanics, where, as Kant makes clear, matter can no longer be considered as a mere movable point (547n.). For the relationship between Kant's three Laws of Mechanics and Newton's three Laws of Motion see note 37 of the Introduction. I shall return in detail to these questions in my chapter on the Mechanics.

[36] Newton is clearly assuming that the quantities in question have the (additive) structure of a traditional continuous magnitude in all of his Definitions. In the second Definition, for example, Newton explains (P404): "[I]f a body is twice as large [i.e., massive] as another and has equal velocity there is twice as much [quantity of] motion, and if it has twice the velocity there is four times as much motion." The second Law is then formulated as follows (P416): "*A change of motion is proportional to the motive force impressed and takes place along the straight line in which the force is impressed*." Newton speaks of "*change of motion*" rather than "change of the quantity of motion" here, but, as in the second Definition, motion is conceived as a *magnitude* in terms of precisely this latter quantity. See Newton's explanation of the Law (P416–17; already quoted in note 32 above).

construction of what he calls the "composition [*Zusammensetzung*]" of motions is, as I have suggested, entirely devoted to this end, and it must therefore, as Kant says (487), "be constituted wholly a priori, and indeed intuitively, on behalf of applied mathematics … [before] the rules for the connection of motions by means of physical causes, that is, forces, can [ever] be rigorously expounded."

But why does Kant find it necessary to proceed in this fashion? Why does he not start with the standard definition of speed as distance divided by time and then incorporate direction in the obvious way? Indeed, Kant suggests just this procedure in his third remark to the second explication (484): "In phoronomy we use the word speed [*Geschwindigkeit*] purely in a spatial meaning C = S/T [*Celeritas est Spatium per Temporum*]."[37] It is clear, nonetheless, that Kant does not take this standard procedure as an adequate solution to his problem of conceptualizing velocity as a magnitude. After presenting his construction of the composition of motions in the single proposition of the Phoronomy, Kant explains what is missing from the standard procedure in his second remark to this proposition:

> If, for example, a speed AC is called doubled, nothing else can be understood by this except that it consists of two simple and equal speeds AB and BC [in accordance with the first case of the proposition]. If, however, one explicates a doubled speed by saying that it is a motion through which a doubled space is traversed in the same time, then something is assumed here that is not obvious in itself – namely, that two equal speeds can be combined in precisely the same way as two equal spaces – and it is not clear in itself that a given speed consists of smaller speeds, and a rapidity of slownesses, in precisely the same way that a space consists of smaller spaces. For the parts of the speed are not external to one another like the parts of the space, and if the former is to be considered as a magnitude, then the concept of its magnitude, since this is *intensive*, must be constructed in a different way from that of the *extensive* magnitude of space. (493–94)

[37] In the traditional theory of magnitude as practiced by the ancient Greeks ratios between inhomogeneous magnitude-kinds are not well defined (see note 29 above). In the seventeenth century, however, such ratios gradually became standard in the theory of motion (beginning with Galileo) and were given meaning, in fact, precisely by the kinematical or dynamical processes that they were intended to describe. Thus, to say that two speeds are equal, for example, is to say that the distances traversed in the respective times are equal. To say that force is proportional to change of quantity of motion or momentum is to say that the same force that will impart a given velocity to twice a given mass will impart twice the velocity to the given mass itself (compare Newton's explanations of the second Definition and Second Law of Motion quoted in notes 32 and 36 above). Kant himself has no problem in general with such mixed or inhomogeneous ratios, and he uses essentially the same procedure, as we shall see, in his own discussion of quantity of motion in the Mechanics (538).

For Kant, therefore, whereas space (as well as time) is an *extensive* magnitude, speed (and therefore velocity) is an *intensive* magnitude. It is precisely this important difference that is obscured by the standard procedure.

Here we must appreciate, to begin with, that Kant is not employing our contemporary distinction between extensive and intensive magnitudes, according to which the former are characterized by a well-defined addition operation, and are thus measurable by a ratio scale, while the latter are not (as in the case of temperature before the absolute temperature scale was introduced). This understanding would make nonsense of Kant's claim, because the point of his construction is directly to exhibit the relevant addition operation for speeds (and velocities), so that they are indeed measurable by a ratio scale and are thus extensive magnitudes in our contemporary sense. What Kant means by an extensive magnitude is rather one that has *extension* in space and/or time, so that, in particular, the relevant addition operation is here given in terms of spatial and/or temporal composition of spatial and/or temporal parts "external to one another."[38] What Kant means by an intensive magnitude, then, is one whose quantitative character is independent of spatial and/or temporal extent: it represents the quantitative *degree* of some quality or "reality" (such as heat, illumination, color, and so on) that may vary continuously at a given *unextended* spatio-temporal point.[39] This does not imply, for

[38] Thus, the general definition of magnitude presented in the second edition version of the axioms of intuition applies to *both* extensive and intensive magnitudes for Kant. The passage quoted in note 28 above, for example, continues more fully as follows (B203):

> Therefore the very perception of an object, as appearance, is only possible by means of the same synthetic unity of the manifold of a given sensible intuition, whereby the unity of the composition of the manifold of the homogeneous is thought in the concept of a *magnitude*; that is, all appearances are magnitudes, and, in fact, *extensive magnitudes*, because they must be represented as intuitions in space or time by means of the same synthesis as that by which space and time as such are determined.

Extensive magnitudes are therefore special cases of magnitudes in general, all of which, in accordance with the traditional theory of magnitudes, must be additive. In the immediately following paragraph Kant defines an extensive magnitude (A162/B203) as one "in which the representation of the parts makes possible the representation of the whole (and thus necessarily precedes the latter)" and then characterizes both space and time as extensive magnitudes in this sense.

[39] Kant gives the examples of heat, illumination, and color in the anticipations of perception (A169–76/B211–18). The crucial point, for Kant, is that the apprehension of a "reality," unlike that of space and time themselves, is always *instantaneous* (A168/B210): "The real in appearance always has a magnitude – but one which can only be met with in apprehension in so far as it takes place by means of mere sensation in an instant, and does not proceed from parts to the whole; it thus certainly has a magnitude, but not an extensive [magnitude]."

Kant, that intensive magnitudes thereby fail to be additive and are thus not measurable by a ratio scale.[40]

This way of conceiving intensive magnitude stems from the late medieval theory of the intension and remission of forms or qualities, the point of which was to distinguish qualities such as heat, which are subject to continuous increase and decrease (intension and remission) over time independently of spatial extent, from extensive magnitudes such as length or weight, which can be measured directly by the spatial combination or composition of spatially extended parts. The thinkers in this tradition developed a system of graphical representation for such continuous intensions and remissions, and, in particular, they could thereby graphically distinguish between uniformly increasing or decreasing changes and non-uniform changes. It was in this context, specifically, that the idea of speed or velocity as an intensive magnitude first arose: that is, the idea of speed as an *instantaneous* quantity subject to truly continuous variation over time. It was precisely this idea, moreover, which then formed the immediate background to Galileo's celebrated treatment of the continuously and uniformly varying instantaneous velocity of falling bodies.[41] Since, as we have seen in our discussion of motion as an enduring state (section 2 above), Kant himself conceives speed or velocity as an instantaneous, continuously varying quantity (explicitly following Galileo's treatment of fall), it is certainly not surprising that he then characterizes it, in the present context, as an intensive rather than an extensive magnitude.

[40] Thus, for example, Kant clearly holds that degree of illumination (in our terms) is measurable by a ratio scale. See Kant's comments on the analogies of experience (A178–79/B221):

> The preceding two principles [the axioms of intuition and anticipations of perception], which I called mathematical, in consideration of the circumstance that they justified the application of mathematics to appearances, extended to appearances with respect to their mere possibility, and taught how, with respect to both their intuition and the real in their perception, they could be generated according to the rules of a mathematical synthesis – and therefore [how], in both cases, numerical magnitudes [*Zahlgrößen*] can be used, and with them the determination of appearance as magnitude [*Größe*]. Thus, for example, I will be able to compose [*zusammensetzen*] the degree of illumination of sunlight out of approximately 200,000 illuminations of the moon and give [this degree] as a priori determined, that is, [I will be able] to construct it.

This passage strongly confirms the view that *both* extensive and intensive magnitudes are additive for Kant.

[41] For a clear discussion of the late medieval tradition in question as part of the background to Galileo's work see Clavelin (1968/1974, chapter 2). In particular, it was precisely within this medieval tradition that the so-called mean speed rule for uniform acceleration was first formulated – according to which the time in which a certain distance is traversed by such a motion (starting from rest) is equal to the time in which the same distance would be traversed by a uniform speed equal to one half of the final (or maximum) speed attained in the uniformly accelerated motion.

For Kant, the problem of conceptualizing speed or velocity as a magnitude therefore involves the construction or exhibition of an addition operation directly on the set of *instantaneous* speeds defined at *a single given spatio-temporal point*.[42] And this, once again, amounts to an inversion of the standard procedure – where we start from finite, spatio-temporally extended speeds or velocities, defined on *finite intervals* of space and time, and then define the instantaneous speed or velocity at a single spatio-temporal point as the limit of (a sequence of) such finite or spatio-temporally extended ratios (distances divided by times). On this standard procedure, then, we would, in effect, have reduced the intensive, instantaneous quantity of speed or velocity to an extensive magnitude (more precisely, to an infinite sequence of extensive magnitudes), and there would be no particular need to follow Kant's procedure of directly exhibiting an addition operation on the set of instantaneous speeds or velocities. So the question we need to ask ourselves, at this point, is why Kant should not be satisfied with the standard procedure of defining instantaneous speed or velocity, in turn, as a limit of spatio-temporally extended ratios of distances over times?

I believe that there is a deeper motivation here – one that is central to Kant's philosophical project. Although Kant does not explicitly invoke this motivation in the context of his present discussion of motion as a magnitude, it nonetheless emerges naturally in the context of the whole of the *Metaphysical Foundations* if we compare Newton's remarks about absolute time in his Scholium to the Definitions with Kant's own views about absolute time. The point, briefly, is that the standard definition of speed in terms of distance divided by time assumes that *time* is a well-defined (continuous) magnitude independently of the definition of speed – and, more generally, that time is a well-defined magnitude independently of the mathematical characterization of motion. Newton appears to endorse such a conception in the famous passage on time at the beginning of the Scholium (P408):

Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name

[42] From a contemporary point of view, the set of instantaneous velocities defined at a single spatio-temporal point constitutes what we call the *tangent space* at the space-time point in question. This tangent space, like the space-time manifold itself, is four dimensional; unlike the space-time manifold, however, it is necessarily (even in a general relativistic space-time) a flat, linear, or vector space. What Kant is asking for, from this point of view, is an addition operation defined directly on each tangent space – independently, as it were, from the way in which the tangent spaces themselves are embedded in the space-time manifold (and thus independently, in particular, of space-time curvature). Although this point of view is anachronistic, it will nonetheless prove useful in appreciating the further steps of Kant's procedure considered below.

is called duration. Relative, apparent, and common time is any sensible measure (whether accurate or nonuniform [*seu accurata seu inaequabilis*]) of duration by means of motion: such a measure – for example, an hour, a day, a month, a year – is commonly used instead of true time.[43]

On this conception, therefore, it appears to make perfect sense to take time as a well-defined magnitude independently of motion and, accordingly, to characterize motion mathematically as a magnitude in terms of the ratios of given distances to given times.

Just as Kant does not accept what he understands as the Newtonian conception of absolute space, however, he also rejects what he understands as the Newtonian conception of absolute time. Although this theme is not equally prominent in the Phoronomy (or, for that matter, in the text of the *Metaphysical Foundations* as a whole), it is centrally important in the argument of the transcendental analytic of the first *Critique*. Kant argues there that, since "absolute time" is not an object of perception, all relations in time – that is, duration, succession, and simultaneity – must be determined in and through perceptible features of the appearances themselves. This determination subjects the appearances to the three analogies of experience (substantiality, causality, and community), so that these appearances, in turn, can now become objects of *experience*. And we have already seen Kant's characterization of the analogies of experience as principles of *time determination* – as an alternative to precisely "absolute time" – in an important passage from his concluding remarks to the analogies (A215/B262) quoted in the Introduction.[44] For Kant, therefore, no temporal relation whatsoever (neither duration, succession, nor simultaneity) can be viewed as pre-existing, as it were, in an absolute time subsisting prior to and independently of our categorical procedures for determining these relations within the appearances themselves. All temporal relations are rather the products of an empirical construction whereby we objectively determine the appearances, as objects of a unified experience, by means of the a priori principles of the understanding.[45]

[43] This passage immediately precedes the corresponding passage on absolute space to which Kant appears implicitly to refer in the first explication of the Phoronomy (as discussed at the very beginning of section 1 above).

[44] See the paragraph following the one to which note 33 of the Introduction is appended. The crucial claim in our present context is that (A215/B262), "because absolute time is no object of perception by means of which appearances could be bound together," it is "the rule of the understanding, by means of which alone the existence of appearances can acquire synthetic unity with respect to temporal relations, [that] determines for each [appearance] its position in time."

[45] With respect to the relation of temporal succession, corresponding to the principle of causality, Kant puts the point this way (A200/B245):

The principle corresponding to the magnitude of time or duration is that of substantiality, and this, according to the first analogy of experience, must be realized by what Kant (A182–84/B225–27) calls "the permanent [*das Beharrliche*]," whose total quantity or quantum "is neither increased nor diminished in nature" (B224). Kant explains (A183/B226) that "only by means of the permanent does the *existent* in different successive parts of the time series acquire a *magnitude*, which one calls *duration*," and he illustrates this concretely in the second remark to the refutation of idealism:

All empirical employment of our cognitive faculties in the determination of time fully agrees with this. It is not only that we can undertake all time determination only by the change of external relations (motion) in relation to the permanent in space (e.g., motion of the sun with respect to objects on the earth), but we also have nothing at all permanent, which could underlie the concept of a substance, as intuition, except merely *matter*, and even this permanence is not derived from outer experience, but is rather presupposed a priori as necessary condition of all time determination, and thus also [of] the determination of inner sense with respect to our own existence by means of the existence of outer things. (B277–78)[46]

For Kant, then, it follows that *time* becomes a magnitude (a mathematical measure of duration) only by means of the perception of outer objects (and thus matter) *moving* in space.[47]

The determination of [temporal] positions cannot be borrowed from the relation of appearances to absolute time (for this is no object of perception); rather, conversely, the appearances must themselves determine their positions in time [relative to] one another and make these [positions] in the temporal order necessary; that is, that which follows there or happens must follow on that which was contained in the previous state in accordance with a universal rule, whereby a series of appearances comes to be, which [rule] generates by means of the understanding precisely the same order and continuous connection in the series of possible perceptions (and makes this [order and connection] necessary) as is found a priori in the form of inner intuition (time), wherein all perceptions must have their place.

[46] This passage appears to point backward to the *Metaphysical Foundations*, with its characterization of matter as the movable in space, and, in particular, to the second explication of the Phoronomy, where motion is defined as "change of … *external relations* to a given space" (see note 11 above). I shall return to this situation below.

[47] In the general remark to the system of principles, after discussing the categories of substance, causality, and community, Kant turns to the category of quantity (B293): "In precisely the same way, it can easily be verified that the possibility of things as *magnitudes* [*Größen*], and thus the objective reality of the category of magnitude [*Größe*], can also only be exhibited in outer intuition and can be subsequently applied also to inner sense by means of it [outer intuition] alone." In §24 of the second edition transcendental deduction, after asserting that "we can make time representable to ourselves in no other way than under the image of a line, in so far as we draw it" (B156; see note 14 of the Introduction), Kant goes on to claim that "we must always derive the determination of lengths [intervals] of time [*Zeitlänge*], or also the positions in time [*Zeitstellen*] for all inner perception, from that which outer things present to us as alterable" (B156).

Newton articulates the distinction between absolute and relative time in his famous Scholium. And he illustrates the distinction between truly uniform absolute time and "sensible measures" of time based on motion by invoking astronomical corrections to these sensible measures:

In astronomy, absolute time is distinguished from relative time by the equation of common time. For natural days, which are commonly considered equal for the purpose of measuring time, are actually unequal. Astronomers correct this inequality in order to measure celestial motions on the basis of a truer time. It is possible that there is no uniform motion by which time may have an accurate measure. All motions can be accelerated and retarded, but the flow of absolute time cannot be changed. The duration or perseverance of the existence of things is the same, whether their motions are rapid or slow or null; accordingly, duration is rightly distinguished from its sensible measures and is gathered from them by means of an astronomical equation. (P410)

Newton is here referring to the standard astronomical procedure, already well understood in ancient astronomy, whereby we correct the "natural" (or local) day defined as the time from one noon (when the sun is at its zenith) to another at a given location on the earth's surface. Such days would be equal if the sun's ecliptic coincided with the equator and the sun moved uniformly on the ecliptic. But, since the ecliptic is tilted relative to the equator and the yearly motion of the sun is not in fact uniform, we correct the time measured by natural days accordingly using what astronomers call the equation of time.[48]

Kant, of course, is familiar with this procedure. He holds, as we have seen, that the ordinary or natural conception of motion based on the idea of a fixed earth must be subject to a series of successive corrections based on wider and wider relative spaces or reference frames extending indefinitely far into the heavens. He is also quite aware, by the same token, that the natural measure of time to which he refers in the passage from the refutation of idealism quoted above (B277–78) is subject to a sequence of analogous corrections by the procedures of astronomy. It is for this reason, in fact, that Kant's language in this passage ("we can *undertake* all time determination") strongly suggests the *beginnings* of a procedure (of time determination) that will certainly be extended beyond this initial point (the motion of the sun relative to objects on the earth).[49] Indeed, we know that Kant himself had envisioned a striking correction involving

[48] The result is mean solar time, measured (ideally) by the mean equatorial sun rather than the true sun. For a detailed explanation of this procedure see Evans (1998, chapter 5, §5.9).

[49] I therefore believe that a suggested emendation – whereby "*wahrnehmen* [perceive]" is substituted for "*vornehmen* [undertake]" – is mistaken.

Newtonian gravitational astronomy, which introduces an irregularity or nonuniformity into the daily rotation of the earth due to tidal friction (see again note 11 above, and compare note 46). Thus, whereas the law of inertia implies that a perfectly rigid and spherical earth will rotate uniformly in perpetuity in empty space, the action of external gravitational forces on the (not perfectly rigid and spherical) earth can – and in fact does – introduce numerous irregularities.[50] Newton is perfectly correct, therefore, to suggest that no observable motion may actually be truly uniform, and Kant, as we have seen, can only agree with Newton on this score.

For Kant, however, this circumstance is not an indication of an absolute time subsisting prior to and independently of our empirical procedures for determining temporal magnitudes from observable motions. It rather implies that empirically observable motions must be subject to a priori principles of the understanding (a priori rules of time determination) in order to count as fully objective experience within a temporally determinate objective world. Applying the relevant principles of the understanding – the analogies of experience – thereby results in a sequence of successive corrections or refinements of our ordinary or natural temporal judgements as the observable motions are progressively embedded within an increasingly precise and refined mathematical conception of temporality. I have suggested, moreover, that Kant takes his version of the mechanical laws of motion to be more specific realizations of the analogies of experience.[51] For Kant, therefore, these laws *define* what we mean by true temporal uniformity. Two temporal intervals are truly equal, in particular, if they are equal according to these laws – if, for example, they represent the times during which an inertially moving body were to traverse equal distances.[52]

[50] Further irregularities in the earth's rotation, also involving external gravitational forces, are the precession of the (non-spherical) earth's axis of rotation under the influence of the sun and the moon (as already discussed by Newton in his treatment of the precession of the equinoxes) and the nutation (or further wobbling) of this axis discovered by Bradley (due to the *differential* action of the sun and moon at different times of the lunar month). As I indicated in note 6 above, however, it is doubtful whether Kant himself had a clear understanding of this particular aspect of Bradley's work.

[51] See the paragraph to which note 37 of the Introduction is appended, together with the two preceding paragraphs.

[52] In terms of the modern conception of an inertial frame of reference formulated in the late nineteenth century (see note 28 of the Introduction) such a frame of reference involves not only a privileged relative space but also a privileged *inertial time scale* (relative to which inertially moving bodies traverse equal times in equal distances in such a frame). This privileged time scale is implicitly defined by the mechanical laws of motion in precisely the same sense as is the privileged relative space. In the even more anachronistic (contemporary) terms of note 42 above

Prior to and independently of the mechanical laws of motion, by contrast, the notion of a mathematically determinate temporal magnitude of duration is simply undefined for Kant. Here, in the Phoronomy, where the relevant laws have not yet been formulated, we thus cannot assume that the relevant characterization of time as a magnitude is available, and, accordingly, we cannot presuppose the standard definition of speed or velocity in terms of distance divided by time. Kant therefore adopts an "inversive" strategy, intended to show how motion (speed or velocity) can be mathematically characterized as a magnitude first, prior to and independently of the relevant laws of motion, and to characterize temporal duration as a magnitude, by means of these laws, only subsequently. In particular, by constructing speed or velocity as a mathematical magnitude in terms of an operation of addition defined on the system of *instantaneous* velocities at any given spatio-temporal point, Kant is able to characterize velocity as an *intensive* magnitude independently of all consideration of spatial and temporal extent (and therefore independently of any limiting process defined on finite spatial and temporal intervals). Later, in the Mechanics chapter, Kant will introduce his version of the mechanical laws of motion and complete his construction of the mathematical theory of motion by indicating how mass and momentum (and therefore force and acceleration) are also possible as mathematical magnitudes. At this point – and only at this point – he will finally be in a position to characterize time itself as such a magnitude.

Thus Kant's position in relation to the Newtonian conceptions of absolute time, space, and motion is ultimately quite nuanced and subtle. On the one hand, Kant dismisses these conceptions, taken literally, as metaphysically impossible and absurd (see again note 4 above). On the other hand, however, the constructive procedures that Kant puts in the place of such metaphysical absurdities depend on (his version of) the mechanical laws of motion, and they are closely modeled on the procedures that Newton himself has introduced for correcting "sensible measures" of time, space, and motion by means of the idealized standards implicit in these laws.[53] Corresponding to the metaphysical differences between Kant

the class of inertial trajectories defines an *affine structure* on the space-time manifold – which induces, in turn, an appropriate notion of space-time curvature. Indeed, the space-time trajectories defined by the law of inertia characterize the *flat* affine structure of what we now call Newtonian space-time. For this latter notion, and its relation to Newton's mathematical theory of motion in the *Principia*, see Stein (1967).

[53] For the relationship between Kant's and Newton's versions of the mechanical laws of motion see again note 37 of the Introduction.

and Newton, finally, there is also an important difference concerning the application of mathematics. Whereas Newton begins by presupposing that all the concepts of his theory of motion are already mathematically well defined independently of the mechanical laws of motion, Kant allows this only for the purely kinematical concept of speed or velocity (defined instantaneously as an intensive magnitude). He then admits the fundamental dynamical concepts (mass, momentum, and force) as mathematical magnitudes only in the context of these mechanical laws. I shall come back to this point in more detail in my discussion of Kant on mathematical and empirical motion in section 6 below.

4 THE CONSTRUCTION OF MOTION AS A MAGNITUDE

Kant intends in the single proposition of the Phoronomy to present the construction of motion as a magnitude and thus, as we have seen, to exhibit an addition operation directly on the set of instantaneous (directed) velocities without presupposing that temporal duration has already acquired the (additive) structure of a mathematical magnitude independently of this construction. Accordingly, this exhibition or construction, for Kant, should be executed purely mathematically in pure intuition, without presupposing anything about either physical forces or the mechanical laws of motion governing such forces. It should be what Kant calls a *mathematical* rather than a *mechanical* construction, which, in the words of the fourth explication, constructs a composite motion by (486) presenting "a motion a priori in intuition, in so far as it arises out of two or several given [motions] united in one movable."

Since, as Kant notes in his remark to the fifth explication, it suffices to explain how a composite motion can be thereby constructed out of two given motions, the proposition considers only this case. Two such motions, Kant claims, can only be added or composed with one another by considering two different relative spaces or reference frames (490): "The composition of two motions of one and the same point can only be thought in such a way that one of them is represented in absolute space, and, instead of the other, a motion of the relative space with the same speed occurring in the opposite direction is represented as identical [*einerlei*] with the latter." Moreover, the ground of the identity in question is the single principle of the Phoronomy. For this principle already asserts the identity or equality of two (apparently different) situations (487): "motion of the body in a space at rest," on the one hand, and "rest of the body and … motion of the space in the opposite direction with the same speed," on the other.

I shall return to a detailed consideration of this principle (Kant's principle of the relativity of motion) in the next section, and I shall only be concerned here with its application to the construction of the composition of motions. In connection with precisely this application, however, it should already be clear that what Kant means by "absolute space" in his statement of the proposition is simply a space initially taken to be at rest as in his statement of the principle. We are thus considering in the proposition two relative spaces or reference frames, both of which, according to the principle, can be considered as either at rest or in motion. The construction effected in the proof of the proposition then proceeds, as I have suggested, in three separate cases: the two motions to be composed occur in the same line and with the same direction; they occur in the same line but in opposite directions; or they occur in two different lines meeting in a point and comprising there a given angle. Kant assumes, for simplicity, that the two motions in question have equal speeds in all three cases, and, in all three cases, what he says about how the construction *cannot* proceed is at least as important as the way in which it *can* proceed.

The first case begins by considering two velocities with equal speeds in the same line and direction represented by two different lines in space. We would delineate them graphically in modern vector representation as two equal lines with arrows at one end indicating the direction in question. (Kant indicates direction by drawing a circle, representing the movable point or body, at the opposite end.) The question Kant then considers is whether one can intuitively construct the composition or addition of the two velocities simply by composing geometrically the two (directed) lines, by adjoining the second to the end of the first creating a single (directed) line double in length. This is certainly how we proceed in modern vector representation, but it is not, according to Kant, sufficient for the construction or exhibition he is after. The reason he gives is that the two parts of the doubled line in question (the original line representing the first velocity and the adjoined line representing the second) do not represent two motions occurring in (or at) the same time: they rather represent two different spaces traversed by a given motion *successively*. More generally, then, we can say that purely spatial operations on (directed) line segments (as in modern vector representation) do not, by themselves, also suffice intuitively to represent the necessary *temporal* relations in questions. They are thus not sufficient, by themselves, for intuitively representing the composition of speeds or velocities, and Kant therefore concludes (490) that "the composition of two velocities in one direction cannot be intuitively presented *in the same space*."[54]

Suppose, however, that the two motions to be composed are defined in two different relative spaces. Suppose, more specifically, that the initial velocity is defined with respect to one relative space, and, instead of attempting directly to double this velocity in the same relative space, I consider a second relative space moving with the same speed but in the opposite direction (along the same line). With respect to this second relative space, then, the first relative space has the same speed as the initial velocity and also the same direction. Moreover, since the point or body under consideration moves with the initial velocity in question with respect to the first relative space, and this first relative space moves with the same velocity with respect to the second, the original point or body moves with double the initial velocity with respect to the second relative space. Thus, it is precisely this velocity (of the original point or body), defined with respect to the second relative space, that intuitively represents the composition or addition of velocities Kant is after. Here, unlike in the representation of purely *spatial* operations on (directed) line segments considered above, we are directly adding or composing motions themselves. And we do this, in particular, by applying Kant's principle of the relativity of motion to infer that the space initially considered to be at rest (the first relative space) can equally be considered to be in motion (with respect to the second relative space). Adding a given velocity to the original point or body is therefore the very same thing as imparting this given velocity to the space initially considered to be at rest – and the very same thing, therefore, as now considering the original point or body from the point of view of the second relative space.[55]

[54] In contemporary vector representation we simply *assume* that all the lines in question represent spaces traversed by motions taking place in (or at) the same time. Kant's question, in these terms, is how do we also intuitively present or exhibit this assumption – in such a way that the (directed) line segments in question thereby represent *velocities*. For it is only if we can intuitively represent the specifically temporal aspect of our composition as well (by intuitively composing not only line segments but also *motions*) that, for Kant, we will have thereby exhibited the real possibility or objective reality of the composition at issue.

[55] From a modern point of view, we can conceive the adjunction or addition of one (directed) line segment to another in this case as a *spatial translation* (in the direction in question) that moves the initial point of the first line segment to the end point of the second. From this same (group-theoretic) point of view, the operation on velocities Kant is attempting to exhibit is then a *Galilean transformation* (along a single spatial direction) – which "boosts" one velocity by a second in the course of "boosting" the original reference frame by the same velocity. In these terms (and the terms of note 54 above), Kant's claim is that only such a Galilean transformation, and not a mere spatial translation alone, can intuitively exhibit the real possibility or objective reality of the concept of the composition of motions.

Kant's second case is that of exhibiting or representing two (equal) speeds in the same line but in opposite directions, so that we are here attempting to add or compose two precisely opposite velocities. The result of such an addition should of course be equal to zero, a state of rest, but how are we intuitively to represent this result? Kant's positive solution to this problem is straightforward, and it in fact follows immediately from his relativity principle and his construction of the first case. Let the point or body in question move with the initial (positive) velocity with respect to a given relative space. Our problem is to add the opposite (negative) velocity to the very same point or body. According to what we already know, however, adding this (negative) velocity to the body is the very same thing as imparting it to the space initially considered to be at rest. We therefore consider a second relative space moving with the initial (positive) velocity with respect to the first relative space, so that, with respect to this second relative space, the first space moves with precisely the second (negative) velocity. Therefore, the point or body itself is in a state of rest with respect to this second relative space. For, according to the statement of Kant's relativity principle, "motion of a body in a space at rest" (the initial positive velocity relative to the first relative space) is the very same thing as "rest of the body and … motion of the space in the opposite direction with the same speed."

More interesting, however, is what Kant says about how the construction of this second case *cannot* proceed. For here, unlike in his parallel remarks on the first case, Kant is not content merely to indicate the limitations of a purely spatial execution of the construction – where, in modern vector representation, we would obtain a single point representing the zero vector. Kant instead says something much stronger, namely, that even the *thought* of combining the two motions in question in a single relative space is impossible (491): "[T]he very thought of representing two such [equal and opposite] motions in one and the same space in precisely the same point as simultaneous would itself be impossible, and thus [so would] the case of such a composition of motions, contrary to the presupposition." The problem here is not simply that we have attempted to represent the composition of motions directly by the composition of (directed) line segments, but rather that the representation of this composition of motions (two equal motions in opposite directions in the same relative space) is itself impossible.

But this does not mean, for Kant, that the thought of composing two equal and opposite motions in the same space is somehow logically contradictory. The problem is rather, as Kant explains in his remark to this case, that the thought or concept we are attempting intuitively to exhibit cannot after all be constructed:

> If, however, the subtraction of one [velocity] from the other is at issue, then this can indeed easily be *thought*, as soon as the possibility of velocity as a quantity through addition is granted, but this concept cannot so easily be *constructed*. For, to this end two opposite motions must be combined in one body; and how is this supposed to happen? It is impossible to think two equal motions in the same body in opposite directions immediately, that is, in relation to precisely the same space at rest. But the representation of the impossibility of these two motions in one body is not the concept of its *rest*, but rather of the *impossibility of constructing* this composition of opposite motions, which is nonetheless assumed as possible in the proposition. This construction is possible in no other way, however, except through the combination of the motion of the body with the *motion of the space*, as was shown. (494)

Kant is perfectly willing to admit, therefore, that the thought of subtracting two equal velocities from one another is logically consistent as soon as the first case, and thus the general possibility of addition, is in place. The problem arising with particular vividness in this second case, therefore, concerns the possibility of an intuition or construction in intuition corresponding to the thought.

But we already know, from Kant's discussion of the third explication (see section 2 above), that there are special problems associated with the representation of rest. In particular, Kant there argues that two equal and opposite motions in perfect uniform reflection do not result in a well-defined state of rest at the turn-around point. One cannot add or compose the two motions at the point in question so as to obtain a resulting state of rest – one obtains, rather, only what Kant calls a "complete lack [*Mangel*] of motion," which, as such, cannot be constructed or intuitively represented at all (485; see the paragraph to which note 21 above is appended). Kant's reason for this view, as we have seen, turns on the fact that the two equal and opposite motions in uniform reflection do not, at the turn-around point, possess a well-defined spatio-temporal tangent (or derivative) at this point. His present claim, accordingly, is that the thought of two equal and opposite motions combined at the same point in the same space suffers from precisely the same defect: it may not result in any well-defined state of motion at all but only a "complete lack of motion" with no intuitive counterpart.[56] Of course, as we also know

[56] In the contemporary terms of note 42 above, therefore, the point is that the addition operation we are attempting to define on each tangent space has an implicit *presupposition*: namely, that a well-defined element of the tangent space always results. The mere thought, in the present case, of combining two equal and opposite motions in a single point does not necessarily fulfill this

from Kant's discussion of the third explication, two equal and opposite motions *can* be combined at a single point (and in a single space) yielding a well-defined state of rest – as, paradigmatically, in the case of a turn-around point described by Galileo's law of fall. But here, in Kant's terms, we are appealing to moving forces (gravity) and mechanical laws of motion (inertia), and we are thus not operating in pure intuition as is required for a genuinely *mathematical* construction.[57]

Kant's third case of the composition of motions is that of two equal speeds proceeding from a given point in two different directions comprising a given angle at that point. This, from the point of view of modern vector representation, is the familiar (and most general) case of vector addition, where two vectors originating at a single point result in a third vector originating at the same point as the diagonal of the parallelogram they determine. From this point of view, therefore, if the two original vectors represent velocities, the diagonal vector thereby determined represents the addition or composition of these velocities. But for Kant, once again, the only way in which we can intuitively exhibit or construct such an addition of velocities in pure intuition is by considering the two original velocities as defined with respect to two different relative spaces. We again consider the first original velocity as defined with respect to one relative space (a space initially considered to be at rest), and, instead of attributing the second original velocity to the given moving point or body with respect to this same relative space, we consider a second relative

presupposition; contemporary (spatial) vector representation, with its resulting single (spatial) point in this case, does not guarantee that a well-defined zero vector – in the *spatio-temporal* tangent space – results. In the terms of note 55 above, only the appropriate Galilean transformation (which is defined on the *spatio-temporal* tangent space) can do this.

57 Kant first addressed these issues in his 1763 essay on *Negative Magnitudes*, which distinguishes (2, 171–72) between two different types of conflict or *opposition*: "*logical* through contradiction," on the one hand, and "*real*, i.e., without contradiction," on the other. In the first case we get "nothing at all (*nihil negativum irrepraesentabile*)," such as "a body that would be in motion and, in precisely the same sense, simultaneously not in motion." In the second case, however, although the two predicates under consideration in a sense cancel [*aufheben*] one another, and therefore also result in "nothing," this is "in a different sense than in the case of contradiction (*nihil privativum, repraesentabile*)." This case is illustrated by "moving force of a body towards one direction" combined with "an equal striving [*Bestrebung*] of precisely the same [body] in the opposite direction," such that "[t]he consequence is rest, which is something (*repraesentabile*)." Such a case – of *real* opposition – corresponds to the example of rest at the turn-around point under the influence of gravity considered in the discussion of the third explication of the Phoronomy, where the (inertial) tendency to preserve the initial upward velocity of projection is exactly counterbalanced, at the turn-around point, by the downward (gravitationally induced) velocity in accordance with Galileo's law of fall. But nothing in the 1763 discussion corresponds to the new notion of a thought (of combining two motions) that is logically consistent but still not constructible in pure intuition. For Kant does not even introduce the notion of pure intuition (or pure sensibility) until the *Inaugural Dissertation* of 1770.

space moving with the same speed but in the opposite direction as the second original velocity. The first relative space then moves precisely with this second original velocity with respect to the second relative space, and the moving point or body moves with the vector sum of the two original velocities (along the diagonal of the parallelogram they determine) with respect to this same (second) relative space. The moral, as before, is that the addition or composition in question cannot be represented "immediately" – that is, in a single relative space or reference frame – but (494) only "by the *mediate* composition of two equal motions, such that one is the motion of the body, and the other the motion of the relative space in the opposite direction, which, however, for precisely this reason, is completely identical [*einerlei*] with an equal motion of the body in the original direction."

In his proof that the operation of addition or composition in question *cannot* be constructed immediately (in a single space) Kant says (492): "[T]his is contrary to the presupposition of the proposition, which indicates by the word 'composition' that the two given motions are to be *contained* in a third, and therefore are to be the same [*einerlei*] as the latter, and are not to produce a third, in that one *alters* the other." In his comment on this proof in the following third remark Kant then gives the reason we might reasonably have expected all along, namely, that such a composition in a single relative space would be a *mechanical* rather than a purely *mathematical* construction:

> Finally, with respect to the composition of two motions with directions comprising an angle, this cannot be thought in the body in reference to one and the same space either, unless we assume that one of them is effected through an *external* continuously influencing *force* (for example, a vehicle carrying the body forward), while the other is conserved unchanged – or, in general, one must take as basis moving forces, and the generation of a third motion from two united *forces*, which is indeed the *mechanical* execution of what is contained in a concept, but not its *mathematical construction*, which should only make intuitive what the object (as quantum) *is to be*, not how it may be *produced* by nature or art by means of certain instruments and forces. (494)

So Kant here finally appeals to the distinction between mathematical and mechanical construction – which has of course been available since Kant's initial introduction of the concept of a construction (in pure intuition) of the composition of motions in the fourth explication. Indeed, as observed in section 3 above, Kant emphasizes the very point at issue in his remark to this fourth explication by asserting (487) that "the rules for the connection of motions by means of physical causes, that is, forces, can never be rigorously expounded, until the principles of their composition in general have been previously laid down, purely mathematically, as basis."

Why, then, does Kant only invoke the distinction between mathematical and mechanical construction in the third case? It applies equally well to all three cases, of course, and Kant calls attention to this in connection with the first case in his third remark:

> [T]wo equal velocities cannot be combined in the same body in the *same direction*, except through external moving causes, for example, a ship, which carries the body with one of these velocities, while another moving force combined immovably with the ship impresses on the body the second velocity equal to the first. But here it must always be presupposed that the body conserves itself in *free* motion with the first velocity, while the second is added – which, however, is a law of nature of moving forces that can in no way be at issue here, where the question is solely how the concept of velocity as a magnitude is to be *constructed*. (494)

Indeed, the same point is implicitly contained in Kant's discussion of the second case as well, since, as we have seen, there is here an implicit contrast with the case of rest at the turn-around point under the influence of gravity, where the (inertial) tendency to preserve an initial upward velocity of projection is exactly counterbalanced at the turn-around point by the downward (gravitationally induced) velocity in accordance with the law of fall. Nevertheless, the fact remains that Kant, in the proof of the proposition itself, explicitly appeals to the need for neglecting the action of forces (the causes of alterations) only in his discussion of the third case.

One reason Kant has for emphasizing the distinction between mathematical and mechanical construction primarily in the third case is that this case, as I have said, corresponds to the standard treatment of the composition of motion in physics. It corresponds, in particular, to Newton's treatment in the first Corollary to the Laws of Motion, where the composition of motions is derived from the composition of forces.[58] Moreover, in physics, as Kant himself points out, the first two cases are not standardly treated separately at all but are instead conceived as special cases of the third. His separate treatment of the first two cases in his proof of the

[58] See again note 32 above. Kant invokes the distinction between mechanical and mathematical (here "geometrical") construction in relation to all three cases in his first remark to the proposition (493):

> Therefore, all attempts to prove the above proposition in its three cases [in a single space] were always only mechanical analyses – namely, where one allows moving causes to produce a third motion by combining one given motion with another – but not proofs that the two motions are identical [*einerlei*] with the third, and can be represented as such a priori in pure intuition.

proposition, and, in particular, his failure explicitly to consider there the distinction between mathematical and mechanical construction, is therefore based on considerations lying outside of a strictly physical treatment – in what he calls "transcendental philosophy" (495; see note 31 above).

Thus, as we have seen, the discussion of the first case focusses on the attempt to represent the composition in question purely spatially or geometrically and argues that this attempt does not properly exhibit or construct the *spatio-temporal* assumptions underlying the composition of *motions*. The moral, for all three cases, is that purely spatial composition or adjunction of (directed) line segments does not, by itself, yield an appropriate intuitive construction for the composition or addition of *velocities*.[59] The discussion of the second case, by contrast, focusses on what we might call the existential presuppositions of the composition in question and argues that the mere concept of the composition of two equal and opposite velocities may not, after all, have a corresponding intuitive object. For, as in the example of perfect uniform reflection, the spatio-temporal trajectory we are attempting to describe may lack an appropriate tangent (or derivative) at a given point (here, at the turn-around point).[60] At the same time, however, the discussion of this case also implicitly points forward to the third case. Since the example of perfect uniform reflection is contrasted (in Kant's discussion of the third explication) with the example of rest at the turn-around point under the continuous influence of gravity, the two opposing motions must indeed be considered in terms of forces and mechanical laws of motion when composed in a *single*

[59] In the terms of note 55 above, what is at issue, in general, is the distinction between the group of isometries or rigid motions operating on three-dimensional Euclidean space, which underlies the notion of congruence or geometrical equality in this space, and the Galilean group operating on the four-dimensional spatio-temporal tangent space, which underlies an analogous notion of congruence or equality on the set of instantaneous velocities. Kant comments on the relevant notion of congruence or equality (now applied to all three cases) in his first remark to the proposition (493):

> Geometrical *construction* requires that one magnitude be *identical* [*einerlei*] with another or that two magnitudes in composition be *identical* with a third, not that they produce the third as causes, which would be mechanical construction. Complete similarity and equality [*völlige Ähnlichkeit und Gleichheit*], in so far as it can be cognized only in intuition, is *congruence*. All geometrical construction of complete identity [*Identität*] rests on congruence. Now this congruence of two combined motions with a third (as with the *motus compositus* itself) can never take place if these two combined motions are represented in one and the same space, for example, in relative space.

(The passage concludes with the quotation from note 58 above.)

[60] This case also results in a general moral applicable to all three cases: namely, the construction in question should intuitively exhibit that what results from composition is always a well-defined speed or velocity (see note 56 above).

frame of reference. It is then left to the third case, finally, to make this connection fully explicit by emphasizing that purely spatial representation in a single reference frame (by means of directed line segments) can express the composition of motions we are after, but only if we are clear from the beginning that we are now presupposing both the concept of (continuously acting) force and the mechanical laws of motion.[61]

5 KANT'S PRINCIPLE OF THE RELATIVITY OF MOTION

Kant's reasoning in his proof of the single proposition of the Phoronomy depends on what he calls a "principle" – a principle of the relativity of motion – that is not supported by a "proof" but only by a following "remark." The principle states (487): "Every motion, as object of a possible experience, can be viewed arbitrarily as motion of the body in a space at rest, or else as rest of the body, and, instead, as motion of the space in the opposite direction with the same speed." The point, as we have seen, is to assert the identity or equality of two (apparently different) descriptions of motion, so that one can be substituted for the other in various cases of the composition of motion. It is appropriate, then, that Kant's discussion of the principle is sandwiched between the fourth explication, where the construction of the composition of motion is first introduced, and the fifth explication, where the composition of motion is officially defined as "the representation of the motion of a point as identical [*einerlei*] with two or several motions of this point combined together [*zusammen verbunden*]" (489). The exhibition of the construction then follows in the proposition.

Kant bases his principle of the relativity of motion on the conception of space and motion that I have considered in detail in section 1 above. According to this conception, which I there characterized as "Copernican," one begins by considering the state of motion of a body in relation to an arbitrary relative space in which one finds oneself (for example, the cabin

[61] In Kant's discussion of the third case in the following third remark he emphasizes that the forces in question must act *continuously* (494), whereas Newton, in his proof of the first Corollary to the Laws of Motion, explicitly invokes two *discretely* or *instantaneously* acting forces (impulses) resulting in uniform rectilinear motions along both sides of the parallelogram (which then combine to yield a uniform rectilinear motion along the diagonal – compare note 33 above). I shall discuss in detail in my chapter on the Mechanics why Kant insists on continuously acting forces. As already suggested at the end of section 2, however, this insistence depends on his desire to assimilate all cases of interaction (including cases of impact) to the case of continuous deceleration and acceleration under the influence of gravity described by Galileo (see note 27 above, together with the paragraph to which it is appended).

of a ship). This initial starting point can then be extended indefinitely, generating a potentially infinite sequence of ever more inclusive relative spaces (the banks of a river on the earth's surface, the solar system in which the earth itself is located, the Milky Way galaxy containing the solar system, and so on). It follows that any given relative space is potentially in motion itself, with respect to a still more inclusive space, so that a body in motion with respect to a given relative space may equally well be viewed as at rest with respect to a second such space.

I have already quoted the core of Kant's argument for his principle in the following remark (487–88) in section 1, beginning with the claim that "[t]o make the motion of a body into an experience it is required that not only the body, but also the space in which it moves, be an object of outer experience and thus material," and concluding that "it must be completely the same [*einerlei*] for all experience and every consequence of experience whether I wish to view a body as moved or at rest (but with the space moved in the opposite direction."[62] Immediately following this passage Kant explicitly draws the conclusion about the identity of two (apparently different) descriptions of motion that is required for his construction of the composition of motion:

> Further, since absolute space is nothing for all possible experience, the concepts are also identical [*einerlei*] whether I say that a body moves in relation to this given space, in such and such direction with such and such speed, or I wish to think the body as at rest, and to ascribe all this, but in the opposite direction, to the space. For any concept is completely identical with a concept whose differences from it have no possible example at all, being only different with respect to the connection we wish to give it in the understanding. (488)

Thus the two concepts in question, "motion of the body in a space at rest" versus "rest of the body and … motion of the space in the opposite direction with the same speed," although they are certainly not *logically* equivalent, are, according to Kant's Copernican conception of motion and space, what we might call *really* or *empirically* equivalent nonetheless.[63]

[62] For the entire passage, along with the passage from Kant's 1758 *New System of Motion and Rest* where the sequence of relative spaces is extended out to the (rotating) Milky Way galaxy, see the paragraph to which note 5 above is appended, together with the preceding paragraph.

[63] That a difference between *logical* possibility (depending on the understanding alone) and *real* possibility (concerning the existence of a corresponding instance or intuition falling under the concept) is at issue emerges from the immediately following sentences (488):

> Further, since absolute space is nothing at all for any possible experience, the concepts are also the same whether I say that a body moves in relation to this given space, in such and such direction and with such and such speed, or I wish to think the body as at rest, and to ascribe all this, but in the opposite direction, to the space. For any concept is entirely the same [*einerlei*]

Kant proceeds to generalize this conclusion, finally, in the following passage (immediately after the last sentence quoted in note 63), so that any *part* of the motion of a body can be redistributed, instead, to an appropriately chosen relative space:

Of this motion of a body in empirical space, I can give a part of the given velocity to the body, and the other to the space, but in the opposite direction, and the whole possible experience, with respect to the consequences of these two combined [*verbundenen*] motions, is completely identical [*einerlei*] with that experience in which I think the body as alone moved with the whole velocity, or the body as at rest and the space as moved with the same speed in the opposite direction. (488)

It is precisely this conclusion, in particular, that Kant needs for the proof of his proposition in general – where, for example, the velocity of a body in an initial relative space can be divided into two *unequal* parts, such that one part remains with the body with respect to a new relative space, while the other is given to the initial relative space (but in the opposite direction) with respect to this same new space. It is in precisely this way, for Kant, that the composition of two unequal velocities is constructed.[64]

Immediately following this last passage, however, Kant introduces a striking qualification:

But here I assume all motions to be rectilinear. For in regard to curvilinear motions, it is not in all respects the same [*einerlei*] whether I am authorized to view the body (the earth in its daily rotation, for example) as moved and the surrounding space (the starry heavens) as at rest, or the latter as moved and the former as at rest, which will be specially treated in what follows. Thus in phoronomy, where I consider the motion of a body only in relation to the space (which has no influence at all on the rest or motion of the body), it is entirely undetermined and

as a concept whose differences from it have no possible example at all, being only different with respect to the connection we wish to give it in the understanding.

The dependence of this conclusion on Kant's Copernican conception of motion and space is made explicit in the next sentence (488):

We are also incapable, in any experience at all, of assigning a fixed point in relation to which it would be determined what motion and rest should be absolutely; for everything given to us in this way is material, and thus movable, and (since we are acquainted with no outermost limit of possible experience in space) is perhaps also actually moved, without our being able to perceive this motion.

[64] I observed above that Kant assumes for simplicity that the two motions to be composed are equal in his proof of all three cases of the composition of motion. Immediately before the proof itself, however, Kant states that the two motions may have "equal or unequal speeds" in all three cases (489–90).

arbitrary how much velocity, if any, I wish to ascribe to the one or to the other. Later, in mechanics, where a moving body is to be considered in active relation to other bodies in the space of its motion, this will no longer be so completely the same [*einerlei*], as will be shown in the proper place. (488)

This qualification is liable to strike the modern reader as puzzling. For the relativity principle for classical physics with which we are familiar is that of so-called Galilean relativity, which states that the mechanical laws of motion are the same in all inertial reference frames – all of which are moving uniformly and rectilinearly with respect to one another. This principle, in turn, is a direct consequence of the law of inertia, according to which a body affected by no external forces moves uniformly and rectilinearly, in a straight line and with constant (possibly zero) speed. What is puzzling about Kant's qualification, from this point of view, is that he entirely neglects the further qualification of *uniformity*; for, without the latter, the principle of Galilean relativity simply does not hold. Just as, for example, the mechanical laws of motion do not remain the same in uniformly rotating frames of reference (as Kant himself appears to intimate), they are similarly not preserved in *linearly accelerated* frames of reference.[65]

Indeed, from the point of view of the principle of Galilean relativity, Kant's qualification of his relativity principle is doubly puzzling. For the former principle is a statement about the mechanical laws of motion: it is precisely these laws that remain the same in all inertial reference frames. But, as explained in section 4 above, Kant takes great pains to separate his construction of the composition of motions (and, therefore, his version of a principle of relativity as well) from all consideration of moving forces and laws of motion. It is in the Mechanics, not the Phoronomy, that the mechanical laws of motion are first introduced. And, as Kant emphasizes in the very passage we are now considering (488), it is in the Mechanics, not the Phoronomy, that bodies in motion under the action of moving forces ("in active relation to other bodies in the space of [their] motion") are first discussed. If the mechanical laws of motion are not yet in question, however, why does Kant bother to qualify his own version of a principle of relativity at all? From a purely kinematical point

[65] Kant is certainly clear about the fundamental difference between *uniform* and *non-uniform* rectilinear motion, for it is precisely this difference that figures centrally in the two cases he distinguishes in his discussion (considered in section 2) of when a genuine state of rest is well defined: namely, at the turn-around point of two oppositely directed uniformly *accelerated* rectilinear motions but not at the turn-around point of two oppositely directed *uniform* (constant *velocity*) motions.

of view, independently of these laws, it would appear that all frames of reference whatsoever are equally valid (including both rotating and accelerated frames). So why does Kant take it upon himself to restrict his version of a principle of relativity to *rectilinear* (but not necessarily uniform) motions?[66]

We can begin to shed light on these questions if we keep firmly in mind the overarching point of Kant's construction of motion as a magnitude, as discussed in sections 3 and 4 above. In the first place, the construction in question is supposed to take place purely mathematically in pure intuition, and this is why the mechanical laws of motion, in particular, are not yet in question. In the second place, however, the operation of composition in virtue of which motion can be considered as a magnitude is defined on the system of *instantaneous* velocities at any given spatio-temporal point, so that motion is here to be constructed as an intensive rather than an extensive magnitude. Indeed, as explained in section 3, these two ideas are intimately connected for Kant. Motion is constructed as an intensive magnitude, by means of an operation of addition defined directly on the system of instantaneous velocities at a point, because we cannot assume, at this stage, that time has the structure of a mathematical magnitude. On the contrary, time becomes a mathematical magnitude, for Kant, only by using motion as a measure – by a procedure of successive approximation in which the mechanical laws of motion, in particular, define the ultimate standard of temporal uniformity. We abstract from the mechanical laws of motion precisely to avoid any presuppositions about the mathematical structure of time as a magnitude, and we instead exhibit an addition operation directly on the system of instantaneous velocities. We thereby hope to achieve a mathematical rather than a mechanical construction of motion as a magnitude, prior to and independently of the mechanical laws, so that the mathematical structure of time as a magnitude can be subsequently determined in *empirical* intuition. This takes place by a procedure Kant calls time determination under the guidance of the analogies of experience, which are realized, in the Mechanics, by Kant's version of the mechanical laws of motion.

[66] That Kant's relativity principle and construction of the composition of motions are completely parallel in this respect is clearly indicated in Kant's third remark to the proposition, which explains (495) that phoronomy "contains no more than this single proposition, carried out through the above three cases, of the composition of motion – and, indeed, of the possibility of *rectilinear motions* only, not curvilinear [; f]or since in these latter the motion is continuously changed (in direction), a cause of this change must be brought forward, which cannot now be the mere space." Thus both the principle and the proposition are subject to the same restriction, and for the same reason.

Kant's principle of the relativity of motion, as an integral part of his construction of speed or velocity as a mathematical magnitude, is similarly restricted to the set of instantaneous velocities at a given spatio-temporal point. But it is thereby restricted to *rectilinear* motions as well; for, by construction, all the instantaneous (linear) velocities at a given spatio-temporal point are necessarily rectilinear.[67] There is no need to add a further restriction to *uniform* rectilinear motion, because all instantaneous velocities defined at a single given spatio-temporal point are necessarily uniform as well – or, perhaps better, the distinction between uniform and non-uniform motion makes no sense since *changes* in the state of motion over time are not yet in question. Indeed, as I have just emphasized, the very notion of temporal uniformity is not yet available, for Kant, but rather requires a subsequent introduction of both moving forces and the mechanical laws of motion in the following Dynamics and Mechanics chapters.[68] And it is for this reason, above all, that Kant's principle of the relativity of motion in the Phoronomy cannot be equated with our principle of Galilean relativity.[69]

What is the real connection, then, between Kant's construction of the composition of motions and accompanying relativity principle, on the one side, and his Copernican conception of the relativity of space and of motion, on the other? This connection now appears quite tenuous, since,

[67] As we shall see in the next section, the reason *angular* velocities cannot yet be considered here is that we are only considering moving (mathematical) points and not yet (three-dimensional) moving bodies. A mere (zero-dimensional) point cannot have an angular velocity of rotation.

[68] By contrast, the notion of purely spatial uniformity – the distinction between rectilinear and curvilinear spatial paths – is already available in pure geometry, and this goes some of the way towards explaining Kant's asymmetrical treatment of curvilinear motion, involving changes of spatial direction (compare note 66 above), and linearly accelerated motion, involving changes solely in (*spatio-temporal*) speed. But a full explanation of Kant's procedure can only emerge gradually in the sequel.

[69] In the terms of notes 42 and 59 above, Kant's principle of the relativity of motion is therefore restricted to the spatio-temporal tangent space at each (space-time) point, where it introduces the Galilean group as a construction of the relevant addition operation defined on the instantaneous velocities at this point. And it follows that Kant's principle is not a relativity principle at all, in our modern sense; for such principles, in our terms, essentially characterize the (spatio-temporally extended) *affine structure* of a given space-time, and therefore characterize spatio-temporal *curvature*. The principle of Galilean relativity, for example, thus characterizes the flat affine structure of Newtonian space-time defined by the law of inertia. But, for Kant, since the law of inertia is not yet in question here, there can similarly be no question of what we now call affine structure. Kant's principle, at this stage, merely characterizes the structure of the spatio-temporal tangent space at each point, and it says nothing at all about the affine structure that may then coordinate these various tangent spaces together. (Compare the situation in special and general relativity: both have the same *infinitesimally* Lorentzian structure on each tangent space, but only the first has a fixed flat affine structure and therefore satisfies Lorentz invariance *locally*.)

from our modern point of view, Kant's principle now looks like no relativity principle at all. In particular, because the law of inertia is not yet in question, for Kant, the equivalence between rest and motion Kant asserts in his principle of relativity does not yet amount to the equivalence between rest and *uniform* motion that we take to be embodied in the law of inertia. So what kind of conception of the relativity of space and of motion, more generally, is Kant here committed to?

I shall only be in a position to explain Kant's conception fully in my chapter on the Phenomenology. But I can begin to clarify the question here by observing that Kant's Copernican conception of space and motion – which is explained in the Phenomenology as a procedure in which "all motion and rest must be reduced to absolute space" (560) – is also independent of the principle of Galilean relativity. As I explained in the Introduction, Kant's Copernican conception ultimately aims at a *single* privileged relative space or reference frame defined by the "common center of gravity of all matter" (563), and it does not acknowledge the equivalence of an infinite *class* of such (inertial) frames, each moving with constant rectilinear velocity relative to the others. Kant begins with our parochial perspective here on the surface of the earth and then moves to an indefinitely extended sequence of "improved" relative spaces – defined, successively, by the center of mass of the solar system, the center of mass of the Milky Way galaxy, and so on *ad infinitum*.[70]

In the Phenomenology, more specifically, Kant begins with the earth, *provisionally* taken to be in a state of rest, then discusses how its state of true rotation (relative to the fixed stars) can be determined (by the effects of centrifugal and Coriolis forces), and finally indicates how we can proceed beyond this stage towards the "common center of gravity of all matter." In the first stage, with the earth provisionally at rest, the mechanical laws of motion are not yet in play, but we do assume provisionally (in the same sense) that we can apply Galileo's law of fall with constant acceleration directed towards the earth's center. We can, in particular, thereby view this law (at least provisionally) as involving the continuous addition of (downward directed) velocities in the sense of the construction in the Phoronomy. In the second stage we then appeal to the law of inertia (for the first time) in concluding that the (centrifugal and Coriolis) forces manifested in the earth's (true) rotation are due to a continuous change

[70] In connection with both this paragraph and the next see note 28 of the Introduction, together with the paragraph to which it is appended. Compare also note 52 above, together with the paragraph to which it is appended.

in direction of each moving point in the rotating earth (outside its axis of rotation) relative to the (ideal) inertial motion of this point along the tangent. In the last stage, finally, we appeal to the equality of action and reaction to infer the following: in the case of *any* two bodies whatsoever, interacting with one another by any moving forces (whether of attraction or repulsion), their motions (momenta) cannot be apportioned arbitrarily between them after all but only in such a way that the two motions (momenta) are equal and opposite (relative, therefore, to their common center of mass). As we shall see, Kant is anticipating precisely this three-stage procedure in his at first sight extremely puzzling qualification of the principle of the relativity of motion in the Phoronomy.

6 MATHEMATICAL AND EMPIRICAL MOTION

Kant repeatedly emphasizes that his construction of motion as a magnitude is to proceed purely mathematically in pure intuition and, accordingly, it is entirely independent of all properly physical considerations involving forces and mechanical laws of motion. Indeed, this construction, in an important sense, is the sole object of the Phoronomy, which can therefore consider the moving body in question as a mere (mathematical) point. This is why, in an important passage already quoted in the Introduction, Kant says that (489) phoronomical motion "can only be considered as the *describing of space*," and that it is for precisely this reason that phoronomy is "the pure doctrine of the magnitude (*Mathesis*) of motion."[71] Moreover, as we have also seen, this same conception of the movable as a mere (mathematical) point is emphasized in Kant's first remark to the first explication (480), according to which "one abstracts in phoronomy from all inner constitution, and therefore also from the quantity of the movable, and concerns oneself only with motion and what can be considered as quantity in motion (speed and direction)."[72]

[71] See note 49 of the Introduction, together with the paragraph to which it is appended. Kant reiterates this emphasis on phoronomy as pure (mathematical) doctrine of the magnitude of motion immediately before the passage, quoted in note 66 above, to the effect that phoronomy is limited to rectilinear motions only (495):

> Phoronomy, not as pure doctrine of motion, but merely as pure doctrine of magnitude of motion, in which matter is thought with respect to no other property than its mere movability, therefore contains no more than this single proposition, carried out through the above three cases, of the composition of motion – and, indeed, of the possibility of *rectilinear motions* only, not curvilinear [ones].

[72] For the full passage, see note 34 above – where I also explain that Kant means to abstract, in particular, from both the volume and mass of the movable matter or body under consideration.

Kant concludes by characterizing his frequent use of the term "body" for the subject of motion (the movable) as merely anticipatory of subsequent developments (480): "If the expression 'body' should nevertheless sometimes be used here, this is only to anticipate to some extent the application of the principles of phoronomy to the more determinate concepts of matter that are still to follow, so that the exposition may be less abstract and more comprehensible."

Nevertheless, the second remark to the first explication (which first introduces Kant's Copernican conception of space and motion) concludes on a rather different note:

> Finally, I further remark that, since the *movability* of an object in space cannot be cognized a priori, and without instruction through experience, I could not, for precisely this reason, enumerate it under the pure concepts of the understanding in the *Critique of Pure Reason*; and that this concept, as empirical, could only find a place in a natural science, as applied metaphysics, which concerns itself with a concept given through experience, although in accordance with a priori principles. (482)

Here Kant suggests that the concept of motion introduced in the first explication is an *empirical* concept, and this passage appears to involve an allusion, in turn, to a parallel passage in (both editions of) the first *Critique*:

> Finally, that the transcendental aesthetic could contain no more than these two elements, namely, space and time, is clear from the circumstance that all other concepts belonging to sensibility, even that of motion, which unites the two, presuppose something empirical. For this [concept] presupposes the perception of something movable. But in space, considered in itself, there is nothing movable. Therefore, the movable must be something that is encountered *in space only by means of experience*, and thus an empirical datum. (A41/B58)

But now we clearly have a puzzle. Motion of a mere (mathematical) point, as in Kant's first remark to the first explication (and parallel passages concerning the construction of motion as a magnitude), is not, presumably, an object of perception and experience. It is rather the object (or subject) of a purely mathematical construction in pure intuition, which must be presented, as Kant says, entirely a priori. *This* concept of motion (of a mere movable point) cannot, therefore, be empirical. So why does Kant emphasize the empirical character of the concept of motion at the end of his second remark?

Section 24 of the transcendental deduction in the second edition of the *Critique*, entitled "On the application of the categories to objects of the senses in general," articulates an important clarification that is very relevant to this question. Kant introduces (B150–52) an operation he calls "figurative synthesis (*synthesis speciosa*)" or "transcendental synthesis of the imagination," in virtue of which the understanding determines the manifold of pure intuition in accordance with its form. He then proceeds to illustrate this operation as follows:

> We always observe this in ourselves. We can think no line without *drawing* it in thought, no circle without *describing* it, we can in no way represent the three dimensions of space without *setting* three lines at right angles to one another from the same point; and we cannot represent time itself without attending, in the *drawing* of a straight line (which is to be the outer figurative representation of time), merely to the action of synthesis of the manifold, through which we successively determine inner sense, and thereby attend to the succession of this determination in it. Motion, as action of the subject (not as determination of an object*), and thus the synthesis of the manifold in space – when we abstract from the latter and attend merely to the action by which we determine *inner sense* in accordance with its form – [such motion] even first produces the concept of succession. (B154–55)

What is immediately relevant to our present concern is Kant's distinction here between two different concepts of motion ("action of the subject" versus "determination of an object"), as further explained in the note:

> * Motion of an *object* in space does not belong in a pure science and thus not in geometry. For, that something is movable cannot be cognized a priori but only through experience. But motion, as the *describing* of a space, is a pure act of successive synthesis of the manifold in outer intuition in general through the productive imagination, and it belongs not only to geometry, but even to transcendental philosophy. (B155n.)

According to this important clarification, then, motion of an object in space (that is, an appearance or object of experience) can indeed be cognized only empirically. But there is also a second type of motion, the describing of a space in pure intuition by means of the productive imagination, which belongs both to pure geometry (as in the fundamental Euclidean constructions of the straight line and the circle) and to the transcendental determination of time.[73]

It appears, therefore, that the consideration of motion introduced in the first explication of the Phoronomy is also twofold. The first remark to

[73] For the connection between the *describing* of a space – in "the *drawing* of a straight line" (B154) – and the transcendental determination of time see note 47 above, together with its reference back to the Introduction.

this explication is concerned with motion as the describing of a space in pure intuition, and we are therefore considering motion merely as "action of the subject" – as a (purely mathematical) activity or operation of the transcendental imagination.[74] In the second remark, however, we are considering motion as an object of possible sensible experience or perception, in the context of Kant's Copernican conception of space and motion. In this context, as explained in section 1, we are specifically considering matter as the object of the outer senses: as the object of what I there called embodied perception in space.[75] It is this second, Copernican concept of motion that is necessarily entangled with the problem of absolute space and absolute motion, and it is this concept that Kant explicitly characterizes as empirical. Our problem is to understand how these two very different types of motion are related. What is the relationship between the mathematical – and even transcendental – concept of motion manifested in the describing of a space (by a mere mathematical point) in pure intuition and the empirical or perceptual concept of motion (of a sensibly given object in space) encapsulated in Kant's Copernican conception of the relativity of motion and space?[76]

I begin by observing that Kant's discussion in the Phoronomy immediately proceeds to mix these two different types of motion together. In particular, although Kant emphasizes in his first remark to the first explication that the movable here is to be considered as a mere mathematical point, and that his use of the term "body" is therefore merely anticipatory, he very quickly deviates from this usage in the second explication. There motion is defined (482) as "change [*Veränderung*] of [a thing's] *external relations* to a given space," and Kant comments as follows:

> I have so far placed the concept of motion at the basis of the concept of matter. For, since I wanted to determine this concept independently of the concept of extension, and could therefore consider matter also in a point, I could allow the common explication of *motion as change of place* to be used. Now, since the concept of a matter is to be explicated generally, and therefore as befitting also moving bodies, this definition is no longer sufficient. (482)

[74] That the motion of a mere (mathematical) point characterized in the first remark to the first explication is the same as the activity of the transcendental imagination illustrated in §24 of the second edition of the *Critique* is strongly suggested by the fact that the Phoronomy, in the passage quoted above (489), also characterizes the relevant kind of motion as "the *describing of a space*."

[75] Moreover, as I also argued in section 1 above, this type of motion is what Kant has specifically in mind in the Preface, where he explains (476) that "[t]he basic determination of something that is to be an object of the outer senses must be motion."

[76] See Pollok (2006) for a more general discussion of the various concepts of motion in Kant – transcendental, mathematical, and empirical.

Thus, while the "common" definition of motion as a change of place (over time) is adequate to the merely mathematical notion of a (zero-dimensional) moving point, it is no longer adequate to that of (three-dimensional) moving bodies.

Kant continues his comment on the new (three-dimensional) definition by discussing "the case of the earth rotating around its axis" (482; see again note 11 above). Such a motion does not count as a *change of place*, because the earth's central point does not change its position. Yet "its relation to external space still changes thereby" (482): for example, it "turns its different sides toward the moon in 24 hours – from which all kinds of varying effects then follow on the earth." Thus, Kant not only explicitly invokes a state of *rotation* here, and thereby suggests an important stage in his progressive Copernican determination of the true motions, he also alludes to the action of gravitational (tidal) forces. We are now very far from a merely mathematical consideration of motion and quite substantially involved, accordingly, with a clearly empirical consideration.

Kant appears to echo this discussion of the new (three-dimensional) definition of motion, moreover, in an important passage from the refutation of idealism in the second edition of the *Critique* (B277–78), according to which "we can undertake all time determination only by the change of external relations (motion) in relation to the permanent in space (e.g., motion of the sun with respect to objects on the earth)" (compare note 46 above, appended to the full passage in question). As I explained in section 2 above, Kant's use of the verb "undertake [*vornehmen*]" here indicates that he is considering the very *first* stage of his progressive Copernican determination of motion, in which the earth is still taken to be at rest (see note 49 above, together with the paragraph to which it is appended). Just as in the Phoronomy, therefore, characterizing motion as a change of external relations to a given (empirical) space belongs to Kant's (similarly empirical) Copernican conception rather than to a purely mathematical one. By contrast, when Kant characterizes motion as a mere change [*Veränderung*] of place in the second edition of the *Critique*, he is always referring to a purely mathematical consideration of a (zero-dimensional) *point* in space.[77]

[77] For example, Kant characterizes motion as an "alteration [*Veränderung*] of place" in the transcendental exposition of time added to the second edition (B48; see the passage to which note 12 of the Introduction is appended). Similarly, in the general remark to the system of principles Kant speaks of "motion, as alteration [*Veränderung*] in space" and of "motion of a point in space, whose existence in different places (as a sequence of opposed determinations) alone makes alteration intuitive to us in the first place" (B291–92; see the paragraph to which note 14 of the Introduction is appended).

Kant's aim in distinguishing between a purely mathematical concept of motion and an empirical concept, both in the *Metaphysical Foundations* and in the second edition of the *Critique*, is not to hold the two concepts rigidly apart from one another. He envisions, rather, a continual process of *transition* from the former to the latter, which is intimately connected, in turn, with a progressive determination of the "true" motions in the universe in accordance with his Copernican conception. Thus, although the primary goal of the Phoronomy is to exhibit motion as a mathematical magnitude by a construction in pure intuition of the composition or addition of two (instantaneous) speeds or velocities, this construction itself rests on a principle of the relativity of motion grounded in precisely Kant's Copernican program. Accordingly, although Kant begins the Phoronomy by considering the motion (change of position) of a mere mathematical point, he quickly makes a transition to the example of the rotating earth – where the determination of this state of "true" (rotational) motion constitutes an essential first step in projecting his Copernican conception into the heavens.[78]

This necessary interpenetration of mathematical and empirical concepts of motion in the Phoronomy is in harmony with Kant's overall aim in the *Metaphysical Foundations*, which, as I argued towards the end of the Introduction, is to explain how the *application* of pure mathematics to the empirical concepts of mathematical physics becomes possible. Kant begins with what he takes to be a purely mathematical concept of motion as a magnitude (a construction in pure intuition), but he then aims to use this initial starting point as a springboard for gradually mathematizing the central empirical concepts of physics (mass, momentum, force, and so on) step by step.[79] Moreover, this characteristically Kantian perspective on the application of mathematics to the empirical world indicates a fundamental divergence (as I suggested in the final paragraph of section 3 above) from Newton's mathematical method.

[78] Kant's examples of motion in the second edition of the *Critique* exhibit a similar pattern. He begins, in the transcendental exposition of time, with the motion of a mere mathematical point (B48), and, in the refutation of idealism, he then arrives at "motion of the sun with respect to objects on the earth" as a "change of external relations" to a given space (B277–78). Moreover, in the two passages beginning, respectively, with "motion as the *describing* of a space" from the second edition transcendental deduction (B154–55) and "motion of a point in space" from the general remark to the system of principles (B291–92), Kant (in both passages) clearly indicates a transition to the argument of the refutation of idealism (see again note 14 of the Introduction, together with the paragraph to which it is appended).

[79] See the last four paragraphs of the Introduction, together with the corresponding notes.

Newton's initial Definitions at the beginning of the *Principia* assume that the central concepts of his theory of motion (velocity, acceleration, mass, and force) are already structured as mathematical magnitudes (see note 36 above, together with the paragraph to which it is appended). The famous Scholium to these Definitions reveals the basis for this assumption (P408–9): namely, that there is an "[a]bsolute, true, and mathematical time" which "flows uniformly" and thereby constitutes duration independently of all "sensible measures," together with a corresponding "[a]bsolute space" which, "without reference to anything external, always remains homogeneous and immovable." Newton therefore begins, in modern terms, with a one-dimensional temporal continuum having the mathematical (metrical) structure of the real numbers and a three-dimensional spatial continuum having the mathematical (metrical) structure of Euclidean space. Motion, in these terms, can thus be characterized as a continuous (and at least piece-wise differentiable) function or mapping from the former into the latter. After Newton introduces his Axioms or Laws of Motion, in the immediately following section, and the argument of the *Principia* unfolds, we then gradually see how all (or almost all) of Newton's initial mathematical presuppositions are justified empirically in their application to the phenomena.[80]

Kant's own account of the application of mathematics to the central concepts of Newton's theory of motion closely follows the procedures that Newton develops to exhibit their empirical application and meaning. But Kant's mathematical starting point is much more minimal than Newton's. Rather than postulating what we would now call a global space-time structure, Kant begins with only the infinitesimal structure of the system of instantaneous velocities at each spatio-temporal point. It is this structure – and this structure alone – that is provided by the construction in pure intuition given in the sole proposition of the

[80] In contemporary terms, Newton begins with the structure $R \times E^3$ for space-time, and this structure is not *completely* justified in its application to the empirical phenomena. In particular, Galilean relativity (which Newton himself formulates as Corollary 5 to the Laws of Motion) entails that a privileged state of rest cannot be empirically distinguished from rectilinear motion with constant speed. Thus, only the slightly weaker structure of what we call Newtonian space-time – which dispenses with a privileged state of rest and retains only the distinction between inertial and non-inertial trajectories given by a flat affine structure on space-time – is, as a matter of fact, empirically justified by the phenomena. For this whole matter see again the classic discussion in Stein (1967) cited in note 52 above. As Stein points out, given that Newton does formulate Galilean relativity, and that the contemporary notion of a Newtonian space-time was only articulated after Einstein's general theory of relativity, this slight excess of structure in Newton's original mathematical articulation of his theory can hardly be viewed as a serious shortcoming.

Phoronomy.[81] At the same time, moreover, this minimal starting point is also embedded in Kant's Copernican conception of space and motion, whereby Kant initiates a route from our sensible experience of motion here on the surface of the earth extending progressively outwards into the furthest reaches of the heavens. Along the way, as we shall see, Kant hopes to account for the mathematical structure of each of the remaining empirical concepts of Newton's theory (mass, momentum, force, and so on) using only the resources that are already fully justified empirically by the Newtonian procedures that are available in the context of the particular stage of the argument of the *Metaphysical Foundations* at which we find ourselves. In following out this strategy, as we shall also see, Kant finds it necessary to provide a metaphysical analysis with roots in the Leibnizean tradition (emphasizing the purely intellectual concepts of substance, causality, action, and interaction) of precisely these Newtonian procedures.[82]

7 MOTION AND THE CATEGORIES OF QUANTITY

The sole proposition of the Phoronomy exhibits the construction of motion as a mathematical magnitude. In the first *Critique* the categories primarily concerned with mathematical magnitudes as such are the categories of quantity (unity, plurality, and totality), and it is precisely this heading of the table of categories (quantity) that the Phoronomy as a whole is supposed to instantiate.[83] At the end of the Phoronomy, in the

[81] As we have seen (again from a contemporary point of view), the operation of composition Kant exhibits on the system of instantaneous velocities at a given spatio-temporal point has the structure of (the addition operation induced by) the Galilean group operating on each (four-dimensional) tangent space. So each such tangent space, in our terms, in fact has the structure of a Newtonian space-time. This by no means implies, however, that Kant's procedure is thereby more empirically justified than Newton's slightly stronger structure for space-time. For, as already emphasized in note 69 above, since Kant's structure is purely infinitesimal, it is not what we would conceive as a structure for space-time at all (either local or global), and, accordingly, Kant, unlike Newton, entirely fails to articulate what we would call Galilean relativity.

[82] One motivation Kant had for embarking on such a *metaphysical* analysis, as we have seen, is that Newton's global mathematical starting point was associated, in both Newton's mind and the minds of his critics, with corresponding global metaphysical views (concerning divine omnipresence throughout all of infinite space, for example), and Kant, following Newton's Leibnizean critics, wished to reject these views. See again the final paragraph of section 3, and compare note 4 above, along with its references back to the Introduction.

[83] As observed in the Introduction (in the paragraph to which note 24 thereof is appended), Kant explains in the Preface that the four chapters of the *Metaphysical Foundations* correspond to the four headings of the table of categories. Corresponding to the first heading, that of quantity, Kant says (477) that "the *first* [chapter] considers *motion* as a pure *quantum* in accordance with its composition [*Zusammensetzung*], without any quality of the movable, and may be called **Phoronomy**." For the relationship between Kant's discussion of magnitude in the Phoronomy and the corresponding principles under the heading of quantity (the axioms of intuition) see notes 28 and 38 above.

third remark to the proposition on the composition of motions, Kant claims that the three categories of quantity thereby correspond to the three cases of the proposition (495): to "*unity* of line and direction, *plurality* of directions in one and the same line, and the *totality* of directions, as well as lines, in accordance with which the motion may occur, which contains the determination of all possible motion as a quantum." Indeed, as already suggested (note 31 above), it is in view of this correspondence with the categories of quantity (and thus with the concepts and principles of transcendental philosophy) that the proposition is divided into three separate cases in the first place (whereas in physics only the third or most general case need be considered).

Yet the correspondence Kant proposes between the three categories of quantity and the three cases of his proposition is puzzling. For the three categories of quantity as applied in relation to Kant's concept of magnitude in general are most naturally understood in a quite different way. In the *Prolegomena*, for example, where Kant presents one of his most explicit discussions of this correspondence, the categories of quantity are illustrated as follows:

> The principle [that] the straight line is the shortest between two points presupposes that the line is to be subsumed under the concept of quantity [*Größe*], which is certainly not a pure intuition, but rather has its place solely in the understanding, and serves to determine the intuition (the line) with respect to the judgements that may be made of it, in relation to their quantity, namely [their] plurality (as *iudicia plurativa**), in that it is thereby understood that in a given intuition a plurality of [*vieles*] homogeneous [elements] is contained. (4, 301–2)[84]

Moreover, the immediately following table of categories presents the three categories of quantity in the sequence (4, 303): "unity (the measure), plurality (the magnitude [*Größe*]), totality (the whole)." In asserting that the straight line is the shortest line (i.e., curve) connecting two given points, we are presupposing that it makes sense to speak of the length of a line in general. This presupposes, in turn, that *measurement* of lines is meaningful, which itself presupposes, finally, that one can choose a unit of measurement (corresponding to the category of unity) and compare the lengths of different lines in terms of the number of

[84] Here and in the appended note difficult issues arise concerning the relationship between the categories of quantity and their corresponding forms of judgement – issues that have been subject to considerable discussion in the literature. But this is not the place to consider these issues.

such units (corresponding to the category of plurality) of which they are composed.[85]

The most natural way of understanding the categories of quantity, therefore, is in terms of their role in measurement. The category of unity involves the choice of a unit of measure; the category of plurality refers to the circumstance that we are considering, in general, a multitude or aggregate of such units; and the category of totality, finally, indicates that we have summed up or counted this multitude to obtain a definite result – the (finite) measure of the object under consideration with respect to the given choice of unit (e.g., the length of a line segment in centimeters). Kant's application of the categories of quantity to motion in the Phoronomy, by contrast, is entirely different. Here the concepts of unity, plurality, and totality are used to characterize whether the motions in question occur along the same or different lines and in the same or different directions, in accordance, as Kant puts it (495), with "all three moments suggested by [the structure of] space." From the point of view of Kant's more general conception of the categories of quantity this particular application is then liable to appear artificial – and it would appear much more natural, instead, to apply the categories of quantity to the measurement of velocity (by the addition or composition of units) along a single line and in a single direction. For it is this case (Kant's first case) that most closely corresponds to the measurement of a line segment.

Kant's full explanation of the application of the categories of quantity in the Phoronomy reads as follows:

> If anyone is interested in connecting the above three parts of the general phoronomic proposition with the schema of classification of all pure concepts of the understanding – namely, here that of the concept of *quantity* [*Größe*] – then he will note that, since the concept of quantity always contains that of the composition of the homogeneous, the doctrine of the composition of motion is, at the same time, the pure doctrine of the quantity of motion [*reine Größenlehre*],

[85] Compare the characterization in the chapter on phenomena and noumena (A242/B300): "No one can explain the concept of quantity [*Größe*] in general except in the following manner – that it is the determination of a thing whereby it can be thought how many times a unit [*Eines*] is posited in it." In his remarks on the table of categories added to the second edition Kant explains (B111) that *totality* is "nothing other than plurality considered as unity" and adds that "the concept of a *number* (which belongs to the category of totality) is not always possible [given] the concepts of aggregate [*Menge*] and unity (e.g., in the representation of the infinite)." (Here Kant clearly limits himself to *finite* numbers.) Thus, a totality must always have a definite (finite) measure (as in the case of a finite line segment, for example), whereas a mere aggregate or plurality (of given units) need not (as in the case of an infinite line).

and, indeed, in accordance with all three moments suggested by [the structure of] space: *unity* of line and direction, *plurality* of directions in one and the same line, and the *totality* of directions, as well as lines, in accordance with which the motion may occur, which contains the determination of all possible motion as a quantum, even though the quantity [*Quantität*] of motion (in a movable point) consists merely in the speed. This remark has its uses only in transcendental philosophy. (495)

Kant's contention is that it is only from the point of view of transcendental philosophy (and thus from the point of view of a *metaphysical* foundation of phoronomy) that his proposition on the composition of motions is to be divided into three separate cases corresponding to the three categories of quantity. Our problem is therefore to understand this contention.

An important clue is provided by the distinction Kant makes at the end of the above passage between considering motion as a *quantum* and as a *quantity*.[86] The distinction between quanta and quantity is central to Kant's philosophy of mathematics, and it marks a contrast, in general, between the objects of intuition to which mathematics is applied in the process of measurement and the more abstract concepts involved in the process of measurement itself. The science of geometry, for example, deals directly with quanta, with (spatial) objects of intuition considered as magnitudes (lengths, areas, and volumes), whereas the sciences of arithmetic and algebra have no such objects of their own but rather concern the general process of counting and measuring applicable to all measurable objects (quanta) whatsoever. Thus, Kant contrasts geometry and algebra in the discipline of pure reason in the first *Critique* as follows (A717/B745): "But mathematics does not only construct quantities [*Größen*] (*quanta*), as in geometry, but also mere quantity [*Größe*] (*quantitatem*), as in algebra, wherein it completely abstracts from the constitution of the object that is to be thought in accordance with such a concept of quantity."[87]

It appears, therefore, that the categories of quantity in general are most directly connected with *quantitas*, with the general procedure for counting and/or measuring any objects of intuition (*quanta*) whatsoever. Thus, in the example from the *Prolegomena*, although we certainly have

[86] I am indebted to Konstantin Pollok for first calling my attention to this aspect of the passage. Compare his own discussion in Pollok (2001, pp. 218–21).

[87] For further discussion of the distinction between *quanta* and *quantity* see Friedman (1992b, chapter 2); for a more recent and rather different account see Sutherland (2004a). Note that Kant sometimes indicates the contrast between *quanta* and *quantity* by using the term "*Quantität*" for the latter, sometimes by using the singular form of "*Größe*" for the latter and the plural form for the former. I often translate "*Größe*" as "magnitude," except where a special connection with the categories of quantity [*Größe*] is at issue.

the length of a particular line segment (as a quantum) in view, the categories of unity, plurality, and totality themselves characterize the general procedure by which *any* objects of intuition (lengths, areas, volumes, weights, and so on) can be measured by counting or numbering *any* arbitrarily chosen unit of measure (a unit length, a unit area, a unit volume, a unit weight, and so on). As thus directly connected with *quantitas*, the categories of quantity abstract from the particular objects of intuition thereby counted or measured and, as Kant says, involve only the result of this enumeration (a number): the answer to the question how large something is.[88] In particular, when we apply the concept of *quantitas* to motions or velocities, we abstract from all consideration of the line and direction in which the motion is proceeding and attend only to the (scalar magnitude) speed with which it proceeds – the answer to the question how fast it is moving. It is this application of the categories of quantity to motion that directly corresponds to the measurement of a line segment (in terms of the scalar magnitude length).

As Kant explains in the passage we are considering, however, he here has in mind a quite different application of the categories of quantity – to motion considered as a quantum or as the particular kind of object of intuition it is.[89] The crucial point, in this connection, is that motions considered as quanta, as particular measurable objects of intuition, are *directed* magnitudes (vector rather than scalar magnitudes), which, accordingly, are necessarily associated with particular lines and directions in space. In considering the addition or composition of such magnitudes, therefore, we must consider whether the two magnitudes to be added are associated with the same or different lines and with the same or different directions with respect to a given line. In the case of two motions associated with a single line, for example, we must consider whether they proceed in the same or in opposite directions (so that one motion is the negative of the other) – which cannot occur in the addition of non-directed (scalar)

[88] Compare Kant's discussion of the difference between geometry and arithmetic – *quanta* and *quantitas* – in the axioms of intuition. Kant explains (A163/B204) that the axioms of geometry "properly speaking concern only quantities [*Größen*] (*quanta*) as such" and continues (A163–64/B204): "But in what concerns quantity [*Größe*] (*quantitas*) – that is, the answer to the question how large [*groß*] something is – there are no axioms in the proper sense, although there are a number of propositions that are synthetic and immediately certain (*indemonstrabilia*)."

[89] See again the passage from the Preface (477) quoted in note 83 above, where Kant says that Phoronomy considers motion "as a pure *quantum* in accordance with its composition." Compare also Kant's characterization of his construction of motion as a magnitude quoted in the paragraph to which note 33 above is appended, which should "only make intuitive what the object (as ***quantum***) *is to be*" (494, bold emphasis added).

magnitudes such as length (all of which are necessarily positive). Similarly, again unlike the case of non-directed (scalar) magnitudes such as length, we cannot add two directed magnitudes associated with different lines comprising an angle simply by adjoining the second magnitude to the end of the first along a single straight line; instead, we must transport the second to the end of the first while preserving the angle between them and then construct the diagonal of the resulting parallelogram. In general, then, we here have a progression comprising three cases of increasing complexity: addition of directed magnitudes along the same line and with the same direction (corresponding to [scalar] addition defined on the positive real numbers), addition of directed magnitudes along the same line but with different (opposite) directions (corresponding to addition defined on the set of positive and negative real numbers), and addition of directed magnitudes on the full set of lines intersecting at a common point (corresponding to [vector] addition defined on a three-dimensional real number space).[90]

Since the primary application of the categories of quantity (in connection with *quantitas*) centrally involves an addition or composition operation, and since the three cases of Kant's proposition on the composition of motions (considered as *quanta*) thus concern three different types of addition operation of increasing complexity, the correspondence Kant invokes between these three cases and the three categories of quantity is not so artificial as it first appears.[91] What is most important for understanding the role of this correspondence in the Phoronomy, however, is the way in which it thereby signals a transition to the following argumentation in the Dynamics. In particular, as observed in section 4 above, the second case (opposite directions along the same line) is closely connected with a balance or opposition between the effects of two different *moving forces*.[92] So let us imagine, in accordance with the third case, the totality

[90] As Stein (1990) points out, the traditional conception of magnitude has the structure, from a modern point of view, of a (continuous) linearly ordered semi-group (admitting only positive elements). This corresponds to Kant's first case and, more generally, to what we now call scalar magnitudes. The second case then corresponds to the structure of a (continuous) linearly ordered group (with both positive and negative elements), and the third to the structure of a continuous multi-dimensional Lie group (associated with a three-dimensional real-number space).

[91] The relevant addition operation on the system of (instantaneous) motions at a given spatio-temporal point must, as we know, have both a spatial and a *temporal* aspect. It is given, in contemporary terms, by the Galilean group, which operates on the set of *space-time* (tangent) vectors in all possible spatial directions at the time in question: see again note 55 above.

[92] This case is thereby closely connected with the issues about "real opposition" first broached in Kant's 1763 essay on *Negative Magnitudes*: see note 57 above, together with the paragraph to which it is appended.

of lines intersecting at a given point. We can then imagine (instantaneous) initial motions directed either towards or away from this given point along any of these lines, and we can further imagine, in accordance with the first two cases, that other (instantaneous) motions – directed either towards or away from the given point – may be added to these initial ones (along a given line). The process of adding a motion directed away from the given point corresponds to the effects of a repulsive force, while that of adding a motion directed towards the given point corresponds to the effects of an attractive force. We thereby obtain a purely kinematical counterpart (and, as it were, a purely kinematical anticipation *quoad effectus*) of the dynamical theory of matter Kant develops in the following chapter: the theory that matter, as that which fills a given space, can be conceived in terms of attractive and repulsive forces acting either towards or away from any given material point.

CHAPTER TWO

Dynamics

8 THE PLACE OF THE DYNAMICS WITHIN THE *METAPHYSICAL FOUNDATIONS*

The Dynamics is the longest and most complicated chapter in the *Metaphysical Foundations*. It is also the best-known part of Kant's treatise, in virtue of presenting a so-called "dynamical" theory of matter as constituted solely out of attractive and repulsive forces emanating from any material point conceived as a center of such forces. This theory, in particular, has played a not inconsiderable role in the development of physics and natural philosophy. For, on the one hand, it is representative of a more general "dynamistic" tendency prevalent in late eighteenth-century natural philosophy, in common with such thinkers as Roger Boscovich and Joseph Priestley, for example. And, on the other hand, it can be taken as an anticipation, of sorts, of the field-theoretic approach to physics developed in the nineteenth century beginning with the work of Michael Faraday. One might view Kant's theory of matter, accordingly, as an important step in a gradual process of transformation from the more "passive" and "mechanical" conception of matter prevalent in the seventeenth century to an "active" and "dynamical" conception characteristic of the nineteenth century, when the concepts of energy, force, and field finally triumph over those of (primitive) solidity, (primitive) impenetrability, and (absolute) indivisibility.[1] Given the length and complexity of the Dynamics chapter, together with the historically important role of

[1] For this kind of view of the development of "dynamism" and Kant's role therein see, for example, Harman (1982a, 1982b), McMullin (1978, chapter 5). The *locus classicus* for dynamistic approaches to matter more generally is Leibniz's *Specimen Dynamicum* (1695), which first developed a metaphysical-physical conception of force as the essence of matter against the background of the Cartesian conception of extension as its essence. But later eighteenth-century versions of this approach, including Kant's, appeal to Newtonian impressed forces (and even action-at-a-distance forces) rather than Leibnizean "living force" (*vis viva*). I shall return to this point at various places in the sequel.

the theory of matter developed there, it is no wonder that commentators on the *Metaphysical Foundations* have been tempted to concentrate primarily, if not exclusively, on this particular part.

A more or less exclusive focus on the dynamical theory of matter has its costs, however. For we thereby tend to forget that the *Metaphysical Foundations* as a whole is characterized, first and foremost, as a (pure) *doctrine of motion*, where each of the four chapters adds a further predicate to the initial idea of matter as the movable. According to the first explication of the Dynamics matter is "the *movable*, in so far as it *fills* a *space*" (496, original emphasis), and this is completely parallel, as noted at the beginning of section 1 above, to the way in which matter as the movable is further characterized in all the other chapters. The Dynamics is thereby inextricably embedded within the project of the *Metaphysical Foundations* as a whole, and it is framed, in particular, within a general theory of motion that is only fully articulated, in turn, in the course of all four chapters. Extracting the dynamical theory of matter from the context of the *Metaphysical Foundations* as a whole, and then focussing primarily (if not exclusively) on this particular aspect of Kant's view, can therefore do irreparable damage to the integrity of Kant's text.[2]

But there is also a more specific dimension to this problem. For, as noted at the beginning of section 3 above, the proof of the first proposition of the Dynamics, where repulsion as a moving force is originally introduced, is based on the single proposition of the Phoronomy. This proposition, in turn, is intimately connected with the general Copernican conception of motion, space, and relativity with which the Phoronomy (and the *Metaphysical Foundations* as a whole) is primarily concerned. Indeed, the single proposition of the Phoronomy is derived from Kant's principle of the relativity of motion – which depends, as explained in section 5 above, on this same conception. Thus, if we fail to consider carefully the relationship of the Dynamics to Kant's Copernican conception of space and motion, we shall fail to understand the proof of its first proposition. The argumentative structure of the Dynamics – and, in particular, the way in which this structure explicitly depends on the preceding argumentation in the Phoronomy – will remain entirely closed to us.

Moreover, as explained in section 7 above, Kant's consideration of motion as a *quantum* yields a kind of kinematical counterpart

[2] A particularly clear example of this tendency is found in Adickes (1924), in which the second part on "the dynamical theory of matter" treats the Dynamics chapter separately, together with a discussion of the pre-critical *Physical Monadology*, whereas the rest of the *Metaphysical Foundations* (exclusive of the Dynamics) is then discussed in the third part on "the theory of motion."

and anticipation of the dynamical theory of matter developed in the Dynamics. In particular, a consideration of the three categories of quantity, corresponding to the three cases of Kant's proposition on the composition of motions, issues in a representation of the possible *effects* of moving forces (of attraction and repulsion) emanating from a central point. What the Dynamics adds to this purely kinematical representation (*quoad effectus*) is simply the concept of a *cause* or *causes* (moving forces) capable of generating these effects. Kant's remark to the first explication thus begins as follows (496):

> This is now the dynamical explication of the concept of matter. It presupposes the phoronomical [explication], but adds a property relating as cause to an effect, namely, the capacity to resist a motion within a certain space; there could be no mention of this in the preceding science, not even when dealing with motions of one and the same point in opposite directions.

In the Phoronomy there can be no mention of causes (moving forces), but there certainly can be a representation of the possible effects (motions) of these causes. Indeed, as I shall explain in my discussion of the first proposition, it is precisely through the necessary connection between the possible effects of moving forces (motions) and these forces themselves that the initial introduction of a moving force (of repulsion) into the *Metaphysical Foundations* is accomplished. Even without yet considering the proof of the first proposition, it is already clear from the structure of Kant's discussion that the argument of the Dynamics presupposes that of the Phoronomy.

This connection between the dynamical theory of matter and Kant's Copernican conception of motion and its relativity represents the most fundamental difference between the exposition in the *Metaphysical Foundations* and in the pre-critical writings where it was originally developed. The basic idea of the theory – that matter is to be conceived "dynamically" in terms of primitive forces of attraction and repulsion rather than "mechanically" in terms of a primitive space-filling property of solidity or impenetrability – is first formulated in the *Physical Monadology* in 1756. It is then prominently repeated, as an "example of the only secure method for metaphysics illustrated by reference to our cognition of bodies," in the 1764 *Prize Essay* (2, 286–87). But in both of these earlier works, in sharp contrast to the *Metaphysical Foundations*, neither general considerations about motion and its relativity nor more specific considerations concerning the composition of motions play any role in the argument. The introduction of a dynamical theory of matter in the pre-critical period is instead entirely based on the problem of reconciling a metaphysical conception of the simplicity of substance arising within the Leibnizean tradition with the needs of a mathematical science of motion arising within the Newtonian tradition.

The difficulty in question is that substance in general, according to metaphysics, is necessarily simple and indivisible, whereas all *material* substances or bodies are spatially extended and therefore, according to geometry, necessarily divisible to infinity. As a result, nothing absolutely simple can be found in space, and so the very idea of specifically material or extended substance is threatened with contradiction. Kant's solution to this dilemma in the pre-critical period is to accept both the necessary simplicity of substance and the infinite divisibility of space but to deny, at the same time, that a material substance or simple body (that is, a physical monad or what Kant also calls an element) fills the space it occupies by being immediately present – substantially present – in all the parts of the space that it occupies. Such a substance is not to be conceived as a piece of solid matter filling the space it occupies through and through, as it were, but rather as a point-like center of repulsive force surrounded by a "sphere of activity" in which this repulsive force manifests its effects. Dividing the sphere of activity of such a monad by no means amounts to dividing the substance of the monad itself, but is only a division of the space in which this (still necessarily simple) substance can manifest its (repulsive) effects on other substances external to it. In this way, by conceiving impenetrability or solidity as the effect of a central force rather than as a primitive, non-dynamical, original property of matter, we can reconcile the metaphysical simplicity of substance with the geometrical divisibility of space.

In the critical period, by contrast, the same difficulty – a collision between the metaphysical conception of the simplicity of substance and the geometrical conception of the infinite divisibility of space – becomes the subject of the second antinomy. It thereby becomes one of the main motivations for the characteristically critical doctrine of transcendental idealism, based on a sharp distinction between appearances or phenomena and things in themselves or noumena. The latter are conceived by the pure understanding alone, independently of sensibility, whereas the former result from applying the pure concepts of the understanding to the empirical material given to sensibility in the pure intuitions of space and time. According to the critical philosophy of transcendental idealism, only this latter *schematized* use of the understanding issues in genuine knowledge. The former – pure or unschematized – use of the understanding results merely in a purely "*intellectual system of the world*," which has been correctly delineated by Leibniz but has no (theoretical) objective reality for us.[3] According to the second antinomy, whereas it is true that substance as conceived by the pure understanding is necessarily simple, this is certainly not true of *substantia phaenomenon*, which is the only substance known to us:

Thus, whereas it may always be true of a whole [composed] of substances thought merely by the pure understanding that we must have the simple prior to all composition of [this whole], this is nevertheless not true of the *totum substantiale phaenomenon*, which, as empirical intuition in space, carries the necessary property with it that no part of it is simple, because no part of space is simple. (A441/B469)

In the critical period, therefore, the conflict between metaphysics in the Leibnizean tradition and the mathematical-geometrical approach to nature associated with Newtonianism is now addressed by relegating the former to a pure world of thought quite independent of natural phenomena, while granting full universal validity, within the realm of natural phenomena, to the latter.

The Dynamics of the *Metaphysical Foundations* emphatically reiterates this critical perspective on the problem. In particular, in the second remark to the fourth proposition Kant summarizes the main argument of the second antinomy in the course of once again rejecting the Leibnizean–Wolffian solution to the collision between metaphysics and geometry. This solution, as Kant presents it, consists in simply denying the infinite divisibility of (physical) space on the ground that our mathematical concept of space, as sensible rather than intellectual, is a confused rather than clear representation. In the remark in question Kant simultaneously heaps scorn on this solution and takes pains to distinguish it from what he takes to be the genuinely Leibnizean view:

The ground for this aberration lies in a poorly understood *monadology*, which has nothing at all to do with the explanation of natural appearances, but is rather an intrinsically correct *platonic* concept of the world devised by *Leibniz*, in so far as it is considered, not at all as object of the senses, but as thing in itself, and is merely an object of the understanding – which, however, does indeed underlie the appearances of the senses. Now the *composite of things in themselves*

[3] See the discussion in the transcendental amphiboly (A270–71/B326–27). It is precisely the error of "confusing an object of pure understanding with an appearance" that constitutes what Kant takes to be Leibniz's fundamental mistake – the fallacious inference or "amphiboly" of attempting to derive conclusions about the spatio-temporal phenomena from that which is thought by the pure understanding alone.

must certainly consist of the simple, for the parts must here be given prior to all composition. But the *composite in the appearance* does not consist of the simple, because in the appearance, which can never be given otherwise than as composed (extended), the parts can only be given through division, and thus not prior to the composite, but only in it. (507–8)

Kant here solves the problem that had originally motivated his pre-critical introduction of the dynamical theory of matter by recapitulating the basic ideas of the transcendental amphiboly (note 3 above) and second antinomy.

Indeed, Kant not only emphatically reiterates the critical solution to the problem of infinite divisibility versus substantial simplicity. In the first remark to the fourth proposition he also explicitly rejects his earlier pre-critical solution presented in the *Physical Monadology*. According to the fourth proposition, matter or material substance (503) is "*divisible to infinity*, and, in fact, into parts such that each is matter in turn." In sharp contrast to the *Physical Monadology*, therefore, a space filled with matter in virtue of repulsive force exerts such a force from *every* point of the space in question, not simply from a point-like center of a space otherwise empty of sources and thus, as it were, of *subjects* of this force. The proof of the fourth proposition puts it this way:

[I]n a space filled with matter, every part of it contains repulsive force, so as to counteract all the rest in all directions, and thus to repel them and to be repelled by them, that is, to be moved a distance from them. Hence, every part of a space filled with matter is in itself movable, and thus separable from the rest as material substance through physical division. Therefore, the possible physical division of the substance that fills space extends as far as the mathematical divisibility of the space filled by matter. But this mathematical divisibility extends to infinity, and thus so does the physical [divisibility] as well. That is, all matter is divisible to infinity, and, in fact, into parts such that each is itself material substance in turn. (503–4)

In asserting that *every* part of a space filled with matter also counts as material substance Kant is clearly diverging from the *Physical Monadology*.

The following (first) remark then explicitly introduces the solution of the *Physical Monadology* in order conclusively to reject it:

For suppose that a *monadist* wished to assume that matter consisted of physical points, each of which (for precisely this reason) had no movable parts, but nonetheless filled a space through mere repulsive force. Then he could grant that space would be divided, but not the substance that acts in space – that the sphere of activity of this substance [would be divided] by the division of space, but not the acting movable subject itself. Thus he would assemble matter out of physically indivisible parts, and yet allow them to occupy a space in a *dynamical fashion*.

But this way out is completely taken away from the monadist by the above proof. For it is thereby clear that there can be no point in a filled space that does not exert repulsion in all directions, and is itself repelled, and thus would be movable in itself, as a reacting subject external to every other repelling point. Hence the hypothesis of a point that would fill a space through mere driving [*treibende*] force, and not by means of other equally repelling forces, is completely impossible. (504)

What is now completely impossible, for Kant, is the idea that matter could fill a space with repulsive force simply by a "sphere of activity" surrounding a point-like center. What is now completely impossible, in other words, is Kant's pre-critical dynamical theory of matter.

It is essential, therefore, to understand the version of the dynamical theory of matter presented in the second chapter of the *Metaphysical Foundation* in the wider context of Kant's critical rather than pre-critical philosophy. This means, in the first place, that the dynamical theory of matter is now introduced in the context of the general Copernican conception of motion and its relativity first outlined in the Phoronomy. It also means, in the second place, that the problem of infinite divisibility versus substantial simplicity is now to be resolved by appealing to the second antinomy and transcendental idealism, according to which *substantia phaenomenon* in space must indeed be infinitely divisible (and is consequently in no way simple). It is therefore necessary to understand, in the third place, how these two originally separate problems – the relativity of motion and the infinite divisibility of material substance in space – become connected.[4]

That the nature of this connection is by no means simple or obvious is undoubtedly one important factor underlying the prevalent tendency to extract the argument of the Dynamics from that of the *Metaphysical Foundations* as whole. But there is a second important factor as well. The argument of the Dynamics, unlike that of the other three chapters, does not directly issue in the application of mathematics to any further central concepts of the mathematical theory of motion. In particular, the concept of *force* that is first introduced in the Dynamics does not there acquire

[4] As observed in section 1 above, Kant's Copernican conception of motion and its relativity also has its roots in the pre-critical period – specifically, in the 1758 pamphlet on motion and rest. In the pre-critical period, however, this conception of motion is not brought into any kind of relation with the dynamical theory of matter (which instead arises directly from the problem of infinite divisibility versus substantial simplicity). It is only in the *Metaphysical Foundations* that these two issues become connected.

a precise mathematical structure. Kant's "dynamics," in this respect, is quite different from Newton's – which, in fact, is primarily concerned with articulating the structure of force as a mathematical magnitude.[5] Following the discussion of the mathematical structure of the concept of speed or velocity in the Phoronomy, the next concept whose precise mathematical structure is articulated by Kant is that of mass or quantity of matter. But this only takes place in the Mechanics chapter. There quantity of matter is "estimated" (537–38) in terms of momentum or quantity of motion ("at a given speed"), and the mathematization of force is then derivative from quantity of motion. Nevertheless, although there is no mathematization of quantity of matter in the Dynamics, there is extensive discussion of this concept – in connection with precisely the question of the infinite divisibility of matter and the constitution of matter out of attractive and repulsive forces. It is precisely here, as we shall see, that the metaphysical concept of substance meets the physical concept of quantity of matter. In the end, as we shall also see, Kant undertakes a profound reconsideration of the most fundamental concept of Leibnizean metaphysics, the concept of substance, in order to undertake an equally profound reconsideration of the Newtonian concept of quantity of matter. Kant thereby develops a metaphysical foundation (in his new sense) for a general concept of matter or material substance that is most appropriate, in his view, for the Newtonian mathematical theory of motion.

9 MATTER AS THE MOVABLE IN SO FAR AS IT FILLS A SPACE

The first explication of the Dynamics characterizes matter as "the *movable*, in so far as it *fills* a *space*" (496), where the filling of space is characterized, in turn, as a resistance to the penetration of other matter into the space in question. Kant continues in his remark to the explication: "This filling of space keeps a certain space free from the penetration of any other movable, when its motion is directed towards any place in this space" (496). The same remark makes an important distinction between *filling* a space and (merely) *occupying* a space:

[5] I owe this point to George E. Smith. In Definitions 3, 4, and 5 of the *Principia*, in particular, Newton introduces the concepts of *inherent* (or inertial), *impressed*, and *centripetal* force, and he introduces three different quantitative measures of the latter in Definitions 6, 7, and 8. The following Laws of Motion, as employed in Newton's overall argument, then underwrite the empirical application of these mathematically structured concepts to the phenomena. I shall have occasion to describe this procedure more fully and to compare it with Kant's at a number of points in the sequel.

One uses the expression *to occupy a space* [*einen Raum einnehmen*] – i.e., to be immediately present in all points of this space – in order to designate the *extension* of a thing in space. However, it is not determined in this concept what effect arises from this presence, or even whether there is any effect at all – whether to resist others that are striving to penetrate within, or whether it means merely a space without matter in so far as it is a complex of several spaces (as one can say of any geometrical figure that it occupies a space by being extended), or even whether there is something in the space that compels another movable to penetrate deeper into it (by attracting others) – because, I say, all this is undetermined by the concept of occupying a space, *filling a space* [*einen Raum erfüllen*] is a more specific determination of the concept of *occupying* a space. (497)

Thus a mere geometrical figure occupies a space (is present in a space) simply by having a spatial extension. However, whereas a mere geometrical figure possesses no causal powers, matter, according to the overall argument of the Dynamics, possesses two (fundamental) causal powers: repulsive force (responsible for impenetrability and resistance) and attractive force (responsible for gravity and weight). The concept of filling a space picks out the first of these effects (impenetrability), and the first proposition then argues that it must be the result of a repulsive force.

I shall consider Kant's argument for this conclusion in detail in the next section. But here I want to observe that Kant also has a second and more general concept of "filling" a space with empirical or sensible content – one that subsumes both impenetrability and weight along with all other possible effects of matter. This is the concept of the *real* in space, or, as Kant also puts it, the real of outer sensible intuition. Indeed, Kant has already appealed to this concept in the second remark to the first explication of the Phoronomy. As explained in section 1 above, Kant there presents a "metaphysical" explication of matter as "any *object of the outer senses*" (481) in terms of a contrast between form (space) and matter (any sensible and empirical object in space): "*Matter*, as opposed to *form*, would be that in the outer intuition which is an object of sensation [*Empfindung*]. Matter would thus be the properly empirical element of sensible and outer intuition, since it can certainly not be given a priori. In all experience something must be sensed [*empfunden*], and that is the real of sensible intuition [*das Reale der sinnlichen Anschauung*]" (481)[6]. The concept of that which is present in space in this sense – as the real of specifically outer sensible intuition, as any object of the outer senses, or as that by which the outer senses are affected (476) – is already contained,

[6] For the entire passage and accompanying discussion see the paragraph to which notes 16 and 17 of my chapter on the Phoronomy are appended.

therefore, in the very general characterization of matter presented in the Preface. This concept is indeed more general than its specific determination in terms of filling a space by means of dynamical resistance added in the first explication of the Dynamics, but it is less general than the mere concept of that which occupies a space by means of its extension. A mere geometrical figure, for Kant, is not an object of empirical intuition at all, and so it certainly cannot be considered as something *real* in space.

The concept of the real as the object of sensible intuition in general (outer and inner) is considered in the anticipations of perception in the first *Critique*, whose subject (according to the second edition) is "the real that is an object of sensation" (B207). This concept, in particular, arises by considering the most general features of perception or "empirical consciousness" in space and time:

> Perception is empirical consciousness, that is, one in which there is simultaneously sensation. Appearances, as objects of perception, are not pure (merely formal) intuitions, like space and time (for these can in no way be perceived in themselves). They therefore contain, in addition to the intuition, the matter [*Materien*] for some or another object in general (whereby something existing in space or time is represented). [They contain,] that is, the real of sensation [*das Reale der Empfindung*], as merely subjective representation, of which one can only be conscious that the subject is affected, and which one relates to an object in general, in itself. (B207–8)

This concept, then, corresponds to the "metaphysical" explication of matter presented in the Phoronomy, although the Phoronomy (like all of the *Metaphysical Foundations*) considers only objects of specifically *outer* sensible intuition – the real *in space*.

It is noteworthy, nonetheless, that Kant also considers this latter (more specific) concept of reality in the anticipations of perception in the course of criticizing (A173/B215) a presupposition common to "almost all investigators of nature" ("for the most part mathematical and mechanical investigators of nature"). The presupposition in question is that "the *real* in space [*das Reale im Raume*] (I may here not call it impenetrability or weight, for these are empirical concepts) is *everywhere identical* and can only be distinguished with respect to extensive magnitude, i.e., [with respect to] aggregate" (A173/B215). This presupposition amounts to the assumption that different kinds of matter can differ in their quantities of matter at the same volume only by containing different proportions of a single uniform type of matter and empty space within this volume. And Kant opposes this assumption to his own conception of a purely *intensive* filling of space by different kinds of matter in different degrees. Kant's contrasting conception of a purely intensive filling of space is centrally relevant to the overall argument of the Dynamics chapter, and I shall examine this important contrast in detail in due course. I shall now simply observe that Kant, in the anticipations, is implicitly distinguishing the concept of the real in space from the more specific concept of that which fills a space introduced in the Dynamics. The former deliberately abstracts from impenetrability and repulsive force (as well as from gravity and attractive force), whereas the whole point of the latter is precisely to introduce these more specific determinations explicitly. Moreover, whereas the former concept is a pure a priori concept appropriate to being considered in the first *Critique* (a combination, as it were, of the pure concept of reality and the pure intuition of space), the latter, as Kant says, is an *empirical* concept.

I observed in the Introduction that the Preface to the *Metaphysical Foundations* explains the content of the empirical concept of matter in terms of "the concept of motion, the filling of space, inertia, and so on" (472); I also observed that the empirical character of the concept of matter is thereby particularly associated with the concept of impenetrability.[7] This makes sense, in the context of the passage from the anticipations of perception that we are considering, for this passage suggests both that the concept of the real in space (the real of specifically outer sensible intuition) is not empirical and that empirical content is first added to this concept by further determinations involving Kant's two fundamental forces – repulsion and impenetrability, on the one side, attraction and weight, on the other.[8] If this is correct, however, it would seem to follow that the concept of matter considered in the Phoronomy is also not yet empirical. For this concept corresponds to that of the real in space, where specific moving forces responsible for impenetrability and weight are not yet in question. To be sure, the concept of the real in space includes some (unspecified) causal powers, for, at the very least, such a reality must affect or causally influence our outer senses.[9] Moreover, it is in precisely this way, as I have suggested, that the concept of the real in space differs from the concept of

[7] See notes 46 and 47 of the Introduction, together with the paragraphs to which they are appended.

[8] Kant never gives gravity or weight in the list of constituent concepts of matter, for this concept, according to Kant, is a derivative concept rather than a primitive constituent concept. It is not analytically contained in the concept of matter but added to this concept synthetically (although still a priori). This important asymmetry will be discussed in section 15 below.

[9] Compare the proof of the anticipations of perception in the second edition, where the intensive magnitude of the real as an object of perception or sensation in general is explained in terms of the "degree of influence on the sense [*Grad des Einflusses auf den Sinn*]" (B208).

that which merely occupies or is present in a space through its extension (as in a mere geometrical figure). Nevertheless, no specific causal powers or moving forces are actually mentioned at this stage, and it is reserved for the next stage, that of the Dynamics, to introduce them.

As explained in section 6 above, however, Kant explicitly characterizes the concept of matter as the movable in space – the concept of matter considered in the Phoronomy – as an *empirical* concept, and he does so in the same (second) remark to the first explication in which matter is considered as the real in space (482): "[S]ince the *movability* of an object in space cannot be cognized a priori, and without instruction through experience … this concept, as empirical, could only find a place in a natural science, as applied metaphysics."[10] Here it is clear that the relevant concept of motion or movability (of an object in space) is indeed empirical, for Kant, so that it is not after all correct that the empirical content of the concept of matter arises only with the dynamical concept of impenetrability. Nevertheless, as I also explained in section 6, Kant's consideration of motion in the Phoronomy is actually twofold: it has both a pure a priori aspect (the motion of a mere mathematical point in pure intuition) and an empirical aspect (the motion of a physical or material body in empirical intuition). In particular, the initial concept of matter as simply the object of outer sensible intuition only becomes empirical by being explicitly brought into relation with Kant's Copernican conception of space and motion.[11] It is precisely by connecting the concept of matter as the object of outer sensible intuition (as the real in space) with *this* conception that Kant's concept of matter first becomes empirical.

In the end, therefore, Kant presents us with what we might think of as a nested sequence of ever more specific concepts of an object in space. The first is simply that of a mere geometrical object or figure constructed in pure intuition – which occupies or is present in space merely through its extension but is associated with no causal powers or relations vis-à-vis other objects in space. The second and more specific concept is that of the real in space – that of an object of outer sensible intuition in general or as that which affects our outer senses. This concept ascribes some (unspecified) causal powers to spatial (and therefore geometrical) objects (at least

[10] For the full passage, together with a parallel passage from the first *Critique*, see the paragraph following the one to which note 72 of my chapter on the Phoronomy is appended.

[11] The argument linking the contrast between the pure motion of a mere mathematical *point* and the empirical motion of a physical *body* to Kant's Copernican conception of space and motion is made in the four paragraphs beginning with one to which note 75 of my chapter on the Phoronomy is appended and concluding with the one to which note 78 of that same chapter is appended.

in so far as they affect us as perceivers), but it is nonetheless not yet an empirical concept. On the contrary, it is still a pure a priori concept (reality plus space as it were) appropriate to consideration in the first *Critique*. However, the next and still more specific concept arises from connecting the last concept (of the real in space) with the concept of motion. This is the concept of matter as the movable in space introduced in the Phoronomy – and, in particular, in the second remark to the first explication, where Kant's Copernican conception of space and of motion is also first introduced. It is at precisely this stage that an *empirical* concept of matter (as the movable in space) first arises. But the progressive unfolding of Kant's Copernican conception throughout the *Metaphysical Foundations* demands that the concept of motion (and hence the concept of matter as the movable in space) become increasingly empirical step by step. The first explication of the Dynamics takes the immediately following step by introducing a further determination, impenetrability or the filling of space, into the concept of matter as the movable in space.

10 REPULSIVE FORCE

The first proposition of the Dynamics is one of the most important in Kant's treatise. It represents the first place where Kant introduces a fundamental force (here the fundamental force of repulsion), and so it is crucial for how the dynamical theory of matter arises in his overall argument. But this initial dynamical proposition also depends on the single proposition of the Phoronomy – which, in turn, is supposed to proceed entirely a priori in pure intuition. So the first proposition of the Dynamics not only involves a transition from the Phoronomy to the Dynamics, but also an extremely striking transition from an a priori representation in pure intuition to a quite different representation – that of a fundamental moving force – in empirical intuition.[12] For precisely this reason, however, the first proposition of the Dynamics has also been one of the most controversial in Kant's treatise. Many commentators have expressed considerable (and understandable) skepticism, in particular, concerning Kant's move from

[12] Recall from my chapter on the Phoronomy that the construction (in pure intuition) presented in the proposition on the composition of motions is supposed to proceed entirely independently, in particular, of all consideration of moving forces. This is one important respect in which it differs from a properly physical or "mechanical" derivation such as Newton's: see, e.g., note 33 of that chapter, together with the paragraph to which it is appended. Recall also from the Introduction that Kant is clear in the Dynamics chapter that particular moving forces (of attraction or repulsion) can only be given empirically: see note 44 of the Introduction, together with the paragraph to which it is appended.

an a priori construction in pure intuition to an a posteriori dynamical force.[13] I shall present my own interpretation of this crucial transition, and thus of Kant's argument in the first proposition, in the remainder of this section. But I first want to make two preliminary observations: one concerning the relevant concept of impenetrability or resistance that serves, as it were, as the explanandum for Kant's introduction of repulsive force, the other concerning the contrasting notion of absolute impenetrability or solidity to which this introduction of repulsive force is explicitly opposed.

My first observation is that the relevant concept of impenetrability or resistance is not that of resistance to motion, in so far as a filled region of space or piece of matter resists being set into motion by the impact or pressure of another piece of matter already in motion. Kant calls such resistance *mechanical*, and he treats this concept in the following Mechanics chapter under the rubric of the communication of motion. The present concept of resistance, by contrast, is that by which matter at rest resists being compressed or reduced in volume by other matter that is impinging, as it were, all around it:

> This filling of space keeps a certain space free from the penetration of any other movable, when its motion is directed toward any place in this space. Now the basis for the resistance of matter exerted in all directions, and what this resistance is, must still be investigated. But one already sees this much from the above explication: matter is not here considered as it resists, *when it is driven out of its place*, and thus moved itself (this case will be considered later, as mechanical resistance), but rather when merely the *space* of its own extension is to be *diminished*. (496–97)

This concept of resistance or impenetrability – by which matter resists compression within the space or volume that it fills – appears, for example, in Locke's famous discussion of solidity in Book II, chapter IV of the *Essay*, which illustrates the idea in question by the way that bodies resist our effort to press our two hands enclosing them completely together.[14]

[13] Perhaps the sharpest expression of this kind of skepticism is found in Tuschling (1971), which argues not only that the first proposition of the Dynamics is a failure but also that Kant came to see this himself in the course of his work on the *Opus postumum*. Indeed, one of the central ideas of Tuschling's interpretation of this latter work is based on the idea of a "phoronomy-critique" in which Kant came to see, because of precisely the failure of the present proposition, that the entire project of the *Metaphysical Foundations* can contain no genuine dynamics and must therefore reduce to pure phoronomy. See Friedman (1992b, chapter 5, §1) for a criticism of Tuschling's idea of "phoronomy-critique" in the *Opus postumum* (although I do not provide a detailed reading of the first proposition of the Dynamics there).

[14] See Locke (1975, Book II, chapter IV, §1): "The Bodies which we daily handle, make us perceive, that whilst they remain between them, they do by an insurmountable Force, hinder the

My second observation is that Kant explicitly distinguishes his preferred concept of impenetrability or resistance to compression (involving repulsive force) from a concept of absolute impenetrability or solidity according to which the matter that fills a space is capable of no penetration or compression whatsoever. In the fourth explication, in particular, Kant distinguishes what he calls relative from absolute impenetrability, where the first yields a dynamical concept of the filling of space and the second a (merely) mathematical concept. He articulates this contrast further and explicitly connects it with repulsive force in the following remark:

> According to the merely mathematical concept of impenetrability (which presupposes no moving force as originally belonging to matter), matter is not capable of compression except in so far as it contains empty spaces within itself. Hence matter as matter resists all penetration utterly [*schlechterdings*] and with absolute necessity. However, according to our discussion of this property, impenetrability rests on a physical basis; for expanding force [*ausdehnende Kraft*] first makes matter itself possible, as an extended thing [*ein Ausgedehntes*] filling its space. (502)

(The second proposition, to which I shall turn below, states [499] that "expansive force [*Ausdehnungskraft*]" of a matter filling a space is identical to "the repulsive force of all of its parts.") Kant's preferred, dynamical concept of impenetrability is thereby distinguished from the concept of (absolute) solidity that he attributes to "Lambert and others" in his remark to the first proposition (497–98) – where, according to this concept, matter exerts resistance to penetration merely through its concept as something real or existent in space quite independently of repulsive force.[15]

As observed in the Introduction, Kant returns to the contrast between mathematical and dynamical impenetrability in the general remark to dynamics in the course of drawing a related contrast between what he calls the mathematical-mechanical and metaphysical-dynamical approaches to

approach of the parts of our Hands that press them. That which thus hinders the approach of two Bodies, when they are moving one towards another, I call *Solidity*." In the remaining sections Locke associates this idea with the filling of space (§2), distinguishes it from pure space (§3), and also distinguishes it from hardness (§4) – since bodies that are not hard, such as water or the air in a football, resist compression in this sense just as much as hard bodies do.

[15] I shall discuss Kant's relationship to Lambert (and others) on this point in the next section. That the concept of solidity is here linked to what Kant, in the fourth explication, calls absolute impenetrability is indicated, among other things, by the circumstance that Kant attributes the concept of solidity he rejects to "the mathematician" (498). Note that this concept of absolute solidity or impenetrability is more specific than Locke's, for Locke attributes solidity in his sense to fluid and elastic matter as well (note 14 above).

natural science.[16] The first, characteristic of the mechanical or corpuscular natural philosophy, seeks to explain matter and its behavior merely from the figures and motions of elementary corpuscles without appealing to fundamental (dynamical) forces. The second, apparently characteristic of a more Newtonian approach to natural science, seeks to explain matter and its behavior precisely by means of (Newtonian) fundamental forces of attraction and repulsion. Indeed, Kant begins the general remark as follows:

> The general principle of the dynamics of material nature is that everything real in the objects of the outer senses, which is not merely a determination of space (place, extension, and figure), must be viewed as moving force – whereby the so-called solid or absolute impenetrability is banished from natural science, as an empty concept, and repulsive force is posited in its stead; and, by contrast, the true and immediate attraction is thereby defended against all sophistries of a metaphysics that misunderstands itself, and, as a fundamental force, is declared necessary for the very possibility of the concept of matter. (523)

This passage not only confirms that the concept of solidity that Kant rejects in his remark to the first proposition is closely connected with that of absolute impenetrability (note 15 above), it also shows that Kant's defense of repulsive force as the explanation for (relative) impenetrability is closely connected, in turn, with his defense of Newtonian gravitation as an immediate action at a distance.[17]

But now Kant's strategy in the first proposition is liable to appear even more implausible. Kant's explanation of impenetrability in terms of repulsive force not only involves a very special concept of impenetrability (relative as opposed to absolute impenetrability), it is also explicitly intended to undermine a more general mathematical-mechanical approach to natural science in favor of what he calls a metaphysical-dynamical approach. It is in precisely this way that Kant's preferred concept of impenetrability is intended to defend a broadly Newtonian style of natural philosophy against the corpuscularian or mechanical natural philosophy. How can all of this possibly be inferred from the single proposition of the

[16] See note 41 of the Introduction, together with the paragraph to which it is appended

[17] The strategy of using repulsive force as the explanation for impenetrability or the filling of space to open the way for a parallel defense of Newtonian action at a distance was also characteristic of Kant's pre-critical formulation of the dynamical theory of matter. See, for example, the discussion in the *Enquiry Concerning the Clarity of the Principles of Natural Theology and Morality* (2, 287–88), where Kant argues that, since repulsive force (acting at a very small distance) is already needed to explain impenetrability or the filling of space, "metaphysics has absolutely no sound reason to rebel against immediate action at a distance." *This* aspect of the theory is therefore equally characteristic of both pre-critical and critical periods.

Phoronomy on the composition of motions? How can a purely mathematical construction – which is presumably uncontroversial and acceptable to all parties to the dispute – possibly decide such a large question?

The first proposition states that "[m]atter fills a space, not through its mere *existence*, but through a *particular moving force*" (497), and the proof immediately follows:

> Penetration into a space (in the initial moment this is called a striving to penetrate) is a motion. Resistance to motion is the cause of its diminution, or even of the change of this motion into rest. Now nothing can be combined with a motion, which diminishes it or destroys it, except another motion of precisely the same movable in the opposite direction (Phoron. Prop.). Therefore, the resistance that a matter offers in the space that it fills to every penetration by other matters is a cause of the motion of the latter in the opposite direction. But the cause of a motion is called a moving force. Thus matter fills its space through a moving force, and not through its mere existence. (497)

I shall postpone further discussion of Kant's contrast between filling a space through mere existence or a moving force until I discuss the following remark (concerning "Lambert and others") in the next section. But it is already clear from my discussion so far that filling a space through mere existence is associated with the concept of solidity Kant is most concerned to reject – the concept of absolute impenetrability admitting no compression or penetration whatsoever.[18] Kant's introduction of a repulsive force is therefore intended decisively to undermine this concept.

The central points in Kant's proof are these: (i) penetration into a space is a motion; (ii) this motion is to be diminished or destroyed (brought into a state of rest) by the resistance or impenetrability in question; (iii) such diminution or destruction is a combination or composition (in the sense of the single proposition of the Phoronomy) of the initial penetrating motion with a contrary motion in the opposite direction; (iv) the cause of this contrary motion is a moving force (here of repulsion). At first sight, however, the crucial step in the argument, the transition to (iv), appears simply to beg the question at issue. Why can a defender of solidity and absolute impenetrability not accept (i)–(iii) while simultaneously rejecting (iv)? Why must the contrary motion in the opposite direction to the initial penetrating motion be the product of a fundamental repulsive force rather than an absolute or "merely mathematical"

[18] See note 15 above, together with the paragraph to which it is appended, and compare the first sentence of the remark in question (497): "Lambert and others called the property of matter by which it fills a space *solidity* (a rather ambiguous expression), and claim that one must assume this in every thing *that exists* (substance), at least in the outer sensible world."

resistance to penetration depending, as Kant suggests in the remark to the fourth explication, on "no moving force … originally belonging to matter" (502)?

The key to understanding Kant's argument, I believe, lies in the remarks Kant makes concerning the representation of rest through the combination or composition of two equal and opposite motions in his discussion of the second case of the proposition on the composition of motions in the Phoronomy. In particular, according to my reading in section 4 above, Kant there implicitly appeals to the definition of rest presented in the third explication of the Phoronomy where the case of perfect uniform reflection at an instantaneous turn-around point is contrasted with continuously decelerating and accelerating Galilean motion under the influence of gravity. The former case lacks a well-defined spatio-temporal tangent (or derivative) at the turn-around point, whereas the latter case exhibits a well-defined spatio-temporal tangent (or derivative) everywhere – including a well-defined state of rest or zero velocity (what Kant calls an enduring presence at the same place) at the turn-around point itself. In particular, it is only in the second case of continuous deceleration and acceleration that the change of motion in question can be properly represented by an addition or composition of velocities in Kant's sense, which always operates on the set of *instantaneous* velocities at a given spatio-temporal point. The crux of Kant's argument for the first proposition of the Dynamics, therefore, is that, if we represent the resistance in question as absolute or "merely mathematical," then we have an instance of the first case of perfect uniform reflection where there is no well-defined velocity at the turn-around point and thus no addition or composition of velocities properly speaking. If, by contrast, we represent the resistance in question as relative or dynamical, then we have an instance of the second case of continuous deceleration and (possible) re-acceleration. Velocity, in this case, is everywhere well-defined (including at the turn-around – or stopping – point), and the change of motion can thus be represented by an addition or composition of velocities (at the turn-around point) in accordance with the second case of Kant's phoronomical proposition.

Kant signals the importance of *instantaneous* composition of velocities in the first sentence of the proof, which inserts a parenthetical qualification into the claim that penetration into a space is a motion (497): "in the initial moment this is called a striving [*Bestrebung*] to penetrate." This notion of a "striving" is Kant's version of the Latin term *conatus*, and it denotes, accordingly, an instantaneous velocity or what we would now call a spatio-temporal tangent (or derivative) at a point.[19] The initial moment of (attempted) penetration, on the model of absolute impenetrability, would necessarily lack such a "striving" or *conatus* and would not, strictly speaking, involve a *motion* (a velocity) at all. On Kant's preferred model of relative impenetrability, by contrast, there is a well-defined "striving" or *conatus* at every point, including at the initial moment of penetration. So it follows, on this reading, that Kant has already ruled out the model of absolute impenetrability in the first *explication* of the Dynamics (496; here presented in full): "*Matter* is the *movable* in so far as it *fills* a *space*. To *fill* a space is to resist every movable that strives through its motion to penetrate into a certain space [*das durch seine Bewegung in einen gewissen Raum einzudringen bestrebt ist*]. A space that is not filled is an *empty space*." Kant's explication of filling a space, in terms of resistance to that which "strives through its motion" to penetrate into the space, already rules out the contrasting model of absolute or "merely mathematical" impenetrability all by itself. For, on this model, there can be *no* "striving through its motion" at the moment of (attempted) penetration. Instead of a deceleration or diminution of a well-defined initial (instantaneous) velocity, there is simply an instantaneous and discontinuous transition at the moment of impact from a uniform velocity directed inwards to either a state of rest or an equal and opposite uniform velocity directed outwards.[20]

Kant's definition of the filling of space in the first explication thereby rules out the opposing model of absolute impenetrability or absolute solidity simply by articulating (explicating) a dynamical as opposed to a mathematical concept of this property. As Kant puts it in the fourth

[19] As observed in note 19 of my chapter on the Phoronomy, the term *conatus* was standardly used by such writers as Huygens and Leibniz to denote just such an instantaneous velocity.

[20] As observed in note 20 of my chapter on the Phoronomy, this is the way in which the impact of "absolutely hard" bodies was standardly described. Although in the Dynamics Kant is dealing with dynamical resistance to penetration (in the sense of compression) rather than mechanical resistance to motion (in the sense of impact), there is nonetheless a close connection between the two. For the incoming "movables" certainly exert impacts and/or pressure against the surface of the resisting space-filling matter. At the end of section 2, I noted that in the Mechanics Kant insists that the change of motion in impact cannot be instantaneous but must rather be described by a continuous deceleration. In this same passage from the Mechanics Kant thereby rejects the idea of an absolutely hard body (552): "Hence an absolutely hard body, that is, one that would, on impact, *instantaneously* oppose a body moved at finite speed, with a resistance equal to the total force of that body, is impossible. Consequently, by means of its impenetrability or cohesion, a matter attains instantaneously only an infinitely small resistance to the force of a body in finite motion." There is a close connection, then, between absolute hardness and absolute impenetrability. I shall return to this connection in my chapter on the Mechanics.

explication (502): "The *filling of space* with absolute impenetrability can be called *mathematical*, that with mere relative impenetrability can be called *dynamical*." So Kant's argument in the proof of the first proposition is not intended to demonstrate that the former, merely mathematical concept is impossible. Indeed, as explained, he continues to discuss the opposition between these two concepts in the general remark to dynamics and beyond. Rather than ruling out the merely mathematical concept of impenetrability all by itself, the first proposition is simply intended to demonstrate how Kant's preferred dynamical concept is intimately connected, in turn, with his conception of the composition of motions already developed in the Phoronomy. I observed in section 2, when discussing Kant's explication of rest as enduring presence, that the contrasting concept of rest as a mere "lack of motion" (as in a case of discontinuous transition) "can in no way be constructed" (486). Rest must therefore, Kant continues, "be explicated as enduring presence at the same place, since this concept can also be constructed, through the representation of a motion with infinitely small speed throughout a finite time, and can therefore be used for the ensuing application of mathematics to natural science" (486). Now, in the context of the proof of the first proposition of the Dynamics, the moral is that motion at the turn-around (or stopping) point *can* be mathematically constructed (as a state of rest) on Kant's preferred dynamical conception of relative impenetrability but not on the opposing model of absolute impenetrability.

As Kant explains in his remark to the proposition, this leads to the rather paradoxical result that the "merely mathematical" concept of absolute impenetrability or solidity erects a limit to mathematical construction (and thus a limit to the application of mathematics to natural science) where the opposing dynamical concept does not:

> Here the mathematician has assumed something, as a first datum for constructing the concept of a matter, which is itself incapable of further construction. Now he can indeed begin his construction of a concept from any chosen datum, without engaging in the explication of this datum in turn. But he is not therefore permitted to declare this to be something entirely incapable of any mathematical construction, so as thereby to obstruct us from going back to first principles in natural science. (498)

In opting for the concept of absolute impenetrability or solidity "the mathematician" has injected a fundamental discontinuity into the transition from motion to rest. The spatio-temporal trajectory representing the effects of such impenetrability necessarily lacks a tangent (or derivative) at the precise instant that these effects are manifested, and so nothing further can be said about the transition in question. In particular, the construction of the addition or composition of motions developed in the Phoronomy cannot be applied here to describe this transition.

In the second remark to the fourth explication Kant therefore characterizes absolute impenetrability as an occult quality:

> Absolute impenetrability is in fact nothing more nor less than an occult quality [*qualitas occulta*]. For one asks what the cause is for the inability of matters to penetrate one another in their motion, and one receives the answer: because they are impenetrable. The appeal to repulsive force is not subject to this reproach. For, although this force cannot be further explicated in regard to its possibility, and therefore must count as a fundamental force, it does yield a concept of an acting cause, together with its laws, whereby the action or effect, namely, the resistance in the filled space, can be estimated in regard to its degrees. (502)

The advantage of Kant's dynamical concept of relative impenetrability, in the end, is that it employs the model of a continuously acting force – which produces a continuous change of motion by the continuous addition of new motions in accordance with the addition or composition of velocities. An exact mathematical force law capable of describing the resulting instantaneous motions at *every* point thereby becomes first possible.[21]

I am now in a position to explain the relationship between the construction of the composition of motions described in the Phoronomy and the proof of the first proposition of the Dynamics more precisely. As explained in section 7 above, the correspondence Kant sets up between the three cases of his proposition on the composition of motions and the three categories of quantity issues in a purely kinematical representation (in pure intuition) of the possible effects of both repulsive and attractive forces emanating from a central point. We begin, in accordance with the third case of the composition of motions, by constructing the set of all lines in space intersecting at a common point (in all possible angles). We can represent initial motions (velocities) along any of these lines directed

[21] Kant's model for such continuous changes of motion is Galileo's description of the constant acceleration due to gravity. A crucial step in Newton's treatment of *any* continuously acting force (constant or otherwise) is the realization that Galileo's description actually applies (in the limit) at the initial moment of any resulting change of motion. See Lemma 10 of Section 1 of the *Principia* (P437–38): "*The spaces which a body describes when urged by any finite force, whether that force is determinate and immutable or is continually increased or continually decreased, are at the very beginning of the motion in the squared ratio of the time.*" Kant takes such (Newtonian) continuously acting forces to be paradigmatic of the successful execution of his preferred dynamical strategy: compare notes 26 and 27 of my chapter on the Phoronomy, together with the paragraph to which they are appended.

either towards or away from the point in question. Then, in accordance with the first two cases of the composition of motions, we can represent the addition of other velocities to these initial velocities in either the same or the opposite directions. The process of adding such velocities directed away from the central point in question represents the effects of a repulsive force; the process of adding such velocities directed towards the central point in question represents the effects of an attractive force.[22]

In the proof of the first proposition of the Dynamics Kant is only considering the effects of a repulsive force. Indeed, he is only considering the process of adding velocities directed away from the given center to (instantaneous) incoming velocities ("strivings to penetrate") directed towards the center – *and this representation of the possible effects of a repulsive force is itself a purely phoronomical representation in pure intuition*. What the first explication and first proposition of the Dynamics add to this representation is simply the concept of a *cause* or *ground* of the effects (motions) in question. This, and no more, is then the content of the concept of a repulsive force, which, as Kant says in his remark to the first explication (497), "presupposes the phoronomical [explication], but adds a property relating as cause to an effect, namely, the capacity to resist a motion within a certain space."[23]

[22] This representation is the basis for Kant's claim in the note to the second explication of the Dynamics (where repulsive and attractive forces are first defined) that only these two types of forces are possible (498–99):

> Only these two moving forces of matter can be thought. For all motion that one matter can impress on another, since in this regard each of them is considered only as a point, must always be viewed as imparted in the straight line between the two points. But in this straight line there are only two possible motions: the one through which the two points *remove* themselves from one another, the second through which they *approach* one another.

I shall return to this important note in my chapter on the Mechanics.

[23] I observed in my discussion of Kant's second case of the composition of motions in section 4 that this case is closely connected to the issues about "real opposition" first addressed in Kant's 1763 essay on *Negative Magnitudes*: see note 57 of my chapter on the Phoronomy. In the terminology of the 1763 essay, accordingly, Kant has now introduced the concept of a "real ground" of the (repulsive) effects in question. It is no wonder, then, that Kant's argument for repulsive force in the 1763 essay is the closest of all his pre-critical arguments to that of the first proposition of the Dynamics (2, 179):

> Every body resists through impenetrability the moving force of another to penetrate into the space it occupies. Since the body is a ground of the rest of the other, which nonetheless has a force for motion, it follows from the preceding [discussion of real opposition and real grounds] that impenetrability just as much presupposes a true force in the parts of the body, by means of which they together occupy a space, as does that force, whatever it may be, whereby another is striving to move into this space [*in diesen Raum sich zu bewegen bestrebt ist*].

These considerations illuminate the sense, finally, in which Kant's preferred concept of (relative) impenetrability, as a manifestation of the effects of a repulsive force, is an *empirical* concept. Because the mere representation of the possible effects (motions) produced by this force is a purely mathematical representation in pure intuition, the real possibility of objects (motions) corresponding to the representation can also be known entirely a priori. It does not follow, however, that the concept of a cause or force capable of producing these effects can similarly acquire real possibility or objective reality in this way. Indeed, as observed in the Introduction, Kant emphasizes that the real possibility of any fundamental force can never be demonstrated a priori in the general remark to dynamics (524; see again note 44 of the Introduction). Accordingly, in the case of Kant's preferred dynamical concept of matter (525), "we lack all means for *constructing* this concept of matter, and presenting what we thought universally as possible in intuition." The real possibility or objective reality of any fundamental force can only be inferred from experience (534): "[N]o law of either attractive or repulsive force may be risked on a priori conjectures, but rather everything, even universal attraction as the cause of weight, must be inferred, together with its laws, from data of experience."

The idea that the concept of a specific force or cause can only obtain objective reality empirically is central to the first *Critique* as well. Kant draws an important distinction between what he calls the *mathematical* principles of pure understanding (the axioms of intuition and anticipations of perception) and the *dynamical* principles of pure understanding (the analogies of experience and the postulates of empirical thought). The former principles are concerned with "appearances and the synthesis of their empirical intuition" (A178/B220) – which synthesis, since it always takes place within the framework of pure intuition, can be a priori determined in a mathematical construction.[24] The dynamical principles, by contrast, are concerned with "*existence* [*Dasein*] and the *relation* among [the appearances] with respect to [their] existence" (A178/B220). And, since "the existence of appearances cannot be cognized a priori" (A178/B221), because "[existence] cannot be constructed" (A179/B221), the latter principles, unlike the former, cannot be *constitutive* with respect to appearances. On the contrary, dynamical principles such as the principle of causality are only *regulative* with respect

[24] Here follows the example of determining the degree of illumination of sunlight in relation to the degree of illumination of the moon quoted in note 40 of my chapter on the Phoronomy.

to appearances, in so far as the existence of a specific cause for a given effect, for example, cannot be constructed a priori but only inferred a posteriori from some actual perception or empirical intuition in accordance with the principle of causality itself. Kant explains (A180/B222–23): "An analogy of experience will thus only be a rule in accordance with which from perceptions unity of experience may arise (not, like perception itself, as empirical intuition in general), and it is valid as [a] principle of the objects (the appearances) not *constitutively* but merely *regulatively*."

The representation of the possible effects (motions) produced by a repulsive force is a purely mathematical representation constructed in pure intuition. The concept of a cause or force that is added to this representation is also, in itself, a pure a priori representation as well – a pure concept of the understanding. The conjunction of the two, however, is the concept of a specific cause or force responsible for a specific effect, and this concept, according to Kant, can only obtain its objective reality empirically through an actual experience in which it is exhibited.[25] However, since experience, for Kant, is always a unified whole, the full exhibition of the objective reality of the concept of a repulsive force must await the full explanation of how all the pure concepts of the understanding are instantiated or realized together in the metaphysical doctrine of body. Kant's empirical concept of (relative) impenetrability is not, as in Locke, a simple idea of sensation.[26] Its empirical character is not the product of an immediate sensory origin but reflects a much more complex process in which pure a priori concepts – both mathematical concepts and pure concepts of the understanding – are progressively realized in experience step by step.

[25] Compare the passage from the postulates of empirical thought quoted in the paragraph to which note 44 of the Introduction is appended (A222/B269): "[I]f one wanted to make entirely new concepts of substances, of forces, and of interactions out of the material offered to us by perception, without borrowing the example of its connection from experience itself, then one would fall into mere phantoms of the brain, whose possibility would have no indications at all."

[26] In particular, Kant's initial introduction of the concepts of impenetrability and repulsive force makes no reference to immediate sensations of pressure such as those produced in squeezing a body between one's hands (compare note 14 above) but only to the purely abstract (purely phoronomical) *motions* by which the filled space in question is, as it were, to be probed. Later, in the remark to his fifth proposition, Kant does consider the immediate character of our sensations of pressure and relate this immediacy to the priority of our concept of impenetrability. I shall consider this remark in detail in section 15, but it here suffices to note that none of these considerations is involved in Kant's original introduction of the concept.

II KANT, LAMBERT, AND SOLIDITY

It is striking that in his remark to the first proposition of the Dynamics Kant explicitly names only Lambert (along with some unspecified "others") as a representative of that concept of solidity or absolute impenetrability he is most concerned to reject. For the relationship between Kant and Lambert was a special one.[27] As is well known, the two came to similar conclusions about the nebular structure of the Milky Way and the universe as a whole at approximately the same time, and Kant calls attention to this agreement (somewhat ruefully) in the Preface to his *Only Possible Basis for a Proof of the Existence of God* in 1763 (2, 68–69).[28] After seeing this work, Lambert wrote to Kant on November 13, 1765 (10, 51–54), expressing the conviction that the two are embarked on a common project of reforming the method of metaphysics and asking for Kant's help in publishing his most recent philosophical work, *Architectonic or Theory of the Simple and Primary* [*Elements*] *of Philosophical and Mathematical Knowledge*. Kant replied on December 31, 1765 (10, 54–57), calling Lambert "the first genius in Germany … who is capable of making an important and enduring contribution to the kind of investigations with which I, too, am primarily occupied" (i.e., the reform of metaphysics), and promising to help Lambert with his publication project.[29] It is no wonder, then, that Kant made a point of sending Lambert a copy of the *Inaugural Dissertation* in 1770, together with an accompanying letter (10, 96–99) expressing his gratitude that "a man of such decisive penetration and universality of insights" has suggested that the two collaborate in "outlin[ing] the plan for a secure structure [for metaphysics] with united testing and investigation."[30] It is similarly no wonder, finally, that Kant had originally intended to dedicate the *Critique of Pure Reason* to Lambert (but had abandoned this plan after Lambert's untimely death in 1777).[31]

[27] Johann Heinrich Lambert was one of the most remarkable scientific thinkers of the mid-century. He made important contributions to mathematics, astronomy, and physical optics, as well as significant contributions to philosophy. For basic information about Lambert (and his relationship with Kant, in particular) see Beck (1969, pp. 402–12).

[28] Kant's rueful tone is due to the circumstance that Lambert's *Cosmological Letters* of 1761 had attracted quite considerable attention, whereas Kant's own *Theory of the Heavens* of 1755 had fallen virtually still-born from the press.

[29] Lambert replied on February 6, 1766 (10, 62–67), thanking Kant for his help, once again expressing his conviction that the two are engaged in a common undertaking, and (further) explaining his ideas concerning the form and matter of cognition (see below). Lambert's *Architectonic* appeared with Kant's publisher Hartknoch in 1771.

[30] Lambert's lengthy reply (10, 103–11) is also of great interest, especially in connection with Kant's first articulation of the transcendental ideality of time in the *Dissertation*.

[31] See Beck (1969, p. 402n. 25).

It is clear, therefore, that Kant and Lambert had the greatest respect for each other, and they even saw themselves as sharing (to one extent or another) a common philosophical project. This project can be described, in extremely general terms, as looking for a way of revising the then dominant Leibnizean–Wolffian philosophy so as to make it more responsive to recent results of the exact sciences, primarily in mathematics and mathematical physics. For both Kant and Lambert, this philosophy appeared excessively concerned with the purely logical form of knowledge expressed in such maxims as the principles of identity and non-contradiction. It therefore appeared incapable, in particular, of giving a satisfactory account of the more material or contentful principles of scientific knowledge such as those of geometry and mechanics. The common project, then, was to preserve the undoubted metaphysical advances of Leibniz and Wolff while simultaneously doing justice to more recent advances in the mathematical sciences. This common project, moreover, constitutes the background against which their fundamental disagreements concerning the concept of solidity can best be appreciated.

From Lambert's perspective, Leibniz had made a decisive contribution to metaphysics by emphasizing the importance of the analysis of concepts, and Wolff had then added the idea that metaphysics, like geometry, should exhibit a deductive form. Yet Wolff had followed the Euclidean model only partially: he had rightly emphasized the importance of definitions and deductive proofs but left the axioms and postulates wholly out of account. For Lambert, by contrast, definitions of the primitive concepts (point, line, and so on) actually play very little role in Euclid's geometry, which is instead driven primarily by the fundamental laws governing these concepts formulated in the axioms and postulates. The postulates, moreover, are especially important, for it is they, and they alone, that establish the real (as opposed to merely logical) *possibility* of objects falling under the primitive concepts by providing the rules for their construction.[32] From this perspective, therefore, the process of conceptual analysis is only important in helping us to arrive at the truly simple primitive concepts and thereby allowing us to exhibit all other (complex) concepts in terms of a list of ultimate simples. These ultimate simple concepts are not further definable, and, as Locke has shown, they must rather be directly obtained from the senses as simple ideas of sensation. What Locke did not see, however, is that a genuine science can be based on these concepts only if we also have *general material principles* governing

[32] For further discussion see Laywine (2010), along with Laywine (1998) and (2003).

them – principles for which Euclid's axioms and postulates again supply the model.[33]

Lambert also follows Locke in taking the concept of solidity, in particular, to be perhaps the most fundamental of these simple concepts (compare note 14 above). We obtain this concept from the resistance manifested to our sense of touch [*Gefühl*], and we use this same sensory basis in then branching out to a number of related simple concepts: extension, motion, force, existence, substance, and duration. Each of these concepts has axioms and postulates appropriate to it (typically involving the fundamental simple concept of solidity), and, accordingly, we are thereby able to articulate a metaphysical basis for all of the mathematical sciences. These include, most notably, geometry or the science of pure extension, chronometry or the science of pure duration, phoronomy or the science of pure motion, dynamics or the science of moving force, and mechanics or the science of machines driven by moving force.[34] Lambert gives five axioms and two postulates governing the concept of solidity. The axioms state (1771, §81): (1) the solid fills a space so far as it goes; (2) the solid excludes other solids from the place where it is; (3) the solid has the three dimensions of space; (4) space can be no more than filled with solids; (5) the solid has an absolute density and is therefore an unchangeable unity. The postulates state: (1) any space can be thought as entirely or partially filled with solids, as little as one likes, but no more than entirely; (2) the solid in a space that is not entirely filled can be thought as brought together into a smaller space that it entirely fills (§81).

To illustrate how the concept of solidity functions in the axiomatization of the other fundamental simple concepts, I consider two examples – the concept of moving force and the concept of existence. Lambert initially gives five axioms for the concept of moving force (1771, §94): (1) the solid is in itself at rest or without motion; (2) the solid is set into motion by

[33] These ideas are developed in the first chapter of Lambert's *Architectonic* (1771) and are also sketched in the two letters to Kant from 1765–66.

[34] Although we originally obtain our ideas of extension, duration, and motion from the senses (primarily from the same tactile sensation of resistance underlying our concept of solidity), we can nonetheless formulate pure sciences governing these concepts by abstracting from sensation and considering, as it were, only empty space and time (empty, in particular, of solidity). In dynamics, however, the concept of solidity *cannot* be abstracted from, and so the axiomatization of this science must explicitly employ the concepts of solidity and (therefore) force. Thus, when Lambert presents his axiomatizations of the various simple concepts in the second chapter of the *Architectonic*, he first presents axioms of geometry, chronometry, and phoronomy independently of the concept of solidity – which then figures essentially in the axiomatization of all subsequent simple concepts (force, existence, substance, and so on). For the sensory basis of *all* the simple concepts see Lambert (1764, *Alethiologie*, chapters 1 and 2).

other solids; (3) every change of motion of the solid is caused by another solid that is in immediate contact with the moved solid; (4) in free space the solid once set into motion preserves its direction and speed; (5) the motion is in proportion to the force whereby the solid is set into motion and follows the direction in which the force is applied. We see from axiom (3) that Lambert here holds to the basic doctrine of the mechanical philosophy that all action between bodies, and thus all moving forces, must arise from the immediate contact of bodies with one another. We see from axioms (4) and (5) that Lambert includes versions of Newton's first two Laws of Motion – limited, in accordance with (3), to forces due to impact and pressure – within what he calls the science of dynamics. Lambert then gives seven axioms governing the concept of existence (1771, §103): (1) existence is an absolute unchangeable unity; (2) without solids and forces, or in general without something substantial, nothing exists; (3) what exists has duration; (4) what exists is at a place; (5) a single solid does not exist simultaneously at more than one place; (6) different solids do not exist simultaneously at the same place; (7) what exists is not simultaneously different, or what exists is one and the same (numerically). We see from axioms (4)–(7) that space functions, with respect to existence, as a principle of individuation for solidity: one and only one solid can exist at a single place at a single time.

Putting these three sets of axioms (governing solidity, force, and existence) together, we arrive at the following picture of the nature of bodies and their fundamental interactions. From the axioms governing the concept of existence it follows that anything real or existent in space must be solid. Moreover, since space is a principle of individuation for the solid, we can conclude that anything real or existent in space must indeed exclude all other such realities from the space that it occupies – otherwise it would not be one solid body but several. Thus, the second axiom governing the concept of solidity follows from the axioms governing the concept of existence, and the impenetrability of bodies follows from their nature as something real or existent in space. But it is also in virtue of this same impenetrability that bodies can be set into motion in the first place, by other bodies already in motion impinging upon them by impact. So the first three axioms governing the concept of moving force follow from the impenetrability of bodies, which itself follows from the concept of the solid as that which is real or existent in space. Hence, all the most essential features of bodies and their interactions (motions) follow from the concept of the solid as that which is real or existent in space. This line of thought therefore supplies us with a full metaphysical justification of the fundamental doctrines of the mechanical natural philosophy – and it appears to be precisely such a justification that Kant is decisively rejecting in his remark to the first proposition addressed to "Lambert and others" (497).[35] It appears to be precisely this picture that Kant is describing when he says that, according to the thinkers in question, "the presence of something *real* in space must already, through its concept, and thus in accordance with the principle of non-contradiction, imply this resistance, and bring it about that nothing else can be simultaneously in the space where such a thing is present" (497–98).[36]

As emphasized in section 9 above, however, one of Kant's principal claims concerning the concept of impenetrability is that it does *not* follow from the mere concept of something real in space. In the anticipations of perception, in particular, Kant takes pains to distinguish this latter concept, as a priori (reality plus space, as it were), from the empirical concept of impenetrability.[37] Here, as suggested at the end of section 9, Kant envisions a nested sequence of ever more specific concepts of an object in space – ranging from the (pure geometrical) concept of that which occupies space through its extension, to the (still pure) concept of that which is real or existent in space, to the (empirical) concept of a movable real existent in space (i.e., matter as the movable in space), and finally to the (even more empirical) concept of that which is impenetrable or fills a space in Kant's preferred dynamical sense. In sharp opposition to the metaphysical justification for the mechanical natural philosophy developed by Lambert, Kant himself is centrally concerned to *separate* the

[35] Warren (2001a, pp. 113–15n. 15) calls attention to a passage from the *Essay* where Locke employs space as an identity condition for bodies and suggests that it would be a contradiction for two bodies to be in the same place at the same time (Book II, chapter XXVII, §2) – "it being a contradiction that two or more things could be one." As Warren points out, this would then give us a conceptual route from the identity conditions for bodies to their impenetrability. The remaining step in the line of thought in question, the route from impenetrability to the nature of moving force, is explicit in Euler's *Letters to a German Princess* (1768–72, letters 77–78). It is therefore very plausible, as Pollok (2001, pp. 229–32) explains, that both Locke and Euler are included among the "others" to whom Kant alludes in his remark.

[36] Warren (2001a, p. 113n. 14) emphasizes that Lambert himself explicitly distinguishes purely formal principles such as the principle of non-contradiction from the more material and contentful conceptual principles he formulates in his axioms and postulates. So it is not clear, in particular, how the line of thought we have been considering could be purely conceptual in the sense of following from the law of non-contradiction. This leads, however, to a more general problem for Lambert – namely, how can his material conceptual principles be genuinely a priori, even in the case of pure geometry? In other words, as I shall explain in more detail below, Lambert, from Kant's point of view, here runs into the problem of how synthetic a priori judgements are possible.

[37] For my earlier discussion of this passage from the anticipations (A173–74/B215–16) see the two paragraphs following the one to which note 6 above is appended.

concept of impenetrability (as empirical) from the (pure a priori) concept of that which is real or existent in space.[38]

In the passage from the anticipations in question (A173–74/B215–17; see again note 37 above) Kant opposes the presupposition (represented by most "mathematical and mechanical investigators of nature") that different kinds of matter can differ in their quantities of matter at the same volume only by containing different proportions of a single uniform type of matter and empty space within this volume. Moreover, in further developing the opposition between his preferred metaphysical-dynamical approach and the contrasting mathematical-mechanical approach in the general remark to dynamics Kant develops an extended criticism of this same presupposition. Thus, immediately following the first sentence of the general remark where he "banishes" the solid or absolute impenetrability from natural science (523; see the paragraph to which note 17 above is appended), Kant goes on to add, as a consequence, "that space, if it should be necessary, can be assumed to be completely *filled*, and in different degrees, even *without dispersing empty interstices* within matter" (523). Kant here further elaborates his preferred conception of a purely intensive filling of space that he had already outlined, as a mere possibility, in the anticipations.[39]

[38] Kant had already sharply separated the concepts of existence and force in his pre-critical writings, through the idea that God's creation of the existence of a substance does not yet amount to establishing the *coexistence* of a number of such substances in a common world. The latter can be established only through a distinct creative act – where, in accordance with a "schema of the divine intellect," God establishes fundamental laws of interaction (manifested phenomenally as attractive and repulsive forces) governing the *relations* among the substances in question. See, in particular, the second principle of §3 ("The Principle of Coexistence") in the *New Exposition of the First Principles of Metaphysical Knowledge* of 1755 (1, 412–16). For further discussion of this central pre-critical doctrine see, e.g., Laywine (1993) and Watkins (2005). Warren (2001a, §§4, 5) provides a helpful discussion of this doctrine as the background for Kant's "dynamistic" objections to the mechanical philosophy and, in particular to the view attributed to "Lambert and others" in the section of the Dynamics chapter currently under discussion. Compare Warren (2001b) for a fuller account.

[39] In more detail, Kant describes the presupposition he opposes in the anticipations as follows (A173/B215):

> Almost all investigators of nature, because they observe a large difference in the quantity of matter of different kinds at the same volume (partly by means of the moment of gravity, or weight, partly by means of the moment of resistance to other moved matters), unanimously conclude from this that the volume in question (extensive magnitude of the appearance) must in all matters be empty in different amounts.

Kant then describes his alternative purely intensive possibility (A174/B216):

> [A]lthough the same spaces may be completely filled by different matters, in such a way that there is no point in either where its presence is not to be found, each reality of the same quality still has its degree (of resistance or weight) which, without diminution of the extensive magnitude or aggregate, can be smaller to infinity, before it is transformed into the empty and vanishes.

By contrast, Lambert's axiomatization of the concept of solidity specifically excludes this idea of a purely intensive filling of space in favor of the contrasting conception favored, according to Kant, by the majority of "mathematical and mechanical investigators of nature." Axiom (5), in particular, states that the solid has an absolute density constituting an "unchangeable unity" – which therefore admits of no compression whatsoever that could change this given (absolute) density. The two postulates then express the idea that there is a maximum of density whereby a space is completely filled, without interspersed empty spaces, so that the only way in which a filled space can be diminished or compressed is by eliminating such empty spaces until it is completely filled.[40] It is important to observe, nonetheless, that Lambert is by no means dogmatic on this issue. For, immediately after presenting his axiomatization, Lambert notes that the idea of a purely intensive filling of space is still possible:

> But another question arising here is the following: Whether a completely filled space could not be still more filled intensively, or [whether] the solid that fills it could be brought into a still smaller space – or whether all solids are in themselves equally dense and are in this respect an absolute and unchangeable unity? These questions concern the second, fourth, and fifth axioms, which are based on the well-known concept of the *impenetrability* of matter that is also assumed in mechanics. Without regard to this concept, however, these questions can nevertheless arise. We have the concept of solidity through touch [*Gefühl*], and this does not provide us with the inner differences thereof. In the concept that we have, there also seems to be no impossibility [in the idea] that the solid could not [*sic*] have different degrees of inner density. The above axioms would thereby have to be changed so that solidity would not be an absolute and unchangeable unity, so that a completely filled space could be filled with more or less dense solids, and so on. This investigation has an influence on the question of empty space, and the determination of moving forces also depends on it, because these are proportional to density. (1771, §91; 1965, pp. 70–71)[41]

The particular axiomatization Lambert has provided, therefore, governs a particular concept of solidity and impenetrability conventionally assumed in mechanics – and, as he suggests, further investigation of nature may

[40] Densities are therefore estimated by volume in relation to a fixed state of absolute (i.e., completely filled) density (Lambert, 1771, §89; 1965, p. 69): "This latter postulate provides the basis for the determination of the degree of density, because the smaller space, which the solid completely fills, relates to the greater, which the solid does not completely fill, as the density in the greater space to the absolute density in the smaller."

[41] Note that among the axioms Lambert suggests might have to be revised is the crucial second axiom, according to which the solid excludes other solids from the place where it is. Although Lambert never discusses this further, it is therefore clear that the revision in question would involve quite fundamental changes to the mechanical natural philosophy more generally.

lead us to revise this concept. Lambert repeatedly returns to this question, accordingly, in the remainder of the *Architectonic*. He ultimately suggests that it is to be decided, in the end, by further researches in chemistry, concerning whether each type of matter can be somehow transformed into all others, as one would expect on the presupposition that different types of matter of different densities consist of a single uniform type of matter of absolute density combined with different proportions of empty space.[42]

It is by no means surprising, therefore, that Kant singles out Lambert in the course of his own defense of a purely dynamical conception of impenetrability. For, as an especially open-minded defender of the opposing conception of absolute impenetrability, Lambert has already left room for Kant's conception. At the same time, from Kant's perspective, this very open-mindedness exposes deep problems and tensions in Lambert's overall view. Lambert intends his particular axiomatization of solidity as a metaphysical justification for precisely what Kant calls the mathematical-mechanical approach to natural philosophy, and Lambert claims, accordingly, that his axiomatization of solidity, as a "grounding-principle [*Grundlage*] of metaphysical truth" (1771, §298; 1965, p. 28), possesses "geometrical necessity" (§313; p. 394). But how can this axiomatization possess this metaphysical (and therefore a priori and necessary) character if it can be revised in the light of further scientific research? This question not only concerns Lambert's particular axiomatization of the concept of solidity, it strikes at the heart of his new conception of metaphysical method more generally. Lambert hopes to reform Leibnizean–Wolffian metaphysics on the model of Euclid's axiomatization of geometry, so that metaphysical axioms and postulates governing the ultimate simple concepts underlying all of our knowledge can then be formulated. Moreover, these axioms and postulates are intended to have an a priori and more than empirical character, despite the fact that such ultimate simple concepts are, following Locke, all derived from experience as simple ideas of sensation. Yet Lambert has no philosophical account of how such explicitly non-analytical or non-definitional a priori judgements are possible.[43]

[42] See Lambert (1771, §§119, 143, 148, 159, 205–8). The last sections consider the question in relation to chemistry – where, to be sure, it is most difficult to decide (§207) since we are by no means acquainted with the structure and mechanisms of the smallest parts of matter (§208).

[43] From a modern perspective, we might conceive Lambert's axiomatizations as providing *implicit definitions* of the fundamental concepts of physics, and we might then be tempted to applaud him as anticipating more recent views, arising in the late nineteenth century and beyond, according to which philosophy ("metaphysics") describes the conceptual presuppositions or frameworks

Here, as suggested, Lambert is squarely faced with the Kantian question of how synthetic a priori judgements are possible.[44] It is precisely here, accordingly, that Kant's reform of metaphysical method diverges most fundamentally from Lambert's. Kant, like Lambert, aims to revise Leibnizean–Wolffian method so as to inject more scientific content – and even more empirical content – into what they both viewed as an excessively formal or excessively logical metaphysical system. In sharp contrast to Lambert, however, Kant does not appeal to Lockean simple ideas of sensation as supplying the ultimate simple concepts for a projected axiomatization of metaphysics modeled on the Euclidean axiomatization of geometry. On the contrary, Kant rather proceeds by further specifying the fundamental concepts of formal logic itself, so as to bring them progressively into relation with both pure and empirical intuition. Kant derives the categories or pure concepts of the understanding from the logical forms of judgement, and he then moves from the pure or unschematized categories to the schematized categories by considering the pure categories in relation to the pure intuitions of space and time. This process of schematization results in the principles of pure understanding, where we encounter genuinely material (not purely logical) metaphysical principles for the first time. These principles are further specified, finally, in the metaphysical doctrine of body, so as to yield precisely the synthetic a priori metaphysical principles of pure natural science. Here, to be sure, we encounter an empirical concept – namely, the concept of matter that Kant is now in the process of articulating. But this is emphatically not a Lockean simple idea of sensation. It is rather a concept progressively articulated from the top down, as it were, so as thereby to make intelligible the application of pure mathematics (which is also synthetic a priori for Kant) to corporeal nature.[45]

Kant's explication of the empirical concept of matter at the beginning of the Dynamics (the concept of the movable in so far as it fills a space) occupies a particular place in this procedure of progressive articulation.

underlying one or another branch of empirical science at a given time. But Lambert himself is far from any such view and is instead immersed in precisely the post-Leibnizean–Wolffian metaphysical problematic dominating late eighteenth-century German philosophy. Indeed, the modern view in question was itself only developed historically as a reaction to – and generalization of – the original Kantian conception of the synthetic a priori.

[44] See note 36 above – and recall that the problem even arises for Lambert's conception of pure geometry. Compare also Beck (1969, pp. 406–7) for Lambert and the problem of the synthetic a priori. It is perhaps not too far-fetched, in light of this, to speculate that Kant had originally intended to dedicate the first *Critique* to Lambert (note 31 above) precisely because of the way in which the problem of the synthetic a priori arises for him.

[45] See note 26 above, together with the paragraph to which it is appended.

His strategy is to begin with the concept of motion already articulated in the Phoronomy with respect to the categories of quantity and to proceed from this concept in further articulating the concept of matter as the movable in space with respect to the categories of quality. But to subsume the empirical concept of matter under the categories of quality, in the first instance, is to subsume it under the concept of the real in space, and it is central to Kant's strategy that the empirical concept of matter as the movable in so far as it fills a space *not* be identified with the (pure) concept of the real in space. On the contrary, this last concept can only obtain *empirical* content by being brought into connection with Kant's Copernican conception of space and motion already articulated in the Phoronomy. And it is then used to derive the fundamental force of repulsion in the first proposition of the Dynamics employing the single proposition of the Phoronomy on the composition of motions. In this way, as explained in section 10 above, we finally arrive at a characteristically Kantian defense of the thoroughgoing application of the mathematics of continuity to motion, along with a characteristically Kantian criticism, in his remark to the first proposition, of the seemingly paradoxical attempt by "the mathematician" (498) – e.g., Lambert – to erect an insurmountable limit to such application at the instant of (attempted) penetration of an absolutely impenetrable or solid body.

12 MATTER AS AN ORIGINALLY FLUID AND ELASTIC MEDIUM

I have observed that it is central to Kant's dynamical theory of matter (523) that "the so-called solid or absolute impenetrability is banished from natural science, as an empty concept, and repulsive force is posited in its stead." I have also pointed out that repulsive force, for Kant, is introduced as the cause or ground of a particular effect: namely, resistance to penetration in the sense of resistance to compression. Finally, I argued that the empirical character of this concept of impenetrability, for Kant, does not rest on its being a simple idea of sensation in the Lockean sense, but rather depends on a much more complex process by which pure a priori concepts – both mathematical concepts and pure concepts of the understanding – are successively realized in experience step by step. The pure concept of the understanding that is at issue here is that of causality, and so the point, in the present case, is that the concept of the cause of the effect we are interested in (impenetrability in the sense of resistance to compression) can only obtain objective reality in experience. Kant makes this point explicitly towards the end of the general remark to dynamics (534): "[N]o law of either attractive or repulsive force may be risked on a priori conjectures, but rather everything, even universal attraction as the cause of weight, must be inferred, together with its laws, from data of experience."

In the case of attractive force, as Kant himself suggests (and as we shall see in more detail below), the empirical basis for the concept of this exercise of causality is Newton's celebrated "deduction from phenomena" of universal gravitation from the observed phenomena of our solar system in Book 3 of the *Principia*. So what, it is now natural to ask, is the analogous empirical basis for the particular exercise of causality represented by Kant's conception of repulsive force? Kant first addresses this question (along with universal gravitation) in the *Theory of the Heavens* of 1755, where the two fundamental forces of attraction and repulsion are also first introduced:

> After setting the world into the simplest [state of] chaos, I have applied no other forces except attractive and repulsive force for the development of the great order of nature – two forces which are equally certain, equally simple, and at the same time equally original and universal. They are both borrowed from the Newtonian philosophy. The first is a law of nature that has now been placed beyond all doubt. The second, for which the natural science of Newton can perhaps not secure as much clarity as the first, I assume here only in that sense which no one disputes, namely, in the case of the finest dissolution [*feinsten Auflösung*] of matter as, for example, in the case of vapours. (1, 234–35).

In the case of repulsive force, in particular, the phenomena Kant has in mind involve primarily what we now call gases, or, as they were standardly called in the eighteenth century, elastic or permanently elastic fluids.[46]

It seems clear, moreover, that the central such phenomenon is that of the expansive force or pressure exerted by such a fluid, by which it strives to expand into a larger volume – and, conversely, resists compression into a smaller volume – in accordance with the well-known law of Boyle and Mariotte stating that (expansive) pressure is inversely proportional to volume. For it is with reference to this law that Newton derives a proposition in the *Principia* that most closely approximates for the case of repulsive

[46] Kant further explains (1, 265) that "this repulsive force manifests itself in the elasticity of vapours, the effluence of strong-smelling bodies, and the diffusion of all spirituous bodies, and is an undisputed phenomenon of nature." Kant presents Newton's derivation of the law of universal gravitation from the observable phenomena of our solar system in a short introductory section to the *Theory of the Heavens* (1, 243–46) entitled "Brief sketch of the necessary fundamental concepts of Newtonian science."

force what his "deduction from the phenomena" of the law of universal gravitation does for the case of attractive force. This is Proposition 23 of Book 2, which shows that if an elastic fluid consists of particles in a state of static equilibrium, such that the outer compression is balanced by repulsive forces acting between each particle and its immediately neighboring particles, then the Boyle–Mariotte law holds just in case the repulsive force in question is inversely proportional to the distance.[47] So the empirical basis for Kant's introduction of the concept of repulsive force, more generally, appears to be precisely the expansive behavior of permanently elastic fluids.

Immediately after introducing the concept of repulsive force in the first proposition of the Dynamics of the *Metaphysical Foundations*, Kant goes on to characterize it as an *expansive* force in the second proposition (499): "Matter fills its space through the repulsive forces of all of its parts, that is, through an expansive force of its own, which has a determinate degree, such that smaller or larger degrees can be thought to infinity." In the first note to this proposition Kant associates the expansive force in question with a fundamental property of matter he calls original elasticity (500): "The expansive force of a matter is also called *elasticity*. Now, since it is the ground on which the filling of space, as an essential property of all matter, rests, this elasticity must therefore be called *original*, because it can be derived from no other property of matter. All matter is therefore originally elastic." In the second note to this proposition, finally, Kant makes it clear that the force by which matter continually strives to expand is the same force by which it resists penetration in the sense of compression: "Beyond every expanding force a greater moving force can be found. But the latter can also act contrary to the former, whereby it would then decrease the space that the former strives to enlarge, in which case the latter would be called a *compressing* force. Therefore, for every matter a compressing force can also be found, which can drive it from the space it fills into a decreased space" (500). The original or fundamental state of matter, for Kant, manifested in its continual striving to expand the space that it occupies due to the mutual repulsion of all of its parts (that is, its

[47] This derivation is not as secure as the argument for the law of universal gravitation of Book 3, because the present Proposition essentially depends on a purely hypothetical model of the micro-structure responsible for the expansive or elastic power in question. Newton himself makes this clear in a typically cautious remark added in the Scholium (P699): "Whether elastic fluids consist of particles that repel one another is, however, a question for physics. We have mathematically demonstrated a property of fluids consisting of particles of this sort so as to provide natural philosophers with the means with which to treat the question."

impenetrability or resistance to compression), is precisely that of a permanently elastic fluid.[48]

In the following third explication Kant explains that a matter is penetrated (500) when "the space of its extension is completely destroyed through compression," and he illustrates such compression by the elastic behavior of atmospheric air:

> When, in the barrel of an air pump filled with air, the piston is driven closer and closer to the bottom, the air-matter is compressed. If this compression could now be driven so far that the piston completely touched the bottom (without the least amount of air escaping), then the air-matter would be penetrated. For the matters enclosing the air would leave no remaining space for it, and it would thus be found between the piston and the bottom without occupying a space. (500)

Such complete penetration is actually impossible, since, according to the third proposition (501): "Matter *can* be *compressed* to infinity, but can *never* be *penetrated* by a matter, no matter how great the compressing force of the latter may be." Indeed, the more that matter is compressed, the greater is the contrary force of expansion by which it resists such penetration: "An original force, with which a matter strives to extend itself on all sides beyond a given space that it occupies, must be greater when enclosed in a smaller space, and infinite when compressed into an infinitely small space" (501). It appears, then, that a central example of an originally elastic fluid, for Kant, is provided by atmospheric air.

It turns out, however, that this is not the case. For, in the second remark to the eighth proposition of the Dynamics, Kant conjectures that the fundamental force of repulsion varies in inverse proportion to the cube of the (infinitely small) distance rather than to the distance itself. He then immediately points out, in accordance with Proposition 23 of Book 2 of the *Principia*, that his original expansive force *cannot* obey the Boyle–Mariotte law:

> From the law of the parts of matter repelling one another originally in inverse cubic ratio to their infinitely small distances, a law of expansion and contraction of matter completely different from Mariotte's law for the air must therefore

[48] It is at this point, and only at this point, that the *principle* of the anticipations of perception comes into play. This principle states (B207) that "in all appearances the real that is an object of sensation has an intensive magnitude, i.e., a degree," so that "every reality in the appearance, no matter how small, has a degree, i.e., an intensive magnitude, which can always be diminished, and between reality and negation a continuous series [*kontinuierlicher Zusammenhang*] of realities is possible" (A169/B211). The principle is instantiated in the Dynamics chapter by the second proposition, the proof of which concludes (499): "[T]he expansive force by which every matter fills its space has a degree, which is never the greatest or the smallest, but is such that beyond it both greater and smaller degrees can be found to infinity."

> necessarily follow; for the latter proves fleeing forces of its adjacent parts standing in inverse ratio to their distances, as Newton demonstrates (*Principia*, Book II, Prop. 23, Schol.). But we may also view the expansive force of air, not as the action of *originally* repelling forces, but as resting rather on *heat*, which compels the proper parts of air (to which, moreover, actual distances from one another are attributable) to flee one another, not merely as a matter penetrating it, but rather, to all appearances, through its vibrations. But that these tremors must impart a fleeing force to the adjacent parts, standing in inverse ratio to their distances, can doubtless be made conceivable in accordance with the laws of communication of motion through the oscillation of elastic matters. (522)[49]

For Kant, the elastic behavior of atmospheric air manifested in the Boyle–Mariotte law is therefore a *derivative* phenomenon. It rests, in the end, on the circumstance that the air-matter itself is dispersed or dissolved in a more fundamental elastic medium wherein the dispersed or dissolved air particles acquire a mutual tendency to flee one another from the "vibrations" or "oscillations" of this medium.

Kant is deliberately non-committal about the nature of this more fundamental elastic medium here – not wishing to entangle his metaphysical treatment of the concept of matter in general with more specific physical questions concerning this or that kind of matter. In particular, he deliberately leaves it open whether the originally elastic matter in which the matter of air is dispersed or dissolved is identical with the matter of heat.[50] Nevertheless, we know that Kant consistently held to a material theory of heat (typically supplemented, as it is here, with vibratory motions of the heat-matter) throughout his intellectual career, and we also know that heat-matter or caloric fluid plays a central role in Kant's more properly physical theorizing about matter. In his little treatise *On Fire* of 1755, for example, Kant invokes an originally elastic fluid both to explain the properties of ordinary fluids such as water and air and to explain the properties of elastic solids. Ordinary fluids such as water and air cannot consist simply of agglomerations of smooth solid particles, Kant argues, because such agglomerations will not, in general, exhibit the essential properties of a fluid – the property, in particular, that fluids always manifest a

[49] A further important difference between Kant's law of repulsion and Newton's (aside from the exponent of inverse proportionality) is that Kant's is a function of an *infinitesimal* or infinitely small distance rather than a finite distance. This is because Kant's model of an elastic fluid, unlike Newton's, is ultimately a continuous rather than a discrete model. Kant's differences with Newton on this central point will be further discussed below.

[50] Compare Kant's comments in the general remark to dynamics (530, my emphasis): "Thus air has a derivative elasticity in virtue of the matter of heat, which is most intimately united with it, and whose own elasticity is *perhaps* original."

horizontal surface under the action of gravity. Similarly, the cohesion of elastic solids, such as metal bands or wires stretched by a hanging weight, cannot simply be explained by the immediate contact of solid or rigid parts – since such contact is necessarily diminished by the expansion in question *without* diminishing the cohesion. In both cases, according to Kant, we therefore need to assume an originally fluid and elastic medium in which any and all solid parts are, as it were, immersed and through which the non-rigid properties in question are to be explained. This fluid is easily identified with the matter of heat (because of the well-known expansive effects of heat), and Kant then argues, in the remainder of the treatise, that this same elastic fluid is also to be identified with the matter of light – the matter, that is, whose vibratory motions constitute light.[51]

In the general remark to dynamics Kant sketches a parallel but more general physical theory of matter. On the one hand, there are a number of "subtle" elastic fluids (to use standard eighteenth-century terminology) that are less visible and more penetrating than ordinary matter.[52] The aether is the most subtle and pervasive elastic matter, which fills all of space with a minimum of density.[53] Moreover, it is most likely the pressure exerted by the aether on all other matters that is responsible for the phenomenon of cohesion.[54] Heat-matter or caloric fluid is responsible, as we have seen, for the heat-dependent elastic properties of atmospheric air and other ordinary elastic fluids. There is also a light-matter whose vibratory motions constitute visible light.[55] And, finally, there are magnetic and (presumably) electric subtle elastic fluids as well.[56] Whether any (or indeed

[51] Here Kant refers to Euler's (1746a) wave theory of light (1, 378), to which Kant consistently adhered throughout his career. I shall return to the importance of Euler's wave theory for Kant at a number of points below.

[52] This general conception of subtle fluids was a dominant theme in eighteenth-century natural philosophy: for an introductory account see Hankins (1985, chapter 3).

[53] Thus the aether (534) is "a matter … that completely fill[s] its space without any emptiness, and yet with an incomparably smaller quantity of matter, at the same volume, than any bodies we can subject to our experiments."

[54] See especially the general remark to phenomenology, where Kant suggests (563–64) that "the *attraction* assumed in order to explain the cohesion of matter [is] only apparent, not true attraction, and [is] merely the effect of a *compression* by external matter (the aether) distributed everywhere in the universe." I shall return to this point in section 17 below.

[55] See the long footnote to the first remark to the eighth proposition of the Dynamics (519–20), where Kant once again appeals to Euler's wave theory of light and argues that the light-matter in question must be an "original fluid" (520).

[56] See the fourth number of the general remark to dynamics (530–32), where Kant considers a kind of "chemical penetration" of matters by both "caloric fluid [*Wärmestoff*]" and "magnetic matter" (532). I shall return to the topic of "chemical penetration" below. An imperceptible "magnetic matter penetrating all bodies" figures in the discussion of actuality in the postulates of empirical thought in the first *Critique* (A226/B273).

all) of these subtle elastic fluids are identical with one another is here left entirely open.

On the other hand, however, the more visible forms of ordinary matter – physical material substances such as water, mercury, (various forms of) air, iron, and so on – also consist of originally fluid and elastic matters of differing specific gravities or densities. These originally elastic materials are all immeasurably denser than the aether and the subtle fluids more generally, and they assume various states of aggregation (solid, liquid, and gaseous) depending on how they interact with these subtle fluids – principally with the aether and the (possibly identical) matter of heat.[57] In all cases, therefore, matter is originally an elastic fluid medium, and it can only assume the less "dispersed" states of aggregation (liquid and solid respectively) as the result of complex physical and chemical processes that are not yet well understood.[58]

This general type of physical theory of matter, giving special priority to the state of permanently elastic fluidity, is, as we have seen, characteristic of Kant's earliest thoughts on the subject beginning in the pre-critical period. It is the heart of the physical theory of matter sketched in the little treatise *On Fire* published in 1755, and, in a somewhat different context, it is also central to the cosmogenetic conception of the evolution of the universe developed in the *Theory of the Heavens* published in the same year.

[57] In the continuation of the passage quoted in note 50 above Kant explains (530) that "the fundamental material of the fluid we call air must nonetheless, as matter in general, already have original elasticity in itself," and he thus makes it clear that the air-material that is dispersed or dissolved in the matter of heat is itself an originally fluid elastic medium. Here the gaseous state is maintained by a combination with the matter of heat, which can also, more generally, result in a transition from the liquid to the gaseous state (as in the production of water vapour or steam). The liquid state, by contrast, arises from a (probably external) compressive or cohesive force that counteracts the original expansive force characteristic of the gaseous state (see note 54 above) – where a liquid, for Kant, is characterized by cohesion with no internal friction permitting displacement without separation of parts. A solid or rigid matter, finally, is characterized, in addition, by precisely such internal friction, whereby the parts of the matter in question resist *both* displacement and separation. This state, too, may also involve the matter of heat, in a process of crystallization or rigidification beginning from the liquid state (as in the production of ice). For Kant's conception of the states of aggregation see the second number of the general remark to dynamics (526–29) – to which I shall return below.

[58] In sharp contrast to the "mechanical" conception of matter and the filling of space that he is most concerned to reject Kant takes the nature of solid or rigid matter to be the least well understood of all (529):

> But why certain matters, even though they may have no greater, and perhaps even a lesser force of cohesion than other matters that are fluid, nevertheless resist the displacement of their parts so strongly, and hence can be separated in no other way than by destroying the cohesion of all parts in a given surface at once, which then yields the semblance of a superior cohesion – how, that is, rigid bodies are possible – is still an unsolved problem, no matter how easily the common doctrine of nature presumes to have settled it.

In the latter work, in particular, the original simple chaos from which the organized structures in the heavens all develop is a maximally "dispersed" or "dissolved" condition of matter in the gaseous or elastic fluid state.[59] The metaphysical theory of matter Kant publishes one year later in the *Physical Monadology* appears intended to be smoothly connected, as it were, with the same physical conception. The concluding proposition of this work states (1, 486) that the elements or physical monads "possess a perfect elastic force … and constitute a medium that in itself and without admixture of a vacuum is primitively elastic." Moreover, this proposition is followed by an even more striking corollary, according to which, since the elements in question are elastic and therefore compressible, "this is the origin of primitively elastic bodies or media, among which may be announced, to begin with, the aether or the matter of fire" (487). It seems clear, therefore, that Kant intends the metaphysical theory of 1756 to fit together smoothly with the physical theory of 1755.[60]

In the Dynamics of the *Metaphysical Foundations*, however, Kant breaks decisively with the *Physical Monadology*. For, in his remark to the third proposition, Kant makes it clear that expansive or elastic force is exerted by *all* of the points in a space filled with matter. In particular, it can now be "postulated" that the expansive force of matter due to the fundamental force of repulsion "must counteract all the more strongly, the more it is driven into a smaller space" precisely because "expansive force, exerted from every point, and in every direction, actually *constitutes* this concept

[59] Here Kant originally characterizes the initial "complete chaos" as a "universal dissipation [*allgemeinen Zerstreung*]" (1, 225) and then as a "complete dissolution and dissipation [*gänzlichen Auflösung und Zerstreung*]" (227). Immediately preceding the passage quoted in note 46 above, Kant introduces the fundamental repulsive force, as one of those "which primarily manifest themselves when matter is dissolved into fine particles [*in feine Theilchen aufgelöset*], as [forces] by which they repel one another and through whose conflict with attraction they bring forth that motion which, as it were, is the enduring life of nature" (264–65). In particular, as matter begins to separate itself out from the original (gaseous) chaos towards centers of attraction containing specifically heavier types of matter, the repulsive force in question deflects the attracted matter away from its rectilinear path towards such a center and thereby imparts a counterbalancing centrifugal tendency.

[60] The eighth proposition of *On Fire* (1, 377) identifies the aether, the matter of heat, and the matter of light. The theory of the *Physical Monadology* is metaphysical rather than physical, because it is entirely motivated by the metaphysical problem of harmonizing the idea of an ultimate simple substance with the infinite divisibility of space – not by properly physical considerations aimed at explaining particular empirical phenomena. Accordingly, it is illuminating to view the *Physical Monadology* as a kind of bridge or transition between the more general metaphysical theory of substance, interaction, and world presented in the *New Exposition* of 1755 (see note 38 above) and the physical theory of matter presented in *On Fire*. In this way, the three early Latin treatises of 1755–56 present, at least in outline, a unified metaphysical-physical system of substance, body, and matter.

[of matter]" (501, emphasis added). In the following fourth proposition, where the infinite divisibility of matter is now officially proved, Kant takes up this conclusion (of the third proposition) as his premise:

> Matter is impenetrable, and, in fact, through its original expansive force (Prop. 3), but this is only the consequence of the repulsive forces of each point in a space filled with matter. Now the space filled by matter is mathematically divisible to infinity, that is, its parts can be distinguished to infinity, although they cannot be moved, and thus cannot be divided (in accordance with proofs of geometry). But in a space filled with matter, every part of it contains repulsive force, so as to counteract all the rest in all directions, and thus to repel them and to be repelled by them, that is, to be moved a distance from them. (503)

Kant now makes it clear, therefore, that the expansive force of matter by which it fills its space "through the repulsive forces of all of its parts" (499) is actually exerted by every part of the *space* in question. The attempt in the *Physical Monadology* to fill the space occupied by an element with a repulsive force having a sphere of activity exerted only by the central point of this sphere – leaving the interior of the sphere, according to Kant's present conception, in fact entirely empty of matter – is thus definitively rejected.[61]

Thus, the essential difference between the dynamical theory of matter as an originally elastic medium presented in the *Physical Monadology* and the at first sight very similar theory of matter presented in the *Metaphysical Foundations* is that the latter theory conceives the elastic medium in question as a true continuum – as a space in which every one of the continuum of geometrical points therein equally exerts the expansive force characteristic of matter's elastic filling of space. By contrast, the *Physical Monadology* does not represent matter as a true continuum at all but instead advocates an atomism of discrete force-centers. The elastic medium constituting the fundamental state of matter consists, in the end, of a finite number of small elastic corpuscles, each of

[61] This move is prefigured in the proof of the second proposition by Kant's transition from a claim about the parts of *matter* exerting repulsive force to the parts of the *space* thereby filled (499): "Therefore, matter fills its space only through repulsive forces, and, indeed, through repulsive forces of all of its parts; for otherwise a part of its space (contrary to the presupposition) would not be filled, but only enclosed." On the conception of the *Physical Monadology*, by contrast, the parts of the space it occupies are not actually parts of the monad or simple material element at all – which, by contrast, is absolutely indivisible. It is not entirely clear, in the proof of the fourth proposition quoted in the text, whether Kant intends to refer to the third proposition or the second. But it ultimately does not matter, for what is crucial to the argument is precisely the transition from parts of matter to parts of space that begins with the second proposition and culminates in the fourth.

which has a small but finite volume arising from the intersection of the fields of attractive and repulsive forces emanating from its center. Since the fundamental particles are elastic rather than hard or perfectly rigid, the medium in question may be compressed and thereby exert expansive forces that counteract such compression. Nevertheless, we are still far from the true continuum view of matter represented in the *Metaphysical Foundations*, according to which matter is originally elastic all the way down, as it were, so that "*every* part of it [namely, 'a space filled by matter' – MF] contains repulsive force, so as to counteract all the rest in all directions, and thus to repel them and to be repelled by them" (503, emphasis added).[62]

This view aligns Kant's theory of matter in the *Metaphysical Foundations* with the mathematical continuum models of fluid and elastic materials that were just being elaborated in the second half of the eighteenth century as part of the beginnings of the discipline we now call continuum mechanics. The simplest case of such a model arises in hydrostatics, where the equilibrium state of a fluid (whether liquid or permanently elastic) is characterized by the condition that every point of the continuum representing the fluid sustains equal pressures in all directions, acting normally on every contained surface element. Euler attempted to derive all of the known principles of hydrostatics from this condition in 1755 and argued, on this basis, that the defining condition in question articulates the essential difference between solidity and fluidity. In particular, whereas we may be able to approximate this defining condition by agglomerations of smaller and smaller solid particles, even an infinitely fine such agglomeration would be inherently unstable and would be easily and permanently displaced from hydrostatic equilibrium by the smallest unbalanced force. (To take an intuitive example, compare a sandy surface with a watery surface after disturbance by the wind.) Therefore, Euler concludes, "it is clear that fluidity cannot be explained by an agglomeration [*amas*] of solid particles, even if they are supposed

[62] In the *Physical Monadology* Kant suggests that the law of attractive force varies as the inverse square of the distance and the law of repulsive force varies as the inverse cube of the distance. The sphere on which the two forces precisely balance one another is then the limit of the monad's impenetrability and determines its definite volume. In the *Metaphysical Foundations*, by contrast, the repulsive force varies as the inverse cube of the *infinitesimal* or infinitely small distance (see note 49 above), and no actual (finite) volume is thereby determined. Therefore, *every* point in a (finite) volume filled with matter must (equally) exert repulsive force. As Kant is perfectly aware, however, this representation leads to serious mathematical difficulties, which I shall consider in section 19 below.

to be infinitely small, completely disconnected from one another, and their number infinitely great."[63]

In the general remark to dynamics Kant makes an analogous point:

> But what is completely decisive with respect to our concept of fluidity is this – that fluid matters can also be defined as those in which every point endeavors to move in all directions with precisely the same force with which it is pressed towards any one of them, a property on which the first law of hydrodynamics rests, although it can never be attributed to an agglomeration [*Anhäufung*] of smooth and solid [*festen*] corpuscles, as can be shown by a very easy resolution of its pressure in accordance with the laws of composite motion, thereby proving the original character of the property of fluidity. (528)[64]

Although not as sophisticated as Euler's argument, this passage does appear to echo Euler's rejection of solid corpuscles in 1755.[65] It also echoes Kant's own position of 1755 in *On Fire*, where he argues that an agglomeration (*cumulum*) of spherical hard particles will not necessarily exhibit a horizontal surface under the pressure of gravity (1, 371–72). However, whereas this essential property of the fluid state is a consequence of the more general property of the equality of pressures in every direction at any given point, the more general property has the advantage, from our present point of view, of making it clear that *every* part of the fluid medium

[63] See Euler (1755, §8). In his comments on this paper Truesdell (1954, p. lxxxi) calls it "a turning point in the history of physics," where, for the first time, "the continuum view of matter is put forward as a basic principle." In particular:

> Euler had come to realize that the error of his predecessors lay not in their adherence to mechanics … but in the wrong notion that mechanics is bound up with little physical particles … In §§5–8 is Euler's final and rather disgusted rejection of corpuscles. Henceforth the *principles of mechanics themselves* are to be applied directly to the bodies of physical experience, and "particle" is to mean only a mathematical point in a continuum model of matter.

[64] As a passage on the following page (considered in the next paragraph) makes clear, "hydrodynamics" is a misprint for "hydrostatics." To say that the property of fluidity is "original" is to say that the material in question is fluid all the way down, as it were, so that *every* part, no matter how small, is characterized by exactly the same fundamental property. Thus, in considering objections to Euler's wave theory of light (see notes 51 and 55 above), according to which it is incapable of accounting for *rectilinear* propagation, Kant replies (520):

> [T]his difficulty flows from an easily avoidable mathematical representation of light matter as an agglomeration of little spheres, which would certainly yield a lateral motion of light in accordance with their varying obliquity to the direction of impact; instead, however, there is no obstacle to thinking the matter in question as an original fluid, and, indeed, as fluid through and through, without being divided into solid [*feste*] corpuscles.

[65] Kant's point about the composition of motions appears simply to amount to the claim that a given finite agglomeration of solid particles (touching one another, therefore, at a finite number of points) will not exert pressures in *all* directions. In Euler's analogous argument, however, we also consider the process of allowing the sizes of the particles to shrink – and their number to increase – to infinity.

is subject to the same conditions. It thus leads to the representation, for both Kant and Euler, of the state of fluidity as a true continuum.[66]

Kant thereby acquires a decisive physical reason for rejecting the matter theory of the *Physical Monadology*. For the space inside the sphere of activity of a physical monad cannot, properly speaking, be characterized as fluid precisely because it does not possess the property Kant now finds definitive, in his proof of the fourth proposition of the Dynamics, of elastic matter in general. In the general remark to dynamics Kant explicitly relates what he has just called the "decisive" property of fluidity to his earlier characterization of elastic matter in general:

> The above-cited second definition of fluidity, on which the fundamental law of hydrostatics rests – namely, that it is that property of a matter whereby every part of it strives to move in all directions with precisely the same force by which it is pressed in any given direction – follows from the first definition, if one combines it with the principle of general dynamics that all matter is originally elastic. For this matter must then be striving to expand in all directions of the space in which it is compressed, with the same force by which the pressure occurs in any direction, whatever it may be, that is (if the parts of a matter can be displaced along one another by any force, without resistance, as is actually the case with fluids) it must be striving to move in all directions. (529)[67]

The moral, in the present context, is that the points within the space occupied by a physical monad are *not* independently movable (in all directions) under the action of an external pressure. On the contrary, only the central point of a repulsive sphere of activity is movable on its own, and the

[66] Although Euler's (1755) paper was certainly well known at the time, I have found no explicit reference to it in Kant's writings. The most relevant passage with which I am acquainted occurs in an appendix that Kant wrote to Samuel Sömmering's *Über des Organ der Seele* (1796), where he explains (12, 33) that a "*fluid* is a continuous matter such that every part within the space that it occupies can be moved from its place by the smallest force," which property is contrasted with the "rigid [*starre*]" matter of "machines." A note to this passage continues: "The *rigid* (*rigidum*) must be properly opposed to the *fluid* (*fluidum*), as Euler also does, the *solid* [*Soliden*] is opposed to the *hollow*" (12, 33). This reference, however, appears to be to Euler's *Mechanica* of 1736 (a copy of which Kant owned), where, in a General Scholium to the first chapter, "On Motion in General," Euler outlines his overall program for first applying the principles of mechanics to "infinitely small bodies, which can be considered as points," then to finite bodies that are rigid [*rigida*], and eventually to the motion of fluids [*fluidorum*]. See Euler (1736, §98), together with the translation and comments on this passage in Truesdell (1954, p. ix). In the 1755 paper Euler contrasts "fluid bodies [*corps fluides*]" with "solid bodies [*corps solides*]."

[67] The first definition states that a fluid allows (frictionless) displacement without separation of its parts (526–27; compare note 57 above). This definition corresponds to what we now call an inviscid fluid, which offers no resistance to shear stress, and it belongs, from a modern point of view, to hydrodynamics rather than hydrostatics. Indeed, in a state of hydrostatic equilibrium there are necessarily no shear stresses (arising from fluid flow) in any case, and the distinction between viscous and inviscid fluids is therefore irrelevant.

behavior of all other points of this sphere entirely depends on that of the central point. In particular, all other points of such a sphere behave as if they were rigidly connected to the central point, and this is quite incompatible with what Kant now takes to be the essential property of fluidity.

Hence, although the fundamental corpuscles of the *Physical Monadology* are elastic rather than hard, and are therefore, in a sense, "compressible," the "elastic medium" that they are supposed to constitute still fails, from Kant's present point of view, to possess the essential property of fluidity.[68] It is not the case that every point within this medium is subject to the same conditions, and it is not the case, in particular, that every such point is independently movable (in all directions). Thus, whereas Kant had indeed claimed in *On Fire* (1755) that fluidity cannot be represented by an agglomeration of hard corpuscles (and had argued, accordingly, for the necessity of a primitively elastic medium), it appears that he had simply not thought this question completely through in connection with the *Physical Monadology* (1756).[69] Now, against the background of a markedly more sophisticated understanding of the state of fluidity, Kant is finally in a position, in the Dynamics of the *Metaphysical Foundations*, to replace an atomism of discrete force-centers with a true continuum view of matter.[70]

[68] When a physical monad is "compressed" by the pressures exerted by external monads, the repulsive sphere of activity of the original monad is not actually changed at all. Rather, the change of "shape" experienced by a monad is entirely due to the *superposition* of its own fields of attraction and repulsion with those of the external monads. It is in this sense that each monad's fields are rigidly connected to it.

[69] The *Physical Monadology* ends with the discussion of elasticity (1, 486–87) quoted in the paragraph to which note 60 above is appended. Indeed it breaks off with the final sentence there quoted (1, 487), and the remaining parts of the larger projected work were never completed. (The *Physical Monadology* represents only the first part – "Specimen 1." – of a larger project entitled "The Use in Natural Philosophy of Metaphysics Combined with Geometry.")

[70] The crucial distinction, from a modern point of view, is between a system with a finite number of degrees of freedom and a system with an infinite number. Thus, although the fields of attractive and repulsive force of the *Physical Monadology* are indeed spread out over a continuum of spatial points, they are capable of no changes not rigidly determined by the motions of their (finite number of) point-centers. This is a consequence of the fact that they represent instantaneous action-at-a-distance forces. By contrast, the fields of matter represented in continuum mechanics can be continuously and non-instantaneously propagated from place to place over time. Typical modern fields of force, such as the electro-magnetic field, for example, also propagate continuously and non-instantaneously, and they, too, possess an infinite number of degrees of freedom. Indeed, the mathematics used to describe such fields arose directly out of continuum mechanics. It is therefore a mistake to view the force-center atomisms of Boscovich and the early Kant as progenitors of the field-theoretic conception of matter developed in the nineteenth century by such thinkers as Faraday and Maxwell. Adickes, for example, misses this point when he equates the "monadological-dynamical" conception of matter of Boscovich and the early Kant with the later (field-theoretic) view of matter developed by "Ampère, Faraday, etc." (1924, p. 196n. 1) – and, accordingly, expresses a very clear preference for Kant's 1756 theory of matter over that of 1786.

13 MATERIAL SUBSTANCE AS INFINITELY DIVISIBLE

Kant's official proof of the continuity or infinite divisibility of matter occurs in the fourth proposition of the Dynamics.[71] This proposition proceeds, not by invoking the issues considered in section 12 above, but rather by introducing an additional pure concept of the understanding into the discussion: namely, the pure concept of substance. Kant's explicit rejection of the earlier metaphysical matter theory of the *Physical Monadology* occurs in the first remark to this proposition, which proceeds (504) by "making the proof of the preceding proposition intuitive." Thus neither Kant's official defense of the continuity of matter nor official rejection of the earlier atomism of discrete force-centers represented in the *Physical Monadology* explicitly engages the considerations from physical or empirical matter theory with which we have just been occupied. They rather meet the *Physical Monadology* on its own metaphysical ground, as it were, by exploring the metaphysical concept of substance and its relation to the geometrical concept of space. They thereby show how Kant's new continuum version of a dynamical theory of matter is to be integrated with the new metaphysical context of the *Critique of Pure Reason*. In particular, the second remark to the fourth proposition then explicitly invokes the doctrine of transcendental idealism and the argument of the second antinomy.[72]

[71] In accordance with standard eighteenth-century usage, Kant simply equates continuity with infinite divisibility. See, for example, the discussion in the anticipations of perception (A169/B211):

> The property of magnitudes according to which no part in them is the smallest possible (no part is simple) is called their continuity [*Kontinuität*]. Space and time are *quanta continua* because none of their parts can be given without being included between limits (points and instants), and therefore only in such a way that each part, in turn, is itself a space or a time.

The modern distinction between continuity and infinite divisibility first became fully clear and explicit in Richard Dedekind's work of 1872.

[72] This is the first place, in fact, where the doctrine of transcendental idealism is officially introduced into the argument of the *Metaphysical Foundations*. In the two previous places where this doctrine is mentioned it is "here in no way in question" (481) and "a digression from our present business" (484). In note 60 above I suggested that the three Latin treatises of 1755–56 are illuminatingly viewed as presenting the outlines of a unified metaphysical-physical system of substance, body, and matter. It is similarly illuminating to view Kant, in the critical period, as revising and reconfiguring precisely this system – where the general or transcendental metaphysics of the first *Critique* now replaces the *New Exposition*, the main body of the *Metaphysical Foundations* now replaces the *Physical Monadology*, and the discussion of empirical matter theory in the general remark to dynamics now replace *On Fire*. Since Kant's critical version of physical or empirical matter theory is not radically changed from this earlier period (although, as suggested in section 12, it has become considerably more sophisticated), it is appropriate that he rather focusses primarily on properly metaphysical considerations here. Moreover, it is precisely these metaphysical

This introduction of the pure category of substance complements the earlier introduction of the pure category of causality in the proof of the first proposition. There, as argued in section 10 above, Kant moves from a purely kinematical representation of the possible effects of a repulsive force to the introduction of this force itself by adding the pure concept of causality: the concept of a repulsive force, at this point, is simply the concept of the ground or cause of the possible effects in question. But the concept of causality, in turn, necessarily leads to the concept of substance for Kant (A204/B249): "[C]ausality leads to the concept of action, and the latter leads to the concept of force and thereby to the concept of substance." In addition (A204/B250): "Where there is action, and thus activity and force, there is also substance, and only in the latter must the seat of the former fruitful source of appearances be sought." Thus, as soon as the concepts of causality and force have been introduced in the proof of the first proposition, we have also posed the question of the "seat" or subject of this force, activity, or causal efficacy – which, for Kant, can only be sought in a substance. The matter or body that manifests this activity, by resisting penetration from incoming movables, must therefore be subsumed under the pure concept of substance as well, and this is precisely what happens in the present proposition.

It is important to appreciate, however, that the concept of substance at issue is the pure rather than schematized category. A schema results, for any pure concept of the understanding, by bringing this concept into relation with the pure intuition of time, and the schema of the pure category of substance, in particular, is the representation of the *permanence* of the real in time.[73] In the fourth proposition of the Dynamics, however, neither temporality in general nor the more specific question of temporal permanence is explicitly discussed. Kant operates instead with the pure or unschematized concept of that which is the final or ultimate subject of predication (503): "The concept of a substance signifies the ultimate subject of existence, that is, that which does not itself belong, in turn, to the existence of another thing merely as predicate."[74] Similarly,

considerations, as we shall see, that ultimately underwrite Kant's explanation of the application of mathematics to the concept of quantity of matter in the Mechanics chapter.

[73] See the characterization in the schematism chapter (A144/B183): "The schema of substance is the permanence of the real in time, i.e., the representation of [the real] as a substratum of empirical time determination in general, which therefore remains while everything else changes."

[74] Compare the parallel characterization in the proof of the first analogy added in the second edition of the *Critique* (B225): "[T]he substratum of everything real – i.e., [everything] which belongs to the existence of things – is *substance*, in which everything belonging to existence can only be thought as determination." As Kant also explains in the second edition, a pure category arises from its corresponding logical form of judgement by thinking such a form as applied to an

the complementary concept of causality at work in the Dynamics is also the pure rather than schematized category. For the latter makes essential reference to temporal *succession*, and we are here operating with the otherwise temporally indeterminate concept of the ground of a given consequent.[75] This makes sense, in both cases, for the determination of existence in time is the topic of the analogies of experience, which are instantiated, in the *Metaphysical Foundations*, in the Mechanics rather than the Dynamics. Nevertheless, the explicit use of the corresponding unschematized categories here alerts us to the fact that the argumentation of the Dynamics is intended eventually to connect with – and even, in a sense, to anticipate – the later argumentation of the Mechanics.

Although the schema of the pure category of substance – and thus the relationship of this concept to the pure intuition of time – is not explicitly in question in the present proposition of the Dynamics, the relationship between the pure category of substance and the pure intuition of *space* is precisely what now has to be settled. Moreover, it is precisely here, from a metaphysical point of view, that Kant's present version of the dynamical theory of matter differs most dramatically from the earlier version represented in the *Physical Monadology*. In the earlier version the relationship between substances and space is conceived in a broadly Leibnizean fashion. Substances themselves, along with their essential, intrinsic, or inner determinations, are not strictly speaking in space at all. Space is rather constituted by the extrinsic or external relations between

intuition (in general) and as thereby *determined* in a certain way. In the case of the pure category of substance and its corresponding categorical form of judgement (B128–29):

> The function of the *categorical* judgement was that of the relation of subject to predicate, e.g., all bodies are divisible. However, with respect to the merely logical use of the understanding it remains undetermined which of the two concepts one wishes to give the function of subject and which that of predicate. For one can also say: something divisible is a body. If I subsume the concept of body under the category of substance, however, it is thereby determined that its empirical intuition must always be considered as subject in experience, never as mere predicate.

[75] See again the characterization in the schematism chapter (A144/B183): "The schema of cause and of the causality of a thing in general is the real such that, if it is arbitrarily posited, something else always follows. It therefore consists in the succession of the manifold, in so far as it is subject to a rule." Concerning the relationship between the pure (unschematized) category and the corresponding logical function of judgement, Kant never gives an explicit explanation for any of the other categories parallel to the explanation that he gives for the category of substance at (B128–29) – where Kant simply adds "and so for all other categories." We may conjecture, however, that, whereas the hypothetical form of judgement corresponding to the pure category of causality is simply the bare logical form, if A then B, the pure category itself adds the condition that if A is posited then B must be posited as well – and it thus adds the representation of necessary connection. The schema then results by adding the representation of temporal succession to such a conditionally necessary positing. But here is not the place to explore this particular issue further.

different substances, as, in Leibnizean terminology, a "well-founded phenomenon" of such external relations. It is for precisely this reason, in fact, that substances viewed intrinsically are necessarily unextended and therefore non-composite or simple. Kant's distinctive variant of this Leibnizean view in the *Physical Monadology* then explains the manner in which an intrinsically unextended simple substance nevertheless *fills* a space by appealing to a "sphere of activity" of repulsive force emanating from a central point. Such a sphere of activity is therefore the phenomenal expression of one species of external relations (phenomenally manifested as repulsive forces) that subsist between the intrinsically non-spatial elementary simple substances or monads.[76]

In the critical period Kant continues to hold (as observed in section 8 above) that this conception of the relationship between substances and space *would* be correct *if* we had only a faculty of pure understanding and there were no independent faculty of pure sensibility. In particular, Kant now argues in the amphiboly of pure reason that substance *as thought by the pure understanding alone* is indeed intrinsically non-spatial, characterized only by internal properties and determinations, and (consequently) necessarily simple.[77] From what Kant takes to be this Leibnizean perspective of the pure understanding alone, therefore, matter (substance) must be prior to the form (space) in which it is phenomenally expressed:

[76] See Proposition VII of the *Physical Monadology* (1, 481):

> For, since space is constituted solely by external relations, whatever is internal to substance, i.e., substance itself, is subject to external determinations but is not strictly speaking spatially defined – it is only those determinations referring to something external that can be properly sought in space. But, you say, substance is to be found in this small space [that is, the sphere of activity] and is everywhere present within it; therefore, if one divides [this] space, does one not divide substance? I answer: this space itself is the circumference [*ambitus*] of the external presence of its element. If one divides [this] space, therefore, one divides the extensive magnitude of its presence. But in addition to external presence, i.e., relational determinations of substance, there are also internal determinations, such that if these latter did not exist the former would have no subject in which to inhere. And the internal determinations are not in space precisely because they are internal. Therefore, they are not themselves divided by the division of the external determinations – nor is the subject itself or the substance divided in this way.

This passage should be compared with the discussion in the "Application" following Proposition XIII ("The Principle of Coexistence") of the *New Exposition* (1, 414–16). For further discussion of this pre-critical conception of the way in which space is related to the internal and external determinations of substances see the references cited in note 38 above – and compare also Langton (1998). But Langton is mistaken, in my view, in then attributing basically the same metaphysical view to the critical Kant as well.

[77] Kant introduces this view early in the amphiboly (A265/B321):

> Only that is internal in an object of pure understanding which has no relation at all (with respect to its existence) to anything different from itself. By contrast, the internal determinations of a *substantia phaenomenon* in space are nothing but relations, and it itself is nothing but a totality

Therefore, in the concept of the pure understanding matter precedes form, and *Leibniz* consequently first assumed things (monads), together with an inner power of representation, in order afterwards to ground their external relations and the community of their states (namely, their representations) on this. Therefore, space and time were [thereby] possible – the former only through the relation of the substances, the latter through the connection of their determinations among one another as ground and consequence. This in fact is how it would have to be if the pure understanding could be related immediately to objects, and if space and time were determinations of things in themselves. (A267/B323)

From Kant's present critical perspective, however, such an "intellectualist" understanding of the relationship between substances and space must now be exactly reversed:

Since, however, sensible intuition is an entirely special subjective condition, which lies a priori at the basis of all perception, and whose form is original, it follows that form alone is given for itself, and it is so far from being the case that matter (or the things themselves which appear) should lie at the basis (as one would have to judge in accordance with mere concepts) that the possibility of matter rather presupposes a formal intuition (time and space) as [already] given. (A268/B323–24)

Matter (substance) is now entirely dependent on form (space), and it is for precisely this reason that we are now committed to transcendental idealism (compare note 72 above, together with the paragraph to which it is appended).

In particular, when, in the present section of the Dynamics, Kant now subsumes the concept of matter under the pure concept of substance, he does so by making essential reference to space and, more specifically, to the concept of (independent) movability in space. The fifth explication characterizes *material substance* (502) as "a thing in space that is movable for itself, i.e., separated from everything else existing outside of it in

of mere relations. We are only acquainted with substance in space through forces that are active in space, either driving others into [this space] (attraction) or stopping their penetration into it (repulsion and impenetrability). We are acquainted with no other properties constituting the concept of the substance that appears in space and which we call matter.

He returns to this point later (A274/B330):

[T]he Leibnizean monadology has no other basis at all than [the circumstance] that this philosopher represented the distinction between the inner and the outer merely in relation to the understanding. Substances in general must have something *inner*, which is therefore free from all external relations and thus all composition. The simple is therefore the basis for the inner in things in themselves. But what is inner in its state cannot consist in place, figure, contact, or motion (which determinations are all external), and we can therefore ascribe no other inner state to the substances than that whereby we ourselves determine our own sense inwardly, namely, the *state of representation*.

space." After pointing out that "[t]he concept of a substance signifies the ultimate subject of existence" (503), the remark to this explication argues that the concept of matter designates precisely such an ultimate subject:

Now matter is the subject of everything that may be counted in space as belonging to the existence of things; for, aside from [matter], no other subject would be thinkable except space itself, which, however, is a concept that contains nothing existent at all, but merely the necessary conditions for the external relations of possible objects of the outer senses. Thus matter, as the movable in space, is the substance therein. (503)

It follows that any *part* of matter must also be substance in turn, so long as it is movable (and therefore separable) independently of all other parts of matter spatially external to it:

But all parts of matter must likewise be called substances, and thus themselves matter, in turn, in so far as one can say of them that they are themselves subjects, and not merely predicates of other matters. They are themselves subjects, however, if they are movable for themselves, and thus are something existent in space outside their connection with other neighboring parts. Therefore, the proper [*eigene*] movability belonging to matter, or any part of it, is at the same time a proof that this movable thing, and any movable part thereof, is substance. (503)

The pure or unschematized concept of substance, as the ultimate subject of predication, therefore leads, when applied to the concept of matter as the movable in space, to the idea that what is properly substantial in space is just matter and all of its independently movable parts.

I have now assembled all the elements for demonstrating the infinite divisibility of material substance that Kant puts together in his proof of the fourth proposition. All that remains, in fact, is to show that every part of a space filled by matter is movable in itself, independently of all other such parts, and this, as we have seen, follows from the third proposition, according to which *every point* in a space filled with matter exerts repulsive force (see the paragraph to which note 61 above is appended). Kant can now conclude (503) that "in a space filled with matter, every part of it contains repulsive force, so as to counteract all the rest in all directions, and thus to repel them and to be repelled by them, that is, to be moved a distance from them," and the infinite divisibility of matter follows straightforwardly:

Hence, every part of a space filled with matter is movable for itself, and thus separable from the rest as material substance through physical division. Therefore, the possible physical division of the substance that fills space extends as far as

> the mathematical divisibility of the space filled by matter. But this mathematical divisibility extends to infinity, and thus so does the physical [divisibility] as well; that is, all matter is divisible to infinity, and, in fact, into parts such that each is itself material substance in turn. (503–4)

The key move is from the infinite (mathematical) divisibility of space to the infinite (physical) divisibility of substance in space, and it is precisely this that is secured by Kant's present explication of material substance.[78]

However, from the perspective of the *Physical Monadology*, that is, from the perspective of an atomism of discrete force-centers, this argumentation is by no means convincing. For it entirely depends on Kant's previous progression of thought in the second and third propositions, where, we have seen, Kant in effect smuggles in the crucial premise in question by sliding from the repulsive force exerted by all the parts of *matter* to a repulsive force exerted by all the parts of the *space* thereby filled (see again note 61 above). But it is precisely this transition that the *Physical Monadology* emphatically rejects. It is by no means surprising, therefore, that the following remark explicitly addresses the contrasting conception of the *Physical Monadology* for the first time. At the beginning of this remark Kant simply reiterates that an atomism of discrete force-centers "is completely taken away from the monadist by the above proof" (504), so that "the hypothesis of a point that would fill a space through mere driving [*treibende*] force, and not by means of other equally repelling forces, is completely impossible" (504). Kant then goes on, however, to add something he apparently takes to be more telling – a constructive illustration intended "to make this and thereby also the proof of the preceding proposition intuitive" (504).

Kant presents the following argumentation and diagram:

[78] Kant emphasizes repeatedly that the infinite divisibility of space is not sufficient, by itself, to secure the infinite divisibility of matter, for the concept of substance, in particular, is not obviously subject to the same conditions as space. See the first remark to the present proposition (504): "The proof of the infinite divisibility of space has not yet come close to proving the infinite divisibility of matter, if it has not previously been shown that there is material substance in every part of space, that is, that parts movable for themselves are to be found there." See also the second remark (505):

> [I]t does not necessarily follow that matter is physically divisible to infinity, even if it is so from a mathematical point of view, even if every part of space is a space in turn, and thus always contains [more] parts external to one another; for so far it cannot be proved that in each of the possible parts of this *filled* space there is also *substance*, which therefore also exists in separation from all else as movable for itself.

Compare, in this context, the discussion of the important differences between substance and space in the remark on the thesis of the second antinomy (A438–41/B466–69).

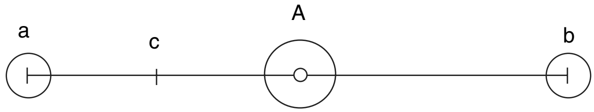

Figure 1

[L]et us assume that A is the place of a monad in space, and ab is the diameter of the sphere of its repulsive force, so that aA is the radius of this sphere:

Then between a, where the penetration of an external monad into the space occupied by this sphere is resisted, and the center A, it is possible to specify a point c (according to the infinite divisibility of space). If now A resists that which strives to penetrate into a, then c must also resist the two points A and a. For, if this were not so, they would approach one another without hindrance, and thus A and a would meet at the point c, that is, the space would be penetrated. Therefore, there must be something at c that resists the penetration of A and a, and thus repels the monad A, the same as it is also repelled by A. But since *repelling* is a [kind of] moving, c is something movable in space and thus matter, and the space between A and a could not be filled through the sphere of activity of a single monad, nor could the space between c and A, and so on to infinity. (504–5)

Yet, at least at first sight, this argumentation, too, appears to be entirely unconvincing. For it appears, once again, simply to beg the question against the monadist. Why, in particular, should the monadist admit that A and a would meet one another if they were not both repelled by c? Why have we not already conclusively ruled out such a meeting by postulating a repulsive force centered on A?

To answer these questions we need to think more carefully and explicitly about what precisely it means for a repulsive force to be exerted by, or centered on, a given spatial point. And here, it turns out, we need to return again to the issues about the relativity of space and motion that occupied our attention in my chapter on the Phoronomy. Indeed, as we have seen, these issues were already implicitly present in the first proposition of the Dynamics, for this proposition depends on the single proposition of the Phoronomy, which itself depends on Kant's principle of the relativity of motion. The concept of repulsive force is introduced by representing an instantaneous motion (a "striving to penetrate") directed towards a central point, and this centrally directed incoming motion is now to be cancelled or counteracted by an equal and opposite contrary motion directed away from the central point. The action of the repulsive force in question, as the cause or ground of the given effect, is represented by precisely this outgoing contrary motion. But what is the relative space or reference frame, we now need to ask ourselves, with respect to which these incoming and outgoing motions are supposed to be defined? For, according to Kant's principle of the relativity of motion, *any* given relative space or reference frame can be taken arbitrarily (at least so far as the Phoronomy is concerned) to be either in (rectilinear) motion or at rest (see section 5 above).

In the construction we are now considering we begin by taking the point A to be at rest and as resisting an instantaneous striving to penetrate at the point a. In this relative space or reference frame there is then an outgoing instantaneous resistance to penetration, directed away from the point A, which cancels (by the addition or composition of motions) the initial incoming motion. Suppose, however, that we view this same situation from the perspective of a relative space or reference frame in which point c is at rest. In this frame of reference we begin with two initial instantaneous incoming motions directed towards c, one at a and one at A, which now appear as cancelled or counteracted by two corresponding contrary motions directed away from c, one towards a and one towards A. (We have divided the initial incoming motion aA into two parts, ac and Ac respectively, and similarly for the corresponding outgoing motions.) Hence, according to Kant's principle of the relativity of motion, there is just as much reason to say that c is resisting a and A – and is thereby exerting a repulsive force directed towards these two points – as there is to say that A is resisting a and is thereby exerting a repulsive force directed towards a. According to all that we know about the relativity of space and motion so far, therefore, as soon as one description of the situation is posited the other must be posited as well, and, as a result, the concept of repulsive force is subject to precisely the same relativity.

Nevertheless, whereas thoroughgoing relativity of (rectilinear) motion may certainly hold sway in the Phoronomy, where we entirely abstract from all forces and causal relations, and we consider the "movables" in question as mere mathematical points, it cannot apply in the same sense when we now bring both forces and causal relations into the picture. Indeed, as explained in section 5, Kant makes this point explicitly in the discussion of his principle of the relativity of motion in the Phoronomy:

> [I]n phoronomy, where I consider the motion of a body only in relation to the space (which has no influence at all on the rest or motion of the body), it is entirely undetermined and arbitrary how much velocity, if any, I wish to ascribe to the one or to the other. Later, in mechanics, where a moving body is to be considered in active relation [*wirksamer Beziehung*] to other bodies in the space

of its motion, this will no longer be so completely the same, as will be shown in the proper place. (488)[79]

The "proper place" in question turns out to be the fourth proposition of the Mechanics, where Kant derives the equality of action and reaction as his Third Law of Mechanics. For, in a long footnote to the proof of this proposition, Kant again asserts that, whereas thoroughgoing relativity of (rectilinear) motion did hold sway in the Phoronomy, where matter "could be considered as a mere moving point" (547), it cannot continue to do so here:

But in Mechanics, where a body is considered in motion relative to another, with respect to which, through its motion, it has a *causal relation* (namely, that of moving the body itself), in that it enters into community with [the body] either in its approach through the force of impenetrability or in its withdrawal through that of attraction, it is no longer indifferent whether I wish to ascribe a motion to one of these bodies, or an opposite motion to the space. For another concept of the quantity of motion now comes into play, namely, not that which is thought merely with respect to space, and consists only in the velocity, but rather that whereby the quantity of substance (as moving cause) must be brought into the calculation at the same time; and here it is no longer arbitrary, but rather *necessary*, to assume each of the two bodies as moved, and, indeed, with equal quantity of motion in the opposite direction. (547)

In particular, when two moving bodies are no longer considered as mere mathematical points, but rather as real physical substances standing in causal relations with one another (whether through attractive or repulsive forces), then the proper relative space or reference frame for considering such causal relations is that determined by the center of mass of the two bodies in question – wherein their respective quantities of motion (given by mass times velocity) are always equal and opposite.

Now this piece of Kant's solution to the problem of the relativity of space and motion belongs to the Mechanics, and I shall only be in a position to articulate it fully when I discuss that chapter in detail. Nevertheless, as I have suggested, Kant is still able in the Dynamics to anticipate elements of that solution when this is particularly relevant. Thus, in the present proposition, Kant has already introduced the (unschematized) categories of substance and causality. Moreover, he has also, at least implicitly, introduced the third relational category as well – the category of community or interaction [*Wechselwirkung*]. For the proof of our present

[79] For the entire passage (Kant's qualification of his relativity principle in the Phoronomy) see the paragraph to which note 65 of my chapter on the Phoronomy is appended.

proposition asserts (503) that "in a space filled with matter, every part of it contains repulsive force, so as to counteract [*entgegen zu wirken*] all the rest in all directions, and thus to repel them and to be repelled by them." The first remark to this proof then claims (504) that "there can be no point in a filled space that does not exert repulsion in all directions, and is itself repelled, and thus would be movable in itself, as a reacting [*gegenwirkendes*] subject external to every other repelling point." Thus, since the proof of the equality of action and reaction in the Mechanics starts from the (transcendental) principle of "general metaphysics" that "all external action in the world is *interaction* [*Wechselwirkung*]," in order to add the further point that "this interaction (*actio mutua*) is at the same time *reaction* [*Gegenwirkung*] (*reactio*)" (544–45), the present discussion in the Dynamics serves to anticipate both the third analogy and the corresponding proposition of the Mechanics.[80]

Let us apply these considerations to the construction that Kant presents in the first remark to the fourth proposition. Although the central point A is initially taken to be at rest, the proper frame of reference for considering this particular exercise of repulsive force is actually centered on a point *between* A and a (and thus at the point c, for example) – wherein both A and the incoming external monad are now viewed as moving (striving to move) towards this intermediate point. Exactly which intermediate point yields the proper frame of reference, however, depends on the masses or quantities of matter of the two monads; for the frame of reference in question is just that in which the respective quantities of motion are equal and opposite.[81] At this stage, however, we are not yet in a position to say anything further, because we are not yet in a position to estimate or quantify these two masses – to treat them, that is, as mathematical magnitudes. Indeed, the concept of mass or quantity of matter is first officially introduced in the second explication and first proposition

[80] It is striking, in addition, that the construction or figure Kant presents in the remark following the fourth proposition of the Dynamics is very close to the analogous construction Kant presents in his proof of the equality of action and reaction in the Mechanics (546). The crucial difference between the two, as explained below, is that Kant is here not yet in a position to estimate the respective quantities of matter or masses of the interacting bodies.

[81] According to Propositions X and XI of the *Physical Monadology*, whereas all physical monads have the same volume, their masses (inertial forces) can vary arbitrarily. Hence, *any* point on the line Aa is a possible candidate for representing the relevant center of mass. Indeed, Kant had already arrived at the idea that the center of mass of an interaction due to repulsive forces determines the proper frame of reference for considering this interaction in the *New System of Motion and Rest* of 1758 (compare note 4 above), which presents essentially the same argument as the fourth proposition of the Mechanics (2, 23–25). Thus it was already clear in 1758 that the central point of a resisting monad cannot in general be at rest.

of the Mechanics, where, in the remark to this proposition, Kant declares (540) that "the quantity of matter is the *quantity of substance* in the movable." Therefore, the full articulation of the concept of material substance begun in the fifth explication of the Dynamics and continued in the following fourth proposition can be completed only in this section of the Mechanics, where the concept of material substance is first exhibited as a magnitude. In particular, according to the same remark to the first proposition of the Mechanics, the quantity of matter of a material substance filling a given space can only be given as the total (infinite and continuous) "aggregate of the movable" in this space (539–40) – in explicit contrast, once again, with the theory of physical monads.[82] So the final refutation of the *Physical Monadology*, from Kant's present perspective, can also be completed only here.[83]

14 MATTER, MOTION, AND THE MATHEMATICAL ANTINOMIES

The second remark to the fourth proposition occupies a pivotal position in the Dynamics, and, by implication, in the *Metaphysical Foundations* as a whole. After demonstrating the infinite divisibility of material substance in the fourth proposition, Kant now officially introduces the

[82] As we shall see, although Kant *defines* the concept of quantity of matter in the first explication of the Mechanics as the "aggregate of the movable" in a given space, he asserts that this quantity can be (mathematically) *estimated* only by "the quantity of motion at a given speed" (537). Thus, the full articulation of the concept of mass or quantity of matter as a mathematical magnitude depends on that of the concept of quantity of motion (momentum), and Kant is preparing the way for this situation here in the Dynamics by interweaving the corresponding metaphysical concepts of substance, action, and interaction. I shall return to this issue in more detail in section 16 below.

[83] As we shall see in more detail as we proceed, Kant considers both the (repulsive and attractive) interactions *between* (different) bodies or matters and those *within* a (single) body or matter – where the latter viewpoint is dominant in the Dynamics and the former in the Mechanics (compare note 20 above). Moreover, both of these viewpoints are also present in the first *Critique*. For example, when Kant in the second edition of the *Critique* considers the relationship between the disjunctive logical form of judgement and the corresponding category of community, he illustrates this relationship by the interactions *within* a given body (B112), as a "connection in a *whole of things*, where one is not *subordinated* to the other as effect [*Wirkung*] of a cause, but is rather *coordinated* with it simultaneously and mutually as cause with respect to its determinations (e.g., in a body whose parts mutually attract and also resist one another)." In the amphiboly chapter, by contrast, Kant claims that "the inner determinations of a *substantia phaenomenon* in space are nothing but relations" and then states that "[w]e are only acquainted with substance in space through forces that are active in space, either driving others into [this space] (attraction) or stopping their penetration into it (repulsion and impenetrability)" (A265/B321; see note 77 above). Here, while illustrating the transcendental amphiboly for the case of the relational categories, Kant refers to both the interactions within a substance and those between substances.

central critical doctrine of transcendental idealism into the special metaphysics of corporeal nature for the first time (see note 72 above). He does so in the second remark by appealing to the argument of the second antinomy, according to which material substance belongs wholly to the realm of appearance (as *substantia phaenomenon*) and is therefore infinitely divisible without consisting of a (completed) infinity of parts (507–8): "[T]he *composite in the appearance* does not consist of the simple, because in the appearance, which can never be given otherwise than as composed (extended), the parts can only be given through division, and thus not prior to the composite, but only in it."[84] Moreover, the second remark separates the main text of the Dynamics into two symmetrical parts, where the first consists of four propositions devoted to the fundamental force of repulsion and the second of four propositions devoted to the fundamental force of attraction. Indeed, the proof of the fifth proposition, where the fundamental force of attraction is first officially introduced, essentially depends (as I explain in section 16 below) on the infinite divisibility of material substance already demonstrated in the fourth. In thus marking the transition between the two main parts of the Dynamics the second remark thereby illuminates both the relationship between the two fundamental forces in the argument of the *Metaphysical Foundations* as a whole and their connection with the doctrine of transcendental idealism.

Finally, and perhaps most importantly, it is in precisely this second remark that the name of Leibniz finally enters into Kant's argument (507–8); indeed, this is the only place in the *Metaphysical Foundations* where Leibniz is explicitly discussed. Kant begins by describing Leibniz as "a great man, who has contributed perhaps more than anyone else to

[84] Compare note 3 above, together with the paragraph to which it is appended and the following paragraph. The second remark recapitulates the solution to the second antinomy by asserting (506) that "*space is not a property of a thing in itself*, and thus matter is not a thing in itself, but merely an appearance of our outer senses in general, just as space is the essential form thereof." It follows, Kant concludes (506–7):

> [The philosopher] is thereby helped out of that difficulty due to the *infinite divisibility* of matter, whereby it still does *not* consist of *infinitely many parts*. Now this latter can perfectly well be thought through reason, even though it cannot be made intuitive and constructed. For what is only actual by being given in the representation also has no *more* given of it than what is met with in the representation – no more, that is, than the progress of representations reaches. Therefore, one can only say of appearances, whose division proceeds to infinity, that there are just so many parts in the appearance as we may provide, that is, so far as we may divide. For the parts, as belonging to the existence of appearances, exist only in thought, namely, in the division itself. Now the division does of course proceed to infinity, but is still never given as infinite: thus it does not follow, from the fact that its division proceeds to infinity, that the divisible contains an infinite aggregate of parts *in itself*, and outside of our representation.

preserving the reputation of mathematics in Germany" (507), and he then proceeds to enlist Leibniz against the "misunderstanding" of his Leibnizean–Wolffian followers – whereby the "metaphysician" or "philosopher" is forced to reject the geometrical proposition of the infinite divisibility of space. At the end of the remark, most surprisingly, Kant attributes his own doctrine of transcendental idealism – "that space, together with the matter of which it is the form, does not contain the world of things in themselves, but only their appearance, and is itself only the form of our outer sensible intuition" (508) – to Leibniz himself. Given that the next (fifth) proposition introduces a fundamental force of attraction that is explicitly modeled on Newtonian universal gravitation, the prominence of Leibniz in the second remark to the present (fourth) proposition is especially significant. It not only casts light on how Kant views the relationship between Leibniz and his Leibnizean–Wolffian followers, it also illuminates the precise way in which the critical version of the dynamical theory of matter, in Kant's eyes, achieves a revolutionary new synthesis of the Leibnizean and Newtonian natural philosophies.

I have argued that Kant's critical version of the dynamical theory of matter is intimately intertwined with his Copernican conception of space and motion first sketched out in the Phoronomy. I have explained, in particular, how the first proposition of the Dynamics, where the fundamental force of repulsion is initially introduced, depends on the single proposition of the Phoronomy – which itself depends, in turn, on Kant's (phoronomical) version of the relativity of motion. I have also argued, at the end of the last section, that the first remark to the fourth proposition of the Dynamics, where Kant's earlier theory of physical monads is officially rejected, reopens the question of the relativity of motion while also pointing ahead towards a further piece of Kant's solution that will only be explicitly presented in the Mechanics. It is reasonable to expect, therefore, that Kant's Copernican conception of space and motion will also play a role here, where we are concerned with both the transition from repulsive to attractive force and the central critical doctrine of transcendental idealism.

The crucial point, in this connection, is that Kant's Copernican conception of space and motion is intimately intertwined with the argument of the *first* antinomy. For (as first explained in section 1 above) this conception is organized around a specific sequence of empirical spaces, relative to which we successively embed our consideration of space and motion within an ever more expansive and comprehensive perspective on the material universe. We begin with a relative space determined by the position of our own body, which may be in motion relative to the earth (for example on a ship); we next take account of the motions of the earth (rotational and orbital) within the solar system; we then take account of the motion of both the sun and the solar system within the wider Milky Way galaxy consisting of a very large number of such suns and solar systems; and so on *ad infinitum*.[85] As we successively expand our perspective in accordance with Kant's Copernican conception, we thereby consider wider and wider concentric regions of space beginning from a comparatively limited region (such as the cabin of a ship). But this is precisely the kind of sequence of ever more comprehensive spaces that is central to Kant's consideration of the existence of a limit to the material universe in space in the first antinomy.[86]

Both Kant's discussion of the relativity of space and motion in the *Metaphysical Foundations* and his discussion in the first antinomy contain clear indications of this situation. In a passage in the remark to his principle of the relativity of motion in the Phoronomy, for example, Kant writes (488): "We are also incapable, in any experience at all, of assigning a fixed point in relation to which it would be determined what motion and rest should be absolutely; for everything given to us in this way is material, and thus movable, and (since we are acquainted with no outermost limit of possible experience in space) is perhaps also actually moved,

[85] See, for example, notes 6 and 7 of my chapter on the Phoronomy, together with the paragraph to which they are appended. Once again (as I pointed out in the preceding paragraph of that chapter), it should be emphasized that this conception of the relativity of space and motion actually dates from 1755–58.

[86] In the discussion of the first antinomy Kant explains his solution this way (A517/B545):

> Here, just as much as in the other cosmological questions, the ground of the regulative principle of reason is the proposition that in the empirical regress *no experience of an absolute limit*, and thus no condition which, as such, is *empirically absolutely unconditioned*, can be met with. And the reason is that such an experience would have to contain a limitation of the appearances by means of nothing, or the empty [here empty space – MF], which the continued regress could encounter by means of perception – which is impossible.

Kant concludes (A521–22/B549–50):

> [T]he regress in the series of cosmic appearances [*Weltercheinungen*], as a determination of the cosmic magnitude [*Weltgröße*], proceeds *in indefinitum*, which means precisely this: the sensible world has no absolute magnitude, but the empirical regress … has its rule, namely, to progress from one term of the series, as a conditioned, always to a still more distant term … and never to be exempt from the expansion of the possible use of its understanding, which is then also the proper and unique occupation of reason with respect to its principles.

For Kant, to say that the regress proceeds *in indefinitum* (rather than *in infinitum*) is to say that there is no guarantee, for example, that material bodies beyond any given limited space will actually be found: there is only the purely regulative demand that we continually *seek* for such bodies.

without our being able to perceive this motion." The parenthetical rejection of an (experienceable) "outermost limit" echoes the heart of Kant's solution to the first antinomy (note 86 above). In this solution, in turn, we find a parallel echo of the Copernican conception of space and motion (A522/B550): "A determinate empirical regress that would progress into a certain type of appearances without ceasing is not hereby prescribed, e.g., that one must ascend … in the series of heavenly bodies [*Weltkörper*] without allowing an outermost sun; rather, only the progress from appearances to appearances [in general] is demanded." Whereas there is indeed no guarantee that the specific sequence of heavenly bodies represented in Kant's Copernican conception can in fact be always continued, and the existence of this specific sequence is certainly not implied by the solution to the first antinomy, it is clear, nonetheless, that there is an intimate connection between the two.

More precisely, the sequence of heavenly bodies (and thus relative spaces) represented in Kant's Copernican conception is a specific instantiation or realization of the more general conception of the empirical regress to wider and wider regions of physical space considered in the first antinomy. The former results from the latter by substituting the specific concept of matter as the movable in space into the more general conception of matter as simply the real in space represented in the first *Critique* – which, as explained, is a *pure* concept.[87] But since the Copernican conception of motion central to the *Metaphysical Foundations* is itself empirical, this concept of motion, as such, cannot figure centrally in the first antinomy. Here Kant touches on the relativity of space and motion only in passing, in an important footnote added to the proof of the antithesis:

> Space is merely the form of outer intuition (formal intuition), but it is not an actual object that can be externally intuited. Space, prior to all things that determine it (fill or limit it) – or, rather, that yield an *empirical intuition* in accordance with its form – is, under the name of absolute space, nothing but the mere possibility of outer appearances, in so far as they either exist in themselves or can still be added to given appearances. Empirical intuition is therefore not composed [*zusammengesetzt*] out of appearances and space (perception and empty intuition). One is not the correlate of the other in the synthesis; rather, they are only combined [*verbunden*] in one and the same empirical intuition, as matter and form of intuition. If one attempts to posit one of these two elements outside of the other (space outside of all appearances), then all kinds of empty

[87] The contrast between the empirical concept of matter and the pure concept of the real in space is discussed in section 9: see, in particular, the two paragraphs following the one to which note 6 above is appended.

determinations of outer intuition arise, which are nevertheless not possible perceptions. For example, motion or rest of the world in infinite empty space, a determination of the relation of the two that can never be observed, is, as a predicate, a mere entity of thought [*Gedankendinges*]. (A429/B457)[88]

Thus, although these remarks constitute merely an application or empirical instantiation of the argument of the first antinomy, they still make it clear that what Kant understands as the Newtonian conception of absolute space and motion is quite incompatible with this argument.[89]

Just as Kant's Copernican conception of space and motion is a specific empirical instantiation of the more general conception of an empirical regress to wider and wider regions of physical space presented in the first antinomy, his conception of the infinite divisibility of material substance in the Dynamics is a specific empirical instantiation of the more general conception of the empirical regress into smaller and smaller parts of matter presented in the second antinomy.[90] Just as the Copernican conception operates with the empirical concept of matter as the movable in space (and not merely with the pure concept of the real in space), the infinite divisibility of material substance demonstrated in the fourth proposition operates with the empirical concept of matter as the source or subject of repulsive force. The second antinomy, by contrast, operates with the more general concept of matter as merely the real in space, and neither repulsive nor attractive force is explicitly at issue.[91] It is for this reason, above

[88] In the amphiboly (A290–92/B347–49) Kant characterizes a "thought-entity [*Gedankending*]" as "the object of a concept to which no specifiable intuition at all corresponds," or, more precisely, as an "empty concept without an object (*ens rationis*)."

[89] Kant explicitly rejects the possibility of an absolute "rectilinear motion of the *cosmic whole* [*Weltganze*], i.e., the system of all matter" in the general remark to phenomenology (562–63). Earlier in this chapter, in the remark to the single explication, Kant explains (554–55) that there are always two "correlates" involved in a determinate predication of motion – the body to which motion is to be ascribed and the empirical space relative to which this motion is defined. In the supposed case of the (rectilinear) motion of the entire world the second "correlate" would be absolute (empty) space. I shall return to this question in more detail in my chapter on the Phenomenology.

[90] Kant first states the solution to the second antinomy for a "whole given in intuition" (A523–24/B551–52) and then applies this solution to "an outer appearance comprised within its limits (body)" (A525/B553): "The divisibility of [such a whole] is based on the divisibility of space, which constitutes the possibility of the body as an extended whole. The latter is therefore infinitely divisible without consisting of infinitely many parts." Kant further explains that in this case there is genuine divisibility *in infinitum* (and not simply *in indefinitum*), for the parts to which we must regress are already given as contained within the same limits.

[91] The heart of the second antinomy is the claim (A435/B463, bold emphasis added) that "everything ***real*** that occupies a space comprises a *manifold* [of elements] external to one another and is therefore composite [*zusammengesetzt*]." Recall from section 9 that, when Kant introduces the concept of the real in space in the anticipations of perception, he specifically distinguishes it from "impenetrability or weight, for these are empirical concepts" (A173/B215).

all, that the pre-critical theory of the *Physical Monadology* is also not considered in the second antinomy.

It is clear, nonetheless, that the main target of the second antinomy is precisely the same as that of the *Physical Monadology* – a monadological conception of the simplicity of (material) substance on which the infinite divisibility of space would then be called into question:

> Against this proposition of the infinite division of matter, the ground of proof of which is purely mathematical, the *monadists* have brought forward objections – which, however, already make them objects of suspicion, in that they are not willing to grant that the clearest mathematical proofs are insights into the constitution of space, in so far as it is in fact the formal condition of the possibility of all matter, and they rather view [these proofs] as only inferences from abstract yet arbitrary concepts, which cannot be applied to real things. (A439/B467)

Kant, in the *Physical Monadology*, had already considered and rejected such a view by rejecting, in particular, the use of a distinction between mathematical or geometrical space and the real or physical space occupied by material bodies in order to raise doubts concerning the infinite divisibility of the latter.[92] In this case, as we know, Kant's strategy was to accept both the simplicity of material substance and the infinite divisibility of space – and then to harmonize the two propositions by the idea of a (finite) sphere of activity of repulsive force emanating from a central point. In the second antinomy, by contrast, Kant simply ignores his pre-critical version of a physical monadology and instead opposes the view in question on more general grounds.

It is important to appreciate, however, that the view of the "monadists" targeted in the second antinomy is not the monadology of Leibniz. In his remark to the antithesis, from which I just quoted above, Kant makes it clear that the monadists he opposes conceive their simple substances as "physical points" out of which the space that they fill is supposed to be

[92] Kant demonstrates Proposition III (that the space filled by bodies is infinitely divisible and thus does not consist of primitive simple parts) by adapting a well-known proof of such infinite divisibility to "physical lines … composed of the fundamental parts of matter" (1, 478), and he then concludes in the Scholium (1, 478–79): "I have adduced this demonstration, which has already been employed by many physicists, and I have adapted it, as clearly as I could, to physical space, so that those who employ a general distinction, based on the difference between geometrical and natural space, should not escape the force of my argument by means of an exception." The demonstration in question could be found in standard physical textbooks by both the Cartesian Jacques Rohault and the Newtonian John Keill: see the editor's explanatory note to Kant (1992, p. 422).

composed (as a "mere aggregation" of such points).[93] But Kant is equally clear in his remark to the thesis that properly *Leibnizean* monads are entirely different:

> I speak here only of the simple, in so far as it is necessarily given in the composite, in that the latter can be resolved into them as its constituents. The proper meaning of the word *Monas* (according to Leibnizean usage) should only extend to that simple which is *immediately* given as simple substance (e.g., in self-consciousness), and not as element of the composite – which one could better call the atom. And, since I want only to prove [the existence of] simple substances in relation to the composite, as its elements, I could call the thesis of the second antinomy transcendental *atomism*. However, because this word has already long been used for the designation of a particular mode of explaining corporeal appearances (*molecularum*), and therefore presupposes empirical concepts, [the thesis] may be called the dialectical principle of *monadology*. (A440–42/B468–70)

Kant is clear, therefore, that properly Leibnizean monads are not to be conceived as "physical points" out of which bodies are supposed to be composed. They are rather mind-like – *and therefore entirely non-spatial* – simple beings, which are given (at least to themselves) in immediate self-consciousness. To be sure, both space and physical bodies in space are in some sense derivative from these beings as "well-founded phenomena." In no sense, however, are they *composed* of such beings.[94]

Hence, the "monadists" whom Kant opposes in the antithesis of the second analogy do not include Leibniz himself but only his eighteenth-century followers under the rubric of the Leibnizean–Wolffian philosophy. For some of the most important of these followers – including Christian Wolff, Georg Bernhard Bilfinger, and Alexander Gottlieb Baumgarten – did indeed conceive the ultimate simple elements of material bodies as "physical points" out of which the spaces occupied by such

[93] See the continuation of the passage quoted above (A439/B467): "If one listens to [these monadists], then one would have to think, aside from the mathematical point, which is not a part but merely the limit of a space, also physical points, which are indeed also simple, but have the advantage, as parts of the space, of filling this [space] through their mere aggregation."

[94] I touched on the Leibnizean notion of "well-founded phenomena" in note 17 of the Introduction. I implied in section 13 that the relationship Kant envisions between space and monads in the *Physical Monadology* is more like the original Leibnizean conception than it is like the "monadology" Kant is describing in the second antinomy: see note 76 above, together with the paragraph to which it is appended. I also observed that Kant explicitly conceives Leibnizean monads as entirely non-spatial in the amphiboly: see note 77 above, together with the paragraph to which it is appended.

bodies are supposed to be composed.[95] Here, however, they ran squarely into the problem of the composition of the continuum and, more specifically, into Zeno's metrical paradox of extension. According to this paradox, in particular, one can never attain an extended region of space by composing any number of unextended simple elements (points), not even an infinite number of such elements. The only way out, therefore, would be to take the elements in question as merely very small extended regions (rather than unextended points) and, as a consequence, to deny the infinite divisibility of (physical) space. For one would otherwise run into the second horn of Zeno's metrical paradox, according to which an *infinite* number of extended (finite) elements could never compose a *finite* extended region (a body).[96] It was precisely this problem that Kant was attempting to resolve in the *Physical Monadology* – and which he is here addressing in the Dynamics by appealing, instead, to the second antinomy.[97]

[95] See the discussion of this issue in De Risi (2007, pp. 301–14). As De Risi points out, it was Wolff's disciple Bilfinger who first used the phrase "Leibnizean–Wolffian philosophy," and De Risi's own main objective is to dispute the picture presented by these thinkers of *Leibniz's* conception of the relationship between monads and space. In the course of his argument De Risi cites some of the same passages from the second antinomy and the Dynamics of the *Metaphysical Foundations* with which I am presently concerned; I am indebted to him, more generally, for helping me to acquire a better understanding of the crucial differences between Leibniz and his Leibnizean–Wolffian followers on this issue.

[96] Zeno's metrical paradox played an important role in eighteenth-century matter theory more generally. For an illuminating treatment, including discussions of Kant and the Leibnizean–Wolffians, see Holden (2004).

[97] Kant's reliance on Zeno's metrical paradox is clear in Proposition IV of the *Physical Monadology* (that a composite divisible to infinity does not consists of primitive simple parts) and, especially, in the Scholium to this Proposition (1, 479): "[I]f a composite were to admit of infinite division, it would follow that all the fundamental parts whatever of a body would be so constituted that, whether they were combined with a thousand, or ten thousand, or millions of millions – in a word, no matter how many – they would not constitute particles of matter." Kant concludes in the Corollary (1, 479): "*All bodies therefore consists of a determine* [i.e., finite] *number of simple elements*." Here Kant emphasizes the first horn of the paradox and concludes, in accordance with the second, that there must be a *finite* number of (finite) simple elements. (Kant then proceeds, in the next proposition, to show that this does not entail that *space* is only finitely divisible, however, since the monad fills the finite space it occupies only by its "sphere of activity" – and not, therefore, by any process of composition.) Euler (1746b) also emphasizes the first horn of the paradox in directly arguing that physical monads (especially in Wolff's version) could not possibly constitute spatial extension. This paper (published anonymously but well known as Euler's at the time) initiated a controversy (and prize question) at the Berlin Academy of Sciences, and it was precisely this controversy that Kant was trying to resolve in the *Physical Monadology*. Euler (1768–72, letters 122–32) then treats the problem in considerably more detail (and also describes the controversy at the Berlin academy in letter 125). So the editor of Kant (1992, p. 422n. 10) appears to be correct in suggesting that Kant is repeating the argument of Euler (1746b) in the *Physical Monadology*, and Pollok (2001, p. 256n. 454) appears to be correct in suggesting that the relevant letters in Euler (1768–72) constitute an important part of the background to Kant's later dismissal of the Leibnizean–Wolffians' physical monads in the critical period.

Kant begins the second remark to the fourth proposition of the Dynamics by distinguishing between the purely mathematical proposition of the infinite divisibility of space and the physical proposition of the infinite divisibility of matter. With respect to the latter proposition, however, the philosopher (as opposed to the pure mathematician) ventures into a "labyrinth, from which it becomes difficult for him to extricate himself" (506). For it seems that matter, as an extended thing in space, must also be infinitely divisible, and then it could not be composed out of ultimate simple parts:

> One would therefore have to conclude either, in spite of the geometer, that *space is not divisible to infinity*, or, to the annoyance of the metaphysician, that *space is not a property of a thing in itself*, and thus that matter is not a thing in itself, but merely an appearance of our outer senses in general, just as space is the essential form thereof. (506)

Kant continues:

> [H]ere the philosopher is caught between the horns of a dangerous dilemma. To deny the first proposition, that space is divisible to infinity, is an empty undertaking; for nothing can be argued away from mathematics by sophistical hair-splitting. But viewing matter as a thing in itself, and thus space as a property of the thing in itself, amount to the denial of this proposition. The philosopher therefore finds himself forced to deviate from this last proposition, however common and congenial to the common understanding it may be. (506)

Kant thus appeals to transcendental idealism and, accordingly, concludes by sketching the solution of the second antinomy.[98]

[98] The reason viewing matter as a thing in itself amounts to the denial of the infinite divisibility of space, it appears, is the argument from Zeno's metrical paradox against the "monadists" reviewed above. It is striking, in particular, that the language Kant uses here clearly echoes that of the second antinomy (A339–441/B467–69):

> If one listens to [these monadists], then one would have to think, aside from the mathematical point, which is not a part but merely the limit of a space, also physical points, which are indeed also simple, but have the advantage, as parts of the space, of filling this [space] through their mere aggregation. But without here repeating the common and clear refutations of this absurdity, of which there are many, [and] how it is entirely pointless, through mere discursive concepts, to pretend to argue away the evidence of mathematics by sophistical hair-splitting, I only remark that, if philosophers here attempt to deal with mathematics by chicanery, this is because they forget that the question has only to do with *appearances* and their conditions.

We have just seen that Kant uses the phrase "sophistical hair-splitting [*vernünfteln*/*Vernünftelei*]" in precisely this connection in the second remark. Kant begins the remark by also using this phrase in conjunction with "chicanery [*schikanieren*/*Schikane*]" (505): "To be sure, mathematics in its internal use can be entirely indifferent with regard to the chicanery of a misguided metaphysics, and can persist in the secure possession of its evident claims as to the *infinite divisibility of space*, whatever objections may be put in its way by sophistical hair-splitting with mere concepts."

Immediately following this sketch, however, Kant does something truly remarkable. He not only proceeds (as he does in the second antinomy) sharply to distinguish Leibniz's view from that of his Leibnizean–Wolffian followers, he also attempts to enlist Leibniz on behalf of the Kantian doctrine of transcendental idealism. Kant begins the argument without yet naming Leibniz explicitly:

> A great man, who has contributed perhaps more than anyone else to preserving the reputation of mathematics in Germany, has frequently rejected the presumptuous metaphysical claims to overturn the theorems of geometry concerning the infinite divisibility of space by the well-grounded reminder *that space belongs only to the appearance of outer things*; but he has not been understood. This proposition was taken to be asserting that space appears to us, though it is otherwise a thing, or relation of things, in itself, but that the mathematician considers it only as it appears. Instead, it should have been understood as saying that space is in no way a property that attaches in itself to any thing whatsoever outside our senses. It is, rather, only the subjective form of our sensibility, under which objects of the outer senses, with whose constitution in itself we are not acquainted, appear to us, and we then call this appearance matter. Through this misunderstanding one went on thinking of space as a property also attaching to things outside our faculty of representation, but such that the mathematician thinks it only in accordance with common concepts [*gemeinen Begriffen*], that is, confusedly (for it is thus that one commonly explicates appearance). And one thus attributed the mathematical theorem of the infinite divisibility of matter, a proposition presupposing the highest [degree of] clarity in the concept of space, to a confused representation of space taken as basis by the geometer – whereby the metaphysician was then free to compose space out of points, and matter out of simple parts, and thus (in his opinion) to bring clarity into this concept. (507)

Since (as we shall see in a moment) Kant immediately goes on to name Leibniz explicitly, it is clear that the "great man" invoked at the beginning of this passage is none other than Leibniz himself.[99] It is also clear from what we have already seen that Kant is here opposing the Leibnizean–Wolffian conception in essentially the same terms as the second antinomy.[100]

[99] Nevertheless, despite what Kant says next, it has been disputed whether the "great man" in question is Leibniz, and alternative possibilities have been suggested (most plausibly, perhaps, Euler and Lambert). See Pollok (2001, pp. 267–73) for an exhaustive consideration of these alternative possibilities and a conclusive demonstration that Kant can only mean Leibniz here.

[100] For the crucial passage from the second antinomy (A339–441/B467–69) see note 98 above, together with note 93 and the paragraph preceding the one to which it is appended. The continuation of this passage reads (A441/B469; partially quoted in the paragraph to which note 3 above is appended):

> But here it is not enough to find the concept of the simple in the pure *concept of the understanding* of the composite; rather, [we must find] the intuition of the simple in the *intuition* of the

Kant finally names Leibniz as his ally in the immediately following sentence:

> The ground for this aberration lies in a poorly understood *monadology*, which has nothing at all to do with the explanation of natural appearances, but is rather an intrinsically correct *platonic* concept of the world devised by *Leibniz*, in so far as it is considered, not at all as object of the senses, but as thing in itself, and is merely an object of the understanding – which, however, does indeed underlie the appearances of the senses. (507)

As observed in section 8 above, Kant then invokes the solution of the second antinomy by distinguishing between "the *composite of things in themselves*" and "the *composite in the appearance*" (507–8; see the paragraph following the one to which note 3 above is appended).

Kant concludes by definitively appropriating Leibniz on behalf of the doctrine of transcendental idealism:

> Therefore, Leibniz's idea [*Meinung*], so far as I comprehend it, was not to explicate space through the order of simple beings next to one another, it was rather to set this order alongside space as corresponding to it, but as belonging to a merely intelligible world (unknown to us). Thus he asserts nothing but what has been shown elsewhere: namely, that space, together with the matter of which it is the form, does not contain the world of things in themselves, but only their appearance, and is itself only the form of our outer sensible intuition. (508)

This appropriation is fitting in the dialectical context of the later Leibnizean–Wolffian philosophy, because Leibniz certainly did not "explicate space through the order of simple beings next to one another." The relation of entities "next to one another," for Kant, is a spatial relation – the relation of being next to one another *in space*. Moreover, the notion that monads could be conceived as "physical points" bearing spatial relations to one another is distinctive of the Leibnizean–Wolffian variant of the original Leibnizean conception, and it is precisely this variant that Kant is opposing here. Since Leibniz, too, both in Kant's understanding of him and in fact, taught that monads are entirely non-spatial and thus belong to a purely intelligible noumenal realm of which both matter and space are appearances or "well-founded phenomena," it certainly

composite (in matter), and this is completely impossible in accordance with the laws of sensibility, and therefore also in the case of objects of the senses. Thus, whereas it may always be true of a whole of substances thought merely by the pure understanding that we must have the simple prior to all composition of [this whole], this is nevertheless not true of the *totum substantiale phaenomenon*, which, as empirical intuition in space, carries the necessary property with it that no part of it is simple, because no part of space is simple.

makes sense for Kant to enlist him as an ally here.[101] Nevertheless, there remain deep and important differences between Leibniz's conception and Kant's – differences of which Kant is well aware. It is certainly not true, for example, that Leibniz considers the intelligible world to be "unknown to us" – or to have "nothing at all to do with the explanation of natural appearances." Most importantly, as I have already observed, Leibniz's conception of matter and space as "well-founded phenomena," according to Kant's own characterization in the amphiboly, is entirely the reverse of the Kantian view.[102]

[101] Kant introduces the relation of entities *next to* one another in the first argument concerning space in the transcendental aesthetic (A23/B38):

> Space is no empirical concept that has been derived from outer experiences. For, in order that certain sensations are related to something outside me (that is, to something in another place in space than the one in which I find myself), and, similarly, in order that I be able to represent them as outside of and next to one another – and thus not merely as different but as in different places – the representation of space must already lie at the basis. Therefore, the representation of space cannot be obtained from the relations of outer appearance through experience; rather, this outer experience is itself only possible in the first place by means of the representation in question.

(The phrase "and next to" in the second sentence is added in the second edition.) Later in the aesthetic (A40/B56–57) Kant describes the view of "some metaphysical students of nature" as the claim that "space and time are taken to be relations between appearances (next to or after one another), abstracted from experience, although in the abstraction represented confusedly." It seems clear, therefore, that it is precisely the Leibnizean–Wolffian philosophy (and not Leibniz himself) that Kant is targeting here: see again note 17 of the Introduction, together with the preceding discussion of Kant, Newton, and Leibniz.

[102] See again the passage on matter and space from the amphiboly (A267/B223; quoted in the paragraph to which note 77 above is appended) according to which Leibniz "first assumed things (monads), together with an inner power of representation, in order afterwards to ground their external relations and the community of their states (namely, their representations) on this," so that "space and time were [thereby] possible – the former only through the relation of the substances, the latter through the connection of their determinations among one another as ground and consequence." This passage should be compared with what Kant says in the second antinomy immediately following the passage quoted in note 100 above (A441/B469):

> Nevertheless, the monadists have been subtle enough to attempt to evade this difficulty by presupposing, not that space is a condition of the possibility of objects of outer intuition (bodies), but rather that the latter, and the dynamical relations of substances in general, are the condition of the possibility of space. But we have a concept of bodies only as appearances, and these, as such, necessarily presuppose space as the condition of the possibility of all outer appearances; and so the [envisioned] escape is in vain, and it has been sufficiently cut off in the transcendental aesthetic above.

Although Kant speaks of "dynamical relations of substances," he is here attributing to the monadists the view that relations between *bodies* (appearances) "are the conditions of the possibility of space." Thus, once again, Kant is here targeting the Leibnizean–Wolffians rather than Leibniz himself. Kant's own view, in the end, is incompatible with the conceptions of both Leibniz and the Leibnizean–Wolffians. See again the passage on the community of substances and space from the general remark to the system of principles (B292–93; quoted in the paragraph to which note 33 of the Introduction is appended), according to which Leibniz's conception of

It is striking, moreover, that Kant attempts an analogous (if not quite so dramatic) assimilation of elements of the Newtonian natural philosophy in this same context. In particular, at the end of his discussion of the infinite divisibility of matter in the *first* remark to the fourth proposition, Kant adds a paragraph on the mathematical representation of repulsive force in a truly continuous conception of matter that serves as a transition to the discussion of the resulting conflict between mathematics and metaphysics in the second remark:

> When mathematicians represent the repulsive forces of the parts of elastic matters as increasing or decreasing, in accordance with a certain proportion of their distances from one another, at greater or lesser compression of these parts (for example, that the smallest parts of the air repel one another in inverse ratio to their distances from one another, because the elasticity of these parts stands in inverse ratio to the spaces in which they are compressed), then one completely misses their meaning and misinterprets their language if one ascribes that which necessarily belongs to the procedure of constructing a concept to the concept in the object itself. For, by the former [procedure], any contact can be represented as an infinitely small distance – which must also necessarily be so in those cases where a greater or smaller space is to be represented as completely filled by one and the same quantity of matter, that is, one and the same quantum of repulsive forces. In the case of something divisible to infinity, no actual distance of the parts may therefore be assumed – they always constitute a continuum, no matter how expanded is the space, even though the possibility of such an expansion can only be made intuitive under the idea of an infinitely small distance. (505)

Here Kant is referring, once again, to Newton's Proposition 23 of Book 2 of the *Principia*, which derives a repulsive force between immediately adjacent particles of an elastic fluid, acting in inverse proportion to the distance, from the Boyle–Mariotte law relating expansive pressure inversely to volume ("to the spaces in which they are compressed").[103] But Kant wants to reinterpret Newton's mathematical representation of an elastic fluid within a truly continuous (rather than discrete) model, so that the repulsive force responsible for original elasticity acts only at an *infinitely small* distance (where an infinitely small distance, Kant suggests, is

the community of substances "needed a divinity for mediation" – while by contrast (emphasis added): "[W]e can make the possibility of community (of *substances as appearances*) conceivable very well, if we represent them to ourselves in space, and therefore in outer intuition. For the latter already contains within itself a priori formal outer relations as conditions of the possibility of real [relations] (in action and reaction, and thus community)."

[103] See notes 47 and 49 above, together with the paragraphs to which they are appended. As explained in the Introduction, Kant's references to (otherwise unspecified) "mathematicians" are typically to the Newtonian tradition, while those to (otherwise unspecified) "metaphysicians" are to the Leibnizean tradition.

equivalent to contact). Yet, just as in the case of his immediately following reinterpretation of the Leibnizean monadology in the second remark, Kant is perfectly aware that Newton's original representation is actually quite different – that, in this case, it is a *discrete* model.[104]

What is the significance of these two complementary attempts at appropriation through reinterpretation of Leibniz and Newton respectively? It is not that Kant has temporarily forgotten the real and important differences between the views he attempts to appropriate and his own.[105] The point, I believe, is rather that Kant is now embarked on a radically new attempt to synthesize elements of the Newtonian and Leibnizean natural philosophies – which, in particular, represents a radical departure from the parallel attempt at synthesis in the pre-critical theory of the *Physical Monadology*. The central idea of this earlier theory is that we can preserve the fundamental monadological commitment to the simplicity of material substance in the face of the geometrical infinite divisibility of space by representing the filling of space by a Newtonian action-at-a-distance force emanating from a central point. In the critical version of the dynamical theory of matter, by contrast, we represent matter by a true continuum model in which the fundamental force of repulsion responsible for original elasticity does not act at a (finite) distance – even at a very small (finite) distant – and is thus a genuine contact force.[106] In the critical theory, as we have seen, material substance is necessarily infinitely divisible, and there is no possibility at all of preserving its ultimate simplicity. Kant

[104] Indeed, Kant makes this point explicitly later in the Dynamics in the second remark to the eighth proposition: see note 49 above, together with the paragraph to which it is appended. In particular, in the passage from the remark in question quoted there (522), he explains that in the case of atmospheric air – unlike in the case of a true *originally* elastic fluid – the "smallest parts," "proper parts," or "adjacent parts" are such that "actual [i.e., finite] distances from one another are attributable."

[105] In the case of the Leibnizean monadology and transcendental idealism, although Kant's discussion explicitly distinguishing the two occurs in the amphiboly of the first *Critique*, it is nevertheless implausible to suppose that Kant has forgotten or otherwise disagrees with this discussion here. Indeed, not only does the second remark to the fourth proposition of the Dynamics describe the Leibnizean view in a way that is clearly reminiscent of the amphiboly (i.e., as [507] "an intrinsically correct *platonic* concept of the world devised by *Leibniz*, in so far as it is considered, not at all as object of the senses, but as thing in itself, and is merely an object of the understanding"), Kant's language also clearly suggests that Leibniz is here being deliberately *reinterpreted*: transcendental idealism is the way in which Leibniz's doctrine that space belongs only to the appearance of things "should have been understood" (507) and is the true meaning of this doctrine "so far as I comprehend it" (508). In other words, Kant here claims to understand the true meaning of Leibniz's view better than Leibniz himself did.

[106] As we shall see, Kant is very clear and explicit about the fundamental divergence from the *Physical Monadology* in the second remark to the eighth proposition. Kant is also clear, once again, that this representation leads to serious mathematical difficulties – to which I shall return in section 19 below.

instead erects a revolutionary new version of the Leibnizean distinction between phenomena and noumena, based on an equally revolutionary distinction between what he now takes to be the distinct faculties of sensibility and understanding. In particular, whereas substance as conceived by the pure understanding alone is indeed necessarily simple, this is not and cannot be true of the only kind of substance that it is now possible for us to know: material substance in space or *substantia phaenomenon*.

Kant strongly re-emphasizes this last point in his critical solution of the second antinomy:

It seems, indeed, that, since a body must be represented as substance in space, it would be distinguished from [space] with respect to the law of infinite divisibility. For, although we can certainly grant that the decomposition of the latter [space] could never remove all composition, in that in this case all space, which otherwise has nothing self-subsistent [*Selbständiges*], would disappear (which is impossible), [the idea] that nothing at all should remain if all composition of matter were to be removed in thought does not appear to be compatible with the concept of a substance, which should properly be the subject of all composition and must [therefore] remain in its elements even if their connection in space by which they constitute a body were to be destroyed. However, in the case of that which is called substance in the *appearance* the situation is not as one would rightly think of a thing in itself through pure concepts of the understanding. The former is not an absolute subject, but rather a permanent image of sensibility [*beharrliches Bild der Sinnlichkeit*], and it is nothing but an intuition, in which there is nowhere anything unconditioned to be found. (A525–26/B554)[107]

In the end, therefore, Kant's new understanding of *substantia phaenomenon* as necessarily infinitely divisible is secured by moving from the pure concept of substance to the schematized concept, according to which material substance is not only defined as the real in space but is also characterized by the *permanence* of the real in time.[108]

[107] Compare note 78 above on the need to distinguish the infinite divisibility of space from the infinite divisibility of the material substance that fills a space. The present passage represents the only place in the antinomies where Kant explicitly emphasizes the importance of this distinction in moving from the former to the latter.

[108] See again the definition of the schema of the pure concept of substance quoted in note 73 above. In Langton's effort to read the pre-critical doctrine of ultimate substances (with absolutely intrinsic properties) into the critical Kant (see note 76 above) she is led, in my view, to misread this passage (A525–26/B554). She maintains that phenomenal substance in the critical period is not a genuine substance properly speaking, because substance (by definition) must have (absolutely) intrinsic properties. Phenomenal substance is therefore merely the most serviceable approximation we can find for genuine substance in matter. Accordingly, Langton suggests a translation and emendation of "*beharrliches Bild der Sinnlichkeit*" as "*sensible abiding picture* [of an absolute subject]" (1998, p. 60). But the term '*Bild*', for Kant, is a technical notion deployed in the schematism chapter. Here it signifies a singular product of the imagination (*Einbildungskraft*) associated with a general concept by its schema – as, for example, a singular

This characterization then leads to the first analogy of experience, which, in the Mechanics of the *Metaphysical Foundations*, becomes the principle of the conservation of the quantity of matter (541–43). Moreover, since the conservation of the quantity of matter, for Kant, essentially depends on the idea that "all quantity of an object possible only in space must consist *of parts external to one another*" (542), it turns out that this principle is intimately connected with the claim, noted at the end of section 13 above, that the quantity of matter of the material substance filling a given space can only be given as the total (infinite and continuous) "aggregate of movables" in this space (539–40).[109] In this way, as we shall see, Kant is led, step by step, to a radical reconsideration of the Newtonian concept of quantity of matter and its relationship to the two fundamental forces – where, in particular, the fundamental force of attraction is implicated primarily with the behavior of matter in the large (and thus with the argument of the first antinomy) and the fundamental force of repulsion with the behavior of matter in the small (and thus with the argument of the second). It is in precisely this way, more generally, that Kant fashions a new interpretation of the metaphysical concept of substance in order to provide a new kind of metaphysical foundation for the Newtonian mathematical theory of motion.

15 FROM REPULSION TO ATTRACTION

Kant's official transition from the fundamental force of repulsion to the fundamental force of attraction takes place in the immediately following fifth proposition, which depends on the infinite divisibility of material substance demonstrated in the fourth. The argument proceeds by a consideration of the conditions for "balancing" the already established repulsive force by another force acting in the contrary direction. If there were only the fundamental force of repulsion, Kant argues, then matter would expand arbitrarily to infinity. Therefore, if nothing counterbalanced this original expansive tendency, matter would eventually reach a state of zero density in which space would actually be empty of matter. This necessary

individual triangle is associated with the general concept of triangle by a Euclidean construction. So an "image [*Bild*]" in this sense need not be a "picture" *of* anything (else) at all. In particular, as we shall see, material substance for the critical Kant is in no sense a picture of noumenal substance; it is rather that which in the phenomenal world *realizes* or *instantiates* the pure concept of substance via its schema.

[109] In the argument of the second antinomy these ideas are reflected in the claim, quoted in note 91 above, that "everything real that occupies a space comprises a *manifold* [of elements] external to one another" (B435/B463).

counterbalancing tendency, Kant argues, can only ultimately be found in a second fundamental force acting in the opposite direction to the fundamental force of repulsion. But this, by the second explication and its accompanying note, is a fundamental force of attraction (see note 22 above). Therefore, the possibility of matter as something filling the space it occupies to a determinate (finite) degree requires a fundamental force of attraction as well as a fundamental force of repulsion.

I shall postpone discussion of this difficult argument (and, in particular, its relation to the infinite divisibility of material substance demonstrated in the fourth proposition) to the next section. For one obtains a better sense of the place of the fifth proposition in the overall argument of the Dynamics as a whole, I believe, if one first considers the discussion in the remark following this proposition. Here Kant steps back from the proposition he has just demonstrated to ask a meta-question, as it were, about his overall procedure. If, as has just been shown, both repulsion and attraction are equally necessary for the possibility of matter, why do we begin with the fundamental force of repulsion or impenetrability – as a constituent or partial concept in the definition or explication of matter – and only subsequently add the fundamental force of attraction by a (synthetic) inference from this definition:

> In this transition from one property of matter to another, specifically different from it, and belonging equally to the concept of matter, *even though not contained in it*, the procedure of our understanding must be considered more closely. If attractive force is originally required even for the possibility of matter, why do we not use it, just as much as impenetrability, as the first distinguishing mark of a matter? Why is the latter immediately given with the concept of a matter, whereas the former is not thought in the concept, but only adjoined to it through inferences? (509)

Kant's answer to this question is intended not only to illuminate his overall procedure in the Dynamics but also to provide an explanation, of sorts, of why so many natural philosophers have found the idea of an original attraction unacceptable.[110]

[110] Kant's own view, more precisely, is that, whereas the proposition that matter fills a space by impenetrability is *analytic* a priori (because part of the official definition of matter: see note 8 above, together with the paragraph to which it is appended), the proposition that matter possesses a fundamental force of attraction is *synthetic* a priori. Here see the examples Kant cites in his definition of analytic and synthetic propositions in §36 of the *Jäsche Logic* (9, 111): "To everything x, to which the concept of body (a+b) pertains, pertains also [the concept of] *extension* (b), is an example of an *analytic* proposition. To everything x, to which the concept of body (a+b) pertains, pertains also [the concept of] *attraction* (c), is an example of a *synthetic* proposition." (That this latter proposition is nonetheless *a priori* follows from Kant's present proof in the

From the point of view of Lockean empiricism there is a simple and straightforward answer to Kant's question. Impenetrability has priority in the definition of matter or body because this concept is a simple idea of sensation by which we are first immediately acquainted with the bodies outside ourselves. Indeed, from this point of view, it is tempting to locate the empirical origins of the concept of force in general in precisely such immediate experiences of resistance to our sense of touch. It is noteworthy, then, that Kant takes this answer to be insufficient (509):

That our senses do not allow us to perceive this attraction so immediately as the repulsion and resistance of impenetrability cannot yet provide a sufficient answer to the difficulty. For even if we had such a capacity, it is still easy to see that our understanding would nonetheless choose the filling of space in order to designate substance in space, that is, matter, and how precisely this *filling*, or, as one otherwise calls it, *solidity*, is then posited to be characteristic of matter, as a thing different from space.

Even if attraction *were* immediately perceivable, like solidity, we would still prefer to begin with the latter. So it cannot be the circumstance that impenetrability is a Lockean simple idea of sensation that explains its priority in the metaphysical foundations of natural science. More generally, I suggest, an empiricist justification of the mechanical natural philosophy – as suggested by both Locke and Lambert, for example – cannot ultimately do justice to the true role of impenetrability in our scientific conception of nature.[111]

Kant's own answer to the question of why the concept of impenetrability has priority is remarkable:

Attraction, even if we sensed it equally well, would still never disclose to us a matter of determinate *volume* and *figure*, but only the striving of our organ to approach a point outside us (the center of the attracting body). For the attractive force of all parts of the earth can affect us no more, and in no other way, than as if it were wholly united in the earth's center, and this alone influenced our sense, and the same holds for the attraction of a mountain, or any stone, etc. But we thereby obtain no determinate concept of any object in space, since neither figure, nor quantity, nor even the place where it would be found can strike our senses (the mere direction of attraction would be perceivable, as in the case of weight: the attracting point would be unknown, and I do not even see how it

Dynamics.) Section IV of the Introduction to the first *Critique* famously illustrates the distinction between analytic and synthetic judgements by the examples "all bodies are extended" and "all bodies are heavy" respectively (A7/B11).

[111] See sections 10, 11, and, in particular, note 26 above, together with the paragraph to which it is appended. Kant's reference to solidity (and thus *absolute* impenetrability) here suggests just such a connection with Locke and Lambert.

could be ascertained through inferences, without perception of matter in so far as it fills space). (509–10)

Far from relying on an empiricist understanding of the concept of force in general, as derived from our immediate experience of resistance to our sense of touch, Kant appeals to the mathematical properties of gravitational force as described by Newton in explaining why this force (in comparison with repulsive force) cannot be epistemically primary.[112]

The property in question figures in a central step in the argument for universal gravitation in Book 3 of the *Principia*. In particular, Proposition 8 of Book 3 treats the planets (including the earth) as spherical bodies with spherically symmetric distributions of mass around their centers, and, on this assumption, shows that the weights of bodies towards different planets (e.g., the weights of falling bodies towards the earth or of the moons of Jupiter towards Jupiter) are given in terms of the inverse-square gravitational force acting between the *center* of the planet and that of the body. Thus, although the gravitational force exerted by a planet on any body is actually compounded out of all the gravitational forces exerted by every part of the planet, the sum of all these forces is precisely the same as if all the mass of the planet were concentrated at its central point. We thereby justify the idealization of the bodies involved in gravitational interactions as isolated point-masses, and, at the same time, we provide support for the idea that terrestrial gravity (responsible for the behavior of falling bodies, pendula, and so on) is in fact identical to celestial attraction (responsible for the motions of the heavenly bodies in their orbits). The crucial argument for this latter identification is the moon test explained in Proposition 4, which shows that the inverse-square acceleration of the moon towards the center of the earth coincides with the acceleration g of gravity when the moon is imagined to descend to the surface of the earth. Proposition 8 adds support for this argument by showing that, although the universal attraction on any body near the surface of the earth is actually compounded out of infinitely many attractive forces, most of which are not directed towards the center, it may still be treated as if it were solely directed towards the center.[113]

[112] It follows that what Kant calls the fundamental force of attraction cannot be epistemically primary either. For gravitation is a manifestation, for Kant, of precisely this force (518): "The *action* of the universal attraction that all matter immediately exerts on all other matter and at all distances is called *gravitation*; the striving to move in the direction of greater gravitation is *weight*."

[113] Newton's comments on Proposition 8 make this clear (P811):

> After I had found that the gravity toward a whole planet arises from and is compounded of the gravities toward the parts and that toward each of the individual parts it is inversely proportional to the square of the distances from the parts, I was still not certain whether that

To illustrate Kant's point, let us imagine that the earth and all of its parts exerted an attractive force but no repulsive force of impenetrability, and let us further imagine that we had the capacity immediately to sense this attractive force. We would then have sensory acquaintance with the direction of the earth's center in space (as we floated nearby, say) but not with any other features of its spatial distribution: neither its surface, nor its volume, nor its (approximately) spherical shape, nor even the spatial position of its center. For all of these other features, Kant suggests, depend on an acquaintance with the earth's (approximately) spherical surface, and our acquaintance with this surface, in turn, can only be mediated by a force of impenetrability – by which, for example, we feel its resistance under our feet to the downward pressure produced by gravity. In the end, therefore, the crucial asymmetry between attraction and repulsion, in this respect, depends on the circumstance that the former is what Kant calls a *penetrating* force, acting from each point of one body on all other points of a second body, whereas the latter is a *surface* force, acting only at the common surface of contact of two bodies.[114] It is only the latter type of force, Kant suggests, that can acquaint us with the surface of a body and thus with all other features of its spatial distribution that depend on this surface. Since attraction, by contrast, is a penetrating force, it cannot possibly afford us sensory acquaintance with any of these features of a body's distribution in space.[115]

> proportion of the inverse square obtained exactly in a total force compounded of a number of forces, or only nearly so. For it could happen that a proportion which holds exactly enough at very great distances might be markedly in error near the surface of the planet, because there the distances of the particles may be unequal and their situations dissimilar.

(That the main example of the problem to be resolved is the case of gravity near the surface of the earth is strongly suggested by Corollary 1 to the preceding Proposition 7.) For the proof of Proposition 8 Newton refers to Propositions 75 and 76 of Book 1, according to which two spheres whose masses are symmetrically distributed around their centers attract one another in inverse proportion to the distance between these centers. Note that the reduction of a compounded inverse square attraction to one proceeding solely from a central point holds only for this special case: in general we obtain a more complicated system of attracting points. Note also, as both Newton and Kant were well aware, the shape of the earth is actually not a perfect sphere but an oblate spheroid. I shall return to this last point in my chapter on the Phenomenology.

[114] See the seventh explication of the Dynamics (516): "I call a moving force whereby matters can act immediately on one another only at the common surface of contact a *surface force*; but that whereby a matter can act immediately on the parts of others, even beyond the surface of contact, [I call] a *penetrating force*."

[115] Compare Reflexion 38 from the mid 1770s (14, 116):

> Because the sensation of a body is only the sensation of a bounded [*terminirten*] space, and therefore of the surface, this sensation cannot be stimulated by forces of the body that are directed towards the center, without the limit [*Grenze*] of possible motion being [thereby] determined, and therefore not by attraction, which proceeds from the circumference towards the center, but only by the boundary [*Grenze*] of the space that resists penetration.

The importance of this asymmetry is explained in the immediately following passage (510):

It is therefore clear that the first application of our concepts of *quantity* to matter, through which it first becomes possible for us to transform our outer perceptions into the empirical concept of a matter, as object in general, is grounded only on that property whereby it fills a space – which, by means of the sense of feeling [*Sinnes des Gefühls*], provides us with the quantity and figure of something extended, and thus with the concept of a determinate object in space, which forms the basis of everything else one can say about this thing.

Kant's idea, therefore, is that our first application of mathematics to the objects of outer experience (our first application of the categories of quantity) occurs when we attribute determinate volumes, figures, and relative spatial positions to these objects – and that this application, in turn, depends on the fundamental force of repulsion as a surface force.[116]

This idea lends confirmation, in the first place, to what was said at the end of section II above. The concept of repulsive force in particular and the concept of matter in general are empirical concepts, for Kant, not because they have an immediate empirical origin as Lockean simple ideas of sensation but because of the central role they play in a complex constructive procedure by which the pure concepts of the understanding, together with pure mathematical concepts, are successively applied to experience step by step. We begin, as Kant suggests, with the categories of quantity, and our first task is to secure the application of pure geometrical concepts – figure, volume, and (relative) spatial position – to the objects of outer experience. This results in their conceptualization as bodies, as pieces of matter with precisely such geometrical properties, and it is this conceptualization, Kant suggests, which then "forms the basis of everything else one can say" about such objects.[117] In particular,

[116] Kant further explains in the remark to the sixth explication that contact in the physical sense (mediated by repulsive force) presupposes contact in the mathematical sense – as "the common boundary of two spaces" (512):

Mathematical contact is the basis for physical contact, but does not yet constitute the latter by itself, since for the one to arise from the other a dynamical relation must also be added in thought – and, indeed, not of attractive, but of repulsive forces, that is, of impenetrability. Physical contact is the interaction of repulsive forces at the common boundary of two matters.

[117] Compare the first number of the general remark to dynamics (525): "A **body**, in the physical sense, is *a matter between determinate limits* (which therefore has a figure). *The space between these limits, considered in accordance with its magnitude* [*Größe*], is the **space-content** [***Raumesinhalt***] (*volumen*)." As I shall explain below, the four numbers of the general remark correspond within (empirical) Dynamics to the four headings of the table of categories – so that this discussion corresponds to the categories of quantity.

all subsequent applications of both pure concepts of the understanding and pure mathematical concepts must proceed from this initial starting point in the categories of quantity, where we thus begin an empirical constructive procedure of progressively rationalizing or objectifying our sensory experience under the concept of a determinate object in space.

Further, and in the second place, Kant's implicit reference to Proposition 8 of Book 3 of the *Principia* provides us with a clear indication of how the procedure in question is to be continued beyond this point. For, according to the corollaries to Proposition 8, we are now able to estimate the *weights* of equal bodies at equal distances from the primary bodies in the solar system (the sun, Jupiter, Saturn, and the earth) in terms of the distances (from their respective centers) and periodic times of the satellites of these primary bodies (Corollary 1). We are thereby able to estimate the *quantities of matter* of these primary bodies from the same geometrical/kinematical information, since, according to Proposition 7, the force of universal gravitation is proportional to the quantity of matter of a body that exerts such a force (Corollary 2). And we are then able to estimate the *densities* of these primary bodies in terms of quantity of matter divided by volume (Corollary 3). The most important result of this procedure for estimating the quantities of matter of the primary bodies in the solar system by the distances and periodic times of their satellites is given in Corollary 2, according to which the ratios of the quantities of matter of the sun, Jupiter, Saturn, and the earth are as 1, 1/1,067, 1/3,021, and 1/169,282 respectively. For it is this result, in Proposition 12, which allows us to determine that the center of gravity of the solar system is always very close to the center of the sun and thereby definitively to settle the decision between the Tychonic and Copernican world systems.[118] So it is also this Newtonian procedure – in the context of Kant's Copernican conception of space and motion – that allows us to establish a (temporarily) privileged

[118] The Tychonic system is geocentric in so far as the earth is still the true center of motion of the entire solar system. It differs from the Ptolemaic system, however, in taking all the other planets to orbit the sun rather than the earth (in effect, on epicycles centered on the solar orbit), while the sun orbits the earth as in the Ptolemaic system. In this way, the Tychonic system, unlike the Ptolemaic, is perfectly consistent with Galileo's telescopic observations of the phases of Venus – and, more generally, it is both optically and kinematically equivalent to the Copernican system in terms of all observations that were possible at the time, including attempted but not yet possible observations of stellar parallax. It is for precisely this reason, in particular, that Newton solves the problem of deciding between these two systems in Book 3 of the *Principia* by a *dynamical* argument crucially involving the determination of the center of gravity. I shall return to this situation in more detail in my chapter on the Phenomenology.

relative space or reference frame within which all the motions in the solar system are to be considered.[119]

Kant's view of how the two fundamental forces are related to one another in this context therefore appears to be the following. We begin by applying geometrical and kinematical concepts to bodies, yielding volumes, figures, (relative) spatial positions, and changes of such positions over time (within a particular relative space) – and this application is mediated by the fundamental force of repulsion as a surface force.[120] So far, however, we are still quite limited in our ability to apply further mathematical concepts to bodies, and, in particular, we are not yet able to apply the Newtonian concept of quantity of matter.[121] Nevertheless, by further observation of their volumes, figures, and changing (relative) spatial positions, we can, by the argument of Book 3 of the *Principia*, establish the laws of the fundamental force of attraction (in its guise as universal gravitation – see again note 112 above). Newton's argument, as applied in Proposition 8 and its corollaries, then allows us to apply the concept of quantity of matter (quite generally) to all of the bodies in question. This application of the mathematical concept of quantity of matter to the objects of our experience is mediated by the fundamental force of attraction as a penetrating force, and it cannot be based, accordingly, on the fundamental force of repulsion alone.[122]

[119] As explained, Kant makes it clear, especially in the *Theory of the Heavens*, that his Copernican conception of space and of motion is then to be successively extended far beyond the center of gravity of the solar system: to the center of gravity of the Milky Way galaxy, the center of gravity of a much larger rotating system of such galaxies, and so on (see notes 6 and 7 of my chapter on Phoronomy, together with the paragraph to which they are appended). It is clear from chapter 2 of Part Two of the *Theory of the Heavens*, "on the differing density of the planets and the ratio of their masses," that Kant is familiar with Newton's results in the corollaries to Proposition 8.

[120] In the case of the heavenly bodies, in particular, this application is mediated by the transmission of light as it is either emitted by such bodies or reflected off their surfaces. I shall return to this point in section 17 below.

[121] For terrestrial bodies near the surface of the earth we have the traditional statical concept of *weight*, estimated in terms of downward pressure exerted in a balance. But this concept cannot be extended to bodies quite generally until it is explicitly linked to universal gravitation by the argument of *Principia*, Book 3. I shall consider the relationship between terrestrial (statical) weight and the universal Newtonian concept of quantity of matter or mass in considerable detail in the sequel.

[122] That the fundamental force of attraction is proportional to the quantity of matter *precisely because it is a penetrating force* – acting between all parts of one matter and every part of another – is stated in the note to the seventh explication following the passage quoted in note 114 above. See also the corresponding comments in the general remark to dynamics, which explain that the fundamental force of repulsion is *not* proportional to the quantity of matter because "through expansive force only those [parts] *at the surface of contact* act, so that it is all the same whether much or little of this matter is found behind that surface" (524). Once again, however, a full understanding ultimately depends on the Mechanics, where quantity of matter

The crucial step in Kant's progressive constructive procedure for successively applying mathematical concepts to empirically given nature is thus a transition from the terrestrial to the celestial realm – whereby we articulate a generally applicable concept of quantity of matter suitable for establishing a privileged frame of reference relative to which both celestial and terrestrial motions may be described within a single unified system of the world. The transition is effected by the moon test described in Proposition 4 of Book 3 and then further supported and extended in Proposition 8. Kant, in the passage we are considering, underscores its importance by mentioning "the attraction of a mountain, or any stone, etc." after discussing the attraction of the earth. For this appears to be an allusion to what we might call the "inverse moon test" described in §3 of Newton's more popular *System of the World* (1728).[123] Here Newton imagines "a stone that is projected … from the top of a high mountain" with progressively greater horizontal velocities until it finally orbits the earth as a heavenly body.[124] It is in this way, for Kant, that we move from an application of mathematics to the bodies of our terrestrial experience mediated primarily by the force of impenetrability (and therefore by the sense of touch) to a much more extensive application of mathematics to the bodies

is first explicated in terms of the aggregate of the movable in a given space (see the paragraph to which note 83 above is appended) and then "estimated" in terms of "quantity of motion at a given speed" (537). The relation of these explications to quantity of matter as determined by universal gravitation is then finally explained in the remark to the first proposition (541). So I shall postpone a full discussion of this complex web of relationships until my chapter on the Mechanics. I shall, however, be able to take a substantial step towards clarifying these relationships in the next section.

[123] It also appears that Kant's eagerness to make the reference to the moon test more explicit may have lead him hastily – and erroneously – to suggest that the attractions of non-spherical and non-uniform bodies (such as a mountain or perhaps a stone) have the same properties as those of spherical uniform bodies (see note 113 above).

[124] See Newton (1934, pp. 551–52), in which the situation is depicted in one of Newton's most famous diagrams. In the Scholium to Proposition 4 of Book 3 of the *Principia* Newton elucidates the argument of the moon test by imagining progressively lower satellites orbiting the earth (P805):

> And if the lowest of them were small and nearly touched the tops of the highest mountains, its centripetal force, by which it would be kept in its orbit, would (by the preceding computation) be very nearly equal to the gravities of bodies on the tops of those mountains. And this centripetal force would cause this little moon, if it were deprived of all the motion with which it proceeds in its orbit, to descend to the earth … and to do so with the same velocity with which heavy bodies fall on the tops of these mountains, because the forces with which they descend are equal.

The "little moon" in question is thus like the projected stone in the *System of the World*, which obeys Galileo's law of fall in its projectile motion.

of both celestial and terrestrial experience mediated primarily by the force of attraction (and therefore independently of the sense of touch).[125]

As I noted at the beginning of this discussion, Kant takes the asymmetry between the two fundamental forces just described to provide an explanation for why so many natural philosophers – especially mechanical natural philosophers – have found the idea of an original attraction unacceptable. He provides the explanation in question at the end of the present remark:

Precisely this circumstance is undoubtedly the reason, despite the clearest proofs from elsewhere that attraction must belong to the fundamental forces of matter, just as much as repulsion, that one nevertheless struggles so much against the former, and one will admit no other moving forces at all except those through impact and pressure (both mediated by impenetrability). For that whereby space is filled is substance, one says, and this is also perfectly correct. However, since this substance discloses its existence to us in no other way than through that sense whereby we perceive its impenetrability, namely, feeling [*Gefühl*], and thus only in relation to contact, whose onset (in the approach of one matter to another) is called impact, and whose persistence is called pressure; it therefore seems as if every immediate action of one matter on the other could never be anything but pressure or impact, the only two influences we can sense immediately, whereas attraction, by contrast, can give us in itself either no sensation at all, or at least no determinate object of sensation, and is therefore so difficult for us to understand as a fundamental force. (510)

So it might appear – at first sight – that Kant's explanation depends primarily on the distance of the fundamental force of attraction from immediate acquaintance by the sense of touch.

We have seen, however, that the circumstance that we have no immediate acquaintance with the fundamental force of attraction is not the primary ground of the asymmetry in question for Kant. It is rather that our first application of the categories of quantity to objects of experience can only be mediated by repulsive force and thus by the sense of

[125] This transition also recalls Newton's discussion of Rule 3 in his "Rules for the Study of Natural Philosophy," according to which those qualities "that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally" (P795). Newton here considers the case of impenetrability, in particular, and explains that "[w]e find those bodies that we handle to be impenetrable, and hence we conclude that impenetrability is a property of all bodies universally" (P795) – which passage, in turn, recalls Locke's discussion of impenetrability or solidity in the *Essay* (note 14 above). Newton then discusses the argument for universal gravitation as "established by [terrestrial] experiments and astronomical observations" and concludes (P796) that "the argument from phenomena will be even stronger for universal gravity than for the impenetrability of bodies, for which, of course, we have not a single experiment, and not even an observation, in the case of the heavenly bodies."

touch. Moreover, Kant is not only giving an account of why the mechanical philosophy (reasonably) begins with the force of impenetrability (and therefore with action by contact in cases of impact and pressure), he also provides a diagnosis of exactly where it goes wrong in permanently restricting itself to this force in all further applications. Kant's own conception of the progressive application of the categories of quantity to the objects of experience in the *Metaphysical Foundations* focusses on the transition from the terrestrial to the celestial realm, and, following Newton, Kant views this transition as essentially mediated by a fundamental force of attraction. Yet, by restricting itself to action by contact and rejecting such an attraction, the mechanical philosophy is severely limited in its ability to follow Newton's argument in Book 3. So it is similarly limited in its ability to follow Kant's argument, developed throughout the *Metaphysical Foundations*, that shows how to extend his Copernican conception of space and motion far beyond a frame of reference fixed at the center of the earth.[126]

16 QUANTITY OF MATTER AND THE TWO FUNDAMENTAL FORCES

Kant's official transition from the fundamental force of repulsion to the fundamental force of attraction is effected in the fifth proposition of the Dynamics, which articulates a "balancing" argument according to which repulsive force on its own would issue in an indefinite expansion of matter towards a state of zero density (508): "[M]atter, by its repulsive force (containing the ground of impenetrability), would, [by itself] alone and if no other moving force counteracted it, be confined within no limit of extension, that is, it would disperse itself to infinity, and no specified quantity of matter would be found in any specified space." In order, then, that a specified quantity of matter may be found in a specified

[126] This diagnosis applies with particular force to empiricist versions of the mechanical philosophy such as those suggested by Lambert and Locke (compare note 111 above). Moreover, it appears that Newton also has such philosophers in mind in the remarks about gravity and impenetrability in his discussion of Rule 3 (note 125 above). By contrast, as George E. Smith has emphasized to me, other advocates of the mechanical philosophy, notably Huygens and Euler, were able to accept much of Newton's argument – and, in particular, to accept the argument of the moon test for identifying terrestrial weight with celestial inverse-square centripetal force. Where they balked was in accepting inverse-square centripetal forces between each particle of matter and every other – and thus, in particular, in accepting that gravitation is what Kant calls a *penetrating* force. I shall begin to explore the significance of this feature of universal gravitation in section 18 below, and I shall return to Newton's (and Kant's) differences with Huygens in my chapter on the Phenomenology.

space (so that this space is actually filled rather than empty), it is necessary that there be a fundamental force of attraction exerting the counteracting effect in question. Conversely, as Kant argues in the following sixth proposition, if we began with the fundamental force of attraction alone, then matter would undergo an indefinite contraction towards a single mathematical point, and, once again, "space would be empty and without any matter" (511). So both fundamental forces are necessary for the existence of a determinate quantity of matter in a given space: "It is now manifest that, whether one takes neither as basis, or assumes merely one of them, space would always remain empty, and no matter would be found therein" (511).

This argument, like so much of the dynamical theory of matter, has its roots in Kant's pre-critical version in the *Physical Monadology*. The corresponding discussion occurs in the second section of this work (1, 483–87), "explicating the most general properties of physical monads, in so far as they are different in different things and contribute to the understanding of the nature of bodies."[127] In particular, the corresponding balancing argument occurs in the first proposition of this section, Proposition x (1, 483): "By the force of impenetrability alone bodies would have no definite volume, if there were not another, equally innate (*insita*) [force of] attraction, which, conjoined with it, defines the limit of [a body's] extension." But, as Kant's wording suggests, the argument is not concerned with explaining the possibility of a determinate *quantity of matter* in a given space but with explaining how an element – a physical monad – acquires a definite *spatial volume*. Indeed, according to the proof of the proposition and the following scholium, a balancing of the two fundamental forces results from the circumstance that the force of repulsion falls off more rapidly than the force of attraction (in the inverse-cube ratio of the distance rather than the inverse-square ratio), so that a determinate spherical boundary is defined beyond which the repulsion vanishes. This balancing, in other words, explains how the repulsive sphere of activity of an element is precisely circumscribed within a definite (that is, finite) volume. Moreover, according to the corollary to our

[127] The first section, "showing that the existence of physical monads is in agreement with geometry," is devoted to the explanation of the filling of a space through the sphere of activity of a repulsive force emanating from a central point. Very roughly, then, the first part corresponds to the first four propositions of the Dynamics of the *Metaphysical Foundations* (dealing with the fundamental force of repulsion), while the second part corresponds to the last four (dealing with the fundamental force of attraction and the relationship between the two).

proposition, it turns out that all elements whatsoever actually have the same (finite) volume.[128]

Kant introduces the concept of mass or quantity of matter in the following Proposition XI, as a measure of a body's force of inertia (*vis inertiae*). The connection is explained, following Newton's discussion in the third definition of the *Principia*, by the circumstance that the mass of a body is a measure of both its resistance to being put into motion by others and its ability to communicate its motion to others (impetus). Kant initially explains the concept of mass or force of inertia in terms of the second:

> A body in motion colliding with another would have no efficacy, and would be brought by an infinitely small obstacle into a state of rest, if it did not have a force of inertia by which it strives to persevere in its state of motion. But the force of inertia of a body is the sum of the forces of inertia of all the elements of which it is composed (and which is of course called its mass); therefore, no element moved with a certain velocity would have an efficacy of motion, if [this velocity] were not multiplied by the force of inertia. (1, 485)

Kant then explains in the second corollary that the concept of mass or force of inertia comprises both aspects: "For the mass of a body is only the magnitude [*quantitas*] of its force of inertia, with which it either resists a motion or, being moved with a given velocity, has a certain impetus of motion" (1, 485).[129] Unlike in the *Metaphysical Foundations*, then, mass or quantity of matter does not arise from a balancing of the two fundamental forces of attraction and repulsion but is rather a *third* fundamental force (*vis inertiae*) by which a body strives to conserve its state of motion or rest. Each element, among its "innate" forces, possesses a force of repulsion, a force of attraction, *and* a force of inertia. The balancing

[128] Kant argues that, although elements of different kinds will have innate forces of repulsion and attraction of different degrees of intensity, the ratios of these different intensities (at equal distances) will always be the same in different monads – since "it is appropriate that all the moving forces of an element that is specifically twice as strong [as another] are stronger in the same ratio" (1, 485).

[129] Compare Newton's famous discussion of "innate force [*vis insita*]" in the *Principia* (P404):

> This force is always proportional to the body and does not differ in any way from the inertia of the mass except in the manner in which it is conceived. Because of the inertia of matter, every body is only with difficulty put out of its state either of resting or moving. Consequently, innate force may also be called by the very significant name of force of inertia [*vis inertiae*]. Moreover, a body exerts this force only during a change of its state, caused by another force impressed upon it, and this exercise of force is, depending on the point of view, both resistance and impetus: resistance in so far as the body, in order to maintain its state, strives against the impressed force, and impetus in so far as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavors to change the state of that obstacle.

of the first two explains an element's finite volume, the third explains its quantity of mass.[130]

In the *Metaphysical Foundations*, by contrast, the concept of mass or quantity of matter, in the sense of resistance, impetus, and inertia, is only fully clarified in the Mechanics chapter – whose distinctive subject matter, more generally, is precisely the communication of motion.[131] For it is here, in the Mechanics, that the concept of mass or quantity of matter is first treated as a magnitude, in terms of precisely the concept of impetus (i.e., momentum = mass times velocity) governing such communication. Nevertheless, as I have explained, Kant is still able in the Dynamics to anticipate the discussion of quantity of matter in the Mechanics. Indeed, Kant explicitly appeals to this concept in the proof of the fifth proposition, according to which a "specified quantity of matter" can be found in a given space only in virtue of a balancing of the two fundamental forces. Moreover, as I explained in my discussion of the remark to this proposition in section 15, Kant there implicitly refers to Newton's Proposition 8 of Book 3 of the *Principia*, where the quantities of matter of the primary bodies in the solar system are determined by the accelerations of their satellites. We can therefore say, more generally, that there are three distinguishable concepts of quantity of matter at work in the *Metaphysical Foundations*: (i) a dynamical concept related to the fundamental force of repulsion through a notion of density linked to the possibility of compression; (ii) a dynamical concept related to the fundamental force of attraction through the idea that the

[130] The contrast between the two works becomes even clearer in this regard when we observe that the *Physical Monadology* introduces the present notion of mass precisely to explain the possibility of different intrinsic densities (masses per unit volume) without assuming empty spaces (1, 485–86):

> Hence, from the fact that a given volume contains a smaller quantity of matter we cannot always validly infer that the matter has a smaller density and greater empty interstices. Each of the two bodies can either possess equal empty interstices, or they can both be completely dense, and one of the two can nevertheless have a much greater mass, the cause of which resides in the nature of the elements themselves.

As we shall see, this same possibility is explained in the *Metaphysical Foundations* by differing ratios of repulsive and attractive force in specifically different matters.

[131] Kant announces the contrast between the Dynamics and the Mechanics in this respect at the very beginning of the latter chapter (536):

> The merely dynamical concept [of matter] could consider matter also as at rest; for the moving force there dealt with had merely to do with the filling of a certain space, without the matter filling it needing to be viewed as itself moved. Repulsion was therefore an originally moving force for *imparting* [*erteilen*] motion; in mechanics, by contrast, the force of a matter set in motion is considered as *communicating* [*mitzuteilen*] this motion to another.

accelerations produced by this force are directly proportional (at a given distance) to the attracting body's quantity of matter; (iii) a mechanical concept related to the communication of motion and therefore to the concepts of impetus and inertia.[132] Although a full clarification of the complex web of relationships among these prima facie quite different concepts must await my discussion of the Mechanics (compare note 122 above), it is still clear, nonetheless, that the first two concepts belong to the Dynamics.

Let us begin, then, with the second concept of quantity of matter, which, since it is essentially connected with the fundamental force of attraction, is at issue in the second part of Kant's balancing argument expounded in the sixth proposition of the Dynamics. Here Kant considers what would happen if there were only a force of attraction and no force of repulsion:

> Attractive force is that moving force of matter whereby it impels another to approach it; consequently, if it is found between all parts of matter, matter thereby strives to diminish the distance of its parts from one another, and thus the space that they occupy together. But nothing can hinder the action of a moving force except another moving force opposed to it, and that which opposes attraction is repulsive force. Hence, without repulsive forces, through mere approach, all parts of matter would approach one another unhindered and would diminish the space that they occupy. But since, in the case assumed, there is no distance of the parts at which a greater approach through attraction would be made impossible by a repulsive force, they would move towards one another until no distance at all would be found between them; that is, they would coalesce into a mathematical point, and space would be empty and thus without any matter. Therefore, matter is impossible through mere attractive forces without repulsive [forces]. (510–11)

The point of this argument is at first sight quite puzzling, however. For, in the first explication of the Dynamics and accompanying remark Kant has already distinguished between filling a space [*einen Raum erfüllen*] and merely occupying a space [*einen Raum einnehmen*], and he has already reserved the former for the action of specifically repulsive forces. He has also, accordingly, defined an empty space as merely one that is

[132] The term "quantity of matter" initially occurs in the *Metaphysical Foundations* in the first remark to the fourth proposition of the Dynamics, where Kant is discussing the expansion and compression of (continuously distributed) matter in terms of the representation of contact as an infinitely small distance – "which must also necessarily be so in those cases where a greater or smaller space is to be represented as completely filled by one and the same quantity of matter, that is, one and the same quantum of repulsive forces" (505; see the paragraph to which note 104 above is appended for the full passage). So it is the first concept that is at issue here.

not filled.[133] It would seem, therefore, that the argument of the sixth proposition could be made much more briefly and simply: if there were no repulsive force then space could not be filled, and it would therefore be entirely empty of matter. So why does Kant resort to this more elaborate argument?

The answer, I believe, is that both fundamental forces are associated with the concept of quantity of matter and, in particular, that the fundamental force of attraction is *directly* associated with the concept in so far as attractive force is proportional to this quantity. Indeed, in the remark to the fifth proposition Kant has just alluded to Newton's procedure for determining the quantities of matter of the primary bodies in the solar system by means of precisely their attractive forces. It would therefore seem, in this context, that we could generate a notion of the quantity of matter in a given space through attractive force alone: simply suppose that the space in question is occupied by a distribution of attracting points, which, by Newton's procedure, thereby allow us to compute a determinate value for the quantity of matter in the space. Kant's present argument, on my reading, is intended to undermine this idea. For, according to Kant, since all the attracting points in such a distribution must also attract one another, the only stable configuration would be one in which they all coalesced into a single point (and therefore had no actual distances from one another). But this configuration, on Kant's view, would be one in which there is no quantity of matter in the space in question after all.

It is essential to Kant's present version of the dynamical theory of matter, in other words, that there can be no such thing as an isolated point-mass. Matter must rather be viewed as a true continuum distributed over all of the points in the space that it fills. Thus, quantity of matter is officially defined, in the Mechanics, as the total (infinite and continuous) "aggregate of the movable in a determinate space" (537). Moreover, it is essential to Kant's conception of the relationship between quantity of matter and the fundamental force of attraction that the latter is a penetrating force acting between each part of one matter and all parts of another – so

[133] For the distinction between filling and occupying a space see the passage from the remark to the first explication (497) quoted at the beginning of section 9 above. The first explication itself concludes (496; see the paragraph to which note 20 above is appended): "A space that is not filled is an *empty space*." At the end of the Dynamics Kant returns to the question of empty space and explicitly makes the point that a space in which there is merely attractive force is *not* filled (535): "Thus attractive force is attributed to matter in so far as it *occupies* a space around itself, through attraction, without at the same time *filling* this space – which space can therefore be thought as empty even where matter is active, because matter is not active there through repulsive forces and hence does not fill this space."

that, in particular, it is thereby proportional to the aggregate of the movable in the two respective spaces.[134] Newton's procedure in Proposition 8 of Book 3 accords with this conception, since, as we have seen, Newton there views mass or quantity of matter as continuously distributed over space and then justifies the idealization in terms of isolated point-masses by the theorem that a sufficiently uniform spherical mass distribution can be treated as if all of its mass were concentrated at the center. Kant, for his part, takes the continuous representation to be fundamental and, accordingly, definitively rejects the possibility of real isolated point-masses.[135] The balancing argument presented in the sixth proposition is therefore intended to show, on my reading, that even a putatively continuous distribution of merely attracting points in a given space cannot, by itself, determine a definite continuously distributed quantity of matter in this space. Unless the putative distribution in question is balanced by a counteracting expansive tendency, it reduces, by Kant's argument, to an isolated point-mass and therefore fails to be continuously distributed after all. So it follows, according to Kant's conception of quantity of matter as the aggregate of the movables in a given space, that there can be no quantity of matter in the space in question.[136]

Let us now turn to the first balancing argument expounded in the fifth proposition, which, as I have suggested, is connected with the dynamical concept of quantity of matter arising from a notion of density linked to the possibility of compression (or expansion) of a space filled with repulsive forces. The heart of the argument runs as follows:

[134] See again note 122 above, together with the paragraph to which it is appended. At the beginning of the general remark to dynamics Kant indicates the connection with the idea of quantity of matter as the aggregate of the movables in a given space by stating (524) that "attraction [as a penetrating force] rests on the aggregate of matter in a given space."

[135] Once again, the definitive rejection of this possibility occurs in the remark to the first proposition of the Mechanics, where Kant asserts that the quantity of matter must always be estimated by the quantity of motion and then adds (540) the qualification "of a body, for [the quantity of motion] of a point consists merely in the degree of the velocity" – which implies that a single moving point cannot have a momentum. Note that this view stands in sharp opposition to that of the *Physical Monadology*, according to which the notions of mass and impetus (momentum) attach to the central point of any element entirely independently of the spatial volume determined by the two fundamental forces of attraction and repulsion. I shall return to this last point in my chapter on the Mechanics.

[136] As we shall see, the general remark to phenomenology develops three different concepts of *empty space*: a phoronomical, dynamical, and mechanical concept (563–64). The second is defined in terms of the dynamical filling of space through repulsive force, whereas the third is defined in terms of mass or quantity of matter. A space in which there is no definite quantity of matter is therefore empty in the mechanical sense, and it is precisely this that has now been shown for the case of a putative distribution of merely attracting points in a given space. By contrast, the notion of empty space figuring in note 133 above is the dynamical concept.

> Impenetrability, as the fundamental property of matter whereby it first manifests itself to our outer senses as something real in space, is nothing but the expansive power of matter (Proposition). Now an essential moving force, whereby the parts of matter flee from one another, cannot, *in the first place*, be limited by itself, for matter is thereby striving instead continuously to enlarge the space that it fills; *in the second place*, [such a force] can also not be determined by space alone to a certain limit of extension, for the latter, although it can certainly contain the ground for the expansive force becoming weaker in inverse proportion to the increase of volume of an expanding matter, can never contain the ground for this force ceasing anywhere, because smaller degrees are possible to infinity for any moving force. Hence matter, by its repulsive force (containing the ground of impenetrability), would, [by itself] alone and if no other moving force counteracted it, be confined within no limit of extension, that is, it would disperse itself to infinity, and no specified quantity of matter would be found in any specified space. Therefore, with merely repulsive forces of matter, all spaces would be empty, and thus, properly speaking, no matter would exist at all. (508)

This argument runs parallel to the converse argument expounded in the sixth proposition. If we imagine a distribution of merely repelling points in a given space, then, because of the repulsive force acting between these points themselves, it would expand itself indefinitely. The only stable configuration of such points, therefore, would be the limiting configuration, as it were, in which the space in question were expanded to infinity (and thus no further expansion is possible). In this configuration, however, the density would be everywhere zero and thus "no specified quantity of matter would be found in any specified space." The present argument, however, is considerably more complicated in its details than the converse argument.

The first important difference is that repulsive force, unlike attractive force, does not act between each part of the distribution in question and all other parts. Repulsive force acts only between adjacent parts that are actually in contact, and, for precisely this reason, it must act at an infinitely small rather than a finite distance. Repulsion in the *Metaphysical Foundations* is not a (Newtonian) action-at-a-distance force, and so it actually has no (finite) spread over space at all. Moreover, this is an essential part of Kant's radically new version of the dynamical theory of matter as a true continuum, based on his definitive rejection, in the preceding fourth proposition of the Dynamics, of the atomism of discrete force-centers articulated in the *Physical Monadology*.[137] In the present argument, therefore, Kant does not represent the repulsive force between adjacent parts

[137] See again the quotation from the first remark to the fourth proposition in the paragraph to which note 103 above is appended. The present rejection of his earlier atomism of discrete point-centers is necessary for Kant's conclusion that the indefinitely expanded distribution in

as a function of distance but immediately moves to a consideration of the expansive force of the distribution as a whole – which, according to the second proposition, arises from "the repulsive forces of all of its parts" (499).[138] It is this expansive force or *pressure* that depends on a function of the (finite) distance – namely, on the "volume of an expanding matter" (508) – and, accordingly, may become weaker as the total volume of the distribution increases. Indeed, Kant's implicit reference, once again, to the Boyle–Mariotte law reminds us that the original or fundamental state of the matter that fills its space through repulsive force is that of an originally fluid and elastic medium. So Kant's point, at this stage of the argument, is that such an originally fluid and elastic medium would, in the absence of a counteracting compressive force, disperse itself indefinitely in virtue of precisely its own internal (expansive) pressure.[139]

The notion of internal (expansive) pressure is then associated, in turn, with a notion of density: in general, the more an elastic fluid is compressed, the greater is its internal pressure, and so the greater, in this sense, is the density of the fluid. So it is density, in precisely this sense, that provides Kant with what he calls (521) the "intensive measure" of the filling of space. Thus, in the general remark to dynamics Kant states (525) that "[t]he degree of the filling of a space with determinate content [volume] is called *density*," and he then points out (525–26) that this notion "in the dynamical system of merely relative impenetrability" is only suitable "for thinking a ratio of matters with respect to their density if we … imagine them as specifically of the same kind, so that one can be generated from the other by mere compression."[140] In the first remark to the

question would have zero density *everywhere*. For, if we imagine a distribution of discrete centers of repulsive force acting at a *finite* distance, then, no matter how far apart such centers become from one another (so that the overall or average density of repulsive force goes to zero), each such center still has the same finite degree of repulsive force in the immediately surrounding small region.

[138] See the paragraph to which note 48 above is appended. Thus the "Proposition" cited in the first sentence of our passage (508) is the second proposition of the Dynamics.

[139] Kant, as we know, will eventually make it clear that the Boyle–Mariotte law holding for atmospheric air does not actually hold for the state of *originally* elastic fluidity. Nevertheless, it follows from the third proposition of the Dynamics that, although the expansive pressure of the state of originally elastic fluidity may not decrease *linearly* with increasing volume (in accordance with the Boyle–Mariotte law), it still decreases monotonically and without limit. Kant's point in the text therefore still holds.

[140] This discussion occupies the first number of the general remark to dynamics, the beginning of which is quoted in note 117 above. The above-quoted definition of *density* immediately follows the first sentence of this number, which concerns the notion of "**space-content** [***Raumesinhalt***] (*volumen*)". This is why I have inserted "volume" in brackets after "determinate content [*bestimmten Inhalt*]" in this quotation.

eighth proposition Kant supposes that the outward expansive pressure of matter (in its originally fluid and elastic state) is balanced by a compression due to the fundamental force of attraction, so that (521) the "degree of compression is [thereby] determined which constitutes the measure of the intensive filling of space."[141] Thus, it is precisely this notion of density or pressure that Kant here associates with the quantity of matter when he asserts that, in a state of everywhere zero density in this sense, "no specified quantity of matter would be found in any specified space" (508). Indeed, at this stage of the argument, Kant simply equates the quantity of matter in a given space with the degree of expansive pressure exerted within it: the greater the state of compression of the matter in a given space, the greater the quantity of matter there.[142]

We have now arrived, however, at the second and perhaps most important difference between the present argument and the converse argument for the case of merely attracting points. In the latter argument the connection with quantity of matter in the sense of mass was already clear, in so far as Newton's procedure for estimating the masses of the primary bodies in the solar system by the gravitational attractions on their satellites forms an essential part of the backdrop to the sixth proposition. In the fifth proposition, by contrast, neither the dynamical concept of quantity of matter associated with universal gravitation nor the mechanical concept of quantity of matter associated with momentum and the communication of motion is yet available. The only concept we have so far is

[141] This assertion is the culmination of an attempted "mathematical construction" of the degree of filling a space corresponding to the balancing construction in the Scholium to Proposition x of the *Physical Monadology*: see again the paragraph to which note 128 above is appended. Thus, the second number of the present remark presents the inverse-square law of attraction, the third number the inverse-cube law of repulsion (for *infinitely small* distances), and the fourth suggests a balancing of the two. I shall discuss this attempted "construction" further in section 19 below. It is already clear, however, that the version in the *Metaphysical Foundations* is concerned with a true continuum view of matter and, accordingly, that the outcome of such a balancing is supposed to be a "measure of the intensive filling of a space" rather than a determinate *spatial volume* for small elastic corpuscles: compare note 62 above, together with the paragraph to which it is appended.

[142] In accordance with the discussion from the first number of the general remark quoted above (525–26), this particular comparison can take place only between matters of the same kind. As we saw in note 132 above, Kant, in the context of the only occurrence of the term "quantity of matter" prior to the present one, considers (505, emphasis added) how a "greater or smaller space is to be represented as completely filled by one and the same quantity of matter, *that is, one and the same quantum of repulsive forces*." Since, according to the second proposition, the expansive force of a matter is given by the repulsive forces of all of its parts (499), the quantum of repulsive forces in a given space is equal to the quantum of expansive force of matter within that space. Compare also the discussion in the remark to the third proposition, which speaks (501) of a given "quantum of [expansive or elastic] force."

a "measure of the intensive filling of space" (521) by repulsive force, based on the degree of compression of an intrinsically expansive medium – so that, in particular, we can say that there is "more" quantity of matter in a given space when "more" (that is, an initially greater volume) of the medium has been compressed into this space. But what is the real connection, we now need to ask, between this concept and the concept of quantity of matter in the more standard sense?

The connection, at this stage, is mediated by Kant's own evolving discussion of the concept of *material substance* in space. The fifth explication of the Dynamics, as observed in section 13 above, characterizes material substance (502) as "a thing in space that is movable for itself, i.e., separated from everything else existing outside of it in space." The remark to this explication states (503) that "[t]he concept of a substance signifies the ultimate subject of existence, that is, that which does not itself belong, in turn, to the existence of another thing merely as predicate." These explications, against the background of the view of expansive force developed in the second and third propositions, then figure essentially in the proof of the infinite divisibility of material substance in the fourth proposition. In particular, since the "original expansive force" of matter "is only the consequence of the repulsive forces of each point in a space filled with matter" (503), we can conclude that *every* part of a space filled with matter is movable in itself and therefore counts as material substance in turn. Precisely because all the points in the space in question exert repulsive forces, giving rise to an expansive force or pressure throughout the material continuum that fills the space, each and every part of this space contains material substance – which is therefore not only mathematically but also physically infinitely divisible.[143]

As we have seen, however, Kant officially characterizes the concept of quantity of matter in the second explication of the Mechanics as the (infinite and continuous) "aggregate of the movable in a determinate space" (537). Moreover, in the remark to the following first proposition (540–41) Kant explains that "the quantity of matter is the *quantity of substance* in the movable" and that this rests, in turn, "on the concept of [substance] as the *ultimate subject* (which is no further predicate of another)." Kant also explains (541) that "the *proper motion* of matter is a predicate that determines its subject (the movable), and indicates the plurality of moved subjects in a matter, as an aggregate of the movable." Thus,

[143] See note 78 above, together with the paragraph to which it is appended; compare also note 61 above, together with the paragraph to which it is appended.

although we are not yet in a position to appreciate the full force of these ideas, we can discern the outlines of the connections Kant intends to establish between quantity of matter in the more standard sense and the action of the fundamental expansive force arising from the repulsion of all the parts of a matter. For it is precisely this expansive force, according to the fourth proposition of the Dynamics, that entails that each part of a space filled with matter is movable in itself and, at the same time, thereby "indicates the plurality of moved subjects in a matter, as an aggregate of the movable." In particular, whenever we have a given space filled with a determinate quantum of expansive force, we thereby have a determinate aggregate of the movable and thus quantity of matter (in the mechanical sense) as well. Hence, when this determinate aggregate of the movables is compressed into a smaller space, the very same quantity of matter – the very same *quantity of substance* – is necessarily found there.[144]

In sum, the connection between the mechanical concept of quantity of matter associated with inertia and momentum, on the one side, and the dynamical concept of quantity of matter associated with repulsive force and the possibility of compression, on the other, is mediated by the concept of the aggregate of the movable in a given space, for Kant – just as much as in the parallel case of the dynamical concept of quantity of matter determined by universal attraction. In both cases we begin with the idea of a continuously distributed collection of points in a given space, all exerting the fundamental force in question, and this collection (as an aggregate of the movable) is then supposed to represent a determinate quantity of matter in that space. Yet in order to do so, it turns out, we must also assume, in both cases, a counterbalancing tendency due to the other fundamental force. A collection of merely attracting points would coalesce into a single point-mass, and there would therefore be no *aggregate* of the movable in the given space after all. A collection of merely repelling points would expand itself indefinitely into a condition of maximal decompression, as it were, in which there would be no internal pressure and therefore a state of zero density everywhere. Therefore, once again, "no specified quantity of matter would be found in any specified

[144] The importance of the concept of substance here is also suggested in the passage from the remark to the fifth proposition discussed at the end of section 15 above, where Kant explains why many (mechanical) natural philosophers have found attraction so difficult to accept. In particular, while the passage from the proof of the fifth proposition that we are now considering says that impenetrability is the fundamental property of matter "whereby it first manifests itself to our outer senses as something real in space" (508), the corresponding passage in the remark says (510, emphasis added) that "*substance* discloses its existence to us in no other way than through that sense whereby we perceive its impenetrability."

space" (508), for the putative finite quantity of matter (as an aggregate of the movables in a given space) is now uniformly distributed over *infinite* space.

Here, however, we encounter a third important difference between the two complementary versions of the balancing argument. For, in the case of a merely expansive distribution of matter considered in the fifth proposition, Kant needs to reckon with a possibility that does not arise in the converse case of a merely attracting distribution. It appears to be possible, in particular, that the expansive tendency of matter in a given space may be counterbalanced by an opposing expansive tendency of other matter located outside this space.[145] Immediately following the passage that we have been examining, therefore, Kant adds some further discussion explicitly considering (and rejecting) this possibility:

> So all matter requires for its existence forces that are opposed to the expansive forces, that is, compressing forces. But these, in turn, cannot originally be sought in the contrary striving [*Entgegenstrebung*] of another matter; for this latter itself requires a compressive force in order to be matter. Hence there must be assumed somewhere an original force of matter acting in the opposite direction to the repulsive force and thus to produce approach, that is, an attractive force. (508–9)

According to this passage, which has no counterpart in the argument of the sixth proposition, anything exerting the required counterbalancing force would have to be matter already, and, as matter in general, some explanation would still be required for how it could fill its space to a determinate degree – that is, with a determinate quantity of matter. The required explanation must therefore ultimately appeal to a further fundamental force of matter: an original force of attraction.[146]

Kant is quite indefinite about the action of the fundamental force of attraction here. Whereas, in the converse case of initially merely attracting points, it is clear that the expansive tendency generated within this

[145] Ordinary cases of expansive fluids are standardly considered as enclosed within vessels, as in the case of atmospheric air enclosed within the barrel of an air pump that Kant discusses in the remark to the third explication (500). Here the impenetrability of the walls of the container provides the necessary counterbalancing force.

[146] Why can the matters in question not mutually determine their degrees of compression simultaneously – in the way, for example, that Cartesian vortices mutually limit one another by the action of their centrifugal forces? The problem with such an explanation, on my reading, is that the concept of centrifugal force already presupposes the concept of mass and, indeed, the concept of inertia that will only be developed in the Mechanics. For the same reason it would not do, at this stage of the argument, to attempt to counterbalance the centripetal tendency of the merely attracting matter considered in the sixth proposition of the Dynamics by an opposing centrifugal tendency of this same matter arising from rotation.

distribution (due to the repulsion of these same points) is quite sufficient to counterbalance its own compressive tendency (due to their attraction), Kant deliberately does not make the parallel claim in the present case and says only that an attractive force "must be assumed *somewhere*" (509, emphasis added).[147] Indeed, in the second note to the eighth proposition Kant explicitly leaves it open whether the counterbalancing compressive tendency arises from the internal parts of the matter in question or rather from some kind of interaction with *external* matter:

> Since every given matter must fill its space with a determinate degree of repulsive force, in order to constitute a determinate material thing, only an original attraction in conflict with the original repulsion can make possible a determinate degree of the filling of space and thus matter; it may now be [the case] that the former derives from the individual attraction of the parts of the compressed matter among one another, or from the uniting of this with the attraction of all cosmic matter [*aller Weltmaterie*]. (518)

In the general remark to phenomenology, finally, Kant indicates that his own view, taken as a physical hypothesis, is that internal expansive force is typically counterbalanced by external pressure. In the context of addressing "the mystery of nature, difficult to unravel, as to how matter sets limits to its own expansive force" (564) Kant cites his own previous discussion of an aether pervading the cosmos at the end of the general remark to dynamics.[148] Moreover, he also suggests, several lines earlier, that (564) the action of this all-pervasive medium is "a *compression* by external matter (the aether) distributed everywhere in the universe, which is itself brought to this pressure only by a universal and original attraction, namely, gravitation." Kant's view of how matter in fact sets limits to its own expansive force (taken as a physical hypothesis) therefore appears to be that the pressure of an external aether – a pressure that is itself due

[147] According to the third proposition of the Dynamics no complete penetration of matter by a finite compressive force is possible, since the original expansive force of matter is exerted ever more strongly with increasing compression without limit. By contrast, the (gravitational) attractive forces among the parts of an elastic fluid are typically much too weak to counterbalance its own expansive tendency – and the more the matter is expanded the weaker this attractive force becomes.

[148] In the general remark to dynamics the aether is introduced (534) as a matter "that completely fill[s] its space without any emptiness, and yet with an incomparably smaller quantity of matter, at the same volume, than any bodies we can subject to our experiments." In the general remark to phenomenology (564) this same aether therefore offers no (appreciable) resistance to (564) the "free and enduring motion of the heavenly bodies" because (564) its "resistance, even in spaces completely filled, can still be thought as small as one likes." Here "resistance" means mechanical resistance in the sense of mass, since, as Kant says (564), he is here considering the possibility of "empty space in *the mechanical sense*": compare note 136 above.

to universal gravitation – typically effects the necessary compression. So, in the end, it is precisely the pressure exerted by an all-pervasive originally elastic medium that is responsible for a body's filling the space it occupies to a determinate degree.[149]

17 TRUE AND APPARENT ATTRACTION: AETHER, LIGHT, AND COSMOS

Kant's view of the way in which compression in an elastic medium may be generated by the action of universal gravitation takes its starting point from a then well-known hydrostatic model of the atmosphere.[150] On this model atmospheric air is an elastic fluid whose internal expansive force or pressure is governed by the Boyle–Mariotte law and, at the same time, also depends on heat. Under the action of the earth's gravity, however, the atmosphere acquires a contrary (compressive) pressure or weight, which, in a state of equilibrium, must precisely balance the expansive pressure at each point. Moreover, this weight or (compressive) pressure depends on the height above the earth's surface: the higher the region of the atmosphere under consideration, the smaller is the weight of the air. It is necessary for a state of equilibrium, then, that the air form concentric layers above the earth's surface – where, at equal distances from the surface, all points of a given layer have the same pressure, and, as the distance increases, the pressure decreases accordingly. The temperature must also be the same at all points of a given layer, for otherwise the expansive force would differ at different points and no longer precisely equal the compressive force: any difference of temperature within a given layer will therefore result in a motion of the air within it, that is, a wind. Thus, both the theory of the barometer and the theory of the winds are based on this general hydrostatic model of atmospheric air in a state of equilibrium.

[149] Immediately before the last quoted passage Kant introduces (563) the problem of "the possibility of the composition [*Zusammensetzung*] of a matter in general … if only this were better understood." His (hypothetical) answer, however, is geared to "the explanation of the cohesion [*Zusammenhang*] of matter" (563). But cohesion, for Kant, is not a truly universal property of matter in general but only of matter in the liquid or solid states: see note 57 above. Thus the limitation of the expansive force of matter in the gaseous or elastic fluid state will, in general, be effected differently. In the case of the aether itself, for example, expansive pressure is counterbalanced by a compression produced directly by attraction. There is more on this below.

[150] This model is developed, for example, in Euler (1755) as the first application of a more general theory of the equilibrium of fluids. Although we do not know for certain whether Kant was acquainted with this particular work (see note 66 above), the "*Vorerinnerung*" to his *Theory of the Winds* of 1756 (1, 491–92) shows that he was familiar with the model.

As observed in section 12 above, it was also standard at the time to view all of space as pervaded by a very rare subtle fluid or aether, which, depending on the circumstances, is responsible for such phenomena as light, heat, electricity, magnetism, and so on. This aether was often conceived, in addition, on analogy with atmospheric air. Euler's wave theory of light developed this analogy in an especially detailed fashion. Just as sound is a (transverse) vibration or pulsation propagated through the air, light is precisely the same kind of vibration propagated through the aether; just as air vibrations can occur at different frequencies giving rise to sounds of different pitch, aether vibrations can occur at different frequencies giving rise to light of different colors; and so on.[151] In his own (hypothetical) explanation of the compressive force responsible for the cohesion of bodies Kant extends this analogy further. The bodies in the universe are dispersed within an all-pervasive aether, which, due to the mutual gravitational attraction of all of these bodies, constitutes a sort of great aetherial "atmosphere" surrounding them. This "atmosphere" thereby acquires a corresponding (compressive) pressure, which, in particular, precisely counterbalances the aether's own inherent expansive pressure (at lesser degrees at greater distances from the source of gravitational force). The resulting inwardly directed weight of the aether counterbalances the expansive tendency of bodies due to their own repulsive force and conserves them, as Kant says, in a state of determinate density.[152]

Kant develops the analogy between the compressive force of atmospheric air as the medium of sound and that of an all-pervasive aether acting as the medium of light in his lectures on theoretical physics from the summer of 1785 (from the same time, therefore, as the *Metaphysical Foundations*). The notes from these lectures, collected as the *Danziger Physik*, describe this analogy in a section on light and colors. After characterizing sound and tones in terms of vibrations in atmospheric air in the

[151] See, for example, Euler (1768–72, letters 19–28). Kant, as we know, was certainly familiar with Euler's theory of light, and he consistently adhered to it throughout his career: see notes 51, 55, and 64 above.

[152] See again the passage from the end of the Phenomenology cited above where Kant explains how there can be no empty space outside the world:

> An empty space *outside the world*, understood as the totality of preeminently attractive matters (the large heavenly bodies), would be impossible for precisely the same reasons [as in the explanation of cohesion mentioned in the paragraph to which note 149 above is appended – MF], since, in accordance with the measure that the distance from them increases, the attractive force on the aether (which encloses all these bodies, and, driven by that force, conserves them in their density by compression) decreases in inverse proportion, so that the latter would itself only decrease indefinitely in density but nowhere leave space completely empty. (564)

preceding section, the notes assert that light and colors are "very similar to sound and tones" and continue:

Light is a similar material to air, only much finer and rarer, because light is propagated so quickly. But the aether must also be elastic to a high degree – it must have [a] force that conserves tension [*Spannung*] within it. The action of the united attraction of all parts of the earth is gravity [*Schwere*], and this is the cause of compressive force. In the case of the aether the cause is also gravity and, in fact, the universally united attraction of all heavenly bodies – this is Euler's theory. Newton did not believe that light is a vibration in an elastic matter, because the existence of light as a particular matter was not yet proved. (29, 146)

Thus, just as atmospheric air is conserved in its state of pressure or tension by the gravity of the earth, the aether is conserved in a corresponding state of pressure by the combined gravitational attraction of all the heavenly bodies together.[153] It is precisely this latter state of pressure or tension in the aether that is responsible for the cohesion of bodies according to the *Metaphysical Foundations*.[154] So it is by no means surprising that the present section of the *Danziger Physik* suggests that the most important objection to the opposed Newtonian theory of light is its inability to explain the phenomenon of cohesion (152): "Here [in the Newtonian theory] there must therefore be immeasurable empty spaces, and here one cannot conceive how bodies are supposed to cohere."

Kant's view of the relationship between universal attraction and an all-pervasive aether – functioning as both the medium for the propagation of light and the ground of the cohesion of bodies – appears to be as follows. On the one hand, universal attraction is a genuine action-at-a-distance

[153] The preceding section developing the theory of sound as vibrations in atmospheric air (29, 146–47) is itself preceded by an earlier section (128–32) in which the elements of hydrostatics are developed under the heading of "weight [*Schwere*]."

[154] Compare again the passage from the second note to the eighth proposition of the Dynamics quoted at the end of section 16 above, where Kant asserts that the counterbalancing compressive tendency of matter (518) may derive "from the uniting of this [the body's own attraction] with the attraction of all cosmic matter [*aller Weltmaterie*]." That this (second) alternative is the preferred one is suggested by the immediately following statement that "the determinate filling of a space by matter in accordance with its measure can, in the end, only be effected by the attraction extending to infinity and imparted to every matter in accordance with the measure of its repulsive force" (518; compare the passage quoted in note 152 above). In remarks to a letter from Jacob Sigismund Beck of September 8, 1792 Kant also leans towards this second alternative, according to which the limitation of the expansive force of matter derives from the external pressure of "a certain original rarity of the universe [*einer gewissen ursprunglichen Dünnigkeit des Universum*]" (11, 362) – which pressure itself derives from the attraction "of the whole universe" (365).

force acting immediately through empty space entirely independently of any and all matter that may lie in between the attracting bodies. This point is strongly emphasized in the very section of the Dynamics we are now considering. The sixth explication defines contact as "the immediate action and reaction *of impenetrability*" and states that "the action of one matter on another outside of [the condition of] contact is *action at a distance* (*actio in distans*)" (511). Kant continues (511–12): "This action at a distance, which is also possible without the mediation of matter lying in between, is called immediate action at a distance, or also the *action* of matter on another *through empty space*." Moreover, the following seventh proposition (whose proof I shall examine in the next section) states (512): "The *attraction essential to all matter* is an immediate action of it on others through empty space."[155]

On the other hand, however, Kant is also committed to the existence of an all-pervasive space-filling aether – which, following Euler and in opposition to Newton, is identified with the medium of light. Although universal attraction is independent of the space-filling aether that in fact pervades the universe, the space through which this force acts at a distance is by no means literally empty. Universal attraction, in the terminology of the seventh explication, is a *penetrating* rather than a contact or *surface* force:

> The repulsive force whereby matter fills a space is a mere surface force, for the parts in contact mutually limit their spaces of action; repulsive force cannot move a part at a distance without the mediation of those lying in between, and an immediate action, passing straight through the latter, of one matter on another by expansive forces, is impossible. By contrast, no intervening matter sets limits to the action of an attractive force, whereby matter occupies a space *without filling it*, so that it thereby acts on other distant matter *through empty space*. Now the original attraction, which makes matter itself possible, must be thought in this way, and it is therefore a penetrating force, and for this reason alone it is always proportional to the quantity of matter. (516)[156]

As Kant puts it in the proof of the following eighth proposition (517): "[T]his [original] attraction is a penetrating force and acts *immediately* at

[155] As observed in note 133 above, Kant returns to this point at the end of the Dynamics (535) by emphasizing that a body *occupies* the space around itself through attractive force without at the same time *filling* this space.

[156] This passage is the continuation of that quoted in note 114 above; see also the paragraph to which this last note is appended. I touched on the relationship between the circumstance that universal attraction is a penetrating force and its proportionality to quantity of matter in the preceding section (see note 134 above, together with the paragraph to which it is appended), and I shall return to this relationship in what follows.

distance, regardless of all matter lying in between, through this space [of its activity], as an empty space."

Nevertheless, although universal gravitation acts independently of the space-filling aether, the properties of the aether depend on the action of universal attraction. In particular, the aether is brought into a state of (compressive) pressure by the action of universal gravitation proceeding throughout all of cosmic space from the "preeminently attractive matters (the large heavenly bodies)" (564; see note 152 above). Moreover, it is precisely this (compressive) pressure, in equilibrium with the internal (expansive) pressure of the aether due to its own repulsive force, that is responsible for both the state of elasticity in virtue of which light vibrations or pulsations are (very rapidly) propagated within the aether and the state of cohesion (and therefore definite density) of the "preeminently attractive matters (the large heavenly bodies)." The heavenly bodies exert attractive forces on one another in proportion to their varying quantities of matter, and they do so entirely independently of an action of the intervening aether. But the aether acquires its state of (compressive) pressure from the action of universal gravitation – a pressure without which the heavenly bodies that are the sources of universal gravitation could not maintain their definite quantities of matter in the first place. Finally, this same state of (compressive) pressure enables the aether, as a highly elastic medium, to function as the vehicle of light propagation, and it is this propagation alone that allows us to achieve perceptual contact with the heavenly bodies so as thereby to infer the existence and properties of universal gravitation from their motions.

As explained in section 15 above, Kant holds that repulsive force is epistemically prior to attractive force, because the former is the basis for our first application of the categories of quantity to matter as we become acquainted with the volumes, figures, and (relative) spatial positions of bodies through the "sense of feeling [*Sinnes des Gefühls*]" (510; see the paragraph to which note 116 above is appended). We then arrive at universal attraction via the argument of Book 3 of the *Principia* by inferring the existence and properties of universal gravitation from the observed relative motions of the heavenly bodies. But these latter observations, in turn, are possible only in virtue of the propagation of light in the all-pervasive aether, for it is only in this way that we can actually attain perceptual contact with the volumes, figures, (relative) spatial positions, and (relative) motions of the heavenly bodies.[157] Such perceptual contact is mediated by repulsive force (i.e., by the pressure of the aether as the light vibrations in question are communicated to our sense organ), just as the more immediate perceptual contact with the bodies in our terrestrial environment is effected through the sense of touch (*Betastung*).[158] In this way, once again, we undertake a transition from a direct perceptual application of the categories of quantity to objects in our immediate terrestrial environment (as, for example, we handle the objects around us and become acquainted with their weights through the downward pressure of gravity) to a less direct application of these same categories to distant objects in the cosmic system (as, for example, we optically determine the relative motions of the heavenly bodies and infer their masses by the law of universal gravitation).[159]

[157] See note 120 above, and compare Kant's discussion of the sense of sight in the *Anthropology*:

> The sense of sight is also [like hearing] a sense of *mediate* sensation by means of a moved matter sensible only for a certain organ (the eye), [i.e.,] by means of *light*, which is not, like sound, merely a wave motion of a fluid element, which is propagated towards all sides in the surrounding space; rather [it is] a radiation, through which a point is determined for the object in space, and by means of which the cosmic system [*Weltgebäude*] becomes known to us to such an immeasurable extent that, especially in the case of the self-illuminated heavenly bodies, if we compare their distance with our measuring rods here on earth, we grow weary of the series of numbers, and we thereby have almost more reason to be astonished by the delicate sensitivity of this organ with respect to the perception of such weak impressions than by the magnitude of the object (the cosmic system), especially if we add to this the cosmos [*Welt*] in the small, as it is presented to the eye by the microscope, e.g., in the case of infusoria. (7, 156)

[158] In the *Anthropology* Kant describes the sense of touch [*der Sinn der Betastung*] as (7, 154–55) lodged in "the finger tips and their nerve endings (*papillae*), in order to collect information about the figure of a solid [*festen*] body by means of contact with its surface … [; t]his sense is also the only one of *immediate* outer perception." In the following section on sight Kant compares the two senses as follows:

> The sense of sight, if not as indispensable as that of hearing, is still the noblest [sense]: because it is the most distant from that of touch [*Betastung*], as the most limited condition of perceptions, and it does not only contain the largest sphere of [perceptions] in space but also feels the affection of its organ least [*sein Organ am wenigsten afficiert fühlt*] (since otherwise it would not be mere seeing), and it thereby comes closer to a *pure intuition* (the immediate representation of the seen object without a mixture of noticeable sensation). (7, 156)

[159] Compare notes 121 and 125 above, together with the paragraphs to which they are appended. I suggest, at the end of the second of these paragraphs that Newton's argument for universal gravitation enables us to "move from an application of mathematics to the bodies of our terrestrial experience mediated primarily by the force of impenetrability (and therefore by the sense of touch) to a much more extensive application of mathematics to the bodies of both celestial and terrestrial experience mediated primarily by the force of attraction (and therefore independently of the sense of touch)." We now see that Newton's argument involves repulsive force as well (at least implicitly), in so far as it relies on our optical acquaintance with the heavenly bodies via light propagated through the aether, but it is nonetheless independent of the sense of *touch* in the sense of the preceding note.

Kant emphasizes the importance of light for our perception of the heavenly bodies – together with the relationship between light and universal gravitation – in the third analogy of the *Critique of Pure Reason*. After appealing (in the second edition) to a mutual interaction between the earth and the moon as the ground for our perception of their simultaneity (B257–58), Kant remarks that such dynamical community can be either "immediate" or "mediate" (A212–13/B259). He then gives the following example of a *mediate* dynamical community:

> It is easy to notice in our experience that only the continuous influences in all positions of space can guide our sense from one object to another, that the light, which plays between our eyes and the heavenly bodies, effects a mediate community between us and them and thereby proves their simultaneity, [and finally] that we cannot empirically change our place ([i.e.,] perceive this change) without matter everywhere making it possible for us to perceive our position, and only this [matter], by means of its mutual influence, can verify their simultaneity [that of the heavenly bodies – MF] and therefore their coexistence, all the way to the most distant objects. (A213/B260)

Thus, whereas the universal gravitational attraction between the heavenly bodies (between the earth and the moon, for example) effects an immediate dynamical community between them, it is the mediate community effected by light propagated through the aether that puts us into perceptual contact with the heavenly bodies in the first place.[160]

[160] Although there is very little explicit discussion of light in the *Metaphysical Foundations*, there is, as observed in note 64 above, one such discussion in an important footnote to the first remark to the eighth proposition of the Dynamics. Here Kant considers Euler's wave theory and the standard objection that light, like sound, should therefore spread out continuously within the medium of its propagation (the aether) and therefore fail to be propagated rectilinearly. This discussion echoes the treatment in the *Danziger Physik* considered above, where the objection is repeatedly brought forward as the most important obstacle faced by Euler's theory. Euler (1746a, §§10–15) had replied to this objection by arguing that in the experiment in which sound enters a box through an aperture it is actually propagated rectilinearly, just as in the parallel case of light; the reason sound nevertheless spreads throughout the box (unlike light) is that it is also communicated through the walls. For discussion see, e.g., Cantor (1983, pp. 117–23). The *Danziger Physik* (29, 151–52) expresses dissatisfaction with this explanation on the ground that light should be communicated through walls (in the all-pervasive, all-penetrating aether) at least as easily as sound. In the corresponding footnote in the Dynamics, therefore, Kant rather suggests that light is propagated rectilinearly in its medium because the aether (520) is "originally" fluid all the way down, as it were, "without being divided into solid [*feste*] corpuscles." By contrast, as Kant explains in the second remark to the eighth proposition (522; see the paragraph to which note 49 above is appended), atmospheric air (the medium of sound) is not a genuinely "original" elastic fluid in so far as it actually consists of proper air particles dispersed or dissolved in the matter of heat. (According to modern wave theory, both sound waves and light waves have an inherent tendency to spread out continuously in space. Rectilinear propagation results from interference effects when a wave passes through an aperture of the appropriate dimensions relative to its wave-length. Since the wave-length of light is much smaller, however, a much larger aperture – such as a large canyon – is needed in the case of sound.)

Now cohesion, according to the second number of the general remark to dynamics, is defined (526) as "*attraction, in so far as it is merely thought as active in contact.*" This type of attraction (526) is "not [a] penetrating but only a surface force," so that "this attraction in contact is no fundamental force of matter, but only a derivative [force]." Moreover, as Kant suggests in the second remark to the seventh proposition, such attraction in contact should thus be characterized as merely apparent (514): "One can call that attraction which takes place without mediation of repulsive forces *true* attraction, that which occurs merely in this [latter] way *apparent* [attraction]; for, properly speaking, the body towards which another is striving to approach merely because it is driven towards [the body] by impact exerts no attractive force at all on the [other]." Merely apparent attraction – attraction in contact – is therefore mediated by repulsive forces and is not, like true attraction, the direct effect of an immediate action at a distance.[161]

Kant's preferred physical explanation of cohesion is thereby linked with apparent rather than true attraction. The discussion from the end of the Phenomenology, where Kant presents this explanation (see notes 149 and 152 above), runs (in full) as follows:

> For, if the *attraction* that one assumes for the explanation of the cohesion of matter should be merely apparent, not true attraction, but rather, say, merely the effect of a *compression* by an external matter distributed throughout the universe (the aether), which is itself brought to this pressure only by a universal and original attraction, namely, gravitation (which view has many reasons in its favor), then empty space within matters would be, although not logically, still dynamically and thus physically impossible – for this matter would itself expand into the empty spaces that one assumed within it (since nothing would resist its repulsive force here) and would always conserve them as filled. An empty space *outside the world*, understood as the totality of preeminently attractive matters (the large heavenly bodies), would be impossible for precisely the same reasons, since, in accordance with the measure that the distance from them increases, the attractive force on the aether (which encloses all these bodies, and, driven by that force, conserves them in their density by compression) decreases in inverse

[161] Compare the discussion of cohesion versus gravitational attraction in the second number of the general remark to dynamics:

> Cohesion is customarily taken for a completely universal property of matter, not as if one were already led to it by the concept of a matter, but because experience verifies it everywhere. But this universality must not be understood *collectively*, as if every matter by this kind of attraction *simultaneously* acted on every other in the universe – gravitation is of this kind – but rather merely *disjunctively*, namely, [it is exerted – MF] on one or the other, whatever kind of matter it may be, that comes into contact with it. (526)

proportion, so that the latter would itself only decrease indefinitely in density but nowhere leave space completely empty. (563–64)

The action of repulsive forces – the pressure of an external aether – is what ultimately explains the cohesion of bodies (i.e., resistance to the separation of their parts: see 527) and, accordingly, the circumstance that they have determinate densities. But only a true attraction acting through empty space can explain the external compression itself.

In the second remark to the seventh proposition of the Dynamics Kant makes a connection between these physical considerations and the balancing argument of the fifth proposition. Immediately before he first introduces the distinction between true and apparent attraction Kant explains (514) that "some or another immediate attraction outside the [condition of] contact, and therefore at a distance, must still be found; for otherwise even the forces of pressure and impact that are supposed to bring about the striving to approach, since they act in the opposite direction to the repulsive force of matter, would have no cause, or at least none originally lying in the nature of matter." Immediately after the passage on true and apparent attraction Kant continues (514): "But even this apparent attraction must finally have a true [attraction] as its ground, for the matter whose pressure or impact is supposed to serve instead of attraction would not even be matter without attractive forces (Proposition 5), and therefore the mode of explanation of all phenomena of approach by *merely apparent* attraction revolves in a circle." Kant's preferred physical realization of the balancing argument presented at the end of the Phenomenology is the hydrostatic model of the aether he inherits from Euler. A balance between the (internal) expansive pressure of the aether and a corresponding compressive pressure is effected by the action of the universal gravitation exerted by the large heavenly bodies on the aether – and thus by a true attraction. It is in virtue of precisely this balancing of forces, in particular, that the aether itself has a determinate density. The (internal) expansive tendency inherent in the more ordinary matter distributed or immersed within the aether (including that of the large heavenly bodies) is then counterbalanced, for Kant, by the external (compressive) pressure of the aether – and thus, in the end, by a merely apparent attraction.[162]

[162] The hypothesis that Kant describes at the end of the Phenomenology as a physical realization of the balancing argument of the fifth proposition of the Dynamics is thereby connected with a characteristic feature of this argument that I emphasized at the end of section 16: namely, Kant deliberately does not say that the attractive force of the internal parts of a matter are sufficient to counterbalance its inherent expansive tendency but only that (509, emphasis added) an attractive force "must be assumed *somewhere* [*irgendwo*]." The second remark to the seventh

18 ORIGINAL ATTRACTION AS IMMEDIATE AND UNIVERSAL

In the seventh proposition, as we have seen, Kant emphatically defends the idea that the fundamental force of attraction (and therefore Newtonian gravitation) is a true action-at-a-distance force acting immediately through empty space (512): "The *attraction essential to all matter* is an immediate action of it on others through empty space."[163] Kant is well aware that such a strong view of gravitational attraction is controversial, and he devotes the following two remarks to responding to opposing views: that immediate action at a distance is impossible because an object cannot act immediately where it is not (first remark), and that Newtonian gravitation is a mere apparent attraction due to the external pressure exerted by an aether (second remark). Further, the view that the fundamental force of attraction acts immediately at a distance through empty space, and is thus what Kant calls a penetrating force (in the seventh explication and accompanying note), is also crucial to the following eighth proposition (516): "The original attraction, on which rests the very possibility of matter, as such a thing, extends in the universe immediately from each part of [matter] to every other part to infinity."[164] The heart of Kant's proof is the claim that the "sphere of activity" of the original

proposition employs a parallel indefiniteness of expression in the claim (514, emphasis added) that "*some or another* [*irgend eine*] immediate attraction outside the [condition of] contact … must still be found." As George E. Smith has emphasized to me, however, there appears to be a counterexample that was well known at the time to Kant's physical hypothesis that the cohesion of the heavenly bodies is generally effected by an apparent attraction due to the pressure of an external aether rather than the true attraction of their internal parts. This is the model of the shape of the earth developed by Newton in the *Principia* (and later perfected by Clairault), which takes the earth to be initially fluid and in a state of hydrostatic equilibrium between its own expansive pressure and the internal universal attraction of each part of the earth for every other part. Indeed, as I shall explain in my chapter on the Phenomenology below, it appears that Kant was quite familiar with this model (at least as developed by Newton). Nevertheless, although I do not have a complete solution to the problem, it is helpful to recall that Kant does not take cohesion to be a universal property of all matter but only to characterize matter in the state of (at least) liquidity, so that matter in the (gaseous) state of permanently elastic fluidity is not cohesive: compare notes 57 and 149 above. What Kant appears to be supposing, therefore, is that matter in the permanently elastic (gaseous) state never has sufficient internal attraction (density) to balance its own expansive tendency, whereas matter in the liquid state may well have sufficient such attraction. In order to explain how the earth maintains the initial state of *liquidity* apparently assumed in Newton's model, in particular, we therefore need to invoke the pressure of an external aether.

[163] Kant also makes the strong claim that the attraction in question is *essential* to matter – in deliberate contrast, as we shall see below, with Newton's own views.

[164] As the context makes clear, the qualification "as such a thing" refers back to the property of filling a space (to a determinate degree) introduced in the first explication of the Dynamics.

attraction cannot be limited by any matter lying within this sphere, "for this attraction is a penetrating force and acts *immediately* at a distance, regardless of all matter lying in between, through this space [of its activity], as an empty space" (517). That the original attraction (and therefore Newtonian gravitation) is universal, acting between each part of matter in the universe and all other parts, is therefore, for Kant, simply a consequence of its already established immediacy.

The proof of the seventh proposition (where immediacy is established) depends on the distinction between action by contact and action at a distance developed in the sixth explication and accompanying remark. The sixth explication states (511) that "contact in the physical sense is the immediate action and reaction *of impenetrability*," and the remark makes this somewhat more precise by distinguishing between "mathematical" and "physical" contact:

> Contact in the mathematical sense is the common boundary [*gemeinschaftliche Grenze*] of two spaces, which is therefore within neither the one nor the other space. Thus two straight lines cannot be in contact with one another; rather, if they have a point in common, it belongs as much to one of these lines as to the other when they are produced, that is, they intersect. But a circle and a straight line, or two circles, are in contact at a point, surfaces at a line, and bodies at surfaces. Mathematical contact is the basis for physical contact, but does not yet constitute the latter by itself, since for the one to arise from the other a dynamical relation must also be added in thought – and, indeed, not of attractive, but of repulsive forces, that is, of impenetrability. Physical contact is the interaction of repulsive forces at the common boundary of two matters. (512)

Physical contact is not merely the interaction of repulsive forces but their interaction at the common boundary of two matters (mathematical contact). This qualification is necessary because repulsive force is what Kant calls a surface force, whereby "matters can act immediately on one another only at the common surface of contact" (516).[165]

With these distinctions in mind, let us consider the proof of the seventh proposition:

[165] Again, this differs sharply from the corresponding treatment in the *Physical Monadology*, where Proposition IX states (1, 483): "Contact is the reciprocal application of the forces of impenetrability of several elements." There is no mention of a common boundary or surface of contact (and therefore no mention of the distinction between mathematical and physical contact), because repulsion acts at a *finite* (although very small) distance in the pre-critical theory. So there is no room for the distinction between contact (or surface) forces and penetrating forces either. The treatment is similar in the *Enquiry* of 1763, where contact is simply defined as "immediate presence … by means of impenetrability" (2, 288).

The original attractive force contains the very ground of the possibility of matter, as that thing which fills a space to a determinate degree, and so contains even [the very ground] of the possibility of a physical contact thereof. It must therefore precede the latter, and its action must thus be independent of the condition of contact. But the action of a moving force that is independent of all contact is also independent of the filling of space between the moving and the moved [matters]; that is, it must also take place without the space between the two being filled, and thus as action through empty space. Hence the original attraction essential to all matter is an immediate action of it on others through empty space. (512)

The force of this brief argument is not immediately clear, however, since it would seem that all of its premises apply equally to the fundamental force of repulsion. Repulsion, too, contains the ground of the possibility of matter filling a space to a determinate degree, and repulsion, too, of course contains a ground of the possibility of a physical contact of matters. But we cannot conclude, in this case, that *repulsion* must be independent of the condition of contact and must therefore act immediately at a distance.

Kant's first remark to the proposition helps to clarify what is at stake. The main target, as noted, is the view that action at a distance is impossible in principle because an object cannot act immediately where it is not. Kant responds by examining the notion of the place where a thing *is* through an example:

If the earth immediately impels the moon to approach it, the earth acts on a thing that is many thousands of miles away from it, and yet immediately; the space between it and the moon may well be viewed as completely empty. *For* even though matter may lie between the two bodies, it still contributes nothing to this attraction. It therefore acts immediately at a place where it is not, which is apparently contradictory. In truth, however, it is so far from being contradictory that one may rather say that everything in space acts on another only at a place where the acting [thing] is not. For if it should act at the same place where it itself is, then the thing on which it acts would not be *outside it* at all; for this *outsideness* means presence at a place where the other is not. If earth and moon were to be in contact with one another, the point of contact would still be a place where neither the earth nor the moon is, for the two are distanced from one another by the sum of their radii. Moreover, no part of either the earth or the moon would be found at the point of contact, for this point lies at the boundary of the two filled spaces, which constitutes no part of either the one or the other. (513)

Therefore, Kant concludes, since even the action by contact of repulsive forces does not take place, strictly speaking, where either the one or the other body properly is, to deny the possibility of immediate action at a distance is simply to deny that any action at all is possible except through repulsive forces. But this would be an entirely groundless assertion based solely on the "confusion of mathematical contact of spaces and physical [contact] through repulsive forces" (513–14).

This example refers us, to begin with, to Kant's earlier discussion of place in the remark to the second explication of the Phoronomy, according to which, in the case of bodies (rather than mere mathematical points), the common definition of motion as alteration of place is not adequate (since the place of a body is a point, and the body might rotate around this point without changing its place). For Kant here (in the Phoronomy) illustrates the relevant notion of place with precisely the example of the earth and the moon:

> If one wants to determine the distance of the moon from the earth, then one wants to know the distance between their places, and for this purpose one does not measure from an arbitrary point of the surface or interior of the earth to any arbitrary point of the moon, but one chooses the shortest line from the central point of the one to the central point of the other, so that for each of these bodies there is only one point constituting its place. (482)[166]

The kinship between this passage and the one that we are now considering (in the Dynamics) then becomes completely clear if we note that in the latter Kant says that even if the earth and the moon were to be in contact with one another, their places would still be "distanced from one another by the sum of their radii" (513) – so that these places are again given by precisely their central points.

Moreover, the relationship between the earth and the moon constitutes a central example, for Kant, of gravitational interaction (see, e.g., the paragraph to which note 160 above is appended), and it is precisely this gravitational interaction that Kant has in mind in the first part of his example, where "the earth acts on a thing that is many thousands of miles away from it" as it "immediately impels the moon to approach it" (513). Moreover, the juxtaposition of this part of the example with the second part – where Kant now constructs an entirely imaginary situation in which the "earth and moon [are] to be in contact with one another" (513) – recalls Newton's well-known discussion of the moon test in Proposition 4 of Book 3 of the *Principia*. Here Newton constructs an example in which "the moon is imagined to be deprived of all its motion

[166] Kant then illustrates his revised definition of motion by the rotation of the earth, whereby it successively turns different parts of its surface towards the moon: see note 11 of my chapter on the Phoronomy.

and to be let fall so that it will descend to the earth" (P804), and the point, as explained in section 15 above, is to show that the gravitational force responsible for keeping the moon in its orbit is identical to terrestrial gravity. Indeed, as I also observed, Kant is quite familiar with Newton's discussion and has already alluded to it implicitly in his remark to the fifth proposition of the Dynamics. Kant was there concerned to underscore the circumstance that Newton thereby effects a crucial transition between the ordinary concept of weight due to terrestrial gravity and the general concept of mass or quantity of matter linked to universal gravitational attraction. I believe, therefore, that a full understanding of Kant's use of the earth–moon example in the first remark to the seventh proposition (and, accordingly, the argument of the seventh proposition itself) depends on Kant's prior discussion of quantity of matter in the balancing argument of the fifth and sixth propositions.[167]

This argument, as explained in section 16 above, hinges on the interplay between three different concepts of quantity of matter at work in the *Metaphysical Foundations*: (i) a dynamical concept related to the fundamental force of repulsion through a notion of density linked to the possibility of compression, (ii) a dynamical concept related to the fundamental force of attraction through the idea that the accelerations produced by this force are directly proportional (at a given distance) to the attracting body's quantity of matter, (iii) a mechanical concept related to the communication of motion and therefore to the concepts of impetus and inertia. The connection between these three concepts is mediated by Kant's official characterization of the concept of quantity of matter in the second explication of the Mechanics as the (infinite and continuous) "aggregate of the movable in a determinate space" (537). In particular, since Kant there

[167] Two further aspects of the first remark to the seventh proposition establish a clear connection with the balancing argument of the fifth and sixth propositions – mediated, in particular, by the remark to the fifth proposition. First, in explaining why we commonly think that attraction must be secondary to repulsion in the first remark to the seventh proposition, Kant repeats the explanation from the fifth proposition for why so many philosophers have rejected the idea of an original attraction:

> [Original attraction] simply does not present itself so immediately to the senses as impenetrability, so as to furnish us with concepts of determinate objects in space. Thus, because it is not felt, but is only to be inferred [*weil sie also nicht gefühlt, sondern nur geschlossen werden will*], it has so far the appearance of a derived force, exactly as if it were only a hidden play of moving forces through repulsion. (513)

Second, Kant's emphasis in the first remark to the seventh proposition that the place of a body is given by its central point corresponds to his insistence, in the remark to the fifth proposition, that the gravitational attraction of the (idealized spherical) earth (and similarly for the moon) acts as if all of its mass were concentrated at its center.

explains that "the quantity of matter is the *quantity of substance* in the movable," and that this rests, in turn, "on the concept of [substance] as the *ultimate subject* (which is no further predicate of another)," it follows that "the *proper motion* of matter is a predicate that determines its subject (the movable), and indicates the plurality of moved subjects in a matter, as an aggregate of the movable" (540–41). So it is the fundamental expansive force of matter that entails that each part of a space filled with matter is movable in itself and thereby "indicates the plurality of moved subjects in a matter, as an aggregate of the movable." It is in this way that the fundamental force of repulsion is linked to the "quantity of substance" in a given space and thus to the quantity of matter in that space.[168]

It is also true that the fundamental force of attraction is linked to the concept of quantity of matter through the circumstance that universal gravitation (at a given distance) is directly proportional to this quantity – which plays a crucial role, in particular, in the second part of the balancing argument involving both the remark to the fifth proposition of the Dynamics and the following sixth proposition.[169] At this stage of the argument for the immediacy of original attraction, however, we are still considering the moon test and are not yet in a position to invoke full universal gravitation. Indeed, we have not yet ruled out the idea that universal gravitation may be a merely apparent attraction due to the external pressure exerted by an aether. And this further step (to immediacy) is only taken in the *second* remark to the present (seventh) proposition. There, as I shall explain, Kant is arguing that the Newtonian procedure for measuring the quantities of matter of the primary bodies in the solar system essentially depends on precisely the immediacy of original attraction that we are now attempting to establish.[170] At this stage of the argument, then, the only concept of quantity of matter officially available is the dynamical

[168] See note 144 above, together with the paragraph to which it is appended. The preceding paragraph explains how this conclusion, in turn, depends on the fourth proposition of the Dynamics, where the infinite divisibility of matter (as a true continuum) is first officially proved.

[169] See the paragraph to which note 136 above is appended, together with the preceding paragraph. As explained, Kant is here relying on Proposition 8 of Book 3 of the *Principia*, where the quantities of matter of the primary bodies in the solar system are determined from the gravitational accelerations produced in their satellites. In the second part of the balancing argument presented in the sixth proposition Kant then relies on the characterization of quantity of matter as the aggregate of the movable in a given space to rule out the possibility of real isolated point-masses.

[170] Whereas the moon test is presented in Proposition 4 of Book 3, the proportionality of gravitational attraction (at a given distance) to quantity of matter is only established in the following Propositions 6 and 7. As we shall see, the second remark to the seventh proposition refers explicitly to Proposition 6 and implicitly to Proposition 7.

concept linked to the fundamental force of repulsion and the possibility of compression at work in the first part of the balancing argument.

How is the argument for the immediacy of original attraction presented in the proof of the seventh proposition then supposed to work? The fundamental force of repulsion, at this stage, is all we have to go on in speaking of the quantity of matter in a given space: it is this force, and this force alone, that is directly linked to the concept of the "*quantity of substance*" by "indicat[ing] the plurality of moved subjects in a matter, as an aggregate of the movable" (541). As we also know from the argument of the fifth proposition, however, this force, by itself, cannot explain how the matter in question can be confined within a definite (finite) "limit of extension [*Grenze der Ausdehnung*]" (508), and it is for this purpose, accordingly, that we also need to posit an original attraction. Indeed, without such a "limit of extension" the matter would disperse itself to infinity and "no specified quantity of matter would be found in any specified space" (508). In this context, therefore, the role of the fundamental force of attraction is precisely to insure that the quantity of matter in question possesses a boundary [*Grenze*] or limiting surface. Therefore, when Kant says that the original force of attraction contains the ground of the possibility of a physical contact of two matters in the proof of the seventh proposition, he is referring, more specifically, to the *mathematical* contact of two bodies at a "common boundary [*gemeinschaftliche Grenze*]" (512), which, according to the preceding remark to the sixth explication, is presupposed by all *physical* contact through repulsive forces.[171] The fundamental force of attraction, but not the fundamental force of repulsion, is thus the indispensable precondition for a *mathematical* contact. So the fundamental force of attraction, but not (of course) the fundamental force of repulsion, must operate independently of *physical* contact.[172] The fundamental force of attraction must be a penetrating rather than a surface force.

[171] The importance of the notion of *mathematical* contact is underscored in the first remark where, in describing an imaginary contact of the moon and earth, Kant concludes (513) that "no part of either the earth or the moon would be found at the point of contact, for this point lies at the boundary of the two filled spaces, which constitutes no part of either the one or the other." Recall that the concept of a body (here that of a heavenly body in particular) is defined as "a matter between determinate limits [*Grenzen*] (which therefore has a figure)" (525), and that this is intimately connected, in turn, with Kant's characteristic account of cohesion in terms of the action of merely apparent attraction.

[172] One way to express this asymmetry between the two fundamental forces is that repulsion is trivially (analytically) the condition for a *physical* contact of two matters while attraction is non-trivially (synthetically) the condition for their *mathematical* contact (compare note 110 above). We can therefore conclude that the latter must operate independently of physical contact but not (of course) that the former does.

Let us now turn to the second remark to the seventh proposition, the main target of which is the view that Newtonian universal gravitation may be explained by the external pressure exerted by an aether. Kant begins, as observed in section 17 above, with the distinction between true and apparent attraction, and with the claim, more specifically, that even apparent attraction must have a true (i.e., penetrating) attraction as its basis. He concludes (514): "[T]he matter whose pressure or impact is supposed to serve instead of attraction would not even be matter without attractive forces (Proposition 5), and therefore the mode of explanation of all phenomena of approach by *merely apparent* attraction revolves in a circle." The matter in question is the aether, and Kant in fact holds, as we have seen, that there exists a space-filling aether functioning as the vehicle of light propagation and exerting an external pressure on all bodies dispersed within it. Indeed, it is this external pressure, for Kant, that explains all bodily cohesion and thus the circumstance, just emphasized, that bodies are contained within definite limits and therefore have (mathematical) boundaries. Kant's point about true and apparent attraction, however, is that precisely this external pressure must itself be a product of true action-at-a-distance attraction acting as a penetrating force. Therefore, universal gravitation – as the action of the fundamental force of attraction (518; see again note 112 above) – cannot be explained, in turn, by the external pressure exerted by the aether.

The remainder of the second remark is taken up by a remarkable excursus in which Kant discusses Newton's views on gravitation and action at a distance explicitly and in considerable detail, including relevant parts of Book 3 of the *Principia*. Kant begins by considering (514) the "common" opinion that "Newton did not at all find it necessary for his system to assume an immediate attraction of matter, but, with the most rigorous abstinence of pure mathematics, allowed the physicists full freedom to explain the possibility [of attraction] as they might see fit, without mixing his propositions with their play of hypotheses." The main such "hypothesis" for explaining attraction without assuming immediate action at a distance is that it is somehow effected by an external aether, and Kant accordingly refers, on the following page, to Newton's speculations concerning a possible explanation of gravitation through the action of an all-pervasive "Aetherial Medium" in the second edition of the *Opticks* (1718).[173] Kant's main response to such possible explanations is that they

[173] Kant quotes from the Advertisement to this edition, using Samuel Clarke's Latin translation (1719), where Newton famously says (515, Kant's emphasis): "[T]o show that I do not take *gravity* for an *essential* property of bodies, I have added one question concerning its cause." In the

make Newton's grounding of the proposition that universal gravitation (at a given distance) is directly proportional to quantity of matter unintelligible (514): "But how could he [Newton] ground the proposition that the universal attraction of bodies, which they exert at equal distances around them, is proportional to the quantity of their matter, if he did not assume that all matter – thus merely as matter and through its essential property – exerts this moving force?" Since, according to Kant, Newton's derivation of this crucial property of gravitational attraction depends on presupposing that it is exerted immediately at a distance by all matter as such (and is thus a true rather than a merely apparent attraction), Kant concludes that Newton's attempts to distance himself from such immediate and original attraction (as expressed, for example, in the second edition of the *Opticks*) inevitably "set him at variance with himself [*ihn mit sich selbst uneinig machte*]" (515).

Kant initiates his argument for this claim by distinguishing between two relevant laws governing the concept of quantity of matter, one a "principle of mechanics" and the other a "principle of dynamics":

> For although between two bodies, when one attracts the other, whether their matter be similar or not, the mutual approach (in accordance with the law of the equality of interaction) must always occur in inverse ratio to the quantity of matter, this law still constitutes only a principle of mechanics, but not of dynamics; that is, it is a law of the *motions* that follow from attracting forces, not of the proportion of the *attractive forces* themselves, and it holds for all moving forces in general. Thus, if a magnet is at one time attracted by another equal magnet, and at another by the very same magnet enclosed in a wooden box of double the weight, the latter will impart more relative motion to the former in the second case than the first, even though the wood, which increases the quantity of matter of this second magnet, adds nothing at all to its attractive force, and manifests [*beweiset*] no magnetic attraction of the box. (514–15)

The relevant law of mechanics is the principle of the equality of action and reaction, which Kant officially introduces in the fourth proposition

numbering standard since the third (1721) edition (whose Queries do not differ substantatively from those of the second edition) Newton's reference is to Query 21, which speculates that the Aetherial Medium is much rarer within the heavenly bodies than in the space outside them and asks (1952, p. 350) whether "in passing from them to great distances, doth it not grow denser and denser perpetually, and thereby cause the gravity of those great Bodies towards one another, and of their parts towards the Bodies; every Body endeavoring to go from the denser parts of the Medium towards the rarer?" Newton's general view of the aether in these Queries is that it is originally the medium for the propagation of heat in a "vacuum" (Query 18), that it also plays a role in the propagation of light through the "vacuum" (Queries 19–21), and that light is not, however, a motion or pressure propagated through the aether (Queries 28 and 29). From now on I cite this (1952) edition of the *Opticks* in the form 'Onn', where 'nn' denotes pages numbers.

of the Mechanics.[174] According to this principle, in an interaction between two bodies due to any forces whatsoever the ratio of the acceleration produced in the first body to the acceleration produced in the second is always inversely proportional to the corresponding ratio of their masses – so that (using modern algebraic notation) if a_A is the first acceleration and a_B the second, and similarly for their corresponding masses, then $a_A/a_B = -m_B/m_A$. This principle, as Kant suggests, picks out the center of mass of the two interacting bodies and determines the accelerations in question relative to this point.[175] In the case of the two magnets, for example, adding a wooden box of double the weight to one will move the center of mass closer to the now heavier body and thereby change the two relative accelerations accordingly – even though the corresponding magnetic *forces*, as Kant says, are themselves entirely unchanged.

The particular case of gravitational force, however, is very special in this regard. Indeed, in the last corollary to Proposition 6 of Book 3 Newton contrasts gravitational force with magnetic force to make precisely this point.[176] The conclusion of Proposition 6 is that the "weight" (i.e., gravitational force) of any body towards any planet is "proportional to the quantity of matter which the body contains" (P806). Newton first refers to the well-known fact that all heavy bodies fall towards the earth with the very same constant acceleration ($g = 32$ ft. per sec.^{2}), from which it follows, in the context of the Laws of Motion, that the weights of such bodies are proportional to their masses – a result that Newton has verified with careful pendulum experiments.[177] He then invokes the moon test, which shows that the moon "falls" towards the earth in accordance with the same constant acceleration (when brought down to the earth's surface), so that the gravitational force on the moon is also proportional

[174] Kant there characterizes this principle (548) as "the *mechanical* law of the equality of action and reaction [*Gleichheit der Wirkung and Gegenwirkung*]." Here in the Dynamics he characterizes it as "the law of the equality of interaction [*Gleichheit der Wechselwirkung*]."

[175] This, as we shall see, is crucial for Kant's proof of the principle in the Mechanics and, in turn, to his eventual solution to the problem of absolute versus relative motion as well. As explained in section 13 above, Kant has already begun to anticipate this solution in the Dynamics: see, in particular, note 82 above, together with the paragraph to which it is appended.

[176] See P810: "The force of gravity is of a different kind from the magnetic force. For magnetic attraction is not proportional to the [quantity of] matter attracted."

[177] These experiments are further discussed in section 24 below. Newton constructs his pendulums (P807) from "two wooden boxes, round and equal" containing specifically different materials (e.g., wood and gold). Kant's example of the magnet in a wooden box, in the present context, appears to involve an allusion to this part of Proposition 6.

to its mass.[178] Finally, Newton invokes Kepler's third or harmonic law of planetary motion, which shows that the accelerations ("accelerative gravities") of the moons of Jupiter, for example, are inversely as the squares of the distances from the center of Jupiter – so that "at equal distances from Jupiter their accelerative gravities would come out equal … just as happens with heavy bodies on this earth of ours" (P807).[179] Moreover, since this same conclusion holds for the motions of satellites relative to any primary body in the solar system (including the motions of the planets relative to the sun), it follows that in all such cases of gravitational attraction the "weight" of an attracted body is directly proportional to its mass.[180] This is a characteristic property of gravitational force in particular, from which it follows that the gravitational acceleration of a body is entirely independent of the mass or quantity of matter of that body and depends only on its distance from the attracting body in question: gravitational force produces what has been very helpfully called an "acceleration field" around any gravitating body.[181]

Thus, in Proposition 6 of Book 3 Newton focusses on a unique and characteristic property of gravitational attraction at the beginning of his demonstration that gravitational force, unlike other forces, has a connection

[178] See P807:

> For imagine our terrestrial bodies to be raised as far as the orbit of the moon and, together with the moon, deprived of all motion, to be released so as to fall to the earth simultaneously; and by what has already been shown [in Proposition 4 – MF], it is certain that in equal times these falling terrestrial bodies will describe the same spaces as the moon, and therefore that they are to the quantity of matter in the moon as their own weights are to its weight."

[179] Kepler's harmonic law states that the periods of revolution in any system of satellites orbiting a primary body are as the 3/2 power of their (mean) distances from the center of this body – from which it follows, in accordance with Corollary 6 to Proposition 4 of Book 1 (see Propositions 1 and 2 of Book 3), that the accelerations of any two such satellites are inversely as the squares of these distances: compare note 191 below. It follows, in particular, that the variation in acceleration from orbit to orbit depends *only* on distance. Kant presents this derivation himself in the short introductory section of the *Theory of the Heavens* mentioned in note 46 above.

[180] Newton buttresses this conclusion in the case of the satellite systems of Jupiter and Saturn by arguing that deviations from equal accelerations (along nearly parallel lines) towards the sun for any of the bodies in such a system (including the primary body) would result in irregularities in their motions in accordance with Corollary 6 to the Laws of Motion.

[181] This terminology derives from Stein (1967). The gravitational *force* is given (in modern notation) by $F_{grav} = Gm_Am_B/r^2$. Yet, since F_{grav} on m_B is equal to m_Ba_B, we have $a_B = Gm_A/r^2$ *independently* of m_B. In modern terms derived from the general theory of relativity this depends on the equality of the "inertial mass" (of B) figuring in the (Second and Third) Laws of Motion with the "passive gravitational mass" (of B) figuring in the expression for gravitational force. In the case of other forces (such as electricity or magnetism, for example) the "passive" quantity figuring in the force law is in general *not* equal to the inertial mass of the affected body (in the case of electrostatic force, for example, it is given by charge).

with the masses or quantities of matter of the interacting bodies in question that goes far beyond the general relationship between force (of whatever kind) and quantity of matter stipulated in the Laws of Motion.[182] It is no wonder, then, that Kant refers explicitly to Proposition 6 immediately after emphasizing this distinction between gravitational attraction and all other forces in the second remark to the seventh proposition of Dynamics. But Kant focusses on an aspect of Proposition 6 that we have not yet considered: namely, on the first two corollaries to this Proposition. Newton there argues (in the first corollary, P809) that the weights of bodies cannot depend on their "forms and textures" (since otherwise they would not all fall at the same rate at equal distances), and (in the second, P809) that "[a]ll bodies universally that are on or near the earth gravitate toward the earth, and the weights of all bodies that are equally distant from the center of the earth are as the quantities of matter in them." Newton further concludes that not even the aether can be entirely devoid of gravity or gravitate less in proportion to its quantity of matter – for otherwise, "since (according to the opinion of Aristotle, Descartes, and others) [the aether] does not differ from other bodies except in the form of its matter, it could by a change of form be transmuted by degrees into a body of the same condition as those that gravitate the most in proportion to the quantity of their matter" (P809).

It is this last conclusion that is specifically quoted by Kant (515).[183] And he then uses it to construct a kind of *ad hominem* directed at Newton:

Thus [Newton] did not himself exclude the aether (much less other matters) from the law of attraction. So what other kind of matter could he then have left over, by whose impact the approach of bodies to one another might be viewed as mere apparent attraction? Therefore, one cannot adduce this great founder of the theory of attraction as one's predecessor, if one takes the liberty of substituting an apparent attraction for the true attraction that he did assert, and

[182] As we shall see below, Newton completes this demonstration in Proposition 7 by establishing the equality of inertial mass with what we now call "active gravitational mass" as well – where, in the terms of note 181 above, this refers to the quantity of matter of the *source* of the acceleration field (i.e., the quantity m_A generating the gravitational acceleration of m_B). It is precisely this quantity, from a modern point of view, which underlies the circumstance that the quantity of matter of a gravitating body can be determined by the gravitational accelerations (at a given distance) that it produces in other bodies. The equality of this quantity with inertial mass then implies that we are thereby determining the (inertial) *mass* of the gravitating body as well. So "active gravitational mass" is the modern terminology for quantity of matter as determined by the accelerations produced by a gravitating body (at a given distance) on other bodies.

[183] Kant does not quote the second corollary quite exactly, and he mixes it, at the very end of his quotation, with the conclusion of the first corollary. This does no harm, however, and helpfully serves to remind us that both corollaries are actually at issue here.

assumes the *necessity* of an impulsion through *impact* to explain the phenomenon of approach. (515)

Yet this argument, although correct as far as it goes, is less than completely satisfying. It is not clear, in the first place, that the argument applies to Newton's own attempted explanation of gravitation through the action of an aether suggested in the second edition of the *Opticks* (to which Kant refers in the next sentence).[184] Nor, in the second place, does the argument, by itself, generate any further insight into the point Kant is here most anxious to clarify: namely, the particularly intimate connection between gravitational attraction (as an immediate action at a distance) and the concept of mass or quantity of matter.

We can find a deeper meaning in Kant's reference to Newton's corollaries to Proposition 6, however, if we note that Kant makes a parallel allusion in the corresponding section of the *Physical Monadology*. In Proposition XI of this work, as observed in section 16 above, Kant introduces the concept of mass or quantity of matter as a measure of a body's force of inertia (*vis inertiae*) – where each element or physical monad, among its "innate [*insita*]" forces, possesses a force of repulsion, a force of attraction, *and* a force of inertia responsible for its mass (see the paragraph to which note 130 above is appended). In the proof of Proposition XII Kant then rejects the customary explanation of the differing specific densities of bodies in terms of differing interspersed amounts of empty space on the ground that it commits us to unverifiable conjectures concerning these empty interstices:

If all elements had the same force of inertia and the same volume, then an absolute vacuum intermixed between their parts would be necessary to understand the difference in the rarities of bodies. For, according to the demonstrations of Newton, Keill, and others, free motion cannot take place in a medium that is completely filled in this way. Therefore, in order to explain the infinite diversity of specific densities of various media, e.g., aether, air, water, and gold, one would

[184] See note 173 above. In Query 21 Newton's own view of the constitution of the "Aetherial Medium" is expressed very cautiously. But he does suggest (O352) that "*Aether* (like our air) may contain Particles which endeavor to recede from one another … and that its Particles are exceedingly smaller than those of Air, or even than those of Light." In this case the aether would consist of its own characteristic type of tiny corpuscles interacting with one another by action-at-a-distance repulsive forces. Here, unlike in the case of the hypothetical aether considered in the corollaries to Proposition 6 of Book 3 of the *Principia*, the aether could not be transformed into any other type of matter, and it would clearly not constitute the space-filling plenum considered in the theories of "Aristotle, Descartes, and others" (P809). For precisely this reason, however, it could also not support any general rejection of action at a distance as such.

have to indulge in an unrestrained passion for conjecture concerning that which is most remote from human understanding. (1, 486)

Kant here alludes to Newton's third and fourth corollaries to Proposition 6. Newton argues (in the third corollary) that not all spaces can be equally filled with matter (since all fluid media whatsoever, by the argument of the first two corollaries, would then have the same specific gravity).[185] He also argues (in the fourth) that there must be a vacuum if all solid parts of bodies have the same density (so that bodies can differ in density only through differing amounts of interspersed empty space).[186] Kant's response (in the *Physical Monadology*) is that bodies can in fact differ in density without presupposing empty interstices, since the elements of bodies can simply differ in their intrinsic masses (forces of inertia).[187]

But the notion of mass in the *Physical Monadology* is not connected with the two fundamental forces of repulsion and attraction: the force of inertia is a distinct third force, which belongs to each element entirely independently of the other two.[188] Accordingly, the concept of mass at this stage of Kant's thought is introduced only in connection with inertia and the communication of motion (see note 129 above, together with the text to which it is appended). From the point of view of the *Metaphysical Foundations*, therefore, we here have only a mechanical concept of the quantity of matter that is not yet linked to any dynamical concept. In particular, the concept of mass or force of inertia has no intrinsic

[185] See Corollary 3 (P810):

> All spaces are not equally full. For if all spaces were equally full, the specific gravities of the fluid with which the region of the air would be filled, because of the extreme density of its matter, would not be less than the specific gravity of quicksilver or of gold or of any other body with the greatest density, and therefore neither gold nor any other body could descend in air. For bodies do not ever descend in fluids unless they have a greater specific gravity.

[186] See Corollary 4 (P810): "If all the solid particles of all bodies have the same density and cannot be rarified without pores, there must be a vacuum. I say particles have the same density when their respective forces of inertia are as their sizes."

[187] According to Proposition XII (1, 496): "The diversity of specific densities in bodies observable in the world cannot be intelligibly explained without a specific diversity in the inertia of their elements." The proof concludes (1, 496): "Therefore, unless one concedes that there are specific differences in the simplest elements, in virtue of which one can construct a lesser or much greater mass filling exactly the same space, physics will always founder, as it were, on the rock of this difficulty." As observed (see note 130 above), Kant had already outlined this solution (which contrasts with that of the general remark to dynamics in the *Metaphysical Foundations*) in his corollaries to Proposition XI.

[188] This independence is underscored by the circumstance that all elements whatsoever have the same *volume*, determined by the balancing of the first two fundamental forces (Proposition X: see note 128 above), while the point of Proposition XII is that elements can have very different *masses* at the same volume.

connection with the fundamental force of attraction, and, accordingly, Kant's pre-critical version of the dynamical theory of matter contains nothing corresponding to the seventh proposition of the Dynamics. There is no proof that the fundamental force of attraction must act immediately at a distance, and no argument for the intimate connection between this force and quantity of matter that Proposition 6 of Book 3 of the *Principia* begins to establish.[189]

By contrast, as I argued in section 16 (and have just emphasized again in connection with the first remark to the seventh proposition), the point of the balancing argument presented in the *Metaphysical Foundation* is to establish precisely this kind of connection between the (mechanical) concept of quantity of matter and the two fundamental forces. Whereas the first part of the argument establishes such a connection between the (mechanical) concept of quantity of matter as the total (infinite and continuous) aggregate of the movable in a given space and the fundamental force of repulsion, the second part establishes a parallel connection between this same (mechanical) concept and the fundamental force of attraction. In particular, since quantity of matter is characterized as the total (infinite and continuous) aggregate of the movable in a given space, and since original attraction, as a *penetrating* force, therefore acts immediately between each part of a given such aggregate and every part of any other, it follows, for Kant, that the action of the fundamental force of attraction is necessarily proportional to quantity of matter. The crux of the second part of the balancing argument, accordingly, is that there is no such thing as an isolated point-mass, while the lesson of the first part (conversely) is that an aggregate of the movable governed only by repulsive force would be contained within no limit of extension and would therefore fail to constitute a determinate quantity of matter in any given space. Just as Kant's reference to the moon test in the first remark to the seventh proposition corresponds to the first part of the balancing argument, his reference to the Newtonian argument for the proportionality of universal

[189] The corresponding section of the *Physical Monadology* suggests only (1, 483) that bodies may *possibly* be "immediately present" to one another even without contact because "the Newtonian school, not without a great semblance of truth, defends the immediate attraction of bodies even at a distance, from which their co-presence even without mutual contact then follows." Similarly, the *Enquiry* challenges only the "metaphysical" arguments *against* action at distance (2, 288): "It is well known that most Newtonians go still further than Newton himself and assert that bodies attract one another even at a distance immediately (or, as they put it, through empty space). I do not challenge the correctness of this proposition, which certainly has much in its favor. But I do assert that metaphysics has not in the least refuted it."

gravitation to quantity of matter in the second remark corresponds to the second part.[190]

It is now time, finally, to examine the conclusion of Kant's second remark:

> [Newton] rightly abstracted from all hypotheses purporting to answer the question as to the cause of the universal attraction of matter, for this question is physical or metaphysical, but not mathematical; and, even though he says in the advertisement to the second edition of his *Optics*, "to show that I do not take *gravity* for an *essential* property of bodies, I have added one question concerning its cause," it is clear that the offense taken by his contemporaries, and perhaps even by Newton himself, at the concept of an original attraction set him at variance with himself: for he could absolutely not say that the attractive forces of two planets, e.g., of Jupiter and Saturn, manifested at equal distances of their satellites (whose mass is unknown), are proportional to the quantity of matter of these heavenly bodies, if he did not assume that they attracted other matter merely as matter, and thus according to a universal property of matter. (515)

Kant is here engaging directly with the argument of Book 3 – specifically, with the procedure Newton develops for estimating the quantities of matter of the individual planets from the distances and periodic times of their satellites in the first two corollaries of Proposition 8.

According to these corollaries, as observed (see the paragraph to which note 119 above is appended), we can use the accelerations of the satellites of the sun, Jupiter, Saturn, and the earth towards their primary bodies to infer that the masses of these primary bodies relative to one another are as 1, 1/1,067, 1/3,021, and 1/169,282 respectively. This procedure follows from the law of universal gravitation. Since the gravitational force between any two bodies is directly proportional (at a given distance) to the product of their masses, the gravitational acceleration produced by one body on the other is directly proportional (at a given distance) to the mass or quantity of matter of the first body.[191] At this stage of the argument, however, Newton is in the process of establishing the law of universal gravitation

[190] These connections between mechanical and dynamical concepts of quantity of matter entirely depend on the view of matter as a true continuum characteristic of the critical version of the dynamical theory of matter, and this, in the end, is why they cannot be present in the *Physical Monadology*. In particular, the conception of mass or force of inertia articulated in this earlier work is perfectly consistent with isolated point-masses: see note 135 above.

[191] The gravitational force is given by $F_{grav} = Gm_Am_B/r^2$. Since F_{grav} on B is equal to m_Ba_B, $a_B = Gm_A/r^2$. We infer accelerations from distances and periodic times using the formula for centripetal (or centrifugal) acceleration. Assuming circular orbits for simplicity, and using the circumstance that here $v = 2\pi rt$, with t the period, $a = v^2/r$. In these terms Kepler's third or harmonic law (note 179 above) states that if r, R and t, T are, respectively, the distances and periods of two concentric orbits, then $r^3/R^3 = t^2/T^2$. It follows that the two accelerations stand in the ratio $1/r^2{:}1/R^2$ and thus that the variation in acceleration depends only on distance.

in the first place, and so the crucial question is how he now establishes, in particular, that the gravitational attraction produced by one body on another (by a primary body on one of its satellites) is indeed proportional (at a given distance) to the former body's mass.

Newton derives this crucial property of gravitational acceleration in Proposition 7 of Book 3. We know, from Proposition 6, that gravitational acceleration is independent of the mass of the body being attracted, so that the acceleration of the attracted body is given by k/r^2, where r is the distance between the two bodies and k is a constant depending only on the attracting body. There is an acceleration field around each attracting body depending only on distance, and the constant k characterizes the attracting body's acceleration field independently of distance. We now want to show that the constant k in question also depends precisely on its mass.[192] Newton derives Proposition 7 from Proposition 69 of Book 1, the proof of which, in our present context, can be elucidated as follows. Consider a system of bodies consisting of two planets and their respective satellites, for example, the systems of Jupiter and Saturn.[193] Let the acceleration field on Saturn's moons be given by $a_1 = k_S/r_1^2$ and the acceleration field on Jupiter's moons by $a_2 = k_J/r_2^2$. We want to show that when $r_1 = r_2$, $a_1/a_2 = k_S/k_J = m_S/m_J$, where m_S and m_J are the masses of Saturn and Jupiter respectively. To do so, we assume that the acceleration fields of our two planets extend far beyond their respective satellites, so that we also have an acceleration $a_J = k_S/r^2$ of Jupiter and an acceleration $a_S = -k_J r^2$ of Saturn, where r is now the distance between the two planets. But, according to the Third Law of Motion, $m_J a_J = -m_S a_S$. Therefore, we have $m_S/m_J = -a_J/a_S = k_S/k_J$, as desired. We are now – and only now – in a position to compare the masses of Jupiter and Saturn by reference to the acceleration fields on their respective satellites, that is, by reference to k_J and k_S.

There are two general features of the argument worth emphasizing. First, we need to extend the acceleration fields of the two planets far beyond the regions of their respective satellite systems and to suppose that

[192] So here, in the terms of note 182 above, we want to establish an equality between the inertial mass of the attracting body A and its active gravitational mass.

[193] See Proposition 69 (P587):

> If, in a system of several bodies A, B, C, D … some body A attracts all the others, B, C, D … by accelerative forces that are inversely as the squares of the distances from the attracting body; and if another body B also attracts the rest of the bodies, A, C, D … by forces that are inversely as the squares of the distances from the attracting body; then the absolute forces of the attracting bodies A and B will be to each other in the same ratio as the bodies A and B themselves to which those forces belong.

In our illustration A is Saturn, B is Jupiter, and C, D … are their satellites.

they affect the two planets as well. We need to assume that gravitational attraction takes place not only between primary bodies and their satellites but also among the primary bodies themselves, and this is a first – and very significant – step on the way to truly *universal* gravitational attraction. Second, we need to apply the Third Law of Motion (the equality of action and reaction) directly to the primary bodies themselves (in our case, to the interaction between Jupiter and Saturn), and we need to do this, moreover, independently of any material medium that may fill the space between these bodies.[194] For, if the interaction between two such bodies were in fact mediated by a space-filling aether, then our application of the equality of action and reaction would need to consider this intervening medium as well. In other words, conservation of momentum might not then hold between the two interacting bodies alone. So what we now need to assume, on the contrary, is that conservation of motion *does* hold between the two interacting bodies – at least to an extremely high degree of approximation.

Kant's point is therefore a deep one. Newton's argument for the proportionality of gravitational attraction (at a given distance) to the attracting body's mass involves a key application of the Third Law of Motion to the interactions among the primary bodies in the solar system. This application, in turn, must take place between any pair of bodies directly, as if no intervening matter were (significantly) involved in the interaction.[195] For Newton could not conclude that gravitational interaction has the characteristic mass dependency in question if he did not treat it de facto as a genuine action at a distance through empty space. So

[194] In a perceptive critical examination of my earlier presentation of this point in Friedman (1992b, chapter 3, §iv) Tanona (2000, §4) rightly observes that the comparison between Jupiter and Saturn does not strictly require that we consider the interaction between the two planets themselves. It is enough to consider parallel interactions between the two planets and the sun (which then provides a common point of reference for the comparison of masses). We still need, however, to extend the acceleration fields exerted on the moons of Jupiter and Saturn far beyond their respective satellite systems (this time to the sun) and then apply the equality of action and reaction directly to the interactions among the primary bodies (this time to the interactions between Jupiter and the sun and between Saturn and the sun respectively).

[195] I say "as if" no intervening were present, because the Third Law could apply even in the presence of such a medium so long as it does not exchange any significant amount of momentum in the interaction in question (as in the case of a rigid rod connecting the two bodies and thereby transferring the momentum lost by one to the other). The fundamental importance of Newton's direct application of the Third Law of Motion to the interactions among the primary bodies in the solar system to the overall argument of Book 3 is clearly emphasized in Stein (1967, pp. 179–80, especially p. 180n. 6). Note that the problem arising here *does* create serious difficulties for the aether model suggested in Query 21 of the *Opticks* (see note 184 above), for we would then need to apply the equality of action and reaction between gravitating bodies and the aether particles.

the argument presupposes, in this sense, that universal gravitation is a true rather than merely apparent attraction and, in this same sense, that gravity is an "essential" or intrinsic property of all matter as such.[196] In this context any attempt to explain the interaction of gravity as a merely apparent attraction – as in Newton's own speculative theory of gravity in Query 21 of the *Opticks* – would indeed set Newton "at variance with himself."

19 THE DYNAMICAL THEORY OF MATTER AND MATHEMATICAL CONSTRUCTION

The eighth proposition of the Dynamics, expressing the universality of the original force of attraction, states that this force "extends in the universe immediately from each part of [matter] to every other part to infinity" (516). The proof of this proposition, as observed, is a direct inference from the property of immediacy already established in the seventh proposition (see the first paragraph of section 18 above). More interestingly, however, the eighth proposition is followed by two notes and two remarks – which are concerned not so much with further features or aspects of the universality of original attraction but with the question whether this force, in combination with the original force of repulsion, now allows us to construct the dynamical concept of matter mathematically.

The first note begins with the suggestion that such a construction should be possible:

From this original attractive force, as a penetrating force exerted by all matter, therefore in proportion to the quantity [of matter], and extending its action to all matter at all possible distances, it should now be possible, in combination with the force counteracting it, namely repulsive force, to derive the limitation of the latter, and thus the possibility of a space filled to a determinate degree – and thus the dynamical concept of matter, as that of the movable filling its space (to a determinate degree), would be constructed. (517)

[196] The significance of the present application of the Third Law of Motion for this question was already appreciated by Roger Cotes – a most acute Newtonian, who asserted in a draft of his Preface to the second edition of the *Principia* that gravity is an essential property of matter. (In response to criticism from Samuel Clarke Cotes changed "essential" to "primary" in the final version.) In particular, in correspondence with Newton in 1712–13 Cotes argues that the Third Law can be applied only "when the Attraction may properly be so-called" but not in cases of merely apparent attraction effected by pressure, and he remarks that neglect of this point causes difficulties with the argument of Proposition 7: see Newton (1975, pp. 391–93). For discussion of the Newton–Cotes correspondence see Koyré (1965, chapter 7). For more on the issue of gravitation as an *essential* property of matter see Friedman (1990).

Yet Kant immediately expresses hesitation concerning whether the intended construction is in fact possible by characterizing it as "a purely mathematical task, which no longer belongs to metaphysics; nor is metaphysics responsible if the attempt to construct the concept of matter in this way should perhaps not succeed" (517). Nevertheless, Kant begins the first remark with the statement that (518) he "cannot forbear adding a small preliminary suggestion on behalf of the attempt at such a perhaps possible construction" and then sketches the familiar mathematical version of the balancing argument based on the differing rates of drop-off of repulsive and attractive force in the remainder of this remark.[197] The following second remark, however, raises a fundamental mathematical difficulty for this attempt. So Kant concludes, accordingly, that he does "not want the present exposition of the law of an original repulsion to be viewed as necessarily belonging to the goals of my metaphysical treatment of matter, nor the latter (for which it is enough to have presented the filling of space as a dynamical property of matter) to be mixed up with the conflicts and doubts that could afflict the former" (522–23).

It is clear, therefore, that Kant is quite ambivalent about the mathematical construction he attempts to sketch in the first remark. But it is important, nonetheless, to consider exactly what would be accomplished by such a construction if it were to be successfully carried out. It can be tempting, in particular, to view the attempted construction as a kind of culmination of the dynamical theory of matter as a whole, aimed at showing nothing less than that the concept of matter Kant is in the process of articulating is in fact really possible and thus has objective reality. This idea is especially tempting if we directly connect the present issue with the remarks Kant makes in the Preface concerning the "possibility of determinate natural things" (470): "[I]n order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the *intuition* corresponding to the concept be given a priori, that is, that the concept be constructed." From these remarks, in the present context, one could easily conclude that the ultimate point of the dynamical theory of matter is to show that the (dynamical) concept of matter in general can be mathematically constructed and thus has objective reality. And it would follow, accordingly, that the success or failure of

[197] As explained, this argument is a variation of an earlier mathematical construction presented in the *Physical Monadology*, with the all-important difference, however, that the present version uses a repulsive force that drops off in proportion to the inverse cube of the *infinitely small* distance: see note 141 above.

the dynamical theory of matter as a whole entirely depends on the success or failure of the present attempt at a mathematical construction.[198]

The ambivalence Kant expresses about the feasibility of this attempt, his deliberate separation of the mathematical task in question from "the goals of my *metaphysical* treatment of matter" (522–23, emphasis added), and the circumstance that the present attempt at construction is expounded (very hesitantly) in a remark to a note rather than in any official proposition of the Dynamics, all count against such a strong reading of the role of this attempted construction. Moreover, as I argued in detail in the Introduction and reiterated at the end of section 10, such a reading is incompatible with both the general remark to dynamics and the corresponding discussion of real possibility in the first *Critique*.[199] Kant holds, in particular, that the real possibility of a fundamental force – whether repulsive or attractive – can never be established a priori by mathematical construction. And he also holds, for the same reason, that his preferred *dynamical* concept of matter (as opposed to the merely "mathematical-mechanical" concept) cannot be constructed in pure intuition (525): "[I]f the material itself is transformed into fundamental forces (whose laws we cannot determine a priori, and we are even less capable of reliably enumerating a manifold of such forces sufficient for explaining the specific variety of matter), we lack all means for *constructing* this concept of matter, and presenting what we thought universally as possible in intuition."[200]

The first remark to the eighth proposition devotes at least as much space to the behavior of light as it does to the two fundamental forces.[201]

[198] For this kind of reading of the role of mathematical construction within the dynamical theory of matter see, e.g., Förster (2000, chapter 3). For Förster, the difficulties Kant outlines in the second remark, together with related mathematical difficulties subsequently encountered in developing a revised version of the dynamical theory of matter in the *Opus postumum*, play a crucial role in motivating Kant's later thoughts on the subject.

[199] For my earlier discussion in section 10, including references back to the Introduction, see note 25 above, together with the paragraph to which it is appended and the two preceding paragraphs.

[200] In his discussion cited in note 198 above Förster attempts to defuse this passage by reading it as merely disputing the possibility of constructing the *specific variety* of matter on the "metaphysical-dynamical" approach and not as denying the possibility of constructing the dynamical concept of matter in general: see Förster (2000, pp. 65–66, together with p. 187n. 21). However, while Förster is correct that Kant is here giving particular emphasis to the problem of specific variety, it is just as clear that he is also saying that the two fundamental forces – "whose laws we cannot determine a priori" (525) – are themselves incapable of mathematical construction in the present sense: we cannot verify their objective reality a priori in pure intuition.

[201] Once again, this contrasts sharply with the parallel discussion in the *Physical Monadology*, where light receives only a single, almost parenthetical mention (1, 484). Although in the pre-critical period, as observed, Kant is certainly committed to a light-aether (see note 60 above, together with the paragraph to which it is appended), it nevertheless plays no role at all in the (pre-critical) balancing argument in the *Physical Monadology*.

Kant begins with a general point about all "forces" or "actions" satisfying a given condition, which he then illustrates with the example of light:

Of any force that acts immediately at various distances, and is limited, with respect to the degree in which it exerts moving force on any given point at a certain distance, only by the magnitude of the space into which it must diffuse so as to act on this point, one can say that in all spaces into which it diffuses, large or small, it always constitutes an equal quantum, but [also] that the degree of its action on that point in this space is always in inverse ratio to the space into which it has had to diffuse in order to be able to act on this point. Thus light, for example, diffuses from an illuminating point in all directions on spherical surfaces, which constantly increase with the squares of the distance, and the quantum of illumination on all of these spherical surfaces (which [become] greater to infinity) is always the same in total – from which it follows that a given equal part of one of these spherical surfaces must become ever less illuminated with respect to its degree, as the surface of diffusion of precisely the same light quantum becomes greater; and the same [is true] in the case of all other forces and laws, according to which they must diffuse either on surfaces or in volumes in order to act on distant objects in accordance with their nature. (518–19)

The idea, in general, is to represent any "force" by a fixed quantum of "action" spreading outwards (uniformly) from a given point, so that its strength manifested at a given distance is inversely proportional to the spaces on which (or throughout which) it has had to diffuse itself in order to reach the distance in question. In the cases of light and the fundamental force of attraction the fixed quantum in question spreads from a given point (uniformly) on concentric spherical surfaces, and the strength of the "action" is thus inversely proportional to the square of the distance. In the case of the fundamental force of repulsion, however, the quantum of "action" spreads from a given point (uniformly) throughout an (infinitesimal) volume, and its strength is inversely proportional to the cube of the (infinitely small) distance.

This kind of derivation of a law of "action," in all cases, is therefore purely geometrical: it depends only on the ratios between certain surface-areas, volumes, and distances. It is for precisely this reason, however, that the proposed derivations are insufficient – at least for the two fundamental forces of attraction and repulsion. For a force (in these two cases) is the cause of a motion, and a motion is a *spatio-temporal* rather than purely spatial magnitude. Indeed, the importance of this point was already emphasized in section 4, where I explained that Kant's construction of motion as a magnitude depends specifically on its spatio-temporal rather than purely spatial aspects.[202] Whatever else they might achieve, therefore, the suggested constructions of the laws of attractive and repulsive forces presented in the first remark to the eighth proposition of the Dynamics do not amount to constructions of these forces as mathematical magnitudes. So they cannot, *a fortiori*, facilitate an a priori proof of the real possibility or objective reality of the dynamical concept of matter.[203]

If the construction Kant presents in the first remark to the eighth proposition is not concerned with a proof of the real possibility or objective reality of the dynamical concept of a matter in general, however, what is, then, its real point and significance? At the end of the general remark to dynamics Kant explains the relationship between his metaphysical investigation of the dynamical concept of matter and the application of mathematical construction to this concept:

For it lies altogether beyond the horizon of our reason to comprehend original forces a priori with respect to their possibility; all natural philosophy consists, rather, in the reduction of given, apparently different forces to a smaller number of forces and powers that explain the actions of the former, although this reduction proceeds only up to fundamental forces, beyond which our reason cannot go. And so metaphysical investigation behind that which lies at the basis of the empirical concept of matter is useful only for the purpose of guiding natural philosophy, so far as this is ever possible, to explore dynamical grounds of explanation; for these alone permit the hope of determinate laws, and thus a true rational coherence of explanations.

[202] It is for precisely this reason that Kant's construction in the Phoronomy involves spatio-temporal relations (motions) involving different relative spaces or reference frames rather than merely spatial relations (between vectors represented merely by directed line segments): see, e.g., notes 54 and 55 of my chapter on the Phoronomy, together with the paragraphs to which they are appended.

[203] Since the corresponding law governing the intensity of light does not describe the cause of a *motion*, the situation is less clear in this case, and Kant may very well have taken it to be a standard part of geometrical optics. The "photometric" law of light intensity – that a given element of spherical surface at a given distance from a central source has a degree of illumination that is inversely proportional to its distance from the central point and directly proportional to its own magnitude (surface area) – was originally due to Kepler. It was well known in the eighteenth century from works of Euler and Lambert. The example of the mathematical (additive or compositional) structure of an intensive magnitude Kant presents in his discussion of the mathematical principles of pure understanding – that the degree of illumination of sunlight stands to the degree of illumination of moonlight in the ratio of approximately 200,000 to 1 (A179/B221) – is an application of this law, based on comparing the distances between the earth (and the moon) and the sun, the distance between the earth and the moon, and the surface area presented to the earth by the moon (conceived as a perfect reflector of sunlight). Compare also the discussion of degree of illumination in the anticipations of perception (A176/B217–18), as well as the discussion of light in the third analogy (A213/B260: see the paragraph to which note 160 above is appended).

> This is now all that metaphysics can ever achieve towards the construction of the concept of matter, and thus for the sake of the application of mathematics to natural science, with respect to those properties whereby matter fills a space in a determinate measure – namely, to view these properties as dynamical, and not as unconditioned original positings [*Positionen*], as a merely mathematical treatment might postulate them. (534)

This passage reiterates the idea that the real possibility of the fundamental forces cannot be further explained a priori (and so, *a fortiori*, it cannot be demonstrated by a mathematical construction). What we can achieve, rather, is a reduction of apparently different forces and powers to the truly fundamental ones. So the proper role of metaphysics, accordingly, is to guide the application of mathematics to natural science in precisely this direction – and to do so, in particular, "with respect to those properties whereby matter fills a space in a determinate measure."

This passage suggests a different reading of the attempted construction that Kant presents in the first remark to the eighth proposition. Taking the laws governing the two fundamental forces of attraction and repulsion as given, the point is to show how a further property of matter – the property of dynamically filling a space to a determinate degree – is then mathematically derivable *from* these forces.[204] Moreover, the significance of such an attempted construction for Kant's present (critical) version of the dynamical theory of matter becomes clear if we contrast it, once again, with the at first sight very similar construction that Kant sketches in the *Physical Monadology*. For, as we have seen, this construction, presented in the Scholium to Proposition x, appeals to an inverse-square law for original attraction and an inverse-cube law for original repulsion to argue that the two laws together determine a definite spherical surface beyond which the original repulsion vanishes. They thereby determine a definite

[204] Förster is therefore correct in the discussion cited in note 200 above to distinguish the "difficulty" in the construction that Kant discusses in the second remark to the eighth proposition from the "impossibility" of constructing the dynamical concept of matter discussed in the general remark. The "difficulty" concerns the construction of "those properties whereby matter fills a space in a determinate measure" from the two fundamental forces, whereas the "impossibility," as I understand it, concerns precisely the construction of the two fundamental forces themselves. So, once again, the construction should not be taken as an (attempted) a priori mathematical proof of the laws governing the two fundamental forces. Rather, Kant is here simply presenting a general mathematical representation relating these forces to their (combined) filling of space in a determinate degree. In the words of the general remark (524) we are merely "judg[ing] a priori about the connection and consequences of these forces" given certain "relations among them we can think without contradiction," but we cannot "presume to suppose one of them as actual" unless we find an instantiation of such a relation in experience. See Friedman (1992b, chapter 4) for a detailed discussion of a parallel passage and issue arising in the *Prolegomena*.

spatial volume or "sphere of activity" for the physical monad in question. But they do not determine a notion of density or quantity of matter as well. Quantity of matter is here introduced in the following Proposition XI as an entirely distinct third force (*vis insita* or *vis inertiae*) that pertains to a physical monad quite independently of the spatial volume already determined (in Proposition X) by the two fundamental forces of attraction and repulsion.

Hence, the mathematical version of the balancing argument presented in the *Physical Monadology* cannot yield what Kant here aims to construct (517): the "concept of matter, as that of the movable filling its space (to a determinate degree)." By contrast, the corresponding version of the balancing argument presented in the *Metaphysical Foundations* attempts to construct the truly dynamical or intensive property of *filling* a given space or volume (to a determinate degree) from the two fundamental forces. Since quantity of matter is now characterized as the (infinite and continuous) aggregate of the movable in a given space, a balancing of the two fundamental forces is supposed to explain how a given such aggregate is then maintained or compressed within a particular (finite) volume – which gives rise (508) to a "specified quantity of matter" contained within a "specified space." As Kant puts it in the conclusion of his new mathematical version of the balancing argument (521): "[S]ince repulsion increases with the approach of the parts to a greater extent than attraction, the limit of approach, beyond which no greater is possible at the given attraction, is thereby determined, and so is that degree of compression which constitutes the measure [*Maß*] of the intensive filling of space."

We here encounter the characteristic distinguishing features of Kant's new mathematical version of the balancing argument. Since Kant's critical version of the dynamical theory of matter conceives matter as a true continuum, given by the (infinite and continuous) aggregate of the movable in a given space, the original force of repulsion must now be represented as a contact force acting only at an infinitely small distance rather than a finite distance. Moreover, since our problem is to explain how a given, intrinsically expansive aggregate of the movable can be maintained within a given space by a counterbalancing compression, the counteracting original attractive force is now thought of as acting, in the first instance, on precisely the all-pervasive aether – which then exerts a compression on other matter by an external pressure. Indeed, the present mathematical version of the balancing argument appears as a remark to the eighth proposition, which asserts that the true and original attraction extends immediately in the universe to infinity. It is preceded, moreover, by the second note to the eighth proposition, which appeals to just this feature of original attraction in suggesting (518) that "a determinate degree of the filling of space" is in fact made possible by "the uniting of [the individual attraction of the parts of the compressed matter among one another] with the attraction of all cosmic matter [*aller Weltmaterie*]."[205] Thus what Kant appears to have in mind, as explained in section 17 above, is that the pressure exerted by the all-pervasive aetherial medium responsible for the propagation of light (following Euler's theory) provides the (apparent) attraction that "conserves [the heavenly bodies] in their density by compression" (564; see note 152 above, together with the paragraph to which it is appended).[206]

So much for the first remark to the eighth proposition. The second remark is devoted to a serious difficulty for Kant's new mathematical version of the balancing argument, which arises directly from the circumstance that the critical version of this argument (unlike the parallel argument in the *Physical Monadology*) involves a fundamental force of repulsion operating only at an infinitely small distance (and therefore throughout an infinitesimal volume). Kant begins:

> I am well aware of the difficulty in this mode of explaining the possibility of a matter in general, which consists in the circumstance that, if a point cannot immediately propel another by repulsive force without at the same time filling the entire volume up to the given distance with its force, then it appears to follow that this volume would have to contain several impelling points, which contradicts the presupposition, and was refuted above (Proposition 4) under the name of a sphere of repulsion of the simple [elements] in space. (521)

[205] See the paragraph to which notes 148 and 149 above are appended. Immediately after the sentence quoted in the text Kant appears clearly to decide in favor of precisely such a "global" understanding of the counteracting (original) attraction (518): "The original attraction is proportional to the quantity of matter and extends to infinity. Therefore, the determinate filling of a space by matter in accordance with its measure [*Maße*] can, in the end, only be effected by the attraction extending to infinity and imparted to every matter in accordance with the measure [*Maße*] of repulsive force." Indeed, it is hard to see why Kant is emphasizing the circumstance that the true and original attraction extends to *infinity* at this point if he is not in fact opting for this alternative. Finally, it is noteworthy that the *Physical Monadology* does not even bother to assert that the original attraction extends to infinity (compare note 189 above, together with the paragraph to which it is appended): here the attraction balancing the fundamental force of repulsion is exerted only by the individual monad itself, and there is no need whatever for it to extend beyond the particular spatial volume thereby determined.

[206] It is for precisely this reason, it appears, that Kant's present use of the mathematical version of the balancing argument is intimately connected with the theory of *light* – and, in particular, with Euler's theory. The treatment in the *Physical Monadology*, by contrast, has almost nothing to say about light (see note 201 above).

The problem, then, is that a finite "sphere of activity" everywhere filled with repulsive force would, by the argument of the fourth proposition (section 13 above), necessarily contain an infinite number of repelling points, whereas Kant is here attempting to represent the elementary law of repulsion exerted by each single point. Kant's solution, accordingly, is to distinguish between "the concept of an actual [i.e., finite] space, which can be given, and the mere idea of a space, which is thought simply for determining the ratio of given spaces, but is not in fact a space" (521). He concludes:

> [S]ince the adjacent parts of a *continuous* matter are in contact with one another, whether it is further expanded or compressed, one then thinks [the] distances from one another as *infinitely small*, and this infinitely small space as filled by its repulsive force to a greater or lesser degree. But the infinitely small intervening space is not at all different from contact, and is thus only the idea of a space, which serves to make intuitive the expansion of a matter as a continuous magnitude. (521–22)[207]

In a truly continuous version of the dynamical theory of matter, therefore, the fundamental force of repulsion must be a function of the infinitely small rather than finite distance, and the difficulty that remains, for Kant, is simply that an infinitely small space is not itself representable or constructible (522): "But the infinitely small intermediate space is not at all different from contact, and is thus only the idea of a space that serves to make the expansion of a matter, as a continuous magnitude, intuitive, although it can in no way actually be conceived in this way." An infinitely small space, as an ideal limiting concept, cannot be given in pure intuition, and so this difficulty, in the end, reduces to a problem common to all such ideal limiting concepts: unlike other (finitary) mathematical concepts, they cannot, in fact, be constructed.[208] It is for precisely this reason that the attempted construction (of a determinate degree of the filling of space) in Kant's new mathematical version of the balancing argument

[207] As suggested in section 14 above, this passage is closely related, in turn, to a similar passage from Kant's first remark to the fourth proposition in connection with the argument of the second antinomy: see note 106 above, together with the paragraph to which it is appended.

[208] Being constructed in pure intuition, for Kant, implies being generated in a *finite* number of steps by an idealized iterative procedure. See Friedman (1992b, chapter 2) for further discussion of this point and for an application, in particular, to Kant's views on irrational numbers. In the context of the mathematical antinomies discussed in section 14 above the point is that the idea of an infinitely small space can only be successively approximated in our investigation of ever smaller spaces filled by matter, just as the idea of a most comprehensive absolute space can only be successively approximated in our investigation of ever larger nested rotating systems. Neither can possibly be mathematically constructed in a finite number of steps.

actually fails. So it is no wonder that he concludes the present (second) remark by officially "declar[ing]" that (522–23) he does "not want the present exposition of the law of an original repulsion to be viewed as necessarily belonging to the goals of my metaphysical treatment of matter, nor the latter … to be mixed up with the conflicts and doubts that could afflict the former."

Further light can be shed on the nature and character of this difficulty by considering the limiting process in question. Kant's view of this process is clearly suggested by his discussion both before and after the passages quoted above. If we include the two sentences preceding the first passage (521–22), we find that Kant invokes the example of atmospheric air:

> [I]f, as is actually the case, we think matter as a continuous magnitude, there is then no distance at all between the points immediately repelling one another, and thus no increasing or decreasing sphere of their activity. But matters can expand or be compressed (like air), and here one does represent to oneself a distance of their adjacent parts, which can increase and decrease. Yet since the adjacent parts of a *continuous* matter are in contact with one another, whether it is further expanded or compressed, one then thinks these distances as *infinitely small*, and this infinitely small space as filled by its repulsive force to a greater or lesser degree. (521–22)

So we begin with the known (Boyle–Mariotte) law relating (expansive) pressure to (finite) volume and arrive at the limiting case of an infinitely small volume by thinking the distances in question (between immediately adjacent parts) as also infinitely small.

Immediately after the second passage quoted above (522) Kant continues:

> If it is said, therefore, that the repulsive forces of the parts of matter that immediately repel one another stand in inverse ratio to the cubes of their distances, this means only that they stand in inverse ratio to the volumes one imagines between parts that are nevertheless in immediate contact, and whose distances must for precisely this reason be called *infinitely small*, so as to be distinguished from every actual distance. (522)

Thus, according to the Boyle–Mariotte law, (expansive) pressure is inversely proportional to (finite) volume. So if we now imagine that the volume in question becomes arbitrarily small, then so does the distance generating the (ever smaller) sphere of activity of repulsive force (as its radius). Since the volume is proportional to the cube of this distance, the (expansive) pressure in the limit is inversely proportional to the cube of the infinitely small distance. Kant's inverse-cube law for repulsive force is arrived at as the limit of the Boyle–Mariotte law as the volume in question becomes infinitely small.

We have now arrived at a further difficulty – one that Kant himself raises in the following paragraph. For, as observed, Kant there notes that his law of repulsive force would give rise to a completely different law of expansion and compression from the Boyle–Mariotte law. Kant invokes the Newtonian derivation in Book 2 of the *Principia*, according to which this law implies a law of repulsive force between the adjacent particles of a fluid in inverse ratio to their (finite) distances. Thus, although he arrives at a law for the fundamental force of repulsion by a limiting process beginning with the Boyle–Mariotte law, Kant cannot then recover this law from the law of repulsive force. The problem is that the Boyle–Mariotte law operates with the concept of *pressure*, not with the concept of a fundamental repulsive force, and the only way we have available, in the present context, for relating these two quite different concepts is precisely the Newtonian derivation.[209] Moreover, since Kant's fundamental repulsive force is a function of the infinitely small distance, it is quantitatively *incomparable* with any finite magnitude – and, in particular, with expansive pressure as a function of (finite) volume. As a result, there is no apparent way that this law of repulsive force can be brought into a determinate relation with the phenomenological regularity expressed by the Boyle–Mariotte law.[210] Kant's critical representation of a fundamental law

[209] As explained in section 16 above, Kant needs to make precisely this transition from the fundamental force of repulsion to the (expansive) pressure of a (continuous) matter distribution in the first part of the balancing argument presented in the fifth proposition, for only the latter depends on a function of (finite) volume: see the paragraph to which note 139 above is appended. The concept of pressure in a continuum model, as first introduced by Euler (see the paragraph to which note 63 above is appended), is a function of a point in the continuum together with a surface element at that point (for precisely which reason it is a *contact* force). Although this dependence on a surface element means that pressure indeed operates infinitesimally (on what we would now call the tangent space at the point), it nonetheless remains an essentially phenomenological magnitude not intrinsically connected with a fundamental force law (whether finite or infinitesimal). In the Newtonian derivation of a law of repulsive force as inversely proportional to the (finite) distance, by contrast, we hypothetically introduce a discrete model of the fluid (as a lattice of separated particles in static equilibrium), where, in accordance with phenomenological hydrostatics, the pressure is taken to be the same at all points and in all directions: see again note 47 above.

[210] In addition, although it may be true, in accordance with the third proposition of the Dynamics, that the original expansive force or (elastic) pressure of matter increases without limit as the matter in question is compressed (see the paragraph to which note 61 above is appended), this fact is not derivable from a fundamental (infinitesimal) law of repulsive force. For the same reason, therefore, Kant's conclusion of the present mathematical version of the balancing argument – relating the "limit of approach" due to original attraction (as a function of finite distance) to the (infinitesimal) drop-off rate of original repulsion – fails, strictly speaking, to make mathematical sense.

of repulsive force remains disconnected from any possible empirical basis for such a law, and so, in Kant's own words, it remains a mere relation (524) that "we can think without contradiction" but cannot "presume to suppose … as actual."[211]

Even if we set these mathematical and empirical difficulties aside, however, an important conceptual difficulty with Kant's attempted construction of a determinate degree of filling of space by the balancing of attractive and repulsive force still remains – a difficulty, once again, of which he is well aware. The concept of quantity of matter (or density) that is at issue in this construction is the dynamical concept associated with repulsive force and the possibility of compression. It is represented (505) as "one and the same quantum of repulsive force" by which "a greater or smaller space is to be represented as completely filled."[212] So the more this quantum is compressed (into a smaller space), the greater the density or the intensive filling of space. In Kant's attempted construction, accordingly, he is supposing that (521) there is a "limit of approach, beyond which no greater is possible by the given attraction," which determines "that degree of compression which constitutes the measure [*Maß*] of the intensive filling of space." The intensive filling of space by a given quantum of repulsive force, in other words, is measured by the smallest volume into which it can be compressed by the given degree of attractive force acting upon it.

The conceptual difficulty that arises here (even waiving the mathematical problem of comparing finite and infinitesimal quantities) is that, according to Kant's discussion in the general remark to dynamics, this notion of density or the intensive filling of space cannot be used to compare matters of specifically different kinds. This discussion contrasts "the system of absolute impenetrability" with "the dynamical system of a merely relative impenetrability" (525) and concludes:

[211] In this respect, in particular, the fundamental force of repulsion is deeply problematic empirically in a way in which the fundamental force of attraction is not. The latter has a firm empirical basis in Newton's "deduction from the phenomena" of the law of universal gravitation – from which it obtains its real possibility and objective reality. The fundamental force of repulsion initially appeared to have an analogous empirical basis in the behavior of gases or permanently elastic fluids, but precisely this basis has now been seen to be illusory: see again the paragraph to which note 47 above is appended, together with the immediately preceding paragraph. I shall return to this difficulty below.

[212] See note 132 above, together with the paragraph to which it is appended. More generally, the relationships among the three different concepts of quantity of matter at issue in Kant's critical version of the balancing argument are discussed throughout section 16 above.

[I]n the dynamical system of a merely relative impenetrability there is no maximum of minimum of density, and yet every matter, however rarified, can still be called completely dense, if it fills its space entirely without containing empty interstices, and is thus a *continuum*, not an *interruptum*. In comparison with another matter, however, it is less dense, in the dynamical sense, if it fills its space entirely, but not to the same degree. But in this system, too, it is inappropriate to think of matters as related with respect to their density, if we do not imagine them as specifically of the same kind, so that one can be generated from the other by mere compression. Now since the latter by no means appears to be necessary to the nature of all matter in itself, no comparison with regard to their density can properly take place between matters of different kinds, between water and mercury, for example, even though it is customary. (525–26)[213]

In his dynamical continuum view of matter, Kant suggests, we can indeed measure the degree of filling of space by the volume into which matter is compressed – but only in comparing matters of specifically the same kind. In this case, the ratio of densities between a given kind of matter compressed into two different volumes is given by the ratio of the volumes themselves. But such a comparison by volumes does not make sense between two specifically different kinds of matter (such as water and mercury), and the dynamical concept of density or the degree of filling a space is simply unsuitable in that case.[214]

Kant makes it clear in the following chapter that precisely this difficulty ultimately leads to the replacement of the dynamical concepts of density, quantity of matter, and the degree of filling a space by corresponding mechanical concepts – including, especially, the mechanical concept of mass or quantity of matter. The first proposition of the Mechanics states (537): "The quantity of matter, in comparison with *every* other matter, can be estimated only by the quantity of motion at a given speed." And the following proof hinges on the difficulty with which we have just been concerned:

[213] Compare note 142 above, together with the paragraph to which it is appended – where Kant's discussion (in the first number of the general remark to dynamics) was already discussed in a preliminary way.

[214] In the system of absolute impenetrability, by contrast, matter is absolutely or perfectly dense when it contains no empty interstices (525): "In accordance with this concept of the filling of space we make comparisons, and call one matter denser than another when it contains less emptiness, until finally that in which no part of the space is empty is called perfectly dense. One can only make use of the latter expression in connection with the merely mathematical concept of matter." In this system, therefore, there are no specifically different kinds of matter in Kant's sense, because any matter can be transformed into any other by an appropriate expansion or compression.

For even if this [estimation] occurs in comparing the given matter with another of the same kind, in which case the quantity of matter is proportional to the size of the volume, it is still contrary to the requirement of the proposition, that it is to be estimated in comparison with *every* other (including the specifically different). Hence matter cannot be validly estimated, either immediately or mediately, in comparison with every *other*, so long as we abstract from its own inherent motion; no other generally valid measure [*Maß*] remains, therefore, except the quantity of motion. (537–38)

I shall return to a detailed discussion of this proposition in the following chapter – and, in particular, to a consideration of exactly how Kant understands the estimation of quantity of matter "by the quantity of motion at a given speed." But it suffices at present to observe that Kant ultimately proposes mathematically to estimate the quantity of matter (and therefore density and the intensive filling of space) by the quantity of *motion* (i.e., momentum) of the matter under consideration. The concept of an intensive (as opposed to a purely extensive) filling of space can, in the end, be given the structure of a mathematical magnitude only by reference to a *spatio-temporal* quantity (motion) and not by reference to merely geometrical quantities (such as volume) alone. The final lesson of the Dynamics, in this context, is that any mathematical construction of either the concept of force or the concept of quantity of matter needs to proceed via a corresponding construction of the concept of motion.

20 THE DYNAMICAL THEORY OF MATTER, THE MECHANICAL PHILOSOPHY, AND CHEMISTRY

The Dynamics ends with a lengthy general remark devoted primarily to a contrast between what Kant calls the mathematical-mechanical approach to natural philosophy and his preferred metaphysical-dynamical approach. Here Kant recognizes two fundamentally opposed points of view. According to the mechanical natural philosophy (or mathematical-mechanical mode of explanation) there is a single, completely homogeneous, absolutely impenetrable, basic type of matter, and differences in density arise from differing mixtures of this basic material with differing amounts of empty space. According to the dynamical natural philosophy (or metaphysical-dynamical mode of explanation), by contrast, matter always entirely fills the space it occupies as a true continuum (as an originally fluid and elastic medium), and differences in density instead arise from different states of compression of matter in proportion to the ratio between the original forces of attraction and repulsion.

Kant begins the general remark (523) by stating "[t]he general principle of the dynamics of material nature," according to which "everything real in the objects of the outer senses, which is not merely a determination of space (place, extension, and figure), must be viewed as moving force." In this way, Kant continues, "the so-called solid or absolute impenetrability is banished from natural science, as an empty concept, and repulsive force is posited in its stead" (523). In addition, "the true and immediate attraction is thereby defended against all sophistries of a metaphysics that misunderstands itself, and, as a fundamental force, is declared necessary for the very possibility of the concept of matter" (523). Kant concludes:

> Now from this it follows that space, if it should be necessary, can be assumed to be completely *filled*, and in different degrees, even *without dispersing empty interstices* within matter. For, in accordance with the originally different degree of the repulsive forces, on which rests the first property of matter, namely that of filling a space, their relation to the original attraction (whether of any [piece of] matter separately, or to the united attraction of all matter in the universe) can be thought as infinitely various. This is because attraction rests on the aggregate of matter in a given space, whereas its expansive force, by contrast, rests on the degree of filling of this space, which can be very different specifically (as the same quantity of air, say, in the same volume, manifests more or less elasticity in accordance with its greater or lesser heating). (523–24)

Kant is here relying on the considerations advanced in the notes and remarks to the preceding eighth proposition, including his comments on the Boyle–Mariotte law for atmospheric air made in the second remark (522; see the paragraph to which note 209 above is appended).

Kant articulates the same contrast between two approaches to natural philosophy later in the general remark:

> But now as to the procedure of natural science with respect to the most important of all its tasks – namely, that of explaining a potentially infinite *specific variety of matters* – one can take only two paths: the *mechanical* [path], by combination of the absolutely full with the absolutely empty, and an opposing *dynamical* path, by mere variety in combining the original forces of repulsion and attraction, to explain all differences of matters. The first has as materials for its derivation *atoms* and the *void*. An atom is a small part of matter that is physically indivisible. A matter is physically *indivisible* when its parts cohere with a force that cannot be overpowered by any moving force in nature. An atom, in so far as it is specifically distinguished from others by its figure, is called a *primary corpuscle*. A body (or corpuscle) whose moving force depends on its figure is called a *machine*. The mode of explaining the specific variety of matters by the constitution and composition of their smallest parts, as machines, is the *mechanical natural philosophy*; but that which derives this specific variety from matters, not as machines, that is, mere instruments of external moving forces, but from

the moving forces of attraction and repulsion originally inherent in them, can be called the *dynamical natural philosophy*. The mechanical mode of explanation, since it is the most submissive [*fugsam*] to mathematics, has, under the name of *atomism* or the *corpuscular philosophy*, always retained its authority and influence on the principles of natural science with few changes from Democritus of old, up to Descartes, and even to our time. (532–33)

What is most striking here is that an important natural philosopher of Kant's own time – namely Lambert – appears to be the best representative of the path Kant wants to oppose.

As explained in section 11 above, Lambert is the most salient representative of the concept of solidity or absolute impenetrability Kant now wants to have "banished from natural science, as an empty concept" (523). It is Lambert, in the first instance, who is most representative, for Kant, of the "mathematical and mechanical investigators of nature" committed to explaining differences in density in terms of differing mixtures of (absolutely) filled with (absolutely) empty space (see note 40 above, together with the paragraph to which it is appended). Lambert also best represents "the mathematician" – who, according to Kant's remark to the first proposition of the Dynamics (498), "has assumed something, as a first datum for constructing the concept of matter, which is itself incapable of further construction." By assuming absolute impenetrability, in particular, Lambert has blocked the application of the construction of the composition of motions in phoronomy from playing any further role in dynamics. By insisting on the purely mathematical concept of the filling of space – and thus on the mathematical-mechanical natural philosophy – "the mathematician," paradoxically, has erected an insurmountable limit to the application of mathematics, in Kant's sense, at the instant of attempted penetration of an absolutely solid body.[215]

This point is especially important, because it illuminates the crucial difference between Kant's own conception of the application of mathematics to physical nature and the more customary conception characteristic of the mechanical philosophy. On the latter conception, beginning with Descartes, the idea is to understand matter itself, as far as possible, in purely mathematical (purely geometrical) terms. The most essential properties of matter are extension and figure, and one attempts to explain even

[215] The problem, as explained in section 10 above, is that the assumption of absolute impenetrability leads to a discontinuity at the turn-around point of attempted penetration, and therefore the mathematics of a continuously acting force (as, paradigmatically, in Galileo's law of fall) is inapplicable here: see note 21 above, together with the paragraph to which it is appended and the preceding paragraph. Compare also the final two paragraphs of section 11 above.

the property by which matter fills a given volume of extension to a certain degree (density) in terms of the (purely geometrical) ratio between absolutely filled and absolutely empty space within this volume.[216] On Kant's conception, by contrast, matter cannot be understood purely mathematically (purely geometrically). For, as we have seen, the original forces of attraction and repulsion – which, according to Kant, constitute matter as such – can by no means be constructed a priori in pure intuition. The concept of matter, for precisely this reason, is an *empirical* (rather than purely mathematical) concept.

Kant thus takes the problem of explaining the application of mathematics to nature to be highly non-trivial: it cannot be accomplished in one fell swoop, as it were, by defining matter in purely geometrical terms. What is required, rather, is an explanation of how it is possible to apply mathematical construction to empirically given properties of matter step by step, beginning with the most fundamental such property – motion – and then proceeding to others such as density, mass, and force. The required explanation, in Kant's view, also involves a full appreciation of how the traditional a priori concepts of metaphysics (such as substance, causality, and so on) are themselves applied to the objects of our empirical intuition.[217] In understanding how the concepts of density and quantity of matter can acquire the structure of mathematical magnitudes, in particular, Kant thinks that a full appreciation of the complexities involved in applying the pure category of substance to our experience of matter is required. We thereby find, in the Dynamics, that material substance is

[216] Of course Descartes does not believe in absolutely empty space (the void) and thinks that all of space is filled as a plenum. Nevertheless, he takes there to be three essentially different types of matter of different degrees of subtlety: the most subtle (and rapidly moving) matter of light (including the sun and the fixed stars), the next most subtle matter constituting the heavenly vortices, and finally the more ordinary matter constituting the earth, planets, and comets: see *Principles of Philosophy* (1644, Part III, §§49–52). Accordingly, Descartes explains condensation and rarefaction (using the example of a sponge) in terms of one matter containing another within its interspersed pores: when these pores are completely eliminated by compression, the remaining (original) matter becomes absolutely dense (Part II, §§5–7). Thus Descartes's conception of density is fundamentally the same as on the more classical version of corpuscularianism postulating atoms and the void. It appears that Kant names Descartes here because of his well-known view that the essence of matter is extension and therefore wholly geometrical.

[217] See again the important paragraph on mathematics and metaphysics from Kant's Preface (which is quoted and discussed in the Introduction), where Kant says that "a complete analysis of the concept of a matter in general" is "a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions of space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*" (473; see note 45 of the Introduction, together with the paragraph to which it is appended and the remainder of the Introduction).

necessarily infinitely divisible, and later, in the Mechanics, that (for precisely this reason) the quantity of material substance (in a given space) can only be estimated mathematically by the quantity of motion at a given speed. Thus it is the metaphysical-dynamical approach, according to Kant, that is best suited for a proper explanation of the application of mathematics to the (empirical) concept of matter. And this is so despite, and even because of, the fact that the mathematical-mechanical approach, in Kant's view, makes matter "most submissive to mathematics" (532–33) in so far as the problem of application is then easily – all *too* easily – solved in one fell swoop.[218]

Kant articulates the sense in which the mathematical-mechanical approach indeed has an advantage over the metaphysical-dynamical approach in his earlier discussion:

> We can indeed certainly judge a priori about the connection and consequences of [fundamental] forces, whatever relations among them we can think without contradiction, but we cannot yet presume to suppose one of them as actual; for to be authorized in erecting an hypothesis it is unavoidably required that the *possibility* of what we suppose be completely *certain*, but with fundamental forces their possibility can never be comprehended. And here the mathematical-mechanical mode of explanation has an advantage over the metaphysical-dynamical [mode], which cannot be wrested from it, namely, that of generating from a thoroughly homogeneous material a great specific variety of matters, according to both their density and mode of action (if foreign forces are added), through the varying shape of the parts, by interspersed empty interstices. For the possibility of both the shapes and the empty interstices can be verified with mathematical evidence. (524–25)

In the mathematical-mechanical approach we postulate only absolutely hard and impenetrable elementary corpuscles of various sizes and shapes: we dispense with all fundamental forces in the Kantian sense. Because size, shape, and absolute impenetrability, for Kant, are purely mathematical concepts, their real possibility (unlike the real possibility of Kant's fundamental forces) can indeed be verified a priori. Whether such

[218] It is important not to be misled by Kant's terminology here. This is especially true if we recall from the end of the previous section that it is the mechanical concept of quantity of matter (in *Kant's* sense of "mechanics") that best facilitates the application of mathematics (quantitative estimation) to quantity of matter. Indeed, it is precisely because Kant adopts the *dynamical* (as opposed to merely "mathematical") concept of density that his *mechanical* concept of quantity of matter is our best (and indeed only) option for a proper mathematization: see note 214 above, together with the paragraph to which it is appended and the remaining two paragraphs of section 19 above.

elementary corpuscles actually exist in nature, however, is a completely different question.[219]

The parenthetical insertion in the above passage (525), where Kant entertains the possibility that "foreign forces [*fremde Kräfte*]" may be "added" to the originally homogeneous, (absolutely) impenetrable matter, is of particular interest. For it raises the question of how Newton's conception of matter fits into Kant's bipartite scheme. In the famous Query 31 of the *Opticks*, for example, Newton begins with "solid, massy, hard, impenetrable, movable Particles" or atoms, but these particles are also animated by further active principles or powers – by forces of attraction and repulsion which may, for all we know, act immediately at a distance.[220] Indeed, precisely this Newtonian conception exerted a profound influence on eighteenth-century matter theory, giving rise to both the "nut shell" view of matter as consisting primarily of empty space and the program for explaining chemical phenomena, in particular, by short-range forces of attraction and repulsion.[221] Thus, the Newtonian version of an approach to natural philosophy based on atoms and the void goes far beyond the standard versions of the mechanical philosophy (such as that of Lambert, for example) by "adding" active powers or active principles to the originally passive solid matter. So where does this distinctively Newtonian version stand vis-à-vis Kant's own dynamical approach?

[219] Indeed, Kant goes on to insist that absolute impenetrability must be rejected as an "empty concept" (525):

> [A] merely mathematical physics pays doubly for this advantage on the other side. First, it must take an empty concept (of absolute impenetrability) as basis; and second, it must give up all forces *inherent* in matter; and beyond this, further, with its original configurations of the fundamental material and its interspersing of empty spaces … such a physics must allow more freedom, and indeed rightful claims, to the imagination in the field of philosophy that is truly consistent with the caution of the latter.

Thus, although there is no doubt that the concept in question (as merely mathematical) is indeed really possible, Kant does not believe that actual concrete instances of this concept can ever (with proper justification) be found in experience.

[220] See Query 31 of the *Opticks* (O400): "[I]t seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, movable Particles, of such Sizes and Figures, and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he form'd them." And among such "other Properties," it is clear, are precisely various forces of attraction and repulsion (401): "It seems to me farther, that these Particles have not only a *Vis inertiae*, accompanied with such passive Laws of Motion as naturally result from that Force, but also that they are moved by certain active Principles, such as that of Gravity, and that which causes Fermentation, and the Cohesion of Bodies."

[221] See, e.g., Thackray (1970). It is clear that Kant was not only well acquainted with the beginnings of this program in the Queries to Newton's *Opticks* but also with its further development throughout the eighteenth century by such writers as 'sGravesande, Musschenbroek, Boerhaave, and Buffon.

We know, on the one hand, that Kant consistently models his two original forces of attraction and repulsion on "the Newtonian philosophy" – in particular, on Newton's "deduction from the phenomena" of universal gravitation in Book 3 of the *Principia* and the analogous derivation of the law of a repulsive force for permanently elastic fluids in Book 2.[222] Yet it is not clear, on the other hand, whether Newton's own conception of these forces makes them "originally inherent" in matter in Kant's sense, since, for Newton, they are not actually constitutive of matter itself – as "solid, massy, hard, impenetrable, [and] and movable."[223] Be this as it may, however, the more interesting question concerns what Kant makes of the Newtonian program for chemistry, which is predicated on the consideration of short-range forces of attraction and repulsion responsible for such phenomena as fermentation and cohesion. This question is especially pressing, because, as just observed, precisely this Newtonian program exerted an overwhelming influence on the matter theory and chemistry of the eighteenth century.

The Preface to the *Metaphysical Foundations* famously asserts that chemistry is not yet a science properly speaking but merely a "systematic art or experimental doctrine":

So long, therefore, as there is still for chemical actions of matters on one another no concept to be discovered that can be constructed, that is, no law of the approach or withdrawal of the parts of matter can be specified according to which, perhaps in proportion to their density or the like, their motions and all the consequences thereof can be made intuitive and presented a priori in space (a demand that will only with great difficulty ever be fulfilled), then chemistry can be nothing more than a systematic art or experimental doctrine, but never a proper science, because its principles are merely empirical, and allow of no a priori presentation in intuition; consequently, they do not in the least make the

[222] See note 47 above, together with the paragraph to which it is appended and the preceding paragraph. Moreover, the criticism Kant makes of the mathematical-mechanical approach in the passage quoted in note 219 above is very much in the spirit of the Newtonian *hypotheses non fingo* (see note 20 of the Introduction). The connection with Newton's *hypotheses non fingo* becomes even clearer in a related passage several pages later (532) – to which I shall return at the end of this section.

[223] See again the passage quoted above (532), where Kant characterizes the dynamical natural philosophy as one that that "derives this specific variety from matters, not as machines, that is, mere instruments of external [*äußerer*] moving forces, but from the moving forces of attraction and repulsion originally inherent in them." Newton certainly postulates one "inherent" or "innate force [*vis insita*]," i.e., the "force of inertia [*vis inertiae*]," and Kant does the same in the *Physical Monadology*: see notes 187–89 above, together with the paragraphs to which they are appended. However, as we shall see, Kant no longer counts the force of an inertia as a genuine force in the *Metaphysical Foundations*, and, in any case, there appears to be nothing in Newton's view corresponding to the Kantian conception of the original forces of attraction and repulsion as *constitutive* of matter.

principles of chemical phenomena conceivable with respect to their possibility, for they are not receptive to the application of mathematics. (470–71)

Kant simultaneously asserts that the way for chemistry to become a science is through the discovery of further Newtonian-style forces of attraction and repulsion governing the microscopic interactions responsible for chemical phenomena, and also that this discovery has not yet occurred (and will "only with great difficulty" ever occur). It follows that chemistry, in particular, is not yet a science, for (unlike in mathematical physics) the phenomena do not as yet have an a priori basis – our principles, so far, are merely empirical regularities.[224]

But it then follows, in accordance with the very conception of real possibility Kant articulates in the general remark, that the short-range attractive and repulsive forces Newtonians invoke to explain chemical phenomena are not even really possible. In general, as explained in section 19 above, a fundamental force, for Kant, can only receive its real possibility or objective reality empirically, in virtue of actual empirical phenomena from which such a force can be inferred. In the case of the true and original attraction, for example, we have the Newtonian "deduction from the phenomena" from the observable relative motions in the solar system. In the case of the fundamental force of repulsion, moreover, we have both our ordinary experience of impenetrability gained in the manipulation of solid bodies and the expansive behavior of gases or permanently elastic fluids – which, according to Kant's argumentation in the first part of the Dynamics, must lie at the basis of our ordinary experience of impenetrability.[225] Both of these forces are really possible, for Kant, because they are given as actual in experience as the objects of irreducibly empirical concepts (see again note 25 above, together with the paragraph to which it is appended). Yet precisely this is what we do not yet have (and may

[224] See the related passage several paragraphs earlier in the Preface (468):

> If, however, the grounds or principles in [a science] are still in the end merely empirical, as in chemistry, for example, and the laws from which the given facts are explained through reason are mere laws of experience, then they carry with them no consciousness of their *necessity* (they are not apodictically certain), and thus the whole of cognition does not deserve the name of a science in the strict sense; chemistry should therefore be called a systematic art rather than a science.

[225] See again notes 46 and 47 above, together with the paragraphs to which they are appended. I here temporarily set aside the problem raised at the end of section 19 above in the case of the fundamental force of repulsion: see note 211 above, together with the paragraph to which it is appended. What we can actually "deduce from phenomena" in this case is only the phenomenological magnitude of (expansive) pressure: see note 209 above, together with the paragraph to which it is appended. I shall return to this point below.

not ever have) in the case of the short-range forces Newtonians invoke to explain chemical phenomena. So it follows, according to Kant, that it is not even legitimate to entertain such forces hypothetically, for, as he says (524), "to be authorized in erecting an hypothesis it is unavoidably required that the *possibility* of what we suppose be completely *certain*, but with fundamental forces their possibility can never be comprehended [a priori]." A fundamental force can be legitimately invoked, for Kant, only when it is not merely hypothetical but already actually exhibited in given phenomena.[226]

The crucial point, in this connection, is that Kant's preferred dynamical mode of explanation is not put forward as one more hypothetical project for inquiring into the inner structure of matter. It is undertaken, from a methodological point of view, on behalf of a purely *experimental* natural philosophy, which is supposed to proceed, accordingly, entirely independently of theoretical hypotheses:

> In order now to introduce a dynamical mode of explanation (which is much more appropriate and conducive to the experimental philosophy, in that it leads directly to the discovery of matter's inherent moving forces and their laws, while restricting our freedom to assume empty interstices and fundamental corpuscles of determinate shapes, neither of which are determinable or discoverable by any experiment), it is not at all necessary to devise [*schmieden*] new hypotheses, but only to refute the postulate of the merely mechanical mode of explanation – *that it is impossible to think a specific difference in the density of matters without interposition of empty spaces* – by simply advancing a mode of explanation in which this can be thought without contradiction. For once the postulate in question, on which the merely mechanical mode of explanation rests, is shown to be invalid as a principle, then it obviously does not have to be adopted as an hypothesis in natural science, so long as a possibility remains for thinking specific difference in densities even without any empty interstices. (533)

It appears that Kant's own representation of specific differences in density – in terms of differing ratios of the fundamental force of repulsion to the fundamental force of attraction resulting in different intrinsic quantities of expansive elasticity – is not put forward as another explanatory hypothesis opposed to the mathematical-mechanical alternative. It is rather intended, in the first instance, as simply a rejection of the supposed necessity of this alternative. Once this supposed necessity is shown to be

[226] This is why, immediately after the passage (534) asserting that "no law of either attractive or repulsive force may be risked on a priori conjectures," Kant adds: "Still less may such [laws] be attempted in the case of chemical affinities otherwise than by way of experiments" (534). Chemical affinities figure prominently among the phenomena for which short-range forces of attraction and repulsion were customarily invoked.

illusory, the way is then cleared for a purely experimental investigation proceeding without reliance on any theoretical hypotheses whatsoever.[227]

To be sure, Kant devotes the bulk of the general remark (525–31) to a discussion, under four numbered headings, of his proposed positive contribution to matter theory and, in particular, to the problem of the specific variety of matter. The discussion begins with a preface (525): "Instead of a sufficient explanation for the possibility of matter and its specific variety from these fundamental forces, which I cannot provide, I will present completely, so I hope, the moments to which its specific variety must collectively be reducible a priori (albeit not conceivable in regard to its possibility. The remarks inserted between the definitions will explain their application." In light of this and Kant's just-quoted remarks concerning the purely methodological import of his metaphysical-dynamical approach, I do not understand his proposed positive contribution as consisting of explanatory hypotheses. It rather consists of definitions of fundamentally phenomenological or experimental properties of various types and states of matter, together with interspersed remarks explaining the application of these definitions. The entire discussion, accordingly, presupposes only the fundamental forces that have already been established in the dynamical theory of matter in general – that is, the two fundamental forces of attraction and repulsion, which, as we have seen, have already been sufficiently established empirically (subject to the proviso in note 225 above).

Thus, the first number introduces the concept of density and again contrasts two different conceptions: "the system of absolute impenetrability" and "the dynamical system of a mere relative impenetrability" (525; see the paragraph to which note 214 above is appended). The second number introduces the concept of cohesion and the related concept of fluidity (whereby parts of a matter can be displaced along one another while still cohering with one another). It proceeds to a rather lengthy and substantial discussion of the elements of hydrostatics, which features an emphatic

[227] Kant makes a parallel point in the passage from the beginning of the general remark. After sketching his own metaphysical-dynamical conception of differences in density, he remarks that the import of this conception is purely methodological (524):

> In all this the advantage of a metaphysics that is here used methodically, to get rid of principles that are equally metaphysical, but have not been brought to the test of criticism, is apparently only *negative*. Nevertheless, the field of the natural scientist is thereby indirectly enlarged; for the conditions by which he formerly limited himself, and through which all original moving forces were philosophized away, now lose their validity. But one should guard against going beyond that which makes possible the general concept of a matter as such, and wishing to explain a priori its particular or even specific determination and variety.

assertion of "the original character of the property of fluidity" as a true continuum (528; see note 64 above, together with the passage to which it is appended). The third number introduces the concept of elasticity (as applied, for example, to solid elastic materials) and distinguishes original and derived elasticity (as in the case of atmospheric air; see note 50 above, together with the paragraph to which it is appended and the preceding paragraph). Nevertheless, in accordance with the first note to the second proposition (530), all matter, "as matter in general," must "already in itself have [an] elasticity, which is original." The fourth number, finally, characterizes the distinction between mechanical and chemical actions of matters (530): the first is the "action of moved bodies on one another through the communication of their motion," while the second takes place "in so far as [matters] mutually alter the connection of their parts also at rest by means of their own inherent forces." The main examples of such chemical actions are solutions and dissolutions.

It is precisely here, in chemistry, that we find the proper methodological goal of Kant's preferred dynamical natural philosophy. This becomes especially clear, once again, if we compare the present discussion in the general remark with the parallel discussion in the *Danziger Physik*. For, in place of the distinction between mechanical and dynamical modes of explanation, the *Danziger Physik* employs a distinction between mechanical and chemical modes. Mechanics has primarily to do with (29, 116–17) the communication of motion ("for example, by impact"); and "these kinds of changes belong to the mathematics of nature" where mechanical forces "change matter not inwardly, but rather [with respect to] figure, connection, separation, and location of the parts." Mechanics treats the motion of light, statics or the theory of simple machines, hydrostatics (117: "how fluid matters can be moved by the pressure of others"), and also universal attraction as manifested in the motions of the heavenly bodies. In chemistry, by contrast, we consider (117) how "the constitution of matter is inwardly changed, that is, becomes specifically otherwise or different." Such changes result in chemical solutions and dissolutions involving (117) affinities or "chemical forces," which are "the constitutive forces of matter whereby new matters are generated," and "chemical forces rest neither on the figure [of the parts] nor on laws of impact or attraction at a distance."

The *Danziger Physik* distinguishes, accordingly, between the mathematical part of physics and the chemical part (29, 97): "The mathematical part of physics teaches the laws of the actions of bodies on bodies and has a priori principles, and is thereby distinguished from chemistry which teaches the laws of the actions of matter on matter" – for "mathematics is not at all sufficient to explain chemical results or one is not yet able mathematically to explain any chemical experiment." Indeed:

> [O]nly the least [amount of] natural phenomena can be explained mathematically – only the smallest part of natural events can be mathematically demonstrated; so, for example, it can indeed be explained in accordance with mathematical principles why snow falls to the earth, but [concerning] how vapours are transformed into drops or dissolved mathematics provides no information, rather this must be explained from universal empirical laws of chemistry. (97–98)

Then, in a manner clearly reminiscent of the Preface to the *Metaphysical Foundations*, the *Danziger Physik* (99) divides "the universal doctrine of nature" into "mathematical physics, chemistry, and natural description," and (101) divides "pure physiology" (or pure natural science) into "mathematics of nature" and "metaphysics of nature."[228]

We are also presented with a division of the "physical mode of explanation" into two subsidiary types:

> 1. The mechanical [mode] where one explains something from already present and available [*vorhandenen*] forces. Thus, for example, Descartes explained the solution of crab's eye [rosary pea] in vinegar mechanically when he assumed the parts of vinegar to be sharp and thus to penetrate into the crab's eye when heat drives the particles into the crab's eye as with a warm blow.
>
> 2. The dynamical [mode] where one places particular not yet present and available forces at the basis. In chemistry one explains almost everything dynamically. The mechanical and dynamical modes of explanation taken together constitute the physico-mechanical [mode]. (29, 105)[229]

[228] The corresponding part of the Preface divides the doctrine of nature into (468) the "historical doctrine of nature" (including "natural description") and natural science, where the latter can now be either "*properly*, or *improperly* so-called natural science." Chemistry is then only "*improperly* so-called natural science," precisely because it cannot (so far) be treated mathematically. For pure (or "rational") physiology compare also the architectonic of pure reason in the first *Critique* (A847–48/B875–76). According to the *Danziger Physik* (29, 101):

> The mathematics of nature (*physiologia pura*) is also called *physica generalis*. It is the basis of all cognition of nature and the noblest part and has the greatest uses. It is now very developed, but it can only be used in the case of mechanical explanations. In the case of chemical [explanations] it does not achieve any uses; but in the first [case] it provides evidence and intuitive conviction.

Compare again the architectonic of pure reason (A847/B875n.), where Kant distinguishes between "rational physiology" (in this case "*physica rationalis*") and "*physica generalis*."

[229] The reference to Descartes appears to be to *Principles*, Part IV, §61, to which I shall return below. Crab's eye or rosary pea (to which Descartes does not specifically refer) is a tropical woody vine having black poisonous seeds used as beads.

It seems clear, then, that what Kant calls the metaphysical-dynamical mode of explanation in the general remark to dynamics of the *Metaphysical Foundations* is focussed primarily on the science of chemistry – as "systematic art or experimental doctrine" (471).

But how is the dynamical mode of explanation supposed to be applied to chemistry as an "experimental doctrine"? The tenth and final section of the *Danziger Physik* ("Of the elements or materials from which bodies are composed") is entirely devoted to chemistry as Kant understood it at the time. It begins as follows:

> It is singular that, although the entire world speaks of pure fire, water, air, etc., we must nevertheless admit that nothing is pure. We therefore see that we have mere actions in us, to which we relate the materials, and with respect to these we call them pure. Our reason makes certain classifications that precede experience and in accordance with which we then order our experiences. Reason employs for this purpose certain forces, about which mechanics best instructs us, and among which the simplest is the lever. The whole of nature is a doctrine of motion and without this merely space remains. But the doctrine of motion presupposes something movable (*onus*) and something moving (*Potentia*). The *Commercium* among the two is called in mechanics *Machina* or *Vehiculum* when it refers to inner connections, and it here appears also that all of our concepts ultimately come to this. In nature we cannot penetrate the differences of things in such a way that we could determine everything, but we must rather follow the classification of the understanding, where it is necessary that it proceed according to universal rules. The earths are the *Onus*, the negative material that has no dissolving force but is rather to be dissolved. Salts and inflammable things are the two *Potentiae* in nature that dissolve everything. (29, 161)

This passage is quite similar to a well-known passage from the regulative use of the ideas of pure reason in the first *Critique*, where Kant speaks (A645–46/B673–74) of "concepts of reason" that "are not extracted from nature; rather, we put questions to nature in accordance with these ideas, and we take our cognition to be deficient so long as it is not adequate to them." He illustrates (A646/B674) with "*pure earth, pure water, pure air*, etc" – which concepts, "with respect to complete purity, thus have their origin only in reason." Such concepts are necessary, he concludes:

> [I]n order properly to determine the share that each of these natural causes has in the appearance; and so we reduce all materials to the earths (as it were mere weight), salts and inflammable beings (as force), and finally to water and air as vehicles (as it were machines, by means of which the foregoing act), in order to explain the chemical actions of materials among one another in accordance with the idea of a mechanism. (A646/B674)

It seems clear, accordingly, that the faculty of reason in its regulative capacity is what directs the experimental progress of chemical inquiry.

It also seems clear, in both passages, that the chemistry Kant has in mind is that of Georg Ernst Stahl, the great developer of the theory of phlogiston in the early years of the eighteenth century.[230] For Stahl, phlogiston (the principle of inflammability) and salts (that is, acids and alkalis) are the true chemical agents. In combustion and calcination phlogiston is released from the body in question, leaving behind an inert earth or calx. Thus, a metal, for example, is a compound of an earth plus phlogiston, so that when the metal is calcined by heating the phlogiston then "burns off." Moreover, a metal can also be dissolved in an acid, whereby the acid combines with phlogiston resulting in a compound of the earth underlying the metal with a substance associated with the acid. For example, sulphuric acid (vitriolic acid or oil of vitriol), for Stahl, is an elementary material not a compound, while sulphur is a compound of the acid plus phlogiston: when a metal is dissolved in sulphuric acid, its phlogiston bonds with the acid resulting in the corresponding metallic sulphate (vitriol). Thus phlogiston and salts (e.g., acids) are the primary agents of chemical affinity, whereas water and air serve as mechanical "vehicles" or "instruments" for literally carrying these agents from place to place. In combustion the presence of atmospheric air serves as a necessary "mechanical" condition for the chemical interaction in question by carrying off the phlogiston that is extracted from the inflammable body (so that when the air becomes "phlogisticated" or thoroughly saturated with phlogiston the body can no longer burn). Similarly, when sulphuric acid diluted in water dissolves a metal, the water carries off the dissolved parts of the metal from its surface (now deprived of their phlogiston and combined with sulphur) and disperses them uniformly within the resulting solution of metallic sulphate.[231]

Stahl's work is important from a methodological point of view as well. In contrast to both the purely mechanical explanations of chemical

[230] See, e.g., Metzger (1930), Partington (1961, chapter 13), Thackray (1970, §6.3). As Thackray emphasizes, in particular, it is important not to overlook Stahl's importance from our modern Lavoisier-inspired point of view. Partington calls Stahl "[o]ne of the outstanding chemists of the eighteenth century" (1961, p. 653), in so far as he was the first to transform the phlogiston theory – "the first real chemical theory" – into a genuine "system of chemical doctrine" (p. 666). That Kant has Stahl foremost in mind here will become completely clear below.

[231] See Kant's further development and illustration of Stahlian chemistry in the *Danziger Physik* (29, 161–69). Compare also side II of R. 45 (approximately 1755–57), together with Adickes's comments (14, 371–96).

phenomena of the Cartesian school and the short-range force conception of the Newtonian school, Stahl advocated a systematic empirical program of chemical experimentation drawing from the chemical and metallurgical traditions rather than from physical theories of the internal structure of matter. And it is in precisely this spirit that the Preface to Stahl (1723) expresses strong doubts concerning the usefulness of "mechanical philosophy" in chemistry.[232] Kant's rejection of the mechanical mode of explanation in chemistry is thus entirely of a piece with Stahl's chemical methodology. So when Kant, in the two passages just quoted concerning the role of reason in chemical method, speaks of mechanics as a guide to chemistry under "the idea of a mechanism," he is not invoking a direct application of mechanics to chemical phenomena as in the mechanical natural philosophy that he explicitly opposes. He is rather suggesting that mechanical experience constitutes the starting point for properly experimental investigation in general, so that chemical experimentation in particular can be appropriately directed and systematically organized using ideas drawn from the science of mechanics (such as the ideas of weight and force).

What Kant has more specifically in mind becomes clearer from a famous passage from the Preface to the second edition of the first *Critique* – which passage, we are now in a position to see, is very closely related to the two Kantian passages quoted above concerning chemical methodology. After describing the great revolutions in the mode of thinking resulting in the inventions of logic and scientific geometry by the ancient Greeks, Kant turns to the more recent such revolution in physics or natural science. Kant is here considering natural science (Bxii) only "in so far as it is based on *empirical* principles," and he accordingly cites "Baco von

[232] See Stahl (1723, p. 2):

> Mechanical philosophy, though it vaunts itself as capable of explaining everything most clearly, has applied itself rather presumptuously to the consideration of chemico-physical matters. In fact, although I will not spurn a sober use of mechanical philosophy, everyone who is not blinded by prejudice will admit that it has brought no progress in these matters. I am not surprised, for it is lost in doubts, and just scratches the shell and surface of things and leaves the kernel untouched, since it is content with deducing general causes of phenomena from the shape and motion of particles, and is uninterested in the nature, properties, and differences between mixed, composite, and aggregate bodies.

Partington (1961, p. 665) translates the first sentence and part of the third; I have filled in the rest with the help of Vincenzo De Risi. The Preface is written by Stahl's student Johann Samuel Carl, who edited the volume from Stahl's lecture notes with his approval. It does not appear in the translation by Peter Shaw (1730), who replaced it with his own Dedication and Advertisement.

Verulam" as the chief apologist for the revolutionary new methodology.[233] He continues:

When Galileo caused balls with a weight he had himself chosen to roll down an inclined plane, or Torricelli caused air to support a weight that he had previously thought as equal to a known volume of water, or, in still later times, [when] Stahl transformed metals into calx and the latter again into metal, in that he extracted something from them and then restored it,* a light broke upon all investigators of nature. They grasped that reason has insight only into that which it itself brings about in accordance with its plan, that it must lead the way with principles of its judgements in accordance with constant laws and compel nature to answer its questions – not, however, [so as] simply to be led around, as it were, by her guiding-strings; for otherwise accidental observations, made in accordance with no previously outlined plan, do not cohere together in a necessary law, which reason nevertheless seeks and requires. Reason, with its principles in accordance with which alone concordant appearances can be valid as laws, in the one hand, and with the experiment it has thought out in accordance with these [principles], in the other, must certainly approach nature in order to be taught by it – not, however, in the character of a student, who takes as dictation everything the teacher says, but rather in that of an appointed judge, who compels the witnesses to answer the questions he puts before them. (Bxii–xiii)[234]

The prominent role of Stahlian phlogistic chemistry makes the close kinship of this passage with our two previous passages completely evident. Moreover, the particular sequence of examples with which the passage begins also suggests, in the present context, a particular trajectory of experimental inquiry under the guidance of the regulative use of reason "in accordance with the idea of a mechanism" (A646/B674).

We begin with Galileo's experiments using inclined planes to infer the rate of fall of heavy bodies under the influence of gravity. Here we find the first steps from statics or the traditional theory of machines to the modern theory of motion. On the one hand, we relate the traditional statical concept of weight as determined by the balance to the new kinematical concept of accelerated motion; on the other, we do this by means

[233] There is a parallel reference to Bacon in the *Danziger Physik* (29, 103): "Baco von Verulam, Chancellor of England, was the first to counsel becoming acquainted with nature through experiment and after him we have become more and more inclined to experimentation. In order to make experiments one employs certain instruments whose use depends on the wit of the observer appropriately to profit thereby."

[234] In the footnote attached to his sequence of examples Kant makes it clear that he is not attempting precisely to follow the actual "history of the experimental method" but is presenting instead what we might think of as an idealized historical narrative or rational reconstruction.

of another closely related mechanical device – the inclined plane.[235] The result is that gravity (in the form of Galileo's uniform downward acceleration) now provides the foundation (together with the law of inertia) for both mechanics in the traditional (statical) sense and the new science of motion. Beginning with Torricelli's experiments with the barometer we then extend our progress to hydrostatics as well. On the one hand, we ascribe a weight or downward pressure to the air that is similarly due to the acceleration of gravity; on the other, we also find, by means of experiments with the air pump, that air has its own expansive elastic force or pressure, which, at a given height, must precisely balance the downward pressure due to gravity in hydrostatic equilibrium.[236] I explained in section 17 above how important this hydrostatic model of the atmosphere is to Kant's critical version of the dynamical theory of matter, in so far as the balance between expansive elastic pressure and gravity provides the model for his more abstract and general critical version of the balancing argument. What I now want to suggest, accordingly, is that the very same hydrostatic model provides the key to Kant's discussion of chemistry in the fourth number of the general remark to dynamics.[237]

The discussion in question is primarily occupied with the concept of what Kant calls absolute (dis)solution or chemical penetration (530): "The (dis)solution of specifically different materials by one another, wherein no part of one can be found that is not united with a part of the other specifically different from it in the same proportion as the whole, is *absolute*

[235] Galileo's use of the inclined plane originated in his earlier work in mechanics, where, in particular, he explained the statical properties of this device by reducing them to the law of the lever: see, e.g., Drake (1978, chapters 3 and 4). Mechanics in this traditional (statical) sense is the systematic study of how loads of a given weight can be raised to a given height, and the five traditional simple machines are the lever, pulley, screw, wedge, and windlass (or winch). In the course of distinguishing between the mechanical mode of explanation and the chemical mode the *Danziger Physik* (29, 117) counts "statics, the doctrine of the equilibrium of rigid bodies, how a matter can be moved by another by means of instruments" as a part of "mechanics" (which now includes the theory of the communication of motion as well) and says that "to this belongs the 5 *potentiae mechanicae*." Compare also the reference to mechanics and the lever in the passage on the role of reason in chemical classification (161; see the paragraph following the one to which note 229 above is appended).

[236] The discussion of hydrostatics in the *Danziger Physik* carefully distinguishes (29, 137) between Torricelli's discovery of the weight of the air via experiments with the barometer and the discovery of the elasticity of the air via experiments with the air pump by Boyle and Guerike. For the resulting hydrostatic model of the atmosphere see note 150 above, together with the paragraph to which it is appended.

[237] The discussion of mechanical and chemical explanation in the *Danziger Physik* (note 235 above), immediately after the discussion of statics (29,117), lists "hydrostatics, how fluid matters can be moved by the pressure of others" (the pressure of air on water, for example), and the discussion then turns (after a mention of universal attraction) to properly chemical explanation.

(dis)solution, and can also be called *chemical penetration*." This concept, it is clear, depends on Kant's view of the materials in question as true continua:

Because in such a case there can be no part of the volume of the solution that does not contain a part of the dissolving medium, this [medium], as a continuum, must entirely fill the volume. Precisely so, because there can be no part of the very same volume that does not contain a proportional part of the dissolved matter, this [matter], as a continuum, must also fill the entire space constituting the volume of the mixture. But when two materials fill one and the same space, and each of them entirely, they *penetrate* one another. (530)

Kant then illustrates the importance of the continuum view in a concrete application:

The volume that the solution occupies can be equal to the sum of the spaces that the materials dissolved in one another occupied before the mixture, or [it can be] smaller, or even greater, depending on the ratios of the attractive forces to the repulsions. Each one in itself and both together constitute in the solution *an elastic medium*. And this *alone* can provide a sufficient reason why the dissolved matter does not again separate from the dissolving medium by means of its weight. For the attraction of the latter [medium] cancels its resistance [by] itself, since it takes place equally towards all sides; and to assume a certain viscosity in the fluid also in no way agrees with the great force that the dissolved materials, such as acids diluted with water, exert on metallic bodies – to which they do not merely adhere, as would have to happen if they [the bodies] merely swam in their medium, but which they [the acids] rather separate with great attractive force and disperse within the entire space of the vehicle. (531)

Thus, for example, when sulphuric acid (vitriolic acid or oil of vitriol) diluted in water dissolves a metal, what is happening, according to Kant, is the following. First, the acid diluted in water is prevented by its own fundamental expansive elasticity (as an originally fluid medium) from immediately descending (in virtue of its greater specific gravity) to the surface of the metal. Second, the resulting solution of metallic sulphate (vitriol) is also maintained in a state of uniform distribution throughout its volume by its own fundamental expansive elasticity. It is in precisely this way that a central chemical process described by Stahlian phlogistic theory is now firmly connected with the mechanical balancing of expansive elasticity and gravity described by hydrostatics.[238]

[238] For the parallel between this process and the phlogistic process of combustion and calcination see the paragraph to which note 231 above is appended. Metallic sulphate solutions are centrally important in Stahlian chemistry, because they provide an important key to the further experimental investigation of chemical affinities as the dissolved metals are precipitated out of solution, in turn, by exposure to another metal – which now bonds with the acid so that the phlogiston of the original metal then returns to it. See Partington (1961, pp. 678–79) for further discussion.

Kant's conception of how chemistry as an "experimental doctrine" proceeds under the guidance of "the idea of a mechanism" does not, therefore, involve any attempt to reduce properly chemical phenomena to mechanics. No attempt is made, in particular, to reduce chemical affinities either to purely mechanical interactions in the Cartesian style or to short-range forces of attraction and repulsion in the Newtonian style. Chemical affinities are rather described only at the experimental or phenomenological level, and mechanics is extended into this domain only in connection with the purely "instrumental" contributions of the "vehicles" of properly chemical interaction such as water and air.[239] Since we already have a well-established mechanical theory – hydrostatics – applicable to these media (as originally fluid continua), the hope is that mechanics, in this sense, can now provide empirical guidance (but not reductive explanation) for further directing the purely experimental progress of chemistry in accordance with the regulative use of reason.

I observed that Kant explicitly rejects Cartesian mechanical explanations of chemical phenomena, such as the explanation in the *Principles of Philosophy* (cited in note 229 above) of the dissolving power of acids in virtue of the (mechanical) penetration of sharply pointed acid particles into the particles of the materials to be dissolved.[240] But the target of the passage just quoted from the fourth number of the general remark appears to be a parallel passage from Query 31 of Newton's *Opticks*:

If a very small quantity of any Salt or Vitriol be dissolved in a great quantity of Water, the Particles of the Salt or Vitriol will not sink to the bottom, though they be heavier in Specie than the Water, but will evenly diffuse themselves

[239] As observed, salts (e.g., acids) and phlogiston are the fundamental properly chemical agents, for Stahl, in so far as primitive (not further explainable) chemical bonding between these agents and other materials (e.g., earths) provides the basis for chemical affinities: see again note 238 above, together with the paragraph to which note 231 above is appended. Moreover, it is important that phlogiston itself is also a phenomenological or experimentally accessible entity, for Stahl, subject to experimental "proof": see Partington (1961, pp. 670–72).

[240] The relevant section of the *Principles of Philosophy*, "Concerning the acrid and acid juices from which are formed vitriol, alum, and other such minerals," reads as follows (1644, Part IV, §61):

> Now of course, particles whose matter is a little more solid, such as those of salt, which are caught and bruised in these pores, are transformed from rod-like and rigid into flat and flexible: just as a cylindrical rod of white-hot iron can be flattened out into a fairly long blade by repeated blows from a hammer. And since these particles are meanwhile agitated by the force of heat and are moving slowly this way and that through these pores; after being struck and rubbed by the hard walls of the pores, they become sharpened like swords, and thus transformed into certain acrid, acid, corroding juices. These juices, subsequently uniting with metallic matter, form vitriol; with stony matter, alum; and form many other substances in the same way; depending on whether they mingle, as they congeal, with metals, stones, or other materials.

into all the Water, so as to make it as saline at the top as at the bottom. And does not this imply that the Parts of the Salt or Vitriol recede from one another, and endeavour to expand themselves, and get as far asunder as the quantity of Water in which they float, will allow? And does not this Endeavour imply that they have a repulsive Force by which they fly from one another, or at least, that they attract the Water more strongly than they do one another? For as all things ascend in Water which are less attracted than Water, by the gravitating Power of the Earth; so all the Particles of Salt which float in Water, and are less attracted than Water by any one Particle of Salt, must recede from that Particle, and give way to the more attracted Water. (O387–88)

Newton is considering the same phenomenon Kant considers in the passage from the general remark: the uniform distribution of acids and the resulting metallic sulphates (both of which are specifically heavier than water) in a dilute solution with water.

In accordance with the program sketched in Query 31 Newton takes this phenomenon as an indication of short-range forces of attraction and repulsion – the same forces responsible for "chymical Operations" in general.[241] Kant, on my reading, is not exactly disputing or rejecting Newton's hypothetical suggested explanation. Kant is not, for example, rejecting the idea that there is a force of repulsion between the parts of the acid or resulting vitriol that is responsible for the phenomenon in question. He is rather insisting that the fundamental expansive forces possessed by all matter as such (as an originally fluid and elastic medium) are already both necessary and sufficient: no further speculations about the forces underlying specifically chemical interactions (affinities) are required.[242]

[241] Several pages earlier Newton asks (O385–86):

Do not the sharp and pungent Tastes of Acids arise from the strong Attraction whereby the acid Particles rush upon and agitate the Particles of the Tongue? And when Metals are dissolved in acid *Menstruums*, and the Acids in conjunction with the Metal act after a different manner, so that the Compound has a different Taste much milder than before, and sometimes a sweet one; is it not because the Acids adhere to the metallick Particles, and thereby lose much of their Activity?

Two paragraphs after the one quoted in the text Newton turns (O388–89) to the properties of "homogeneal hard Bodies" and suggests that "their Particles attract one another by some Force, which in immediate Contact is exceedingly strong, at small distances performs the chymical Operations above-mention'd, and reaches not far from the Particles with any sensible Effect."

[242] Adickes (1924, pp. 228–29n. 1) plausibly suggests that Kant is here drawing on the passage from Newton's Query 31 and refers us to a related passage in R. 45 (1775–77) (14, 344): "When the parts of the salt are more strongly attracted to the water than among one another, then what is acting is the natural repulsion." Kant's thought, I believe, is that Newton's second possible explanation of the endeavor to recede in question (in terms of an attractive force between the salt and the water) presupposes that there is some counterbalancing expansive force producing this endeavor – just as, in the case of bodies ascending in water due to lesser specific gravity, it is the expansive elasticity of water that is actually bearing them up. But the expansive force

The general point lying behind these considerations is that continuum models of matter of the kind Kant is advocating are not intended as explanatory theories of the inner structure of matter in competition with hypothetical accounts of the Cartesian or Newtonian type.[243] In particular, representing matter by a continuum model does not rule out in advance possible future explanation of the properties thereby represented by a discrete or atomistic model. In hydrostatics, for example, we represent the fluid in question as a continuum of geometrical points, and we represent the equilibrium state, accordingly, as a balance between the expansive pressure defined at each point and the uniform downward acceleration of gravity (also defined at each point). But this notion of pressure is an empirical or phenomenological quantity. Accordingly, we now simply leave the possible deeper physical explanation of this quantity to one side – just as, in a Galilean or Newtonian description of gravity, we similarly leave to one side the question of the possible physical cause of the downward acceleration in question. Continuum models of matter are thus fundamentally phenomenological representations, which are intended to describe the empirical behavior of matter as rigorously as possible without either endorsing or rejecting any deeper explanatory models that may or may not later be discovered.[244]

One important advantage of such models lies in their great open-endedness and flexibility. By allowing each point in the continuum to move independently in principle (subject to the laws of mechanics) we allow ourselves the maximum possible freedom in descriptively representing a variety of material behaviors: viscid and inviscid fluids (see note 67 above), elastic and inelastic solid bodies, rigid and non-rigid materials, and so on. We thereby allow ourselves maximal freedom in representing the variety of possible relationships between the smaller parts of

bearing up the dissolved salt or vitriol cannot be that of water alone (since they are specifically heavier than water); it must rather, on Kant's view, be the result of their own fundamental expansive forces as originally fluid and elastic media in general.

243 There is a crucial difference, however, between the Newtonian and Cartesian programs. Newton explicitly takes his hypothetical speculations in the Queries to the *Opticks* to be preliminary or conjectural and, accordingly, explicitly distinguishes them (as mere "Queries") from those propositions that he takes to be "proved" or established from mathematics and experiments (such as the "Propositions" established earlier in the *Opticks*). Kant, moreover, is fully cognizant of this crucial difference – a point to which I shall return at the end of this section.

244 See note 63 above, together with the paragraph to which it is appended, and recall that Truesdell (1954, p. lxxi) emphasizes that continuum models of the kind first generally elaborated by Euler "are to be applied directly to the bodies of physical experience" – entirely independently, that is, of speculations about the hidden inner structure of matter prominent in the earlier "mechanical" tradition.

the material and the material as a whole without in any way prejudging the properties of such (possibly) elementary parts – which, in turn, may also be either elastic or inelastic, rigid or non-rigid, and so on.[245] All such properties – including hardness, solidity, and rigidity – are defined, as it were, from the outside in, by starting with a maximally free continuum and then adding further constraints on possible internal motions. This is essentially what Kant himself is doing (albeit in non-mathematical terms) in the definitions of fluidity, solidity, rigidity, and so on articulated in the second number of the general remark.

In the mathematical-mechanical approach to natural philosophy that Kant opposes, by contrast, we take the notion of an absolutely hard solid particle as primitive and self-explanatory, and we then attempt (hypothetically) to arrive at the observable large-scale properties of matter from the inside out. We either appeal to the sizes and figures of such primitive corpuscles and purely mechanical action by contact (as in standard versions of the mechanical philosophy) or add short-range action-at-a-distance forces regulating their interactions (as in the Newtonian approach). In either case, however, the notion of an absolutely hard solid particle has the effect of erecting an ultimate barrier to the further progress of both theoretical and experimental inquiry, beyond which (or below which) it is simply impossible to penetrate.[246]

Kant's conception of the necessary regulative use of reason (here applied to the successive regress into smaller and smaller parts of matter considered in the second antinomy) is diametrically opposed to all such ultimate barriers to the further progress of inquiry. Kant makes this especially

[245] This great flexibility is closely related to the circumstance that continuum models essentially employ an infinite number of degrees of freedom: see note 70 above.

[246] This much is just as true in the case of the "solid, massy, hard, impenetrable, movable Particles" of Query 31 of the *Opticks* (note 220 above) as it is in the mathematical-mechanical mode of explanation constituting Kant's primary target in the general remark. Newton arrives at such particles by the following considerations (O389):

> All Bodies seem to be composed of hard Particles: For otherwise Fluids would not congeal; as Water, Oils, Vinegar, and Spirit or Oil of Vitriol do by freezing … And therefore Hardness may be reckon'd the Property of all uncompounded Matter. At least, this seems to be as evident as the universal Impenetrability of Matter. For all Bodies, as far as Experience teaches, are either hard, or may be harden'd; and we have no other Evidence of universal Impenetrability, besides a large Experience without an experimental Exception.

Here Newton appears to be taking hardness as primitive and self-explanatory, so that fluid bodies becoming hard (as in freezing) can then count as experimental evidence for primitively (and absolutely) hard elementary corpuscles. For Kant, by contrast, fluidity is the more primitive or "original" state, while the introduction of hardness or rigidity into this state is one of the least understood aspects of nature: see note 58 above, together with the paragraph to which it is appended.

clear in §8 of the antinomies chapter, entitled "The regulative principle of pure reason in relation to the cosmological ideas" (here applied simultaneously to both the first and second antinomies):

> The principle of reason is thus properly only a *rule*, which prescribes a regress in the series of conditions of given appearances, according to which it is never permitted to stop with something absolutely unconditioned. It is therefore not a principle of the possibility of experience and the empirical cognition of objects of the senses, and thus not a principle of the understanding; for every experience is enclosed within its limits (of the given intuition), [and it is] also not a *constitutive principle* of reason for extending the concept of the sensible world beyond all possible experience, [but it is] rather a principle of the greatest possible continuation and extension of experience, according to which no empirical limit may count as an absolute limit, [and it is] thus a principle of reason, which, *as a rule*, postulates how the regress is supposed to be pursued by us, [but it does] *not anticipate* what is given *in the object* prior to all regress in itself. (A508–9/B536–37)

Accordingly, Kant continues (A510/B538), this rule "cannot say *what the object is* but only *how the empirical regress is to be undertaken*, in order to arrive at the complete concept of the object."

The discussion concludes: "The idea of reason will thus only prescribe a rule for the regressive synthesis in the series of conditions, according to which it proceeds from the conditioned, by means of all subordinated [intermediate] conditions, towards the unconditioned, although this latter is never reached. For the absolutely unconditioned is in no way found in experience" (A510/B538). Applied to the second antinomy, in particular, it follows (A527/B555) that "how far the transcendental division of an appearance reaches, in general, is not at all a matter of experience, [but is] rather a principle of reason never to take the regress in the decomposition of the extended, in accordance with the nature of this appearance, as absolutely completed." Although we cannot say in advance what the nature of the smaller parts of any given material must be, we must treat this material, in all of our empirical inquiries, as thoroughly divisible to infinity – and thus as a true continuum.[247]

[247] As explained in section 13 above, the crucial step in Kant's proof that matter is not merely mathematically but also physically infinitely divisible is the claim (504–5) that "every part of a space filled with matter is movable for itself, and thus separable from the rest as material substance through physical division" (compare note 78 above, together with the paragraph to which it is appended). As explained in section 12 above, this is a crucial ingredient, in turn, in his (critical) conception of matter as a true continuum (compare the paragraph to which note 67 above is appended). From Kant's point of view, therefore, the absolutely hard impenetrable particles he opposes are merely mathematically but *not physically* divisible.

The remark to the first proposition of the Dynamics introduces the concept of absolute solidity or impenetrability that Kant is most concerned to reject. He suggests (498) that the concept of solidity embraced by "the mathematician" – taken as a "first datum" – thereby "obstruct[s] us from going back to first principles in natural science." A few pages later, in the remarks to the fourth explication, he (502) characterizes "the mere mathematical concept of impenetrability" as one according to which "matter as matter resists all penetration utterly and with absolute necessity" and rejects such absolute impenetrability as an "occult quality." The general remark to dynamics later returns to this theme (532): "In the doctrine of nature, the absolutely empty and the absolutely dense are approximately what blind accident and blind fate are in metaphysical science, namely, an obstacle to the governance of reason, whereby it is either supplanted by feigning [*Erdichtung*] or lulled to rest on the pillow of occult qualities." So where "the governance of reason" is "lulled to rest" in the doctrine of nature, it is clear, is at the "first datum" or ultimate limit erected by the concept of absolute solidity or impenetrability. Kant's dynamical natural philosophy or metaphysical-dynamical mode of explanation – as applied, more particularly, to matter theory and contemporary chemistry – is thus the only approach, in his eyes, that consistently conforms to the necessary regulative demands of the faculty of reason.[248]

This last-quoted passage from the general remark, together with everything else we have seen, makes it clear that Kant's methodological disagreements with the Newtonian program in contemporary matter theory and chemistry are by no means as deep as his parallel disagreements with standard versions of the mechanical natural philosophy represented by such thinkers as Lambert or Descartes. As I have suggested,

[248] See again the important passage at the end of the general remark where Kant (534) contrasts the "true rational coherence [*Verfunftzusammenhang*] of explanations" provided by "dynamical grounds of explanation" with the "unconditioned original positings [*Positionen*]" of a "merely mathematical treatment." In light of the empirical difficulties afflicting Kant's conception of the fundamental force of repulsion (see note 225 above, together with the references therein to earlier discussions), one might legitimately worry that Kant is in conflict with his own experimental method – and that this force, in particular, is just as much an "unconditioned original positing" as Lambert's absolutely solid or Newton's absolutely hard elementary corpuscles. Nevertheless, although Kant is certainly not as clear as one would wish, there is some reason to understand his conception of the (infinitesimally acting) fundamental force of repulsion as simply a commitment to the Boyle–Mariotte law governing the *phenomenological* quantity of pressure in arbitrarily small volumes (see the two paragraphs following the one to which note 208 above is appended). On this reading, Kant's conception of the original force of repulsion would be consistent with his experimental method after all, but the (infinitesimal) law supposed to govern this force would still be problematic from his own point of view. I shall return to this topic at the end of my chapter on the Phenomenology.

Kant's preference for avoiding speculative hypotheses and pursuing "experimental philosophy" instead is very much inspired by Newton and Newtonianism.[249] This becomes even more evident, in fact, in the context of the two sentences immediately preceding the last-quoted passage (532): "Here is not the place to uncover hypotheses for particular phenomena, but only the principle in accordance with which they are all to be judged. Everything that relieves us of the need to resort to empty spaces is a real gain for natural science, for they give the imagination far too much freedom to make up by feigning [*Erdichtung*] for the lack of any inner knowledge of nature." Although Kant rejects the Newtonian commitment to a primitive notion of absolute hardness or solidity, his own conception of experimental philosophy (like that of so many eighteenth-century thinkers) is directly inspired by Newton – and, in particular, by Newton's critique of the hypothetical methodology embraced by the proponents of the mechanical philosophy known to him. The crucial difference, in this context, is that Kant, following Euler, thinks that a continuum approach to matter represents a better way to pursue experimental philosophy than an atomistic approach.

21 THE DYNAMICAL THEORY OF MATTER AND THE CATEGORIES OF QUALITY

Kant begins the general remark to dynamics with a "general principle of the dynamics of material nature" having two applications (523): first, the notion of absolute solidity or impenetrability is "banished from natural science, as an empty concept, and repulsive force is posited in its stead," second, "the true and immediate attraction is thereby defended against all sophistries of a metaphysics that misunderstands itself, and, as a fundamental force, is declared necessary for the very possibility of the concept of matter." Kant does not intend simply to reject what he refers to as metaphysics here, for the entire point of the general remark is to defend what he calls the "metaphysical-dynamical" approach to natural philosophy against a contrasting "mathematical-mechanical" approach. Kant rather intends to clear up the "misunderstandings" that can easily afflict such metaphysics – especially when they lead to a rejection of Newtonian attraction as a true and immediate action at a distance. This

[249] For Lambert and Descartes see note 216 above, together with the paragraph to which it is appended and the two preceding paragraphs. For the influence of the Newtonian method on Kant more generally see notes 222, 232, and 243 above, together with the paragraphs to which they are appended.

last point suggests, in particular, that the metaphysics in question is precisely Leibnizean metaphysics.

I explained in the Introduction that Kant consistently identifies the side of metaphysics with Leibniz and the Leibnizeans, whereas the side of mathematics is identified with Newton and the Newtonians. Kant consistently suggests, beginning with the *Physical Monadology*, that the former side typically errs in summarily dismissing action at a distance, while the latter fails properly to appreciate the urgent need for metaphysics in addition to mathematics. In the Preface to the *Metaphysical Foundations* Kant explicitly takes Newton to represent the side of the "mathematical" natural philosophers who "solemnly guard against all claims of metaphysics upon their science" (472) while failing to recognize that "true metaphysics is drawn from the essence of the faculty of thinking itself, and it is in no way feigned [*erdichtet*] on account of not being borrowed from experience."[250] The aim of the *Metaphysical Foundations*, accordingly, is to apply Kant's revised version of metaphysics in the Leibnizean tradition – derived from the form and principles of the pure understanding – to explain how it first becomes possible to apply mathematics in the Newtonian style to our actual experience of sensible nature.[251]

As explained in sections 8 and 13 above, Kant's official diagnosis of the fundamental misunderstanding lying at the heart of Leibnizean metaphysics is presented in the amphiboly of the first *Critique*. It consists in the supposition that one can obtain knowledge of the "inner nature of things" from the pure understanding alone, entirely independently of sensibility:

> Lacking such a transcendental topic, and thus deceived by the amphiboly of the concepts of reflection, the celebrated Leibniz erected an *intellectual system of the world*, or rather he believed that he could know the inner nature of things, in that he compared all objects only with the understanding and the isolated formal concepts of its thought. Our table of the concepts of reflection provides us with the unexpected advantage of placing before our eyes the distinctive features of his system in all of its parts, together with the guiding principle of this peculiar mode of thinking, which rests on nothing but a misunderstanding. (A270/B326)

[250] See notes 15–18 of the Introduction, together with the paragraphs to which they are appended.

[251] I explained at the end of the last section that, although he is opposed to the Newtonian conception of absolutely hard bodies, and diverges from the Newtonian explanatory program based on short-range attractive and repulsive forces, Kant's true target in opposing the mathematical-mechanical natural philosophy in the general remark is represented by standard versions of the mechanical philosophy (which Newton also opposes) as represented by Lambert or Descartes. Here the addition of "mechanical" is therefore crucial.

Leibnizean metaphysics is correct, from Kant's point of view, in beginning with the pure functions of thought. The misunderstanding in question lies in omitting the schematism of the pure understanding, which alone enables these pure forms of thought to be applicable to both pure and empirical sensible intuition. We still need an account, in the words of the Preface to the *Metaphysical Foundations* (472), of how these "pure actions of thought" can "bring the manifold of *empirical representations* into the law governed connection through which it can become *empirical* **cognition**, that is, experience."

According to the "transcendental topic" outlined in the amphiboly the distinctive features of Leibnizean metaphysics arise as follows. In connection with the pure concepts of quantity, considered independently of spatio-temporal intuition, the only grounds for the identity or difference of things must be purely conceptual. We thereby obtain the principle of the identity of indiscernibles, according to which two things characterized by all the same purely conceptual marks or characteristics – having all the same "inner determinations" – are necessarily identical. In connection with the pure concepts of quality, also considered independently of spatio-temporal intuition, reality can be conceived only in terms of logical affirmation, opposition only in terms of logical negation. There is no possibility, therefore, of what Kant calls real opposition, by which two different realities can cancel one another like positive and negative numbers (see note 23 above). In connection with the pure concepts of relation, again considered independently of spatio-temporal intuition, substances must be conceived as having purely inner natures, entirely without relation to anything existing outside themselves, and also entirely without composition. Such substances are therefore necessarily ultimate simple elements or *monads*, conceived on the analogy of ourselves as thinking substances (see note 77 above).

Kant brings all these considerations together in connection with the pure concepts of modality, discussed under the heading of matter and form:

> [I]n relation to things in general unlimited reality was viewed as the matter of all possibility, the limitation of this [reality] (negation) as the form by which one thing is distinguished from others in accordance with transcendental concepts. The understanding requires, namely, that something first be given (at least in concept), in order to be able to determine it in a certain way. Therefore, in the concept of the pure understanding matter precedes form, and *Leibniz* consequently first assumed things (monads), together with an inner power of representation, in order afterwards to ground their external relations and the community

of their states (namely, their representations) on this. Therefore, space and time were [thereby] possible – the former only through the relation of the substances, the latter through the connection of their determinations among one another as ground and consequence. This in fact is how it would have to be if the pure understanding could be related immediately to objects, and if space and time were determinations of things in themselves. If, however, they are only sensible intuitions, in which we determine all objects simply as appearances, then the form of intuition (as a subjective constitution of sensibility) precedes all matter (the appearances), and therefore space and time precede all appearances and all data of experience, and rather make them possible in the first place. The intellectualist philosophy could not bear [the idea] that the form should precede the things themselves, and determine the possibility of these [things] – a perfectly correct censure if it assumed that we intuit the things as they are (although with a confused representation). But since sensible intuition is an entirely special subjective condition, which lies a priori at the basis of all perception, and whose form is original, it follows that form alone is given for itself, and it is so far from being the case that matter (or the things themselves which appear) should lie at the basis (as one would have to judge in accordance with mere concepts) that the possibility of matter rather presupposes a formal intuition (time and space) as [already] given. (A266–68/B322–24)

The pure understanding, independently of sensibility, conceives all determination of things as purely logical: as a process by which given concepts or predicates are successively either affirmed or denied of a thing. This purely logical model of determination results in a purely intellectual system of the world consisting of ultimate simple substances, complete concepts, or Leibnizean monads, in which space and time are entirely derivative "well-founded phenomena" of these substances. In our (human) experience, by contrast, all objects (of experience) are given in the pre-existing forms of space and time. All determination of such objects – all empirical judgements about them – are only possible on the basis of their (external) relations to one another within these pre-existing sensible forms.[252]

My conclusion at the end of the last section was that the assumption of absolute hardness erects an ultimate barrier to the regulative use of reason as it attempts to guide our progressive experimental investigation into ever smaller constituents of matter. This particular assumption is not at issue in Kant's criticism of specifically Leibnizean metaphysics. For Leibniz, like Kant, eschews perfect hardness and instead adopts an elastic

[252] Compare notes 76 and 77 above, together with the paragraphs to which they are appended. I was there focussing on the relationship between the pure concept of substance and spatial intuition; I am here focussing on the relationship between the pure understanding in general and spatio-temporal intuition.

fluid continuum model of matter. Moreover, Leibniz's own conception of matter, more generally, represents the most important "dynamistic" conception (which places the concept of force at the center) prior to Kant's (see note 1 above). Kant's criticism of Leibnizean metaphysics, therefore, is not that it erects a barrier to the regulative use of reason in our progressive investigation into ever smaller constituents of matter. It is rather that it depicts this progressive regulative procedure as necessarily converging, as it were, on a certain prior conception of what the ultimate elements lying at the basis of matter must be like – that they are mind-like simple substances or monads conceived by the pure understanding alone on analogy with itself. This prior conception of the ultimate direction of empirical inquiry into the structure of matter can, like the conception of absolute hardness, give a possibly wrong direction to the progress of empirical inquiry. It may, for example, make us skeptical of immediate attraction at a distance.[253] But Kant's point, I believe, goes deeper than this. It is rather that the Leibnizean conception of the ultimate nature of matter upsets all empirical inquiry as such by failing to appreciate the necessary schematism of the pure concepts of the understanding.

This is one reason that it is especially important, as explained in section 14 above, to distinguish Kant's objections to the (Leibnizean–Wolffian) "monadists" in the second antinomy from his objections to Leibniz's own view. For these "monadists" find their ultimate simple elements in "physical points" out of which the extension of matter is supposed to be composed by "mere aggregation" (A439/B467). Such physical points (by the argument of Zeno's metrical paradox) must themselves have finite size, but, nevertheless, they (as ultimate simples) must also be indivisible. So these simple elements, as *spatial*, are actual parts of physical nature, and, accordingly, they erect an ultimate barrier to our inquiry into the inner constituents of matter in a way that is precisely analogous to the absolutely hard solid corpuscles assumed in the mathematical-mechanical tradition. Nevertheless, Kant is perfectly clear in the second antinomy that

[253] For Leibniz's rejection of true and immediate attraction on the grounds that it cannot be explained on the basis of the nature – presumably the *inner* nature – of bodies see, for example, Leibniz (1717, Leibniz's Third Letter, §17). From a metaphysical point of view, in particular, it appears that Leibniz's rejection of inter-substantial in favor of intra-substantial causality is the crucial factor lying behind his rejection of true and immediate attraction. This, in any case, is how Kant understands the Leibnizean conception of the community of substances in the amphiboly (A274–75/B330–31), as "a *pre-established harmony*, [which] could not be a physical influx," since "the state of representations of one substance could stand in no active connection at all with that of the other." For discussion of the opposition between pre-established harmony and physical influx in the pre-critical period see Laywine (1993), Watkins (2005).

the "monadists" in question are not Leibniz himself – precisely because Leibniz conceives his monads as essentially non-spatial, mind-like simple beings that are given to the pure intellect (at least in the case of ourselves) in immediate self-consciousness.[254]

Another way to put Kant's criticism of this last version of the monadology, therefore, is that the properly Leibnizean conception of the ultimate constituents of matter is fundamentally incompatible with Kantian transcendental idealism:

> Matter is *substantia phaenomenon*. I seek what inwardly belongs to it in all parts of the space it occupies, and in all the actions it exerts, and these can certainly only ever be appearances of outer sense. I therefore in fact have nothing absolutely inner but only comparatively so, which itself consists in turn of outer relations. But the absolutely inner [nature] of matter, in accordance with the pure understanding, is also a mere fancy; for [matter] is no object of the pure understanding, and the transcendental object, which may be the ground of this appearance we call matter, is a mere something, of which we would not even understand what it might be if someone were to be able to tell us … If the complaint – *that we have no insight at all into the inner* [*nature*] *of things* – means that we do not conceive by the pure understanding what the things that appear to us may be in themselves, then it is entirely unjust and unreasonable. For it demands that we can still cognize things without senses, and thus intuit them, and, therefore, that we have a completely different faculty of cognition from the human, not merely in degree but even in kind with respect to intuition; [it therefore demands] that we not be humans, but rather beings such that we ourselves cannot declare that they are even possible, much less how they are constituted. Observation and analysis of appearances penetrate into the inner [structure] of nature, and we do not know how far this may proceed in time. But we would never, by all of this, be able to answer those transcendental questions which go beyond nature, even if the whole of nature were uncovered – since it is not even given to us to observe our own mind with any other intuition except that of our inner sense. (A277–78/B333–34)

The idea that we have knowledge of our own mind, by the pure understanding, as it is in itself, and that we can project this knowledge, again by the pure understanding, onto the ultimate monadic constituents of matter, is, according to Kant, Leibniz's most fundamental mistake. In particular, it entirely misconceives the route from pure understanding to empirical knowledge. This route, for Kant, begins with pure logical forms of thought, schematizes them in terms of our spatio-temporal forms of pure intuition, and only then employs the now schematized categories in

[254] See notes 93–98 above, together with the paragraphs to which they are appended.

grasping empirical sensible appearances – including above all, matter as *substantia phaenomenon*.[255]

In the case of the Dynamics, however, the relevant pure concepts of the understanding are the categories of quality – reality, negation, and limitation. Kant explains their relevance to the overall argument of the Dynamics in a brief general note immediately preceding the general remark:

> If we look back over all our discussions of the subject, we will notice that we have therein considered [the following]: *first*, the *real* in space (otherwise called the solid), in the filling of [space] through *repulsive force*; *second*, that which in relation to the first, as the proper object of our outer perception, is *negative*, namely, *attractive force*, whereby, for its own part, all space would be penetrated, and thus the solid would be completely destroyed; *third*, the *limitation* of the first force by the second, and the determination of the *degree of filling* of a space that rests on this. Hence, the *quality* of matter, under the headings of *reality*, *negation*, and *limitation*, has been treated completely, so far as pertains to a metaphysical dynamics. (523)

This correspondence between the categories of reality, negation, and limitation and the argument of the Dynamics, unlike the parallel correspondence in the case of the Phoronomy (section 7 above), seems, at first sight, to be quite straightforward. But our consideration of the relationship between the Dynamics and the amphiboly has made it possible to recognize some important and unexpected subtleties.

In the first place, the amphiboly does set up a correspondence between the categories of quality and the fundamental forces of attraction and repulsion – a correspondence Kant first introduces under the heading of agreement and opposition:

> If reality is represented only by the pure understanding (*realitas noumenon*), then no conflict between realities is thinkable – that is, such a relation where, combined in one subject, they cancel [*aufheben*] one another's consequences, like 3 – 3 = 0. By contrast, the real[ities] in the appearance (*realitas phaenomenon*) can certainly be in conflict among one another, and, united in the same subject, one can destroy [*vernichten*] *the consequence of the other* wholly or in part – as two moving forces in the same straight line, in so far as they either attract [*ziehen*] or repel [*drücken*] a point in contrary directions.

Kant returns to the subject in a bit more detail several pages later:

[255] As also observed in section 14 above, Kant attempts to appropriate Leibniz himself on behalf of transcendental idealism in the Dynamics of the *Metaphysical Foundations*. For my earlier discussion of this attempt see notes 99–109 above, together with the paragraphs to which they are appended.

> [T]he principle that realities (as mere affirmations) never logically conflict with one another is an entirely true proposition concerning the relations of concepts, but it does not have the least meaning either with respect to nature or, in general, with respect to any [possible] thing in itself (of which we have *no* concept). For real opposition takes place wherever we have A – B = 0, that is, where one reality, combined with the other in a subject, cancels [*aufhebt*] one of the actions of the other – which all impediments and reactions in nature continually show us, and which similarly, as resting on forces, must be called *realitas phaenomena*. General mechanics can even present the empirical conditions of this conflict in an a priori rule, in that it attends to the contrariety of directions – a condition of which the transcendental concept of reality knows nothing at all. (A273/B329)

It is clear, then, that a conflict of realities – what Kant calls real opposition – is paradigmatically instantiated by a conflict of moving forces.

As Kant suggests, the basis for our ability to represent such a conflict in pure intuition (as opposed to the pure understanding) in an "a priori rule" is the representation of a single straight line that can be traversed in two opposite directions. This leads us back, therefore, to the single proposition of the Phoronomy on the composition or addition of velocities and, in particular, to the second case of this proposition (section 4 above). As explained in section 10 above, this case of the composition or addition of velocities (which considers two equal velocities oppositely directed along a single straight line) is the basis, in turn, for the first proposition of the Dynamics. Here Kant introduces the fundamental force of repulsion as the cause or ground of an outgoing motion directed away from a central point along a straight line – a motion whose role is precisely (497) to "diminish or destroy [*vermindert oder aufhebt*]" a contrary incoming motion directed towards the central point along the same straight line (as a "striving to penetrate").[256] Real opposition along a single straight line, as considered in "general mechanics," is not only the basis for the balancing of attractive and repulsive forces considered in the general note (and in the fifth and sixth propositions of the Dynamics). It is also the basis for the first proposition of the Dynamics, where a moving force (here the fundamental force of repulsion) is initially introduced into the overall argument of the *Metaphysical Foundations* as a whole.[257]

[256] See notes 22 and 23 above, together with the paragraphs to which they are appended. As explained in note 22, in particular, this same geometrical representation (of a single straight line that can be traversed in two opposite directions) is the basis for Kant's claim, in the note to the second explication of the Dynamics, that there are only two possible types of moving forces – attractive and repulsive.

[257] It is likely, then, that the "a priori rule" of "general mechanics" to which Kant refers in the amphiboly passage (A273/B329) is the principle of the composition of motions demonstrated in the Phoronomy (where, in general, we consider the totality of straight lines intersecting at a

In the second place, however, there is an even more important (yet less apparent) connection between the categories of quality and the argument of the Dynamics. This connection emerges in the continuation of the last-quoted passage (A273/B329) from the amphiboly:

> Although Herr von Leibniz did not in fact announce this proposition ["that realities (as mere affirmations) never logically conflict with one another"] with the pomp of a new principle, he still used it to [arrive at] new assertions, and his successors explicitly incorporated it into their Leibniz-Wolffian system. According to this principle, for example, all evils are nothing but consequences of the limitations of created [beings], that is, negations, because these are the only things that conflict with reality (in the mere concept of a thing in general it is also actually so, but not in the things as appearances). Similarly, [Leibniz's] disciples find it not only possible, but also natural, to unite all reality, without any fear of conflict, in a [single] being, because they are acquainted with no other [kind of conflict] than that of contradiction (by which the concept itself of a thing is destroyed [*aufgehoben*]), but not with that of mutual interruption [*Abbruch*], where one real ground destroys the action of the other, and for which we find the conditions for representing such a state of affairs only in sensibility. (A273–74/B329–30)

Consideration of this passage reveals an unexpected connection, in particular, between the categories of quality and the application of the regulative use of reason to matter theory in the general remark to dynamics (see section 20 above).

The passage refers to a certain conception of how the sum total of all reality (or all possibility) is united in a single being (namely God) – a conception arising naturally in the Leibnizean–Wolffian tradition. It arises, in fact, in Kant's own pre-critical works: notably, in the *New Exposition of the First Principles of Metaphysical Knowledge* of 1755 and *The Only Possible Basis for a Proof of the Existence of God* of 1763. In the

point). A closely related principle of the *communication* of motion is demonstrated in the fourth proposition of the Mechanics (as we shall see below). But this involves an interaction between two different substances (moving bodies) and not, as discussed here, a case "where one reality, combined with the other in a [single] subject, cancels [*aufhebt*] one of the actions of the other." Kant also appears to have in mind the situation considered in the first proposition of the Dynamics a bit later in the amphiboly (A282/B338):

> [T]here is absolutely no conflict in the concept of a thing if nothing negative was combined with an affirmative [concept], and mere affirmative concepts can effect no cancellation [*Aufhebung*] at all in combination. But in sensible intuition, wherein realities (e.g., motions) are given, we find conditions (contrary directions), from which we abstracted in the concept of motion in general, [and] which make possible a conflict, which certainly is not logical, namely, from pure positives [to make possible] a zero = 0.

Critique of Pure Reason Kant now reflects on this conception from his new (critical) point of view in an important section on the transcendental ideal. We begin from the idea or principle of *complete determination*, according to which every thing is characterized either affirmatively or negatively by each one of the totality of all possible predicates (A571–72/B599–600). When, however, we consider this totality of all possible predicates not merely logically but also transcendentally, we can distinguish mere logical negation represented simply by the word 'not' from properly transcendental negation signifying a genuine lack of being or reality (A574–75/B602–3). In this way, we can distinguish positive properties or realities, such as light or knowledge, from merely negative properties or (transcendental) negations, such as darkness or ignorance (A575/B603): "[A]ll concepts of negations are thus also derivative, and the realities contain the data and, as it were, the matter, or the transcendental content, for the possibility and complete determination of all things." We thereby arrive at the idea of an All or totality of reality (*omnitudo realitatis*), such that all true or transcendental negations are viewed as limitations of this single whole.[258]

However, since there is no possibility of conflict (i.e., contradiction) in the sum total of all possible realities (as true or transcendental affirmations), they can all be thought as united in a single individual being that is itself completely determined:

But, through this total sum [*Allbesitz*] of reality, the concept of a *thing in itself*, as completely determined, is also represented, and the concept of an *ens realissimum* is the concept of an individual being, because, among all possible contradictorily opposed predicates, one – namely, that which belongs to being as such – is found in its determination. Therefore, it is a transcendental *ideal*, which is the basis for the complete determination necessary for everything that exists, and constitutes the highest and complete material condition of [a thing's] possibility – [a condition] to which all thought of objects in general must be traced back according to their content. It is also the only true ideal of which human reason is capable, because only in this single case is an intrinsically universal concept of a thing completely determined by itself, and [thereby] cognized as the representation of an individual. (A576/B604)

[258] In more detail (A575–76/B603–4):

If, therefore, a transcendental substratum as the basis for complete determination is [posited] in our reason, which, as it were, contains the entire store of material, and therefore can be taken [to contain] all possible predicates of things, then this substratum is nothing but the idea of an All of reality (*omnitudo realitatis*). All true negations are then nothing but *limitations*, which they could not be called if the unlimited (the All) were not the basis.

This individual being – the *ens realissimum* – is God, the object of transcendental theology.[259]

Whereas, in the pre-critical period, this line of thought was developed into a proof of the existence of God (as the necessary ground of the possibility of all things), it is at just this point that Kant now stops short:

> Here, however, this use of the transcendental idea would already overstep the limits of its determination and admissibility. For reason takes it as a basis for the complete determination of things in general only as the *concept* of all reality, without requiring that all this reality is objectively given and itself constitutes a thing. This last [conception] is a mere fiction [*Erdichtung*], by which we unite and realize the manifold of our idea in an ideal, as a particular being – for which we have no warrant even to assume the possibility of such an hypothesis, so that all conclusions that flow from such an ideal also have nothing to do with the complete determination of things in general (for which alone the idea was necessary), and do not have the least influence on this. (A580/B608)

Thus, although the *idea* of a totality of all reality (*omnitudo realitatis*) is a necessary and legitimate idea of reason associated with the corresponding principle (of reason) of complete determination, the transcendental *ideal* of an individually existing thing corresponding to this idea (the *ens realissimum*) is an illegitimate hypostatization with no objective reality whatsoever (at least from a purely theoretical point of view).

Kant does not rest content, however, in simply rejecting the legitimacy of the inference from idea to ideal. He also proceeds, characteristically, to construct a diagnosis of the "dialectical illusion" involved in this inference:

> The possibility of objects of the senses is a relation of these [objects] to our thought, wherein something (namely, the empirical form) can be thought a priori, but that which constitutes the matter, reality in the appearance (which corresponds to sensation), must be given – without which it cannot be thought at all and thus its possibility cannot be represented. Now an object of the senses can only be completely determined when it is compared with all predicates of appearance and represented either affirmatively or negatively by them. However, because that which constitutes the thing itself (in the appearance), namely, the real, must be given, without which it cannot even be thought; but that wherein

[259] In more detail (A580/B608):

> If we now further follow up this idea [of the *ens realissimum*], in so far as we hypostasize it, we will be able to determine this primordial being through the mere concept of the highest reality as a unique, simple, all-sufficient, eternal being, and so on – in short, [we will be able to determine] it in its unconditioned completeness through all predicates. The concept of such a being is that of *God*, thought in its transcendental meaning, and thus the ideal of pure reason is the object of a transcendental *theology*, as I have also explained above.

the real of all appearance is given is the single all-embracing experience; it follows that the material for the possibility of all objects of the senses must be presupposed, as given in a totality, and the possibility of empirical objects, their differences from one another, and their complete determination rests on the limitation of this [totality] alone. (A581–82/B609–10)

The relevant whole or totality of reality in the *empirical* use of the understanding is thus nothing other than the whole or totality of experience itself – a unified whole containing both the form and (all) the matter of experience.

But here we encounter a natural dialectical illusion:

However, that we here hypostasize this idea of the totality of all reality comes about as follows: we dialectically transform the *distributive* unity of the empirical use of the understanding into the *collective* unity of experience as a whole, and we think a single thing [in place of] this whole of appearance, containing all empirical reality within itself, which is then confused, by means of the already mentioned transcendental subreption, with the concept of a thing standing at the summit of the possibility of all things, for whose complete determination it contributes the real conditions. (A582–83/B611)

The dialectical illusion in question thus arises in two distinct steps. In the first we move from distributive to collective unity within the empirical use of the understanding. In the second we confuse the (collective) unity of experience with a non-empirical or transcendental unity embracing all things or objects in general (not simply objects of experience).

The second step involves precisely the "transcendental subreption" already diagnosed in the amphiboly: we attempt to use the pure understanding alone, independently of sensibility, to obtain knowledge of all objects in general.[260] We already know, therefore, why this is illegitimate. We also already know, in accordance with the passage with which I began (A273–74/B329–30), that empirical realities – realities in the appearance – *can* conflict with one another after all. The first step, however, involves a different and more subtle mistake. For we are still operating entirely within experience and thus still presupposing, in accordance with the

[260] See the text connecting the two last-quoted passages (A582/B610):

Now, no other objects but those of the senses can in fact be given to us, and nowhere except in the context of a possible experience. Therefore, nothing is an object *for us* if it does not presuppose the totality of all empirical reality as a condition of its possibility. In accordance with a natural illusion we now view this as a principle that must hold for all things in general, whereas it is properly only valid for those that are given as objects of our senses. Thus, by omitting this limitation, we take the empirical principle of our concepts of the possibility of things, as appearances, for a transcendental principle of the possibility of things in general.

argument of the amphiboly, that the spatio-temporal "empirical form" of experience is given prior to its (sensory) matter.[261] Even so, Kant suggests, we are making a natural dialectical mistake by moving from the *distributive* unity of the empirical use of the understanding to the *collective* unity of experience as a whole.

This last mistake, Kant makes clear, is no less than the "transcendental illusion" diagnosed in the dialectic as a whole. It is the mistake of viewing the indefinitely successive progression of the empirical use of the understanding as a finished and complete infinite totality – in such a way, in particular, that we confuse the *regulative* use of the ideas of reason with the properly *constitutive* use of the concepts of the understanding:

> Therefore, reason properly has only the understanding and its purposive operation as object, and, as the latter unites the manifold in the object through concepts, the former, for its part, unites the manifold of concepts through ideas, in that it posits a certain collective unity as the goal of the activities of the understanding, which are otherwise occupied only with distributive unity.
>
> I accordingly assert [that] the transcendental ideas are never of constitutive use, in such a way that concepts of certain objects would thereby be given, and, understood in this way, they are merely sophistical [*vernünftelnde*] (dialectical) concepts. By contrast, they have an excellent and indispensably necessary regulative use, namely, to direct the understanding towards a certain goal, at the prospect of which the lines of direction of all of its rules converge in a point – which, although it is in fact only an idea (*focus imaginarius*), that is, a point from which the concepts of the understanding actually do not proceed, in that it lies entirely beyond the limits of possible experience, it nevertheless serves to provide these [concepts of the understanding] with the greatest [possible] unity together with the greatest [possible] extension. (A644/B673)

When the pure concepts of the understanding are considered with properly constitutive force, as conditions of the possibility of experience, they provide for *distributive* unity of experience: namely, the conditions that make possible each and every given experience. By contrast, the *collective* unity of experience – the unity of all experience as a single (complete) totality – is comprehended by no such constitutive conditions at all (and therefore not by the pure concepts of the understanding) but only by merely regulative principles based on ideas of reason.[262]

[261] See, in particular, the paragraph to which note 252 above is appended. The passage quoted there begins by asserting (A266/B322) that "in relation to things in general unlimited reality was viewed as the matter of all possibility, the limitation of this [reality] (negation) as the form by which one thing is distinguished from others in accordance with transcendental concepts."

[262] Compare the parallel passage in §40 of the *Prolegomena* (4, 327–28):

> The empirical use to which reason limits the understanding does not fulfill its own entire determination. Each individual experience is only a part of the entire sphere of its domain, but the

In regard to the question presently at issue (complete determination) we know (A582/B609) that "an object of the senses can only be completely determined when it is compared with all predicates of appearance and represented either affirmatively or negatively by them." Hence, in order to implement the idea or principle of complete determination in experience, we must consider the totality of all empirical concepts or empirical predicates. This totality is never given as a completed whole, however, but only step by step as we successively introduce more and more empirical concepts during the indefinite progress of empirical inquiry. In this way, we generate an indefinitely progressive specification of our empirical concepts by logical division (into progressively lower species, sub-species, sub-sub-species, and so on) starting from any given super-ordinate genus:

Now the understanding cognizes everything only by means of concepts: therefore, so far as it reaches in the division, [it cognizes things] never by means of mere intuition, but only always in turn by means of *lower* concepts. The cognition of appearances in their complete determination (which is only possible through the understanding) requires an unceasing progressive specification of [the understanding's] concepts, and a progress to ever remaining differentiations, from which we abstract in the concept of the kind, and even more in that of the genus. (A656/B684)

Since this process of specification is never actually completed in experience (which always, at any given stage, has only a finite number of empirical concepts in view), the demand for complete determination can only be a requirement of reason. It bids us to seek unceasingly for ever lower empirical concepts and thus ever finer empirical differentiations.[263] So the demand for complete determination, in this context, can only be merely regulative.[264]

absolute whole of all possible experience is not itself experience, and is therefore a necessary problem for reason, for the mere representation of which it requires entirely different concepts than these pure concepts of the understanding – whose use is only *immanent*, i.e., extends to experience in so far as it can be given, whereas the concepts of reason extend to the collective unity of the whole of possible experience, and thereby [extend] beyond every given experience and become *transcendent*.

[263] The above passage continues (A657/B685):

This law of specification can also not be borrowed from experience; for the latter can yield no such far-reaching prospects. Empirical specification soon comes to a stop in the differentiation of the manifold, if it is not guided by the already preceding law of specification, as a principle of reason, to seek such [differentiation], and always to anticipate it if it is not immediately obvious to the senses.

[264] See the corresponding discussion in the transcendental ideal (A573/B601): "Complete determination is thus a concept which we are never able to present *in concreto* in accordance with its totality, and it is therefore based on an idea, having its seat solely in reason, which prescribes to the understanding the rule of its complete employment."

Now the main question considered in the general remark to dynamics, as discussed in the previous section, is the problem of the specific variety of matter: explaining how the universal (empirical) concept of matter in general is further specified into species, sub-species, and so on. Only the most general and universal properties of all matter as such (gravity resting on the fundamental force of attraction, and original elasticity resting on the fundamental force of repulsion) can be characterized a priori in a properly "metaphysical" treatment, whereas all further specification into a variety of types and species of matter is left to an empirical or "physical" treatment.[265] Nevertheless, Kant (programmatically) attempts "completely to present, so I hope, the moments to which [matter's] specific variety must collectively be reducible a priori" (525) in the four numbered headings of the general remark constituting his positive contribution to matter theory. He considers the (specific) density of matter under the first number, the differentiation of matter into solid, (expansive) fluid, and liquid states under the second number, the distinction between original and derivative elasticity (especially with respect to solid elastic materials) under the third number, and chemistry (as systematic art or experimental doctrine) under the fourth number.[266] It is precisely chemistry in this sense, I have argued, that represents the true subject matter and goal of Kant's preferred dynamical natural philosophy – and, in this sense, the

[265] See the second note to the eighth proposition of the Dynamics (518):

> The *action* of universal attraction, which all matter immediately exerts on all [other matter] and at all distances, is called *gravitation*; the striving to move in the direction of greater gravitation is *weight* [*Schwere*]. The action of the general [*durchgängigen*] repulsive force of the parts of any given matter is called its *original elasticity.* Hence this [property] and weight constitute the sole universal characteristics of matter that are comprehensible a priori, the former internally, and the latter in external relations; for the possibility of matter itself rests on these two [properties]: *Cohesion*, if this is explicated as the mutual attraction of matter limited solely to the condition of contact, does not belong to the possibility of matter in general, and cannot therefore be cognized a priori as bound up with this. This property would therefore not be metaphysical but rather physical, and so would not belong to our present considerations.

[266] I suggested in note 117 above that these four numbers correspond, within empirical dynamics, to the categories of quantity, quality, relation, and modality respectively. We would thereby have a further differentiation of matter, under the categories of quality, with respect to the whole table of categories. Thus, the first number discusses the differentiation of matter in so far as it fills a space in accordance with its quantity (of specific density), whereas the second number discusses the differentiation of matter in so far as it fills a space in accordance with its quality (as solid, liquid, or gaseous). The correspondence in the case of the last two numbers is not quite so obvious. But, as we shall see, it appears that the discussion of elasticity corresponds to the categories of relation in virtue of the importance of this concept in the Mechanics (elastic versus inelastic collisions), whereas the discussion of chemistry corresponds to the categories of modality in virtue of the correspondence between these categories, in turn, and the regulative use of reason. I shall discuss this last point in detail at the end of my chapter on the Phenomenology.

true subject matter and goal of Kant's preferred treatment of the problem of the specific variety of matter. Moreover, Kant sees such chemistry, in the present context especially, as the paradigmatic application of the regulative use of reason, and it is here, in particular, that the problem of the specific variety of matter is ultimately resolved. It is not resolved by theoretical explanatory hypotheses, for Kant, but by a progressive and open-ended experimental program guided throughout by the regulative use of reason. Here, following Kant's preferred continuum model of matter, the constituents of matter are investigated from the outside in, as it were, with no prior assumptions at all concerning what the ultimate direction of this procedure must be.[267]

In sections 10, 11, and 15 above I emphasized the fundamental differences between Kant's empirical concept of impenetrability and the Lockean understanding of impenetrability as a simple idea of sensation. The former does not arise by abstraction from an immediately given sensation of resistance manifest to our sense of touch. It is rather the product of a much more complex procedure in which pure a priori concepts – both mathematical concepts and pure concepts of the understanding – are successively realized in experience step by step.[268] So the concept of impenetrability, for Kant, is only possible in the first place by embedding all properly sensory content (including that provided by the sense of touch) into a pre-existing a priori structure constituted by the pure intellectual concepts of quality and force (where the latter, in turn, presupposes the a priori sensible representation of a geometrical line in space). Moreover, the empirical concept of matter in so far as it fills a space (via impenetrability) also points forward to a further articulation of this concept under the categories of substance and causality that can only be completed in the Mechanics.

I am now in a position to add that this same empirical concept, considered under the categories of quality, not only presupposes the pre-existing a priori structures of pure sensibility and pure understanding. It also presupposes the complementary a priori structure of pure reason – where, in accordance with an indefinitely open-ended procedure directed by the regulative use of reason, an indefinite number of *further empirical concepts*

[267] So it is no wonder, in particular, that the passage about empirical specification quoted in note 263 above is immediately followed by another example from Stahlian phlogistic chemistry – the differentiation (A657/B685) of "absorbent earths" into "calx and muriatic earths" – that is closely related to the examples discussed in section 20 above: compare, for example, note 231 above, together with the paragraph to which it is appended and the preceding paragraph.

[268] See notes 26, 45, and 111 above, together with the paragraphs to which they are appended.

(further specifications of the empirical concept of matter in general) are also progressively introduced into experience. In the end, what corresponds to our immediately given sensory content in accordance with the categories of quality is precisely this indefinitely extended totality, conceived (A143/B182) as "the transcendental matter of all objects as things in themselves (thinghood, reality)" – conceived, that is, under the transcendental idea of an *omnitudo realitatis*.[269] This indefinitely extended totality of empirical concepts, for Kant, is all that ultimately remains of what he understands in the amphiboly (A273–74/B329–30) as the "Leibnizean" principle "that realities (as mere affirmations) never logically conflict with one another."

[269] To see the connection between the passage just quoted (from the discussion of the schematism of the pure categories of quality) and the transcendental idea of an *omnitudo realitatis*, observe that the discussion (from the transcendental ideal) quoted in the paragraph to which note 258 above is appended also contains the following assertion (A574–75/B602–3):

> A transcendental negation, by contrast, signifies non-being in itself, to which transcendental affirmation is opposed – which [latter] is a something whose concept in itself already expresses a being, and is therefore called reality (thinghood), because through it alone, and so far as it reaches, objects are something (things), while the opposed negation signifies a mere lack, and, where this alone is thought, the cancellation [*Aufhebung*] of all things is represented.

But further discussion of the relationship between the categories of quality and the transcendental idea of an *omnitudo realitatis* will have to wait for another occasion.

CHAPTER THREE

Mechanics

22 MOVING FORCE AND THE COMMUNICATION OF MOTION

The first explication of the Mechanics states (536): "Matter is the movable in so far as it, as such a thing, has moving force." The following remark explains the sense of "moving force" now at issue by contrast with the moving forces already considered in the Dynamics: "This is now the third definition of matter. The merely dynamical concept could consider matter also as at rest; for the moving force there dealt with had merely to do with the filling of a certain space, without the matter filling it needing to be viewed as moved itself" (536). Thus, as noted in section 9 above, the dynamical resistance exerted by the force of impenetrability as a resistance to *compression* was already distinguished in the Dynamics from the mechanical resistance to *motion* at issue in the present discussion: the former concerns whether a space filled by matter may be diminished in volume independently of any possible motions of this space, while the latter concerns precisely such possible motions.[1] The moving force at issue in the Mechanics is intrinsically connected from the beginning with the motion of the matter exerting this force – a point Kant underscores in the statement of the first explication by emphasizing that matter "has" the force in question precisely "*as* such a thing" (emphasis added), that is, as movable.[2]

[1] See the passage quoted in the paragraph to which note 13 of my chapter on the Dynamics is appended (496–97): "Matter is not here considered as it resists, *when it is driven out of its place*, and thus moved itself (this case will be considered later, as mechanical resistance), but rather when merely the *space* of its own extension is to be *diminished*."

[2] "*Materie ist das Bewegliche, so fern es, als ein solches, bewegende Kraft hat.*" Bax (Kant 1891, p. 214) obscures the point at issue by rendering this as "Matter is the movable, in so far as it is something having a moving force." Ellington, who generally follows Bax rather closely, has the same rendering here (Kant 1970, p. 95). The (sole) explication of the following Phenomenology is parallel in form (554): "*Materie ist das Bewegliche, so fern es, als ein solches, ein Gegenstand der Erfahrung sein kann.*" And here, as we shall see, the issue is clearly whether the *motion* of matter can be

Kant further develops the point in the next sentence of the remark by drawing a sharp distinction between the "imparting [*erteilen*]" of motion considered in dynamics and the "communication [*mitteilen*]" of motion considered in mechanics (536): "Repulsion [in the Dynamics] was therefore an originally moving force for *imparting* motion; in mechanics, by contrast, the force of a matter set in motion is considered as *communicating* this motion to another." When one piece of matter directly collides with a second in a straight line, for example, the former loses as much motion (i.e., momentum) through the resistance of the impacted matter as the latter gains from the motion of the former. In this sense, the motion lost by the former is communicated to the latter, so that, as explained in section 16 above, mechanical resistance and the communication of motion (impetus) are simply two sides or aspects of a single unitary phenomenon.[3] Indeed, as we shall see, the phenomenon in question is so unitary, for Kant, that he repeatedly emphasizes throughout the Mechanics that *both* bodies undergoing such an interaction – both the impacting body and the resisting body, for example – must necessarily be viewed as moving. In particular, Kant's official derivation or "construction" of the communication of motion in the fourth proposition is entirely based on the idea that both bodies must be viewed as moving. As Kant explains in the second remark to this proposition (551): "Nothing can resist a motion except the contrary motion of another [body], but never its [state of] rest."

Kant's paradigmatic example of the communication of motion, here and throughout the Mechanics, is thus the phenomenon of impact, whereby two colliding bodies resist one another by their forces of impenetrability and, at the same time, mutually transfer motions to one another by impetus. Kant is perfectly clear, however, that communication of motion is by no means confined to the phenomenon of impact. On the contrary, it takes place equally, and symmetrically, in the phenomenon of attraction, whereby two bodies affect one another's motions by the fundamental force of universal gravitation:

determined in experience. Bax (1891, p. 233) renders this explication as "Matter is the movable, in so far as it can be an object of experience as such." Ellington (1970, p. 118) has "Matter is the movable insofar as it can as such be an object of experience."

[3] See the paragraph to which note 129 of my chapter on the Dynamics is appended, together with Newton's famous discussion of "innate force [*vis insita*]" or "force of inertia [*vis inertiae*]" quoted in the note (P504): "[T]his exercise of force is, depending on the point of view, both resistance and impetus: resistance in so far as the body, in order to maintain its state, strives against the impressed force, and impetus in so far as the same body, yielding only with difficulty to the force of a resisting obstacle, endeavors to change the state of that obstacle."

It is clear, however, that the movable would have no moving force *by means of its motion*, if it did not possess originally moving forces, by which it is active in every place where it is found, prior to any inherent motion of its own; and no matter would impress proportionate motion on another matter lying in the straight line *ahead of it* in its way, if both did not possess original laws of repulsion; nor could a matter, by its motion, compel another *to follow* in the straight line *behind it* (to drag it along behind [*nachschleppen*]), if both did not possess attractive forces. Thus all mechanical laws presuppose dynamical laws, and a matter, as moved, can have no moving force except by means of its repulsion or attraction, on which, and with which, it acts immediately in its motion, and thereby communicates its own inherent motion to another. (536–37)

That Kant has universal gravitation specifically in mind is confirmed by the next sentence (537) – where "the communication of motion by means of attraction" is illustrated parenthetically by the at first sight rather fanciful possibility that "a comet, with stronger attractive power than the earth, were to drag the latter along behind it in passing [*im Vorbeigehen vor derselben sie nach sich fortschleppte*]."[4]

The priority Kant gives to the "originally moving" dynamical forces of repulsion and attraction with respect to the mechanical moving force involved in the communication of motion is of fundamental importance. For the concept of moving force was typically used ambiguously in Kant's time. On the one hand, it could mean a force a moving body has *in virtue of its motion*, which can then have effects on other bodies through impact. This was the primary meaning of "moving force" in the tradition of the mechanical philosophy, where the main issue concerned whether force in this sense is measured (in modern notation) by mv or mv^2 – on either alternative, then, a body *at rest* can neither possess nor exert moving force.[5]

[4] That the example is not as fanciful as it first appears emerges from a comparison of this passage with Lambert's *Cosmological Letters* (1761), where the same example is prominently featured in the first letter. Lambert (1761, p. 4) begins by asking "[W]hat would happen to us if a large comet would come so close to the earth that the sea would rise over the earth's surface, or even drag the earth along with it [*mit sich fortrisse*]?" He then points out (p. 6) that astronomy has become so complicated, after Copernicus, "that we must now be concerned that a comet may come and drag the earth along with it [*mit sich fortschleppen*] to beyond the fixed stars." Kant's language in the above passage echoes this last quotation from Lambert. I shall return to a discussion of the relationship between Kant's treatment of motion in the *Metaphysical Foundations* and Lambert's *Cosmological Letters* in my chapter on the Phenomenology.

[5] I am referring to the famous *vis viva* controversy, which initially arose, within the tradition of the mechanical philosophy, as a dispute between Cartesians and Leibnizeans. By the early to mid eighteenth century, however, it had been transformed into a dispute between Leibnizean–Wolffians and Newtonians, where the measure of Newtonian *impressed* force is given by the change in momentum experienced by the body affected by such a force. Kant's first published work, *Thoughts on the True Estimation of Living Forces* (1747), attempts to mediate the controversy against this background. I shall consider some of Kant's allusions to this issue in the Mechanics below.

On the other hand, however, a moving force could be a force a body has *for causing or producing motion* in another body. This was the primary meaning of "moving force" in the Newtonian tradition, where forces of repulsion and attraction could be exerted by bodies on one another independently of the state of motion of the body exerting (and in this sense possessing) such a force. This feature of the concept of moving force in the Newtonian tradition, moreover, was intimately bound up with the question of action at a distance – with the possibility, for example, that one body may exert a force of attraction on another independently of any communication of motion by contact.[6]

We know that Kant, from the beginning, places himself squarely on the side of the Newtonian tradition.[7] I observed in the Introduction that Kant's initial appeal to the fundamental forces of attraction and repulsion in the pre-critical period is explicitly intended to defend what Kant takes to be the Newtonian contention (1, 476) that "universal attraction or gravitation is hardly explainable by mechanical causes but shows that it is derived from forces inherent in bodies that are at rest and act at a distance."[8] The possibility of forces acting at a distance and that of forces acting at rest (i.e., independently of the state of motion of the body exerting the force) are therefore closely connected for Kant.[9] This same point is underscored by Kant's emphasis on the symmetry between the two originally moving forces of attraction and repulsion in the concluding sentence (containing the parenthetical example already quoted) of the present remark:

[6] For a classic discussion of the evolution of the concept of moving force through Newton see Westfall (1971).

[7] As observed in note 5 above, Kant's first essay on *Living Forces* attempts (as do all the works of this early period) to synthesize and accommodate both the Leibnizean and Newtonian traditions – and here, in particular, to retain an important place for *vis viva*. Nevertheless, in line with the fundamental ambiguity in the contemporary concept of moving force, Kant still insists there that the primary concept is that of the Newtonian tradition: moving force is that in one substance which *causes* motion in another, whether or not the first substance is moving (or even movable) in turn. I shall discuss the evolution of Kant's concept of moving force from the pre-critical to the critical period in further detail as we proceed.

[8] See the passage – from the *Physical Monadology* – quoted in the paragraph to which note 17 of the Introduction is appended. Recall also that Kant first introduces the two fundamental forces of attraction and repulsion in the *Theory of the Heavens* as "both borrowed from the Newtonian philosophy" (1, 234; see the paragraph to which note 46 of my chapter on the Dynamics is appended).

[9] Several pages later, in the context of a discussion of *vis viva* and the Leibnizean distinction – developed in Leibniz (1695) – between "living" and "dead" forces, Kant characterizes "the originally moving forces of dynamics" as "those forces whereby matter acts on other [matters] even when one completely abstracts from their inherent motion, and even from their striving to move [themselves]" (539). It is "more appropriate," Kant says, to use the term "dead" forces for these– "if in fact these terms for living and dead forces still deserve to be retained" (539).

I will be forgiven if I do not here further discuss the communication of motion by attraction (e.g., if perhaps a comet, of stronger attractive power than the earth, were to drag the latter along behind it in passing), but only that by means of repulsive forces, and thus by pressure (as by means of tensed springs) or impact; for, in any event, the application of the laws of the one case to those of the other differs only in regard to the line of direction, but is otherwise the same in both cases. (537)

In all cases of the communication of motion, therefore, the two originally moving dynamical forces of attraction and repulsion are prior to the mechanical moving force involved in this communication. Repulsion and impenetrability are in no way privileged over attraction at a distance here, where the only relevant difference is that of "the line of direction."[10]

In what sense, however, does Kant think that the actions of the two fundamental dynamical forces of attraction and repulsion are symmetrical, and what are the laws of the communication of motion that he holds to apply in exactly the same way in the two cases (differing only in regard to "the line of direction")? As far as the first question is concerned, Kant initially characterizes attractive and repulsive forces in the second explication of the Dynamics and explains why only these two can be thought in the following note:

Only these two moving forces of matter can be thought. For all motion that one matter can impress [*eindrücken*] on another, since in this regard each of them is considered only as a point, must always be viewed as imparted [*erteilt*] in the straight line between the two points. But in this straight line there are only two possible motions: the one by which the two points *remove* themselves from one another, the second by which they *approach* one another. But the force causing the first motion is called *repulsive force*, whereas the second is called *attractive force*. Therefore, only these two kinds of forces can be thought, as forces to which all moving forces in material nature must be reduced. (498–99)

These are dynamical moving forces in the sense of *causes* of motion, which "impart" motion to – or "impress" motion on – another matter. And the reason that the two bodies ("matters") can be considered merely as points, so that only the straight line between them is relevant, is that the notion of a force or cause of motion is initially introduced in the Dynamics via the representation of the addition of two motions (velocities) directed

[10] Ellington (Kant 1970, p. 96) misleadingly suggests that repulsion is privileged in this respect by rendering the last clause of the above passage as "the application of the laws of repulsion in comparison with the case of attraction differs only with regard to the line of direction, but otherwise is the same in both cases." Here Bax (Kant 1891, p. 215) is more accurate.

either towards or away from a given central point developed in the Phoronomy.[11]

Turning now to the second question, it is clear that the relevant laws of the communication of motion are the three Laws of Mechanics that Kant will derive in the body of the Mechanics (as his second, third, and fourth propositions). The most important, in this context, is Kant's "*Third mechanical law*" (544): "In all communication of motion, action and reaction are always equal to one another." Moreover, it emerges from the proof of this law (which I shall consider in detail below) that what Kant has in mind are the *changes in the quantity of motion* (momentum) experienced by the two interacting bodies. If we view the two bodies, in particular, as points interacting along the straight line connecting them, and we consider the interaction from the perspective of the center of mass of the two bodies along this line, then the changes of momentum produced in the two in virtue of their interaction are always equal and oppositely directed. It is in this sense, as Kant explains at the conclusion of his proof, that the same laws hold symmetrically for both kinds of force:

> [T]he communication of motion by impact [*Stoß*] differs from that by attraction [*Zug*] only in the direction in accordance with which the matters resist one another in their motions; and so it follows that *in all communication of motion* action and reaction are always equal to one another (every impact can only communicate the motion of one body to another by means of an equal contrary impact, every pressure by means of an equal contrary pressure, and every attraction [*Zug*] only through an equal contrary attraction [*Gegenzug*]. (547)[12]

Thus, whereas what Kant calls a dynamical moving force exerted by one body on another is the cause of a change of motion (velocity) in the other, the mechanical moving force communicated between the two bodies is a measure of the change of motion (momentum) thereby transferred from one body to the other (compare note 11 above). In the communication of motion by (direct linear) impact, for example, the impacting body loses as much motion (momentum) as the impacted body gains, and we here

[11] See note 22 of my chapter on the Dynamics, together with the paragraph to which it is appended, for how the first Proposition of the Dynamics is thereby connected to the construction of the composition of motions in the Phoronomy. Recall, however, that the notion of mass is not yet available in the Phoronomy, so that the motions considered there have only speed and direction. How mass (and therefore momentum) comes in is precisely the problem of the Mechanics.

[12] *Zug* can be more literally translated as "traction," but it is linked etymologically to the verb *ziehen* ("to draw or pull") and thus to Kant's word for "attraction" (*Anziehung*). In the second explication to Dynamics (498–99) cited above Kant uses *Anziehungskraft* for attractive force, *Zurückstoßungskraft* for repulsive force, and remarks (498): "The latter will also sometimes be called *driving* [*treibende*] force, the latter *drawing* [*ziehende*] force."

have a case of the communication of motion by means of repulsive forces (impenetrability). Similarly, in the case of the communication of motion by attraction, both bodies, according to Kant, must be conceived to be in motion (i.e., accelerating), and the change of momentum of the first body (by the equality of action and reaction) is precisely counterbalanced by the change of momentum of the second. In all cases of interaction by the fundamental dynamical forces of repulsion and attraction, therefore, there is, at the same time, a transfer of momentum, and it is just this momentum that is the operative (mechanical) moving force. In this sense, as we shall see, Kant's overall argument in the Mechanics can be seen as providing a reinterpretation of the concept of moving force arising in the tradition of the mechanical philosophy in the context of a basically Newtonian conception of force and its relation to motion.[13]

Yet there is, for Kant, an important problem here. He initially introduces the dynamical concept of moving force as the cause of a change of motion (with respect to speed and/or direction) using a phoronomical representation of the addition of two motions (velocities) directed either towards or away from a given central point. It is precisely this representation that underlies his contention that attractive and repulsive dynamical forces differ only with regard to the line of direction connecting the two interacting bodies – represented as points. But the fundamental *mechanical* law governing this interaction is the equality of action and reaction, which essentially involves the further (mechanical) concept of momentum or quantity of motion. This concept, in turn, is intimately related to the concept of mass or quantity of matter, and the latter, for Kant, only applies to an aggregate of the movables in a given (extended) space: it is not meaningful, for Kant, to apply the concept of quantity of matter to an isolated point-mass.[14]

For Kant, therefore, one needs to say considerably more to connect the dynamical concept of moving force as the cause of motion with the mechanical concept of moving force as the *momentum* exchanged in any dynamical interaction (whether attractive or repulsive). The only mathematical representation we can associate with the former is the purely

[13] According to Newton's Third Law of Motion, in particular, if one body exerts an impressed force on another, the second must exert an equal and opposite impressed force on the first, and these two impressed forces, by the Second Law, then give rise to equal and opposite changes in momentum. The Second Law of Motion thereby establishes a correlation between dynamical moving forces, in Kant's sense, and mechanical moving forces, whereby the latter provide a quantitative measure of the former.

[14] I have emphasized this throughout my chapter on the Dynamics: see, for example, note 135 of that chapter, together with the paragraph to which it is appended.

phoronomical construction of the addition of velocities, whereas the latter requires the mathematical representation of two new magnitudes: quantity of matter and quantity of motion. It is for precisely this reason that Kant embarks on a lengthy discussion of these magnitudes in the second explication and first proposition of the Mechanics before he arrives at his three Laws of Mechanics. It is for precisely this reason, as well, that Kant's First Law of Mechanics, somewhat unexpectedly, is the conservation of the total quantity of matter. Here, as we shall see, Kant further develops his fundamental reconsideration of the metaphysical concept of substance begun in the Dynamics and thereby continues his project of uniting metaphysics in the Leibnizean tradition with mathematical physics in the Newtonian. It is for this reason, as we shall also see, that Kant's three Laws of Mechanics differ from Newton's three Laws of Motion.[15]

23 QUANTITY OF MATTER AND QUANTITY OF MOTION

Kant's discussion of the concept of quantity of matter in the second explication begins as follows (537): "The *quantity of matter* is the aggregate of the movable in a determinate space." Kant does not immediately link the concept of quantity of matter with impetus (momentum), the communication of motion, or the concept of inertia. For these concepts, considered mathematically, could be well defined in the case of isolated or unextended point-masses, whereas Kant's view, as I have explained, is that the concept of quantity of matter only has genuine physical significance for extended distributions or aggregates of moving points continuously spread out over a definite spatial volume. Kant's present (critical) conception, as I have also explained, thereby contrasts sharply with that of the *Physical Monadology*, where the concept of mass or quantity of matter is

[15] Again, the most obvious differences between the two sets of laws is that Kant includes the law of the conservation of matter and omits Newton's Second Law relating impressed force to changes in momentum. As pointed out in note 13 above, the Second Law thereby provides Newton with a quantitative measure of impressed force. For Kant, however, we need first to consider how mass or quantity of matter acquires the structure of a mathematical magnitude. We need carefully to consider the transition from (unextended) points to (continuously extended) bodies along with the parallel transition from the merely phoronomical concept of cause of motion (represented *quoad effectus*) to genuinely active *substantial* causes. Eric Watkins (1997, 1998) has argued in detail for the importance of the Leibnizean tradition in Kant's formulation of his Laws of Mechanics, and Marius Stan (in press) has built on Watkins's work to develop an insightful account of the Leibnizean background to Kant's formulation of the equality of action and reaction. Stan's work, in particular, has finally convinced me of the Leibnizean purview of Kant's Laws of Mechanics, although I still believe (and I shall here try to show) that Kant thereby aims to provide a metaphysical foundation for specifically Newtonian mathematical physics nonetheless.

introduced in terms of the concept of inertia and, accordingly, is explicitly independent of spatial volume.

Kant's emphasis here on the need for considering a space-filled volume in connection with the concept of quantity of matter echoes Newton's first definition of the *Principia* (P403): "*Quantity of matter is a measure of matter that arises from its density and volume jointly*" – from which it immediately follows that an unextended point must have zero quantity of matter. The concern with density and the possibility of compression that is central to Kant's conception of matter as an (infinite and continuous) aggregate of the movables is also prominent in Newton's comments on this definition: "If the density of air is doubled in a space that is also doubled, there is four times as much air, and there is six times as much if the space is tripled. The case is the same for snow and powders condensed by compression or liquefaction, and also for all bodies that are condensed in various ways by any causes whatsoever" (P403). In particular, we here appear to have an instance of what the first number of Kant's general remark to dynamics calls the dynamical concept of density, according to which the same matter compressed into a smaller space can be characterized as more dense than in a less compressed state. But this notion of density, for Kant, is only suitable "for think[ing] of a ratio of matters with respect to their density if we … imagine them as specifically of the same kind, so that one can be generated from the other by mere compression" (526; see note 142 in my chapter on the Dynamics, together with the paragraph to which it is appended). For Kant, then, the Newtonian characterization of quantity of matter in the first definition of the *Principia* does not, by itself, provide us with a universally well-defined concept suitable for comparing specifically different types of matter – such as air and snow (i.e., water), for example.[16]

[16] Kant's own example in the general remark to dynamics is a comparison of density "between water and mercury" (526). On what Kant calls the absolute or mathematical concept of density, by contrast, there is a maximum of density given by absolutely hard and impenetrable particles – so that one matter is less dense than another only by containing more empty space. As explained in section 11 above, it is precisely this concept of density that is articulated in Lambert's version of the mechanical philosophy, where density can be uniformly estimated in terms of the volume of a completely filled space (the density of an incompletely filled space relates to that of a completely filled space inversely as their volumes), and each type of matter can be transformed, in principle, into all other types. On such a conception, then, Newton's initial characterization of quantity of matter would be universally well defined for all types of matter after all. Moreover, as observed in section 18 above, just such a conception is suggested by Newton's argumentation in the corollaries to Proposition 6 of Book 3 of the *Principia*, where, beginning from the assumption that bodies (including the aether) differ from one another only in their "forms and textures" (and thus can be "transmuted by degrees" into one another), together with the assumption that "the solid particles of all bodies have the same density and cannot be rarified without pores," Newton concludes that "[a]ll spaces are not equally full" and "there must be a vacuum" (P809–10).

It is for precisely this reason, in fact, that Kant introduces the concept of quantity of motion towards the end of the second explication (537): "The *quantity of motion* (estimated mechanically) is that which is estimated by means of the quantity of the moved matter and its velocity together; *phoronomically* it consists merely in the degree of velocity."[17] For he goes on to assert in the first proposition that only the quantity of motion can provide us with a universally applicable concept of quantity of matter (537): "The quantity of matter can be estimated in comparison with *every* other [matter] only by the quantity of motion at a given velocity." The following proof makes the problem of comparing densities in specifically different types of matter fully explicit:

> Matter is infinitely divisible; so its quantity cannot be immediately determined *by an aggregate* of its parts. For even if this occurs in comparing the given matter with another of the same kind, in which case the quantity of matter is proportional to the size of the volume, it is still contrary to the requirement of the proposition, that it is to be estimated in comparison with *every* other (including the specifically different). (537–38)

For example, if I compare two samples of air at the same degree of compression (in cylinders contained by pistons supporting equal weights, say), I can conclude that the quantities of matter of the two are as their volumes. Moreover, if I compare two equal volumes at different degrees of compression (by doubling the weight supported by one piston, say), I can conclude that the densities are as the compressions (weights supported). But, according to Kant, I cannot do this with air and water, for example, because the two different continuously space-filling elastic fluids have specifically different expansive forces, by which they *differentially* resist the corresponding compressive forces (weights).[18]

[17] The note to the following first proposition puts it this way (538): "The quantity of motion of a body is in compound ratio to the quantity of its matter and its velocity; i.e., it is one and the same whether I make the quantity of matter of a body twice as great and retain [the same] velocity, or I double the velocity and retain precisely this mass." Compare Newton's second definition and comments thereto (P404):

> *Quantity of motion is a measure of motion that arises from the velocity and the quantity of matter jointly*. The motion of a whole is the sum of the motions of the individual parts, and thus if a body is twice as large as another and has equal velocity there is twice as much motion, and if it has twice the velocity there is four times as much motion.

[18] Again, on the absolute or mathematical concept of density we could universally compare quantities of matter directly by volume after all – since all completely filled spaces have quantities of matter directly proportional to their volumes, and the quantity of matter of an incompletely filled space is equal to that of its completely filled parts (see note 16 above).

Kant concludes his proof by claiming that only the quantity of motion (at a given velocity) can yield a universally applicable comparison of quantities of matter:

> Hence matter cannot be validly estimated, either immediately or mediately, in comparison *with every other*, so long as we abstract from its inherent motion. Therefore, no other generally valid measure [of matter] remains except the quantity of its motion. But here the difference of motion, resting on the differing quantity of matters, can be given only when the velocity of the compared matters is assumed to be the same; hence, etc. (538)

It is not immediately clear, however, why we can conclude that "no other generally valid measure remains except the quantity of motion." How do we know that there is really no other possibility except in terms of quantity of motion? Indeed, how did motion and its quantity become relevant to the quantitative comparison of different space-filling aggregates of matter in the first place?

The answer, on my reading, is that Kant is arguing against the backdrop of the three concepts of quantity of matter I distinguished in the context of the balancing argument of the Dynamics in section 16: (i) a dynamical concept related to the fundamental force of repulsion through a notion of density linked to the possibility of compression; (ii) a dynamical concept related to the fundamental force of attraction through the circumstance that accelerations produced by this force are directly proportional (at a given distance) to the attracting body's mass; (iii) a mechanical concept related to the communication of motion and thus to the concepts of impetus (or momentum) and inertia. Kant's present argumentation takes its start from the first concept of quantity of matter, in so far as this quantity is given by the aggregate of the movable in a given (continuously extended) space. Kant here echoes Newton's definition of quantity of matter as the product of volume and density, and the relevant concept of density, from Kant's point of view, is the dynamical concept essentially tied to repulsive (i.e., expansive) force. Since comparisons based on this concept of quantity of matter, as we have seen, are contrary to the hypothesis of the proposition, and since only the first dynamical concept is in play, Kant concludes that only the third, mechanical concept of quantity of matter remains.[19]

[19] I am here ignoring the second dynamical concept based on the fundamental force of attraction. Kant considers comparisons of quantity of matter involving this concept in the following remark to the present proposition. The key point, which I shall discuss in detail below, is that this comparison, in the end, must also be viewed as mechanical (541): "[T]he estimation here, in fact, still takes place mechanically, although only indirectly so."

Kant has introduced this set of alternatives implicitly in the first explication and accompanying remark. For Kant's point there, as explained, is to distinguish the mechanical concept of matter introduced here from the dynamical concept introduced in the previous chapter. According to the former concept, matter is the movable in so far as it has moving force in the mechanical sense: force exerted in its motion in the guise of momentum to be transferred to another. According to the latter (536): "[One] could consider matter also as at rest; for the moving force there dealt with had merely to do with the filling of a certain space, without the matter filling it needing to be viewed as moved itself." Such a moving force, characteristic of the dynamical concept of matter, can be viewed as exerted by one matter on another matter "even when one completely abstracts from their inherent motion" (539; see note 9 above). In the present proof of the first proposition we are seeking a universally valid method for estimating the quantity of matter, and we have just seen that the dynamical concept of matter, by itself, cannot provide us with one. Kant can rightfully conclude, against this background, that only the mechanical concept remains, so that "matter cannot be validly estimated, either immediately or mediately, in comparison *with every other*, so long as we abstract from its inherent motion."[20] The premises for the proof of the first proposition therefore include both the first and second explications.

Kant's contention that quantity of matter can only be validly estimated in general by reference to quantity of motion (estimated mechanically) thus implies that the concept of quantity of matter is intimately connected with the concept of moving force in the mechanical sense (the momentum transferred from one body to another in the communication of motion). Kant emphasizes this connection in the full text of the second explication:

The *quantity of matter* is the aggregate of the movable in a determinate space. This same [quantity of matter], in so far as all its parts in their motion are considered as acting (moving) simultaneously [*zugleich*], is called *mass* [*Masse*], and one says that a matter *acts in mass*, when all its parts, moved in the same direction, *simultaneously* [*zugleich*] exert their moving force externally. A mass of determinate figure is called a *body* (in the mechanical meaning). The *quantity of motion* (estimated mechanically) is that which is estimated by the quantity of

[20] In the case of the fundamental force of attraction, once again, it turns out that comparisons of quantity of matter based on this force do not abstract from the inherent motions of the two compared bodies after all. For, as I shall explain in detail below, the equality of action and reaction is necessarily involved here (compare note 19 above, together with note 13 and the paragraph to which it is appended).

the moved matter and its velocity together [*zugleich*]; *phoronomically* it consists merely in the degree of velocity. (537)

The only way in which a body can manifest its quantity of matter, then, is by exerting an action, represented by its (mechanical) moving force, on another body.[21] Indeed, the importance of the concept of action in this context has already been emphasized in the remark to the first proposition by Kant's claim (537) that "a matter, as moved, can have no moving force except by means of its repulsion or attraction, on which, and with which, it acts immediately in its motion, and thereby communicates its own inherent motion to another." As explained in the preceding section, Kant is here considering the *dynamical* actions exerted by the fundamental forces of attraction and repulsion, which, it turns out, can only be quantitatively measured in the context of the *mechanical* communication of motion.[22] The measure of such action, we now see, is precisely the momentum thereby transferred.[23]

The relationship between quantity of matter and the action one matter exerts on another (as measured by the momentum thereby transferred)

[21] Here Kant specifies a body "in the mechanical meaning" (537), where all of its parts "moved in the same direction, *simultaneously* exert their moving force externally." I shall return to the meaning and importance of this restriction in the following section.

[22] Since, as I shall further discuss in connection with Kant's remark to the first proposition, "the quantity of matter is the *quantity of substance* in the movable" (540), we here have a more specific realization of the general interconnections among the concepts of causality, substance, action, and force articulated in the first *Critique* to which I referred in section 13. See again the discussion in the second analogy (A204–5/B249–50):

> Causality leads to the concept of action, this leads to the concept of force, and thereby to the concept of substance … I can [therefore] not leave untouched [here] the empirical criterion of substance, in so far as it appears to manifest itself, not through the permanence of the appearance, but better and more easily through action. Where there is action, and therefore activity and force, there is also substance, and in the latter alone must the seat of the former fruitful source of appearances be sought.

I shall return to this passage, and the notion of the empirical criterion of substance, below. The noun "action" in this passage from the Second Analogy is *Handlung*, while the verb "to act" in the above passages from the Mechanics is *wirken*. The noun *Wirkung* can also mean "effect," of course, as our passage from the second analogy implies a few sentences later (A205/B250): "Action [*Handlung*] already signifies the relation of the subject of causality to the effect [*Wirkung*]." The two notions are thus intimately related to each other, and *Wirkung*, in a transitive context, can often be translated as "action." When Kant speaks of "action" in the above passage from the Mechanics, therefore, he is implying a dynamical (rather then merely phoronomical) notion of causality "seated" in a substance.

[23] See note 15 above, together with the paragraph to which it is appended. It is precisely here, for Kant, that the *dynamical* notion of moving force – which is grounded, metaphysically, in the concepts of substance, causality, and action – acquires a determinate mathematical measure in terms of *mechanical* moving force (in the communication of motion). As I shall explain in more detail below, it is in precisely this way, in the end, that Kant provides a counterpart to Newton's Second Law of Motion, which specifies a general quantitative measure for "impressed force."

is the subject of the following note to the first proposition. Kant begins with a more formal characterization of the concept of quantity of motion, and he links this characterization to the general mathematical concept of magnitude:

> The quantity of motion of bodies is in compound ratio to that of the quantity of their matter and their velocity, that is, it is one and the same whether I make the quantity of matter in a body twice as great, and retain the same velocity, or double the velocity, and retain precisely this mass. For the determinate concept of a magnitude is possible only through the construction of the quantum. But in regard to the concept of quantity, this is nothing but the composition of the equivalent [*Zusammensetzung des Gleichgeltenden*]; so the construction of the quantity of a motion is the composition of many motions equivalent to one another. (538)

Kant is here invoking the general mathematical concept of magnitude already invoked in his earlier discussion of speed or velocity as a mathematical magnitude in the Phoronomy (section 3 above). In the present context, however, we are considering two new mathematical magnitudes: quantity of matter and quantity of motion. So it follows, according to the general concept of magnitude, that we must now specify an operation of composition or addition suitable for the construction of any such quantity – as a quantum – from a number of "smaller" such quanta.[24]

As explained in section 3 above, speed or velocity is what Kant calls an *intensive* magnitude. Speeds or velocities characterize mere moving points, independently of volume or spatial extent, and the addition or composition of such magnitudes therefore proceeds independently of the composition of spatio-temporal parts "external to one another" (493).[25] In the Mechanics, however, we must explicitly deal with bodies or extended space-filled volumes. Indeed, the first explication emphasizes precisely this in defining quantity of matter as the aggregate of the movable in a determinate space, where, as we have seen, Kant echoes the Newtonian definition of quantity of matter as the product of density and volume. Moreover, since volume is a paradigmatically *extensive* magnitude, it follows that quantity of matter, as a quantum or magnitude, must have an essentially extensive aspect – in so far as the quantity of matter of a whole space-filled volume is the sum of the quantities of matter of its spatial

[24] See the paragraph to which note 29 of my chapter on the Phoronomy is appended, together with the following paragraph. For the distinction between *quantity* and *quantum* see note 87 of this same chapter, together with the paragraph to which it is appended.

[25] See note 38 of my chapter on the Phoronomy, together with the paragraph to which it is appended.

parts.[26] So it follows that quantity of motion (mass times velocity) has an extensive aspect as well: the quantity of motion of a space-filled volume is the sum of the quantities of motion of its spatial parts. We can, as Kant states in the first sentence of his note, double the quantity of motion of a body either by doubling its velocity or by doubling its "size" (i.e., quantity of matter).

We are considering, then, an aggregate of moving parts continuously spread out or extended throughout a particular spatial volume and all moving in the same direction with the same speed. We can increase the quantity of motion of such an aggregate either by increasing the velocity common to all of its parts or by keeping this velocity constant and adding more parts (external to one another) to the aggregate instead. In this way, Kant continues, we might appear to have constructed the pivotal concept of quantity of motion purely phoronomically:

> Now according to the phoronomical propositions, it is one and the same [*einerlei*] whether I impart to a single movable a certain degree of velocity, or to many equal movables all smaller degrees of velocity, which result from the given velocity divided by the aggregate of movables. From this there arises, in the first place, a seemingly phoronomical concept of the quantity of a motion, as composed of many motions of movable points, external to one another yet united in a whole. If these points are now thought as something that has moving force *by means of its motion*, then there arises from this the mechanical concept of the quantity of motion. (538)

It might appear, then, that a mathematical construction of the quantity of motion (momentum) could result from two already given mathematical constructions: the construction of velocity as an intensive magnitude through the composition of motions given in the Phoronomy, together with a geometrical composition of an aggregate of parts external to one another as an extensive magnitude.

But this idea actually makes no sense, because we have left out of consideration the crucial factor – the masses (quantities of matter) of the parts in the aggregate in question. Adding or subtracting parts of our space-filling aggregate of movables will result in the addition or subtraction of mass only if we assume that these parts already have well-defined masses. Yet this is not an assumption that we can coherently make at this

[26] Kant emphasizes the essentially extensive aspect of the concept of quantity of matter in the following remark by asserting (539–40) that "matter has no other magnitude except that which consists in the *aggregate* of the manifold *external to one another*." According to the remark to the second proposition (542): "[A]ll magnitude of an object possible merely in space must consist *of parts external to one another*." I shall return to these passages below.

stage, since the construction of mass or quantity of matter as a mathematical magnitude is precisely what is now at issue. We know, to be sure, that the mass of the whole is the sum of the masses of its spatial parts, but it by no means follows that we can then estimate the quantity of matter by an operation of geometrical composition on this aggregate of parts. Whereas quantity of matter does have an essentially extensive aspect, it also has an intensive aspect – which is given by the *density* distributed within the volume in question. Kant's "seemingly phoronomical" procedure for constructing the concept of quantity of motion therefore begs precisely the question at issue, for it is just this crucial intensive aspect of the concept of quantity of matter that we do not yet know how to construct.

Kant himself explains the shortcomings of his suggested (seemingly phoronomical) construction in the following sentence (538–39): "In phoronomy, however, it is not appropriate to represent a motion as composed of many motions *external to one another*, because the movable, since it is here represented as devoid of all moving force, yields no other difference in the quantity of motion, in any composition with several of its kind, than that which consists merely in the velocity." Kant here provides two related considerations. On the one hand, phoronomical composition is not itself extensive in any sense: it does not involve an addition of further parts external to one another but consists solely in a purely intensive composition of two or more velocities in a *single point*. On the other hand, the points under consideration exert no moving forces at all (neither dynamical nor mechanical), so that, in particular, no other conception of quantity of motion can possibly arise except that which involves only velocity. As explained, a truly dynamical – as opposed to merely phoronomical – representation of causal action, for Kant, essentially involves the metaphysical concept of substance. And, since mechanical moving forces presuppose dynamical moving forces, the mechanical concepts of quantity of matter and quantity of motion essentially involve this concept as well. So Kant's diagnosis of the shortcomings of his "seemingly phoronomical" construction ultimately rests on precisely these metaphysical considerations.[27]

[27] See again notes 14 and 15 above, together with the paragraphs to which they are appended. With respect to the issue raised in the latter note, Kant's explicit consideration of a "seemingly phoronomical" construction, in this context, has the advantage of highlighting the need for a quantitative measure of truly dynamical forces that goes beyond the merely phoronomical consideration of such forces *quoad effectus*. It is precisely this measure that Kant is now in the process of articulating.

In the end, the (mechanical) concept of quantity of motion can only be mathematically estimated (but not mathematically constructed) by the actions or effects that one body exerts on another in the context of the communication of motion. Kant explains in the next sentence (539): "As the quantity of motion of a body relates to that of another, so also does the magnitude of their action [*Wirkung*], where this is to be understood as the *entire* action."[28] The (mechanical) concept of quantity of motion, together with the (mechanical) concept of quantity of matter, can only be exhibited as a magnitude in the context of Kant's three Laws of Mechanics – the very laws, in particular, that govern the communication of motion. Indeed, as explained in section 11 above, Kant suggests just such a procedure in the first *Critique* as well, in the course of his discussion of the "intensive" filling of space by matter in the anticipations of perception. Such intensive differences in quantity of matter, Kant suggests (A173–74/B215–16), are estimated or determined "partly by means of the moment of gravity, or weight, partly by means of the moment of resistance to other moved matters." Moreover, given Kant's parallel opposition between "weight" and "impenetrability" in this same passage, it appears that determining quantity of matter from the "moment of resistance to other moved matters" involves precisely the communication of motion (momentum) by impact.[29]

I shall return to Kant's conception of how quantity of matter is estimated or determined in the next section. But I first want to recall that,

[28] The final qualification concerning the *entire* action, as Kant explains in the remainder of the note, relates to the *vis viva* controversy (see notes 5, 7, and 9 above). Kant wants to use transfer of momentum (mv) rather than *vis viva* (mv^2) as a measure of the action or effect in question, and he therefore uses the time integral of the Leibnizean "dead force" or infinitesimal change of momentum rather than the space integral of this quantity. In modern terminology and notation, whereas the space integral (work) of mdv/dt is $\frac{1}{2}mv^2$ (kinetic energy), its time integral is just mv (momentum). Kant expresses the point as follows (539):

> Those who merely took the quantity of a space filled with resistance as the measure of the entire action (for example, the height to which a body with a certain speed can rise against gravity, or the depth to which it can penetrate into soft matters) came out with another law of moving forces for *actual* motions – namely, that of the compound ratio of the quantity of matters and the squares of their velocities. But they overlooked the magnitude of the action in the given time, during which the body traverses its space at a lower velocity; and yet this alone can be the measure of a motion that is exhausted by a given uniform resistance. Hence there can no difference, either, between living and dead forces, if the moving forces are considered mechanically, that is, as those which bodies have insofar as they themselves are moved, whether the velocity of their motion be finite or infinitely small (mere striving towards motion).

[29] See note 39 of my chapter on the Dynamics, together with the paragraph to which it is appended. In this sense, what I have called the intensive aspect of the concept of quantity of matter (density as opposed to volume) is determined precisely through such interactions.

beginning with the Dynamics, we have been considering matter as the *real* in space. We also know, from the fifth explication of the Dynamics, that Kant characterizes *material substance* (502) as "a thing in space that is movable for itself, i.e., separated from everything else existing outside of it in space." In the remark to the first proposition of the Mechanics Kant returns to this theme and states (540) that "the quantity of matter is the *quantity of substance* in the movable" – where "the quantum of substance here signifies nothing but the mere aggregate of the movables that constitutes the matter." In the remark to the following second proposition Kant continues (542, bold emphasis added): "[S]ince all quantity of an object possible merely in space must consist *of parts external to one another*, it follows that these [parts], if they are ***real*** (something movable), must necessarily be substances." In the case of the aggregates of movables external to one another at issue in the Mechanics, then, such aggregates, together with all their movable parts, must be subsumed under the concept of substance.

Once again, however, the concept of substance is intimately connected with the concepts of causality, action, and force: one cannot have causality, action, and force without a seat of such powers in a substance, and one cannot have a substance that does not manifest itself, in turn, in exercises of causality, action, and force. But, according to Kant's argument in the note to the first proposition, no properly phoronomical composition of motions can possibly give rise to a mechanical moving force. So it follows, as Kant suggests, that no aggregate of movables in the sense of the first explication of the Mechanics – no quantity of matter in the sense of this same explication – can possibly arise from properly phoronomical construction. For such an aggregate is supposed to represent precisely the quantity of *substance* and therefore must be a seat of causality, action, and force.

Kant returns to the relationship between the phoronomical consideration of matter as a mere moving point and the mechanical consideration of matter as both an active cause and a quantum of substance in the important footnote to the fourth proposition on the construction of the communication of motion. Kant explains (547n.) that in phoronomy "the quantity of motion … of the body was nothing but its velocity (for which reason it could be considered as a mere moving point)," while in mechanics:

> [W]here a body is considered in motion relative to another, with respect to which, through its motion, it has a *causal relation* (namely, that of moving the body itself) … another concept of the quantity of motion now comes into play, namely, not that which is thought merely with respect to space, and consists

only in the velocity, but rather than whereby the quantity of substance (as moving cause) must be brought into the calculation at the same time.[30]

Quantity of matter, as the quantity of substance, can only be estimated or manifest itself in the communication of motion, and it is for precisely this reason, as we shall see, that it can and must be conceived as a spatially extended aggregate of the movable rather than a property of mere movable points.

24 ESTIMATING QUANTITY OF MATTER

According to Kant's first proposition, quantity of matter can be estimated, in general, only by "the quantity of motion at a given velocity" (537): that is, in terms of momentum. But this does not mean that we simply consider the momentum of a given moving body and then divide by the velocity to obtain its mass. Rather, it is absolutely necessary, for Kant, that we consider possible exchanges of momentum in the communication of motion, whereby one moving body acts with its (mechanical) moving force on another and thereby transfers some of its quantity of motion. In other words, it is only in terms of *causal interactions* between bodies that their quantities of matter can be quantitatively estimated or determined. Kant emphasizes this point in the second explication introducing the first proposition, where he defines the concept of *mass* as a quantity of matter "in so far as all its parts in their motion are considered as acting (moving) simultaneously" (537), so that "a matter *acts in mass*, when all its parts, moved in the same direction, *simultaneously* exert their moving force externally." For Kant, the concept of quantity of motion – and therefore the estimation of quantity of matter – is entirely predicated on this definition of mass.

Kant returns to this point in his remark to the first proposition, where he proposes (539), "in order to avoid prolixity, [to] combine the explanation of the previous three statements into one remark." The context makes it clear that the three statements in question are the definition of quantity of matter as the aggregate of the movable in a determinate space presented in the first sentence of the second explication, the definition of mass (just considered) presented in the second sentence, and the first proposition itself concerning the estimation of quantity of matter by

[30] We encountered this important footnote in my discussion of the infinite divisibility of material substance in the Dynamics (section 13 above). The context there (see the paragraph to which note 79 of my chapter on the Dynamics is appended, together with the following paragraph) involved the relativity of motion and the equality of action and reaction. I shall return to these issues in my discussion of the fourth proposition of the Mechanics below.

quantity of motion. Kant's discussion of the first statement concerns the circumstance that quantity of matter is thereby a (partly) extensive rather than a (wholly) intensive magnitude. This circumstance, in turn, is intimately linked with the following discussion of the relationship between quantity of matter and quantity of substance in the second full paragraph of the remark, and I shall return to it in the following section. I shall concentrate here on Kant's discussion of the second two statements: the definition of mass and the estimation of quantity of matter by quantity of motion. I shall only be in a position to explain how all three statements are related to one another in the sequel.

The discussion of the definition of mass begins with the relationship between the action of bodies (in the mechanical sense) and that of fluids:

As to the concept of mass in this same explication, one cannot take it, as is customary, to be the same as that of quantity [of matter]. Fluid matters can act by their own inherent motion in mass [*in Masse*], but they can also act in flow [*im Flusse*]. In the so-called water hammer the impacting water acts in a mass, that is, with all its parts simultaneously; and precisely the same thing happens when water enclosed in a vessel exerts pressure by its weight on the scale on which it stands. (540)

When a fluid acts in mass, therefore, it acts precisely as a body in the mechanical sense – "with all its parts simultaneously." So we here have a case of the (mechanical) communication of motion, whereby a fluid mass acting as a whole either exerts moving force (transfers momentum) by impact or exerts pressure by its weight.[31] In such a case, Kant suggests, the fluid mass in question acts, in effect, as if it were a solid body (540): "[W]e also customarily understand by the word *mass* the quantity of matter in a *solid body* (the vessel in which a fluid is contained also stands proxy for its solidity)."[32]

[31] The water hammer consists of a liquid (usually water) hermetically sealed in a glass tube from which all the air has been removed (commonly by boiling the liquid so that the resulting steam forces the air out and then sealing the tube). When such a tube is inverted, the liquid, due to the lack of intervening air, then rushes immediately to the other end and impacts it quite forcefully, resulting in a loud noise and sometimes the breaking of the tube. Note that the two cases considered – impact and the pressure of weight – correspond to the two methods Kant suggests for determining quantity of matter in the anticipations of perception: see the paragraph to which note 29 above is appended.

[32] Recall that, according to the first number of the general remark to dynamics, a body in the dynamical (Kant says "physical") sense is "*a matter between determinate limits* (which therefore has a figure [*Figur*])" (525; see note 117 of my chapter on the Dynamics). Kant then explains in the second number that water droplets naturally assume a "spherical shape [*Kugelgestalt*]" (527), so that liquids, in this sense, can also form bodies in the dynamical sense. We now see that liquid masses may also count as bodies in the mechanical sense – namely, when they act "in mass" with all their parts simultaneously. In neither the dynamical nor the mechanical sense, therefore, must a "body," in general, be solid.

In more typical cases of fluid action, however, a quantity of (fluid) matter does not act in mass, in so far as the parts of the fluid exert their action successively rather than simultaneously. In such cases, Kant suggests, a determination of quantity of matter can only take place indirectly:

By contrast, the water of a millstream does not act on the paddle of an undershot waterwheel in mass, that is, with all its parts impinging on this paddle simultaneously, but only one after the other. Thus, if the quantity of matter, which is moved with a certain velocity and has moving force, is to be determined here, one must first look for the *water body*, that is, that quantity of matter which, if it acts in mass with a certain velocity (with its weight), can bring about the same effect. (540)

(There follows the sentence just quoted about solid bodies and water enclosed in vessels.)

A passage from the *Danziger Physik* concerning "the theory of the impact of fluid matters" clarifies what Kant has in mind. It begins by explaining that the force exerted by fluid matters in motion is proportional to the square of the velocity.[33] And so it follows, by Galileo's law of fall (according to which the square of the velocity attained in fall is proportional to the distance fallen), that we can then find an equivalent to this force in terms of weight (29, 144):

The force with which the water impacts on something is as the magnitude of its surface and the height it would have had to fall if it were to attain the same velocity. If the surface against which it impacts is 1 square foot and the height that it had to fall, in order to attain its velocity, is also one foot, then it exerts as much pressure as a cubic foot of water weighs, namely 64 pounds. (29, 144)[34]

In order to find the moving force exerted by a fluid acting in flow, then, we must first find an equivalent "fluid body" acting in mass – e.g., by weight.[35] Quantity of motion in general, and therefore quantity of matter,

[33] According to the *Danziger Physik* (29, 143): "At a doubled velocity of the water twice as much water with twice as much velocity impacts on the body it meets, and thus 4 times as much [force]." Kant defends the same view, with reference to experiments by Mariotte, in §146 of the essay on *Living Forces* (1, 168–69). As Kant there explains, this does not entail an endorsement of the Leibnizean measure of moving force but precisely the opposite.

[34] The value of 64 pounds for a cubic foot of water is taken from Wolff: compare the earlier discussion in the *Danziger Physik* (29, 138), together with the editor's note on p. 708.

[35] This is the context for the implicit contrast between undershot and overshot waterwheels in the passage from the remark to the first proposition. Whereas an undershot wheel is driven by water flowing beneath it impacting on its paddles with its flow, an overshot wheel gathers water flowing above it into buckets (vessels) which then produce rotation through their weight. It was well known in the eighteenth century that overshot wheels are more efficient than undershot – as was confirmed by experiments by John Smeaton in 1759. Both Bax and Ellington miss the reference to an "undershot waterwheel [*unterschlägigen Wasserrades*]" here. Bax (Kant 1891, p. 218) has

can only be determined or estimated via the communication of motion of bodies (in the mechanical sense) acting in mass.

As I have suggested, Kant appears to envision two possible methods for using (mechanical) communication of motion to determine quantity of matter: interactions involving the impact of bodies (in the mechanical sense) and the phenomenon of weight. The first depends on the conservation of momentum in all cases of impact (elastic as well as inelastic) and allows us to determine the ratio of the masses of the two colliding bodies from the ratio of their changes in velocity: using modern notation, $m_1/m_2 = -\Delta v_2/\Delta v_1$, where Δv_1 and Δv_2 represent the changes in velocities. The second method, employed using a balance, say, also depends on the conservation of momentum – now applied to a static situation where the downward mechanical moving force (pressure) exerted by one heavy body counterbalances the same mechanical force exerted by another.[36] The two methods are equally good in theory, and both depend on mechanical moving force (momentum) in just the way Kant suggests. In practice, however, Kant gives decided preference to the second method and thus to the determination of quantity of matter in terms of weight.[37]

Kant indicates this preference in his discussion of how quantity of motion is related to quantity of matter in the present remark to the first proposition:

Finally, there is something peculiar in the [first] proposition together with its appended note: according to the former, the quantity of matter must be estimated by the quantity of motion at a given velocity, but, according to the latter, the quantity of motion (of a body, for that of a point consists merely in the degree of velocity) must, at the same velocity, in turn be estimated by the quantity of the matter moved – which seems to revolve in a circle and to promise no determinate concept of either the one or the other. This alleged circle would be an actual one, if it were a reciprocal derivation of two identical concepts from one another. However, it contains only the explication of a concept, on the one hand, and

"the waterwheel that strikes it"; Ellington (Kant 1970, p. 100) has "the struck waterwheel." I am indebted to Peter McLaughlin for first setting me straight on this terminology.

[36] The two cases thus correspond to Kant's initial restriction in his discussion of the communication of motion to impact and pressure in the remark to the first explication: see the paragraph to which note 10 above is appended. Newton describes all cases of impact (elastic as well as inelastic) using his Third Law of Motion (from which the conservation of momentum is derived in Corollary 3) in the Scholium to the Laws of Motion (P425–27), and he then explains the dependence of all static forces involving machines (including the balance or lever) on this same law (P428–30).

[37] This point is well emphasized and discussed in considerable detail in Carrier (2001). My discussion here takes this paper as its starting point. Differences with Carrier will be noted as I proceed.

that of its application to experience, on the other. The quantity of the movable in space is the quantity of matter; but this quantity of matter (the aggregate of the movable) *manifests itself* in experience only by the quantity of motion at the same [*gleicher*] velocity (for example, by equilibrium [*Gleichgewicht*]). (540)

Thus, whereas quantity of matter is defined as the aggregate of the movable in a given space, it can only be estimated or determined ("manifest itself in experience") by interactions involving transfer of momentum. The concept of quantity of matter is prior to that of quantity of motion in the order of definition, while the latter is prior to the former in the order of empirical application.

The parenthetical illustration at the end of the passage is a reference to static equilibrium and thus to the determination of quantity of matter by means of the balance.[38] It is for precisely this reason, in fact, that Kant insists that the velocities involved in the comparison of quantities of motion here are the same or equal to one another.[39] Whereas this condition need not hold in the comparison of quantities of matter by impact, it does hold for the determination of quantity of matter by means of weight. For, according to Galileo's law, all bodies fall with the same acceleration and thus acquire equal velocities in equal times. Moreover, the practical equivalence of quantity of matter (mass) and weight rests on precisely this fundamental property of gravitational force – as is stated quite explicitly in the *Danziger Physik*:

All bodies have the same gravity [*sind gleich schwer*], but not the same weight [*Gewicht*]. Gravity means the velocity with which a body falls, and it is the same for all … But they have unequal forces at unequal quantities of matter, although equal velocities. Weight is the moving force, it [takes place] through the mere gravity. – Gravity is the striving to fall, the finite [*sic*] smallest part of fall. – Since the bodies receive the same velocity through gravity, the bodies relate to one another as the quantities of matter; by weighing we merely see where their moving force is. We say that they have the same quantity when their moving

[38] Compare a parallel passage in the essay on *Negative Magnitudes*, where, after illustrating a conflict of two moving forces in a state of rest by the balancing argument for attraction and repulsion, Kant continues (2, 199): "In precisely the same way, the weights on the two arms of the balance are at rest, if they are placed on the lever in accordance with the laws of equilibrium [*Gleichgewicht*]."

[39] According to the first proposition itself, the comparison in question is via "the quantity of motion at a given velocity" (537). But the proof of this proposition, using the same language as the present remark, clearly indicates that the relevant velocities are assumed to be equal (538): "Therefore, no other generally valid measure [of matter] remains except the quantity of its motion. But here the difference of motion, resting on the differing quantity of matters, can be given only when the velocity of the compared matters is assumed to be the same [*als gleich angenommen wird*]; hence, etc. "

force is the same. The moving forces relate to one another as the quantities, if the velocities are the same, and, since this is so, the moving forces relate to one another as the quantities. However, that the velocities are the same was first discovered by Galileo – the ancients did not know this – and it was confirmed by the air pump. If the velocities are the same, then weighing manifests the quantity of matter according to its proportion; – otherwise not, for weighing shows only the proportion of the moving forces. (29, 142)[40]

Thus, in modern notation, the weights (pressures) exerted by two masses m_1 and m_2 on the two (equal) arms of the balance are given by m_1a_1 and m_2a_2. Since, by Galileo's law, $a_1 = a_2$, it follows that these two weights are as the corresponding masses.[41] If the two bodies are in equilibrium, then (by the equality of action and reaction) the two weights – and therefore the two masses – must be equal.

One important reason underlying Kant's decided preference for employing weight rather than impact as a method for determining quantity of matter via the communication of motion, therefore, is simply that, in the former case, we have a ready method for determining the velocities involved by Galileo's law of fall. We can then use these known velocities, in the context of the conservation of momentum (or, equivalently, the equality of action and reaction), to determine the corresponding quantities of matter.[42] But there is also a deeper reason underlying Kant's preference – one that is fundamentally connected with the overall argument of the *Metaphysical Foundations* as a whole. Whereas both weighing by means of static equilibrium and the determination of quantity of matter by means of impact are terrestrial procedures, performed in experimental

[40] The importance of this passage is rightly emphasized in Carrier (2001, §3). Carrier also suggests that the remark about the air pump appears to be an allusion to Newton's discussion in the General Scholium (P939): "The only resistance which projectiles encounter in our air is from the air. With the air removed, as it is in Boyle's vacuum, resistance ceases, since a tenuous feather and solid gold fall with equal velocity in such a vacuum."

[41] As Carrier (2001, §3) also rightly emphasizes, this means that Kant's reference to equal "velocities" can, in this context, be read in terms of equal *infinitesimal* velocities or accelerations. Indeed, Kant made this clear in the note to the first proposition, where, in characterizing the moving force of a body, he says (539) that "it may be that the velocity of its motion is finite or infinitely small (mere striving towards motion)." In the general remark to mechanics (to which I shall return below) Kant characterizes such an infinitely small velocity as a "moment of acceleration" (551).

[42] This does not mean that there is any inherent impossibility in independently determining the relevant changes of velocity in cases of impact, but it is in general no easy or trivial task from a practical point of view. It is important to appreciate, in this connection, that when Newton confirms the law of equality of action and reaction by precise experiments on impact in the Scholium to the Laws of Motion (note 36 above), he uses balls colliding together as a result of *pendular* motion – and he is thereby able to determine the velocities before and after impact from the chords of the arcs traversed in accordance with Galileo's law of fall.

settings on the surface of the earth, Kant's Copernican conception of space and motion requires that the theory of "mechanical" motion be systematically extended from the terrestrial to the celestial realm. And the crucial point, from this point of view, is that only the determination of quantity of matter by means of weight, in accordance with Newton's procedure in the *Principia*, allows us to perform the necessary extrapolation.

As explained in section 15 above, the remark to the fifth proposition of the Dynamics alludes to the Newtonian determination of the masses of the primary bodies in the solar system in Proposition 8 of Book 3. The argument of this Proposition is that, although the gravitational attraction on any body near the surface of the earth is compounded out of the gravitational attractions of all the points contained within the earth's volume, it still behaves (in the case of an idealized spherical earth) as if it were solely directed towards the earth's center. And this provides further support, in particular, for the moon test of Proposition 3. In the corollaries to Proposition 8 Newton then determines the quantities of matter of the primary bodies in the solar system (the sun, Jupiter, Saturn, and the earth) from the attractive forces they exert on their satellites. For, according to Proposition 7, these forces (accelerative gravities) are proportional, at a given distance, to precisely the corresponding quantities of matter. Thus, once we have identified the downward pressure exerted by terrestrial gravity with the attractive force of universal gravitation by the moon test, we are then in a position to equate quantity of matter (weight) determined by static equilibrium in the terrestrial realm with quantity of matter (mass) determined by universal gravitation in the celestial realm. It is precisely this universal extension of the initially terrestrial concept of weight that constitutes the crucial step in extending Kant's Copernican conception of space and motion from an initial relative space centered on the earth to a more adequate relative space defined with respect to the center of mass of the solar system.[43]

As explained in section 18 above, the second remark to the seventh proposition of the Dynamics returns to this Newtonian procedure and connects it with Kant's evolving argument even more explicitly. Kant is here

[43] See note 121 of my chapter on the Dynamics, together with the paragraph to which it is appended. Galileo's law of fall only holds approximately from the point of view of universal gravitation; for, according to the inverse-square law, the acceleration of gravity is not exactly constant at different heights (and, moreover, the lines of fall are not parallel to one another but are directed towards the earth's center). In the celestial context, more generally, the weights of bodies, unlike their masses, are now seen to depend on both their distances from a given gravitating (primary) body and this gravitating body itself: weight, unlike mass, is thus an essentially relational property of bodies.

concerned primarily with Propositions 6 and 7 of Book 3. The first establishes that the weight of any attracted body (i.e., the gravitational force acting on that body) is always directly proportional at a given distance to the *attracted* body's mass, whereas the second establishes, correlatively, that the acceleration of the attracted body is also directly proportional at a given distance to the *attracting* body's mass.[44] The second conclusion, in turn, then figures essentially in the corollaries to Proposition 8, where the masses of the primary bodies in the solar system are determined from the accelerations (accelerative gravities) of their satellites.[45] Kant wants to show that this latter kind of mass-dependence, in particular, presupposes the universality and immediacy of gravitational attraction, and he illustrates his point by a central case of the corollaries to Proposition 8 (515): "[T]he attractive forces of two planets, e.g., of Jupiter and Saturn, manifested at equal distances of their satellites (whose mass is unknown), are proportional to the quantity of matter of these heavenly bodies." Kant claims, finally, that this crucial property of gravitational attraction ultimately rests on the closely related fact, central to his own dynamical theory of matter, that (516) the fundamental force of attraction "is a penetrating force, and for this reason alone it is always proportional to the quantity of matter."

It is no wonder, then, that Kant returns to these considerations in the present remark to the first proposition of the Mechanics. Towards the end of this remark Kant argues that the quantity of matter represents the quantity of substance and that the resulting quantum of substance is given by the aggregate of movables in the matter in question rather than the magnitude of any dynamical force it might exert (whether attractive or repulsive). It thereby becomes clear (541) that "the quantity of substance in a matter must only be estimated mechanically, i.e., by the quantity of

[44] See notes 181, 182, and 192 of my chapter on the Dynamics, together with the paragraphs to which they are appended. Proposition 6 is thus a generalized (and corrected) version of Galileo's law of fall, whereas Proposition 7 is a distinctively Newtonian conclusion depending on both this generalized Galilean law and the equality of action and reaction directly applied to universal gravitation. In the terminology of the notes just cited, the first expresses the equivalence of inertial mass with passive gravitational mass, whereas the second expresses the equivalence of inertial mass with active gravitational mass.

[45] This quantity is thus what Newton calls the "accelerative quantity of centripetal force," which "is proportional to the velocity it generates in a given time," in Definition 6 (P407):

> [An] example is the force that produces gravity, which is greater in valleys and less on the peaks of high mountains and still less (as will be made clear below) at greater distances from the body of the earth, but which is everywhere the same at equal distances, because it equally accelerates all falling bodies (heavy or light, great or small), provided that the resistance of the air is removed.

its inherent motion, and not dynamically, by the magnitude of the originally moving forces."

I shall consider the connection between quantity of matter and quantity of substance in the next section, but the important point here is that Kant immediately goes on to qualify his conclusion with respect to the fundamental force of universal attraction:

> Nevertheless, *original attraction*, as the cause of universal gravitation, can still yield a measure of the quantity of matter, and of its substance (as actually happens in the comparison of matters by weighing), even though a dynamical measure – namely attractive force – seems here to be the basis, rather than the attracting matter's own inherent motion. But since, in the case of this force, the action of a matter with all its parts is exerted immediately on all parts of another, and hence (at equal distances) is obviously proportional to the aggregate of the parts, the attracting body also thereby imparts to itself a velocity of its own inherent motion (by the resistance of the attracted body), which, in like external circumstances, is exactly proportional to the aggregate of its parts; so the estimation here is still in fact mechanical, although only indirectly so. (541)

What is at issue, therefore, is the procedure discussed in the second remark to the seventh proposition of the Dynamics for determining the quantity of matter of a body (here a celestial body) by means of the attractive force it exerts on others – more precisely, via the acceleration it produces in others. Kant's central claim, just as in the earlier context, is that the acceleration in question is proportional (at a given distance) to the quantity of matter of the body exerting the force, because universal gravitation is a penetrating force acting immediately from each part of one body on all parts of another. But Kant now explicitly links the dynamical procedure based on universal gravitation with the mechanical determination of quantity of matter by weight, and, more generally, he claims that this dynamical procedure, too, is essentially mechanical – "although only indirectly so."

It is important to see, however, that the way in which the determination of quantity of matter by universal gravitation is mechanical is fundamentally different from the way in which the ordinary (terrestrial) determination of quantity of matter by weighing is. In the latter case the moving force in question is statical pressure (exerted by a body on the arm of a balance, say, or against the elastic tension of a spring), and conservation of momentum (or, equivalently, the equality of action and reaction) is applied to a system of such pressures in static equilibrium (two bodies exerting equal downward pressures on the two equal arms of a balance, say, or an equilibrium between the downward pressure of a body and the elastic tension of a spring). In this case, moreover, there is as yet no question of locating the cause of the downward pressure of gravity – or, more generally, the corresponding downward acceleration governed by Galileo's law of fall – in a dynamical force of attraction exerted by the earth. There is similarly no question of applying conservation of momentum (the equality of action and reaction) to the dynamical interactions between the gravitating bodies near the surface of the earth and the earth itself. It is only when we proceed to subsume terrestrial gravity under universal gravitation, in accordance with Newton's moon test, that these questions first arise. And it is only at this point, therefore, that a mechanical exchange of momentum between bodies interacting by the dynamical force of universal gravitation (including the earth and all the bodies it attracts) comes into play.[46]

Returning to Kant's passage on universal gravitation (541), let us consider the determination of the quantity of matter of Jupiter from the acceleration produced by Jupiter's force of attraction on one of its moons. Inferring the mass of Jupiter from the moon's acceleration is a dynamical procedure in Kant's sense, because no motion of Jupiter itself is yet being considered.[47] What Kant is pointing out in the remark to the first proposition

[46] Here is where I differ from Carrier, who argues (2001, §6) that the discussion of estimating quantity of matter in the Mechanics is limited to the terrestrial procedure of weighing and, accordingly, criticizes earlier work of mine for putting exclusive emphasis on celestial procedures using universal gravitation. But Carrier (§§2, 4) also reads the present passage from the remark to the first proposition as showing how conservation of momentum (the equality of action and reaction) enters into the terrestrial procedure of weighing via the gravitational interaction between falling bodies and the earth. This is a mistake, I believe, for these procedures do not yet take into account the motion of the earth but rely only on conservation of momentum applied to the pressures exerted in static equilibrium. When we do take into account the motion of the earth, moreover, we have already made the key extrapolation from terrestrial to celestial estimations of mass. Whereas Carrier rightly criticizes earlier work of mine for placing exclusive emphasis on celestial estimations and for neglecting the central importance of weighing in the Mechanics, he is guilty, in my view, of neglecting the equally important *transition* that Kant suggests between terrestrial and celestial procedures.

[47] The alert reader will have noticed that I am using the modification "dynamical" in this paragraph and the preceding one in two different senses or contexts: dynamical versus statical, on the one side, dynamical versus mechanical, on the other. The traditional contrast between statical and dynamical situations involves the difference between cases of equilibrium with no actual motions of the bodies in question (as in a balance of weights) and cases of equal and opposite momenta between actually moving bodies (as in impact or gravitational interaction). The Kantian contrast between dynamical and mechanical refers to the difference between forces exerted by one body on another body independently of the first body's state of motion (as in the production of gravitational acceleration in one body by the gravitational force exerted on it by another) and forces (momenta) transferred from one body to another body in virtue of the (changing) states of motion of both. The connection between these two contexts, in the present discussion, is that the (statical) pressure exerted by weight was traditionally taken to be a paradigmatic "mechanical" moving force (depending on the *infinitesimal* downward velocity of the body exerting this pressure), and, when we subsume this pressure under the theory of universal gravitation, it is now seen to be the effect of a "dynamical" force (in both senses of the

is that Jupiter's force of attraction, by the resistance of its moon to this force and the equality of action and reaction, produces a corresponding acceleration – and therefore change of momentum – in Jupiter itself. *But this change of momentum, in the given circumstances, is also proportional to Jupiter's mass*. Just as the moon falls towards Jupiter, Jupiter falls towards its moon – and Jupiter's "weight" towards this moon (like all gravitational forces) is, at a given distance, directly proportional to Jupiter's mass.[48]

A precisely parallel result holds if we consider the acceleration of *terrestrial* gravity from the point of view of universal gravitation. Just as a falling body near the surface of the earth accelerates towards the earth (and the earth's moon, according to the moon test, similarly accelerates towards the earth), the earth experiences a corresponding acceleration towards the falling body (and towards the moon). We can infer the mass of the earth from the acceleration of terrestrial gravity (or, in accordance with the moon test, from the acceleration of the moon), but it is also true that the earth has a "weight" towards any falling body (including the moon). This "weight" (like all gravitational forces) is, at a given distance, directly proportional to the earth's mass. So determining the quantity of matter of an attracting body by the acceleration it produces in a body thereby attracted is *mechanical* in Kant's sense, because it ultimately rests, like all mechanical comparisons, on an exchange of momentum or quantity of motion between the two interacting bodies. The determination is only "indirectly" mechanical, however, because the change of momentum of the attracting body itself, despite the fact that it is indeed proportional to this same body's quantity of matter, is not what is actually measured.[49]

term). We make a parallel transition, at the same time, from an application of conservation of momentum (the equality of action and reaction) to two bodies in contact to an application of this same principle to bodies exchanging momentum at a distance.

[48] The gravitational force acting on Jupiter is given by $m_J a_J = Gm_J m_M/r^2$, where m_M is the mass of the moon in question. Holding m_M and r constant, then, it follows that $m_J a_J$ is proportional to m_J. This is precisely analogous, in the context of universal gravitation, to the way in which the gravitational force or weight of a falling body near the earth's surface is proportional to its mass. A tricky issue of interpretation/translation arises in Kant's passage (541), according to which the attracting body "imparts to itself a velocity of its own inherent motion … which, in like external circumstances, is exactly proportional to the aggregate of its parts [i.e., its mass – MF]." Grammatically "which" could refer to either velocity or (quantity of) motion, but only the latter makes conceptual sense (since the attracting body's *velocity* is certainly not proportional to its mass). Both Ellington (Kant 1970, pp. 101–2) and Carrier (2001, p. 123) read "which" as referring to the velocity (incorrectly, in my view), while Bax (Kant 1891, 219) preserves the grammatical structure (correctly, in my view) by leaving "which" ambiguous.

[49] It is here that the alternative possibility apparently ignored in the proof of the first proposition is explicitly incorporated into Kant's argument: see notes 19 and 20 above, together with the paragraphs to which they are appended. In the cases under consideration, one should observe, there is no ready way to determine the change of momentum experienced by the *attracting* body in

The general lesson of this part of the remark to the first proposition, therefore, is that terrestrial determination of quantity of matter by static equilibrium is to be placed in the larger framework of universal gravitation, which provides us, in particular, with a method for extending the terrestrial concept of quantity of matter to the celestial realm. Indeed, the passage from the *Danziger Physik* quoted above, which makes the dependence of terrestrial weighing on Galileo's law of fall explicit, continues by invoking the larger framework provided by universal gravitation in the context of Kant's Copernican conception of space and motion:

All things are moved around with the earth. All rest and motion is thus merely relative with respect to another body ... In the case of other planets gravity is different in accordance with the magnitude of the planet and the distance from the central point. On the sun, for example, a human being would not be able to stand, for its gravity would slam him to the ground. Universal gravity is the great band that binds together all heavenly bodies. (29, 143)[50]

Thus, it is precisely by embedding the traditional statical concept of weight within the framework of universal gravitation that we are finally in a position to provide a generally valid measure of its quantity applicable to all matter as such – wherever it may be found in the universe. We do this, moreover, by the at the time controversial extrapolation of conservation of momentum (Newton's Third Law of Motion) from situations of static equilibrium in contact to dynamical equilibrium at a distance (see note 47 above). But we are then in a position, as explained in section 15 above, to offer a powerful argument against the then dominant mechanical natural philosophy on behalf of this extrapolation.[51]

the context of the *Principia*. We know neither the velocity (i.e., acceleration) produced in Jupiter by the resistance of its moon nor that produced in the earth by the resistance of a falling body (or of the earth's moon), and we know neither the mass of Jupiter's moon (as Kant points out in the passage from the second remark to the seventh proposition of the Dynamics) nor that of the earth's moon – at least by the interactions under consideration.

[50] Carrier (2001, §3) does not quote this passage (compare note 40 above), but he does (p. 129) quote an analogous passage from the *Opus postumum* (21, 408): "[Quantity of matter] can only be measured by weighing, i.e., by the compression of an elastic matter (e.g., a steel spring), or, and primarily, by the balance (with equally long lever-arms). The weight indicating this quantity of matter is a pressure that it exerts in virtue of being attracted by the earth's body as a heavenly body." Here the transition from the terrestrial to the celestial realm is also evident.

[51] See note 126 of my chapter on the Dynamics, together with the paragraph to which it is appended. Carrier (2001, §6) is correct, therefore, in stressing that the primary focus of the Mechanics is on terrestrial phenomena – the very phenomena involving pressure and impact to which the mechanical natural philosophy gives precedence. But Kant, on my reading, is concerned to place these same phenomena in the wider (celestial) context of universal gravitation emphasized in both the second part of the Dynamics and (as we shall see) the following Phenomenology – in order to show, at the same time, how the theory of "mechanical" motion must also be placed in this wider context.

I have suggested that Kant's treatment of the estimation of quantity of matter is inspired by Newton's discussion in the *Principia*. Newton's first definition characterizes quantity of matter as the product of density and volume and then links the concept of density to the possibility of compression. How is this quantity of matter to be determined? Newton continues (P404): "It can always be known from a body's weight, for – by making very accurate experiments with pendulums – I have found it to be proportional to the weight, as will be shown below." The experiments in question are described in Proposition 6 of Book 3, where Newton begins (P806): "Others have long since observed that the falling of all heavy bodies toward the earth (at least on making an adjustment for the inequality of the retardation that arises from the very slight resistance of the air) takes place in equal times, and it is possible to discern that equality of times, to a very high degree of accuracy, by using pendulums." Newton begins, therefore, from Galileo's law of fall, from which it follows that the weights of falling bodies are as their masses. And this implies that we can uniformly correlate the traditional statical concept of weight or pressure with the new mechanical concept of mass or "force of inertia" that Newton first introduces in Definition 3 and further articulates in the Laws of Motion.[52] The experiments described in Proposition 6 of Book 3 confirm this fundamental consequence of Galileo's law in the course of arguing that the static quantity of weight or pressure is always proportional to the new mechanical quantity of mass.[53]

Newton thus begins, as Kant does, by estimating quantity of matter in the context of Galileo's law by means of weight. The remainder of Proposition 6, as we have seen, proceeds by extending this result from heavy bodies near the surface of the earth to the motions of the heavens. Newton first invokes the moon test, which shows that the moon "falls" towards the earth with the very same acceleration of gravity – so that the weight of the moon towards the earth is also proportional to its mass. He

[52] For Newton's notion of mass or force of inertia see again note 129 of my chapter on the Dynamics. According to the first sentence of Newton's discussion (P404): "This force is always proportional to the body and does not differ in any way from the inertia of the mass except in the manner in which it is conceived." Immediately before the sentence from the first definition quoted in the main text above Newton explains that he means "this quantity [i.e., quantity of matter – MF] whenever I use the term 'body' or 'mass' in the following pages" (P404). So Newton intends that mass or quantity of matter should be that quantity operative in impetus, the resistance of inertia, and the communication of motion – the third, mechanical concept of quantity of matter that I have distinguished in my chapter on the Dynamics.

[53] Newton's pendulum experiments also involve important subtleties (to which my attention was directed by George E. Smith) that reveal important differences between Newton's treatment and Kant's. I shall return to these subtleties below.

next invokes Kepler's third or harmonic law, which shows that the accelerations of the moons of Jupiter, say, are such that "at equal distances from Jupiter their accelerative gravities would come out equal … just as happens with heavy bodies on this earth of ours" (P807). Moreover, since this also holds for the motions of satellites relative to any primary body in the solar system (including the sun), it follows that in all such cases of gravitational attraction the weight of an attracted body is directly proportional to its mass. Finally, Newton buttresses this conclusion, in the case of the satellite systems of Jupiter and Saturn, by arguing that any significant deviations from equal accelerations towards the sun for any of the bodies in such a system (including the planets in question) would result in detectable irregularities in their motions in accordance with Corollary 6 to the Laws of Motion (P807–8). It is by no means surprising, therefore, that Kant, writing against the background of just this Newtonian discussion, should similarly proceed by starting with the terrestrial estimation of mass or quantity of matter by means of weight – and then embedding it within the theory of universal gravitation so as to effect a far-reaching extrapolation of the concept of quantity of matter into the celestial realm.

Both Newton and Kant begin from what I have called a dynamical concept of quantity of matter linked to the possibility of compression, move to the traditional statical concept of weight, connect this concept of weight with the new mechanical concept of mass or "force of inertia" via Galileo's law of fall, and finally extrapolate this last concept of mass into the celestial realm in the context of universal gravitation. Of particular importance, in both cases, is the transition from the traditional statical concept of weight to the new mechanical concept of mass. For precisely this transition underlies the progression from statics to dynamics (in the sense of note 47 above) that is essential to the new mathematical science of motion. How, exactly, is this transition supposed to be negotiated, and how, in particular, is it supposed to be connected, in turn, with the traditional concept of quantity of matter as the "bulk" or "amount" of matter in a given space?[54]

[54] I am indebted to George E. Smith for emphasizing the importance of three different (but related) concepts at play here: weight (*pondus*), bulk (*moles*), and mass (*masse*). The last concept is Newton's distinctive contribution, and it is defined (implicitly) by the Laws of Motion (especially the Second Law). As we shall see, Newton introduces a version of the second concept in the first definition of the *Principia* and immediately links it to the first concept via his pendulum experiments. The link is forged precisely via the third concept in the context of the Second Law of Motion. What I have called the dynamical concept of quantity of matter linked to the possibility of compression corresponds to the second concept, whereas my (and Kant's) mechanical concept corresponds to the third. NB: Newton uses the term *quantitas materiae* in his first definition, which corresponds to his "amount of matter [*copia materiae*]" in the crucial discussion

Newton begins, as I have suggested, with the traditional concept of "bulk" or "amount" of matter, defined as the product of volume and density, and he immediately links it to the concept of weight by reference to his "very accurate experiments with pendulums" (P404). These experiments are described in Proposition 6 of Book 3, where Newton continues the sentence already quoted above (P806) as follows:

> I have tested this with gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I got two wooden boxes, round and equal. I filled one of them with wood, and I suspended the same weight of gold (as exactly as I could) in the center of oscillation of the other. The boxes, hanging by equal eleven-foot cords, made pendulums exactly like each other with respect to their weight, shape, and air resistance. Thus, when placed close to each other [and set into vibration], they kept swinging back and forth together with equal oscillations, for a very long time. Accordingly, the amount of matter [*copia materiae*] in the gold (by book 2, prop. 24, corols. 1 and 6) was to the amount of matter in the wood as the action of the motive force upon all the gold to the action of the motive force upon all the [added] wood – that is, as the weight of one to the weight of the other. And it was so for the rest of the materials. In these experiments, in bodies of the same weight, a difference of matter that would be even less than a thousandth part of the whole could have been clearly noticed. (P806–7)

This passage, as I have also suggested (note 53 above), involves some important subtleties.

The most important point is that Newton is not simply appealing to Galileo's law of fall in a vacuum. On the contrary, as is indicated by his reference to air resistance, he is not neglecting the medium (here the air) within which the pendulum experiments are conducted. Indeed, when we turn to the relevant part of Book 2, we find that Proposition 24 initiates §6 ("Concerning the motion of simple pendulums and the resistance to them"), which continues the discussion of hydrostatics from §5 ("The density and compression of fluids, and hydrostatics").[55] In particular,

below (P806–7) of the relationship between quantity of matter and weight. The term *moles*, however, was introduced in the (mechanical) tradition preceding Newton (Descartes, Huygens, and Leibniz) and used as a measure of the *resistance* to changes in motion (especially in collisions). The idea was that precisely this "bulk" or "amount" of matter thus played the role for which Newton later introduced the concept of mass. Newton's contribution was to introduce this latter concept quite generally via his three Laws of Motion governing all possible interactions (not simply collisions) and, at the same time, to show how it could be used to forge an empirically well-grounded connection between the two earlier concepts of quantity of matter and weight in the context of his theory of universal gravitation.

[55] Thus, in particular, Proposition 24 (initiating Section 6) immediately follows Proposition 23 (concluding Section 5). I have remarked on the significance of Proposition 23 for both Newton and Kant several times above: see notes 47, 49, and 103 of my chapter on the Dynamics, together with the paragraphs to which they are appended.

Proposition 24 presupposes the discussion of "relative" or "specific" weights ("gravities") in Corollaries 4–6 of Proposition 20. Newton is thus presupposing the well-known hydrostatical method (due to Archimedes) for directly determining the differences in specific density among various kinds of matter (wood, silver, and gold, for example) by immersing samples of a given volume in the same fluid (e.g., water): the differences in density (the different specific weights) are proportional to the different volumes of the fluid thereby displaced. These notions are then deployed in Corollary 6 to Proposition 24 in extending the already known result for motion in a vacuum (Corollary 1) to motion in air now considered as a buoyant medium (heavy elastic fluid).[56]

The proof of Proposition 24, as Newton makes clear, entirely depends on the Second Law of Motion. Given that the motive force acting on the pendulum bob is given by its weight (see note 56 above), and that the times during which a given force acts are proportional to the velocity generated by the force and the body's mass (Second Law), it follows that the masses of two bobs oscillating in pendulums of the same length are proportional to their weights when their times of oscillation are equal. This reasoning depends on equating mass in the sense of the Second Law with quantity of matter in the sense of the first definition. Newton stipulates, in Definition 2, that momentum is the product of velocity and quantity of matter as already defined in Definition 1; he then asserts, in his comments on Definition 3, that the "innate force of matter" or "force of inertia" is "always proportional to the [quantity of matter of the – MF] body and does not differ in any way from the inertia of the mass" (P404; compare note 52 above). According to the Second Law, therefore, "[a] change of [quantity of] motion is proportional to the motive force impressed"

[56] The statement of Proposition 24 (P700) is explicitly limited to a vacuum and so are Corollaries 1–5. In Corollary 6, however, Newton relaxes this restriction (P701):

> But in a nonresisting medium also, the quantity of matter in the bob of a simple pendulum is as the relative weight and the square of the time directly and length of the pendulum inversely. For the relative weight is the motive force of a body in any heavy medium, as I have explained above [in the Corollaries to Proposition 20], and thus fulfills the same function in such a nonresisting medium as absolute weight does in a vacuum.

Newton is still not considering air resistance, however, but only the effect of the buoyancy of the air on the "motive force" (i.e., weight or gravity) of the bob. Newton goes on explicitly to consider the resistance of the medium in the following propositions, culminating in a (not entirely successful) investigation of the resistance of air (and other fluid media) in the concluding General Scholium.

(P416), or, as we would now put it, force is determined by the product of mass and acceleration (change of velocity in a given time).[57]

In the end, therefore, Newton's reasoning depends on embedding Galileo's law of fall within the more general context of his three Laws of Motion (especially the Second Law) – just as in the discussion where I originally made the connection between mass and the statical concept of weight (see the paragraph to which note 41 above is appended). By relaxing the restriction to motion in a vacuum, however, Newton accomplishes significantly more. For, in the first place, he also considers forces acting on the pendulum bob other than weight or gravity: the buoyant force of the air (due to the pressure of its weight) and its resistance to the bob. Newton thereby allows for applications of the Second Law to forces other than weight – and, in particular, for precise tests of the quantitative relation between weight (measured statically by a balance) and mass (determined dynamically via the Second Law) in a variety of experimental situations.[58] In the second place, however, by embedding his consideration of weight within an explicitly hydrostatical context, Newton indicates that the concept of quantity of matter with which he began also has a precise (measurable) empirical meaning (at least in the terrestrial realm) in terms of the Archimedean method for determining relative densities of specifically different types of matter (e.g., silver and gold) by the volumes displaced in a fluid medium. So defining quantity of matter as the product of density and volume is thus in no way circular. Nor does it assume any particular hypothesis about the inner structure of matter: for example, that specifically different types of matter can differ in their

[57] Newton emphasizes the dependence of his reasoning on the Second Law at the very beginning of his proof of Proposition 24 (P700):

> For the velocity that a given force can generate in a given time in a given quantity of matter is as the force and the time directly and the matter inversely. The greater the force, or the greater the time, or the less the matter, the greater the velocity that will be generated. This is manifest from the second law of motion.

[58] As pointed out in note 56 above, Newton considers only the buoyant force of the air in Corollary 6 of Proposition 24. Nevertheless, he goes on to consider air resistance in the following propositions and General Scholium, and the wording of Proposition 6 of Book 3 explicitly mentions such resistance. Indeed, at the very end of the General Scholium to §6 of Book 2 (P722–23), Newton develops a remarkable empirical argument (again using pendulums) against the Cartesian opinion that "a certain aetherial medium" also exerts resistance to the motions of bodies by penetrating into them through their pores. Newton finds that any such resistance would be extremely small in comparison with the resistance (of the air) on their external surfaces. And this puts him in a position to argue (as he also appears to suggest in Proposition 6 of Book 3) that, since the force of air resistance (whatever it may be) acts equally on both the gold and wood filled bobs, it follows from the observed equality of their periods of oscillation that their masses must be equal.

quantities of matter at the same volume only by containing different proportions of a single uniform (absolutely dense) type of matter and empty space within this volume.

I explained in my chapter on the Dynamics that Kant's consideration of the concept of quantity of matter in the anticipations of perception of the first *Critique* is aimed precisely to counter this last hypothesis – which, according to Kant, has been embraced by "[a]lmost all investigators of nature" (A173/B215; see note 39 of my chapter on the Dynamics, together with the paragraph to which it is appended). But this hypothesis, Kant continues, "for which they have no basis in experience," is thus "merely metaphysical" (A173/B215). Kant counters with the alternative hypothesis of a purely intensive filling of space, such that, in spaces "completely filled by different matters … each reality of the same quality still has its degree (of resistance or weight) which, without diminution of the extensive magnitude or aggregate, can be smaller to infinity, before it is transformed into the empty and vanishes" (A174/B216). He concludes:

> My intention is here not at all to assert that this is actually how it is with differences of matters in accordance with their specific gravities, but rather only to show from a principle of pure understanding that the nature of our perceptions makes such a mode of explanation possible, and that one falsely assumes that the real in the appearance is equal in its degree and differs only with respect to its aggregation and extensive magnitude – and even to assert this, allegedly, on the basis of an a priori principle of the understanding. (A174–75/B216)

The wording of this passage suggests that Kant is proceeding from the same empirical phenomena as Newton: observed differences of weight, including the well-known phenomena of differing specific gravities in specifically different types of matter, together with observed differences in "moment[s] of resistance to other moved matters" (A173/B215).

The problem is then to correlate the statical concept of weight with the new mechanical concept of mass, and the concept of quantity of matter is supposed to function as the bridge between them. Moreover, Kant begins his discussion of quantity of matter in the Mechanics of the *Metaphysical Foundations* with a definition very similar to Newton's in characterizing quantity of matter as "the aggregate of the movable in a determinate space" (537). But precisely because this characterization, in Kant's view, does not yield a universally applicable measure of quantity of matter for specifically different types of matter, he proceeds in the first proposition to claim that quantity of matter can only be validly estimated, in general, by the quantity of motion at a given velocity. Finally, since Kant has already defined quantity of motion, following Newton, as the product of quantity of matter and velocity, Kant's first proposition, in this context, does much the same work as Newton's analogous use of the Second Law of Motion: it serves to introduce quantity of matter (defined in terms of density and volume) into the mechanical theory of the communication of motion – and thereby to link it to the new concept of mass.

Newton builds a bridge between quantity of matter and mass via his pendulum experiments and Second Law of Motion. Kant, by contrast, has not yet formulated his three Laws of Mechanics, and he does not ever explicitly formulate the Second Law of Motion in any case (see note 15 above, together with the paragraph to which it is appended). In addition, Kant's First Law of Mechanics (second proposition) is a quantitative conservation law for the total quantity of matter, which serves as a basis for the quantitative concept of matter as a magnitude prior to his formulation of the laws of inertia and the equality of action and reaction. As I have suggested, he thereby embeds the definitions of quantity of matter, quantity of motion, and mass within a broader metaphysical conception involving a general system of relationships among the concepts of substance, action, and force (see notes 22 and 23 above, together with the paragraph to which they are appended). Causal actions or forces can only be exerted by substances, and material substances, for Kant, must both be spatially extended and have purely intensive specific densities. Mechanical causal interactions involving the communication of motion, therefore, must take place between spatially extended bodies comprising continuous aggregates of the movable with definite volumes and determinate densities estimated by "the quantity of motion at a given velocity" (537; first proposition). Indeed, the fundamental laws governing the communication of motion, for Kant, are themselves dependent on precisely this conception of material substance and the resulting characterization of quantity of matter.

25 MATERIAL SUBSTANCE AND THE CONSERVATION OF MATTER

At the end of the preceding section I explained that a significant conceptual problem, for both Kant and Newton, is to articulate a connection between the statical concept of weight and the dynamical (for Kant "mechanical") concept of mass by means of the traditional concept of quantity of matter as the "amount," "aggregate," or "bulk" of matter in a given space. One important dimension of the problem is that, whereas both quantity of matter and mass are supposed to be intrinsic properties of matter possessed entirely independently of its particular spatio-temporal location, weight is an explicitly relational property limited to terrestrial bodies (and fluids) in the vicinity of the earth. To be sure, Newton's argument for universal gravitation embeds the relational property of weight into the more general context of his Laws of Motion, thereby indicating its essential connection with mass. But this argument, as I have explained, depends on identifying mass in the sense of the Laws of Motion with quantity of matter in the sense of Definition 1. Although this identification is empirically well supported by careful pendulum experiments, there remains a significant conceptual tension between quantity of matter as traditionally conceived (as an intrinsic property of matter having nothing to do with either its spatio-temporal position or its state of motion) and the new dynamical concept of mass (characterized, for Newton, by precisely his three Laws of Motion).

This tension comes out especially clearly if we conceive quantity of matter in accordance with the then standard practice of characterizing its magnitude by the ratio of absolutely dense matter in a given space to the total volume it occupies – containing, in addition to the absolutely dense matter, a larger or smaller amount of absolutely empty space. What is supposed to be the connection, in the end, between this ratio and the empirically determined magnitudes associated with either the terrestrial concept of weight or "the moment of resistance to other moved matters" (A173/B215)? Of course neither Newton nor Kant adopts the above characterization of quantity of matter. Newton appeals to an empirically determined notion of density (related to weight) in Definition 1 (see again the paragraph to which note 58 above is appended), and Kant explicitly rejects the entire conception underlying the standard practice of combining "the absolutely full with the absolutely empty" (532). Yet the conceptual tension remains, and it can only be fully resolved, in Kant's eyes, by embedding his opposing conception of quantity of matter as the aggregate of the movables in a given space into the more general metaphysical context of his critical account of the category of substance – including, in this case, the more specific concept of *material substance*.[59]

[59] In Query 31 of the *Opticks* Newton famously proposes (as a conjecture) a version of hard-body atomism on which quantity of matter, at the most fundamental level, would be computed from the volume of absolutely dense matter in a given space (compare note 220 of my chapter on the Dynamics, together with the paragraph to which it is appended and the preceding paragraph). It would then remain an open question how this fundamental quantity is related to the various empirical measures to which Newton appeals. Kant, on my reading, is attempting to avoid this question entirely on behalf of a more phenomenological theory of matter and its quantity that is *conceptually* tied, from the beginning, with precisely these empirical measures. We shall see how this plays out as my analysis proceeds.

Kant begins his discussion in the remark to the first proposition of the Mechanics by implicitly contrasting the conception of material substance in question with his own earlier conception in the *Physical Monadology*:

That the quantity of matter can only be thought as the aggregate of movables (external to one another), as the definition expresses it, is a remarkable and fundamental proposition of general mechanics. For it is thereby indicated that matter has no other quantity than that consisting in the *aggregate* of the manifold [parts] *external to one another*, and hence has no *degree* of moving force at a given velocity that would be independent of this aggregate and could be considered merely as intensive magnitude – which would be the case, however, if matter consisted of monads, whose reality in every relation must have a degree that can be larger or smaller, without depending on an aggregate of parts external to one another. (539–40)

The point of Kant's theory of physical monads was to allow absolutely simple substances to fill the space they occupy – and to interact with one another by physical forces – without sacrificing their absolute simplicity to the necessary (infinite) divisibility of spatial extension. By contrast, his present conception of material substance, as explained in the fourth proposition of the Dynamics, implies that such substances (physical bodies) must be precisely as composite as space itself. In particular, the quantity of matter in a given space must have an essentially extensive aspect, whereby the quantity of matter of the whole is the sum of the quantities of matter of its spatially extended parts.[60]

At the end of this remark Kant makes the connection between the concept of quantity of matter and the new (critical) conception of material substance fully explicit:

It is to be noted, further, that the quantity of matter is the *quantity of substance* in the movable, and thus not the magnitude of a certain quality of the movable (of repulsion or attraction, which are adduced in dynamics), and that the quantum of substance here means nothing else but the mere aggregate of the movables that constitutes matter. For only this aggregate of the moved can yield, at the same velocity, a difference in the quantity of motion. That, however, the moving force a matter has *in its inherent* [*eigenen*] motion alone manifests [*beweise*] the quantity of *substance*, rests on the concept of the latter as the ultimate *subject* in space (which is in turn no predicate of another) – which, for precisely this reason, can have no other magnitude than that of the aggregate of homogeneous [parts] external to one another. (540–41)

[60] See the paragraph to which note 26 above is appended. As I have emphasized, the concept of mass or "force of inertia" in the *Physical Monadology* is explicitly independent of spatial volume: it is precisely a (merely) intensive magnitude in the sense of the above quotation.

As explained in section 13 above, Kant's proof of the infinite divisibility of material substance in the fourth proposition of the Dynamics rests on the pure or unschematized concept of substance as the ultimate subject of predication. Applied to matter, in particular, this means that any matter filling a space counts as substance in this sense, together with all of its independently movable parts (532): "[T]he inherent [*eigene*] movability belonging to matter, or any part of it, is at the same time a proof that this movable thing, and any movable part thereof, is substance." But, since each part of a space filled with matter exerts repulsive force, "every part of a space filled with matter is movable for itself, and thus separable from the rest as material substance through physical division" (532). Therefore, material substance is physically (and not just mathematically) infinitely divisible.

The present passage from the remark to the first proposition of the Mechanics echoes this earlier argument from the Dynamics, but it also introduces an essentially new element: the concept of quantity of motion (momentum) as "the moving force that a matter has *in its inherent* motion" (541). In the Dynamics matter was considered as the ultimate subject of motion – and also as the ultimate subject of force in the dynamical sense. Here, however, matter is being considered as the ultimate subject of force in the mechanical sense: as interacting with other matter by means of the communication of motion and thereby exchanging quantities of motion (momenta) with others. Moreover, Kant is explicitly appealing to the idea that quantity of matter is estimated or determined as a magnitude by the quantity of motion at a given velocity, so that, in particular, "only [the] aggregate of the moved can yield, at the same velocity, a difference in the quantity of motion" (540–41). Given a quantity of matter moved with a given velocity, Kant suggests, the only way in which the quantity of motion – and therefore the quantity of matter – can then be varied is by varying the "aggregate of the movables" (540).

These considerations are crucial for the proof of the following second proposition – Kant's "*First Law of Mechanics*" (541): "In all changes of corporeal nature the total quantity of matter remains the same, neither increased nor diminished." In particular, Kant's proof appeals to both the first analogy already proved in the first *Critique* and the relationship between quantity of matter and quantity of substance just noted in the second remark:

(From general metaphysics we take as basis the proposition that in all changes of nature no substance either arises or perishes, and here it is only shown what substance is to be in matter.) In every matter the movable in space is the ultimate subject of all accidents inhering in matter, and the aggregate of these movables external to one another is the quantity of substance. Hence the quantity of matter, with respect to its substance, is nothing else but the aggregate of substances of which it consists. Therefore, the quantity of matter cannot be increased or diminished except in such a way that new substance thereof arises or perishes. Now substance never arises or perishes in any change of matter; so the quantity of matter is also neither increased nor diminished thereby, but remains always the same, and, indeed, as a whole – that is, in such a way that somewhere in the world it persists in the same quantity, although this or that matter can be increased or diminished by addition or separation of parts. (541–42)

According to the first analogy no substance can either arise or perish – only the changing accidents or determinations of a substance are transitory. But, according to the discussion in the second remark, all movable parts of a spatially extended aggregate of movables count as substances in turn, and the quantity of matter in such an aggregate represents precisely the quantity of substance – i.e., a "plurality of moved subjects" (541) – there. The only way in which such a quantity of matter can be increased or diminished, therefore, is by the addition or separation of spatially external parts, none of which can either arise or perish absolutely. The total quantity of matter, as the total quantity of substance, can thus be neither increased nor diminished.[61]

Kant's characterization of quantity of matter as a spatially extended aggregate of movables external to one another is therefore essential to his proof, because it is only on this assumption that an increase or decrease in the (total) quantity of matter would result in a corresponding increase

[61] The first analogy in the first edition states (A182): "All appearances contain the permanent (*Substance*) as the object itself and the changeable as its mere determination, i.e., a way in which the object exists." Kant then explains (A186/B229) that "the determinations of a substance, which are nothing but particular ways for it to exist, are called *accidents*," so that "change can therefore only be perceived in substances, and arising or perishing absolutely, without it concerning merely a determination of the permanent, can be no possible perception at all" (A188/B231). In the second edition Kant reformulates the first analogy (B224): "In all change of the appearances substance is permanent [*beharrt*], and its quantum in nature is neither increased nor diminished." At the beginning of the proof of the second analogy he then recapitulates the first (B232–33): "That all appearances in the time-series are all only *changes*, i.e., a successive being and non-being of the determinations of the substance which is permanent, and thus that the being of substance itself following upon its non-being, or its non-being following upon its existence – in other words, the arising or perishing of substance itself – does not take place, has been shown in the previous principle." That Kant now reformulates the principle of the first analogy as a quantitative conservation law is clearly connected with his intervening discussion of the quantitative conservation law for quantity of matter in the Mechanics of the *Metaphysical Foundations*.

or decrease in the (total) quantity of substance. Kant emphasizes precisely this point in the following remark:

What is essential in this proof to the characterization of *substance*, which is possible only in space and in accordance with its conditions, and thus possible only as object of the *outer* senses, is that its quantity cannot be increased or diminished without substance arising or perishing – because, since all quantity of an object possible merely in space must consist of *parts external to one another*, it follows that these [parts], if they are real (something movable), must necessarily be substances. (542)[62]

It is essential, then, that "matter has no other quantity than that consisting in the *aggregate* of the manifold [parts] *external to one another*, and hence has no *degree* of moving force at a given velocity [i.e., quantity of matter – MF] that would be independent of this aggregate and could be considered merely as intensive magnitude" (539–40). For a merely intensive change would not result in a substantial change: the parts (degrees) of an intensive magnitude count as "realities" or determinations of substance rather than as substances in turn.

Kant proceeds to illustrate this point by drawing a sharp contrast between matter, as the substantial object of outer sense, and the soul, as the (supposed) substantial object of inner sense:

By contrast, that which is considered as object of inner sense can have a quantity, as substance, which *does not consist of parts external to one another*; and its parts are thus not substances; and thus their arising or perishing need not be the arising or perishing of a substance; and their augmentation or diminution is therefore possible without violating the principle of the permanence of substance. So *consciousness*, namely, and thus the clarity of representations in my soul – and therefore the faculty of consciousness, apperception, and even, along with this, the very substance of the soul – has a *degree*, which can be greater or smaller, without any substance at all needing to arise or perish for this. But since, from its gradual diminution, the complete disappearance of the faculty of apperception would finally have to result, the very substance of the soul would still be subject to a gradual perishing, even if it were of a simple nature; for this disappearance of its fundamental force [*Grundkraft*] could result, not by division (separation of substance from a composite), but rather, as it were, by expiration [*Erlöschen*] – and this, too, not in a moment, but by a gradual waning of its degree, whatever the cause of this might be. (542)

[62] Kant insists on the spatiality of all substance in a marginal note to the statement of the first analogy (A182) in the first edition (23, 30): "Here the proof must be so developed that it applies only to substances as phenomena of outer sense, and therefore from space – which, together with its determinations, exists at all times."

Precisely because the soul, unlike matter, is not spatially extended, the only candidate for the quantity of substance in this case would be a merely intensive quantity (like degree of consciousness) – which, for precisely this reason, could be continuously diminished "without violating the principle of the permanence of substance." We therefore cannot infer from the principle of the permanence of substance (i.e., the first analogy) to the permanence of the soul as the object of inner sense.[63]

Kant inserted a passage into the second edition paralogisms of pure reason that is completely parallel to this one. In the later passage, entitled "Refutation of Mendelssohn's proof of the permanence of the soul" (413), Kant explains that "this acute philosopher" had attempted to repair a gap in the "customary argument" according to which, since the soul is simple, it cannot cease to exist through division. But why can it not simply disappear? Mendelssohn addresses this possibility (on Kant's reading) by appealing to the principle of continuity (B413–14): "[A] simple being cannot cease to exist, because, since it cannot be diminished at all and thereby lose something in its existence step by step, so as to be continually transformed into nothing (in that it has no parts and thus no plurality within itself), no time would be found between one moment, when it is, and the other, when it is no more, which is impossible." Kant objects:

> But he did not consider that, even if we admit this simple nature for the soul, in that it contains no manifold [of parts] *external to one another*, and thus no extensive magnitude, we can still not deny it (no more than any existing thing) intensive magnitude, i.e., a degree of reality with respect to all of its capacities, or, indeed, of all that constitutes its existence – which could diminish through all infinitely many smaller degrees, and thus the supposed substance (the thing whose permanence is not otherwise already established) could be transformed into nothing, even though not by division, but still by gradual waning (remission) of its forces (and thus by relaxation [*Elangueszenz*], if I may be permitted to use this expression). For even consciousness always has a degree, which can always be diminished, and thus so does the faculty of being conscious of oneself and all other faculties. – Therefore, the permanence of the soul, as mere object of inner sense, remains unproved, and indeed indemonstrable, although its permanence in life, where the thinking being (as human being) is equally an object of outer sense, is clear in itself – which, however, is far from sufficient for the rational psychologist, who undertakes to prove its absolute permanence even beyond life from mere concepts. (B414–15)

[63] In a further marginal note to the first edition (A183) Kant puts the point this way (23, 31): "In the soul there is no quantum of substance possible. Therefore also nothing which one could determine by any predicate and call permanent."

The principle of continuity, then, lends no support to the (absolute) permanence of the soul, for it could be continuously "transformed into nothing" by a continual diminution of its merely intensive magnitude. The soul, as "supposed" substantial object of inner sense ("the thing whose permanence is not otherwise already established"), cannot, in the end, be a substance at all.[64]

Yet it is not immediately obvious how this argument is supposed to cohere with the proof of the second proposition of the Mechanics. The way the latter proof works is by taking the permanence of substance – as contrasted with the changing character of its accidents or determinations – as already demonstrated in the first analogy. Kant then shows that, since all parts of a spatially extended aggregate of movables are themselves substances in turn, none of these parts can simply go out of existence without violating the already demonstrated principle of the permanence of substance. This kind of argument cannot apply to a merely intensive magnitude, Kant claims, because the parts (degrees) of such a magnitude are realities or determinations of substance rather than substances in turn. Now, however, in the refutation of Mendelssohn in the second edition paralogisms (along with the parallel passage in the remark to the second proposition of the Mechanics), Kant concludes that the permanence of the soul is not demonstrable because its "degree of reality" (B414) as an intensive magnitude can be continually diminished. But why should it follow from this that "the very substance of the soul" (542) could vanish? Why can we not distinguish the substance of the soul from its realities or determinations and conclude from the principle of the permanence of substance that the former certainly cannot disappear even if the latter can be continuously decreased? Perhaps the relevant degree of reality can be continuously decreased but never actually vanish – or, more to the point, perhaps it can take on a value of zero (during sleep in the

[64] In the case of matter, by contrast, Mendelssohn's envisioned use of the principle of continuity can be applied successfully. The total quantity of matter cannot be *diminished* at all and could therefore only "disappear" by a discontinuous jump (543, bold emphasis added):

> By contrast, the concept of a matter as substance is the concept of the movable *in space*. It is therefore no wonder if the permanence of substance can be proved of [matter], but not of the [soul], since in the case of matter it already results from its *concept* – namely, that it is the movable, which is possible only in space – that what has quantity therein contains a plurality of the real *external to one another*, and thus a plurality of substances; and hence the quantity of matter can be diminished only by division, which is not disappearance, **and the latter would also be impossible in matter according to the law of continuity**.

case of degree of consciousness, for example) without the underlying substance also needing to disappear.[65]

Properly to address these concerns we need to clarify the content of the *schematized* category of substance. Kant is not operating here with the merely logical notion of a subject of predication or determination – even if we take this, in accordance with the pure or unschematized concept of substance, as the ultimate subject of determination "which does not itself belong, in turn, to the existence of another thing merely as predicate" (503). The schema of the concept of substance, by contrast, is "the permanence of the real in time, i.e., the representation of [this real] as a substratum of empirical time determination in general, which therefore remains while everything else changes" (A144/B183).[66] An instance of the schematized concept of substance is therefore precisely a reality that can serve as "the substratum of empirical time determination" or, as Kant puts it in the first analogy, "as the *substratum* of the empirical representation of time itself, in which alone all time determination is possible" (A183/B226). Only a reality that is permanent in this sense can be a substance in accordance with the schematized category (*substantia phaenomenon*).[67]

[65] I am here responding to an objection to an earlier formulation of my interpretation raised by Eric Watkins (1998, pp. 548–49), and I am also indebted to conversations with him concerning this objection. Watkins expresses the point of his objection by distinguishing a weak from a strong version of the conservation of substance (p. 549):

> Substance is conserved in one, relatively weak sense simply if it cannot perish (i.e., it is necessarily permanent). If something is conserved in this sense, it can be the substratum of all time-determination. Substance is conserved in another, much stronger sense if its *quantity* remains unchanged over time. Physical monads or substances having [only] intensive magnitudes are conserved in the weak, but not the strong sense.

I shall argue in what follows that, according to Kant, only the stronger sense of conservation is sufficient for time determination.

[66] See notes 73 and 74 of my chapter on the Dynamics, together with the paragraph to which they are appended. The concept of substance at work in Kant's proof of the infinite divisibility of material substance in the Dynamics involved only the unschematized category, while the proof of the conservation of the quantity of material substance in the Mechanics essentially involves the schematized category. As I shall explain in what follows, this turns out to be intimately connected with the fact (noted above) that the proof of conservation involves an essentially new element not present in the Dynamics: momentum or "the moving force a matter has *in its inherent* motion" (542).

[67] See also the following passage (A183–84/B226–27):

> [T]his permanent in the appearances is the substratum of all time determination … and in this permanent all existence and all change in time can be viewed as a mode of existence of that which remains and endures [*bleibt und beharrt*]. Thus in all appearances the permanent is the object itself, i.e., the substance (phaenomenon), and everything that changes or can change belongs to the manner in which this substance or these substances exist, and thus to their determinations.

An instance of the schematized category of reality, moreover, is an intensive magnitude filling a time, which, by affecting our senses, produces a corresponding intensive degree of sensation in inner sense.[68] As precisely such an intensive magnitude, moreover, any instance of this category must have a degree or "quantum of reality" such that continuous transitions between this (finite) degree and zero are always possible:

> Now every sensation has a degree or magnitude, whereby it can fill the same time, i.e., inner sense with respect to [the] representation of an object, more or less, until it ceases in nothing (= 0 = *negatio*). Therefore, there is a relation and connection, or rather a transition from reality to negation, which makes every reality representable as a quantum; and the schema of a reality, as the quantity of something in so far as it fills time, is precisely this continuous and uniform generation of it in time, in that one continually descends in time from the sensation having a certain degree until it disappears or ascends from negation up to this quantity. (A143/B182–83)

On the one hand, therefore, any reality is an intensive magnitude possessing a degree that can be continuously diminished to zero. Yet, on the other hand, it is just such a reality that, as permanent, must realize the schematized category of substance.[69]

How can these two apparently conflicting demands be met simultaneously? Kant's critical version of the dynamical theory of matter as a spatially extended and continuous aggregate of movables supplies a concrete instance. We are here concerned with the real in *space*, which, accordingly, fills both space and time. The question at issue is exactly how this filling of space is to be conceived. In the theory of the *Physical Monadology* a monad fills the space it occupies by the intensive magnitude of its repulsive force without needing to be spatially extended – we have a purely intensive quantum of reality which, independently of anything that may be happening in the space around it, could continuously decrease to zero. Such a quantum of reality, therefore, cannot possibly instantiate the schematized category of substance. In the theory of the

[68] The principle of the anticipations of perception in the first edition states (A166): "In all appearances sensation, and the *real* which corresponds to in the object (*realitas phaenomenon*), has an *intensive magnitude*, i.e., a degree." In the second edition we have (B207): "In all appearances the real, which is an object of sensation, has an intensive magnitude, i.e., a degree."

[69] Observe that the passage just quoted (A143/B182–83) immediately precedes the definition of the schema of the concept of substance quoted above (A144/B183). In reference to the issue raised in the paragraph to which note 65 above is appended, therefore, it follows that the object of the *schematized* category of substance – *substantia phaenomenon* – is not a hidden "substrate" behind what can be sensibly perceived but precisely "a permanent image of sensibility" (A525–26/B554; see note 108 of my chapter on the Dynamics, together with the paragraph to which it is appended).

Metaphysical Foundations, by contrast, any matter that fills a space does so as a spatially extended continuum characterized by a balancing of repulsive and attractive forces, which yield, at each point of the continuum, a determinate density whereby matter fills the space (at each point) to a definite degree.

The crucial feature of this model, in our present context, is that the density expressing the intensive aspect of the concept of quantity of matter is also necessarily implicated with the essentially extensive aspect of this concept: increase or decrease of density is always correlated with a contraction or expansion of the space filled by the quantity of matter in question. Hence, although the intensive magnitude of density (and thus the quantity of matter) may indeed decrease towards zero in any given (finite) spatial region, this is necessarily accompanied by a compensating expansion of the initial quantity of matter into a more extended region, such that the total quantity of matter in both regions remains constant.[70] The critical version of the dynamical theory of matter as a spatially extended and continuous aggregate of movables thereby provides us, for the first time, with concepts of quantum of reality and quantum of substance that simultaneously satisfy the two apparently conflicting demands of the permanent as the substratum of all time determination.

It is helpful, at this point, to contrast Kant's conception with its two main competitors at the time: a hard-body atomism that conceives the density of matter within a given space as the ratio of absolutely dense matter interspersed with the void to the total volume in question, on the one side, and the force-center atomism of Boscovich and the early Kant, on the other. According to the former matter has only extensive quantity, according to the latter only intensive quantity. The purely extensive conception does not connect with the schematized category of substance at all, because it does not conform to the schematized category of reality.[71] By contrast, the purely intensive conception does conform to the category of reality, but it does not conform to the schematized category of substance. In particular, no distribution of a merely intensive magnitude or quantum

[70] In the limit when the matter in question expands to infinity we would have zero density at every point as the same initial (finite) quantity of matter has now become continuously distributed over infinite space. But this is precisely what the first stage of the balancing argument rules out: see note 137 of my chapter on the Dynamics, together with the paragraph to which it is appended and the preceding paragraph.

[71] Kant's deeper reasons for rejecting the conception of density and thus quantity of matter characteristic of hard-body atomism (which reasons I am now in the process of exploring) are suggested in note 59 above.

of reality that is independent of spatial extension can jointly satisfy *both* schematized categories. In the case of physical monads, for example, mass or quantity of matter is quite independent of the degree of repulsive force, in so far as the former is associated with the "force of inertia" and not with the filling of space. Nevertheless, both of these forces are purely intensive magnitudes, and there is nothing in the *Physical Monadology* that corresponds to density in the *Metaphysical Foundations*.[72]

So for Kant, once again, only a continuously distributed reality extended in space can possibly realize the schematized category of substance. But what exactly does it mean for such a distribution of reality to function as the substratum of all time determination? At the beginning of the proof of the first analogy in the second edition Kant explains that the substratum of all time determination goes proxy, in the objects of perception, for time itself:

> All appearances are in time, in which, as substrate (as permanent form of inner intuition), *simultaneity* as well as [temporal] *sequence* can alone be represented. The time, therefore, in which all change of appearances is to be thought, remains and does not change; because it is that in which succession or simultaneity can be thought only as its determinations. But time in itself [*für sich*] cannot be perceived. Therefore, there must be found in the objects of perception, i.e., in the appearances, the substrate which represents time in general, and in which all change or simultaneity can be perceived by the relation of appearances to it in apprehension. (B224–25)

[72] Falkenstein (1998) raises an interesting difficulty for Kant's argument against the purely intensive conception. The worry is that, since Kant holds that specifically different types of matter are characterized by different values of their intrinsic repulsive forces (expansive pressures), we can imagine that precisely this value (the degree of an intensive magnitude) may vary arbitrarily – so that the intrinsic specific density of matter may decrease to zero (thereby yielding zero quantity of matter). The answer to this worry, I believe, is that Kant's First Law of Mechanics does not allow for changes of intrinsic specific densities – changes that would transform one type of matter into another – as physically possible changes of matter. The only physically possible density changes, therefore, result from compressions or expansions of a given type of matter, which, as explained, do not and cannot change the total quantity of matter. Falkenstein considers this rejoinder and rejects it on the ground that it would make Kant's supposedly a priori theory too dependent on empirical considerations – in this case the choice between competing conceptions of the fundamental structure of matter. It is characteristic of my reading, by contrast, that Kant's articulation of the explicitly *empirical* concept of matter in the *Metaphysical Foundations* is committed to just such a choice between the three leading competing conceptions of the time. The view that any kind of matter can be transformed into any other is characteristic of hard-body atomism, whether that of the mechanical philosophy or that (conjecturally) proposed by Newton: for the former see the paragraph to which note 42 of my chapter on the Dynamics is appended; for the latter see again note 59 above (and compare, in this context, note 184 of my chapter on the Dynamics, together with the paragraph to which it is appended and the preceding paragraph).

In the following paragraph (in both editions) Kant adds that the magnitude of temporal duration is also one of the relations to be determined in this way.[73] In his comments on all three analogies (again in both editions) Kant sums up his view by way of conclusion:

These, then, are the three analogies of experience. They are nothing else but the principles for the determination of the existence of appearances in time with respect to all of its three modes, the relation to time itself as a magnitude (the magnitude of existence, i.e., duration), the relation in time as a series (successively), and finally [the relation] in time as a totality of all existence (simultaneously). (A215/B262)[74]

In general, then, a distribution of reality functions as the substratum of all time determination by providing a perceptible representative of the temporal relations of duration, succession, and simultaneity. It provides an enduring perceptible distribution of some intensive magnitude in connection with which, and by means of the analogies of experience, just these temporal relations may be empirically determined.

Kant clarifies his official definition of the schema of the category of substance ("the permanence of the real in time") as follows (A144/B183): "Time does not flow away, but rather the existence of the changeable flows away in it. Time, therefore, which is itself unchangeable and enduring [*bleibend*], is that to which what is unchangeable in existence in the appearance, i.e., substance, corresponds, and only in [substance] can sequence and simultaneity with respect to time in the appearances be determined." He immediately goes on to suggest that the first task of time determination performed by this distribution of reality is the determination of succession in accordance with the principle of causality: "The schema of cause and of the causality of a thing in general is the real, upon which, if it is arbitrarily posited, something else always follows. It therefore consists in the succession of the manifold, in so far as it is subject to a rule" (A144/B183). Once a permanent distribution of reality corresponding to the schematized category of substance is in place, it then becomes, as Kant suggests in the passage from the beginning of the proof of the

[73] See the following claim (A183/B226): "Only by means of the permanent does the *existent* in different successive parts of the time series acquire a *magnitude*, which one calls *duration*."

[74] The full passage is quoted in the paragraph preceding the one to which note 34 of the Introduction is appended. Kant's argument, as I point out there, is intimately connected with his reconsideration of Newtonian absolute time – a point to which I shall return below. In his discussion in the general principle of the analogies (A177/B219) Kant gives the three "modes of time" as "*permanence*, *sequence*, and *simultaneity*."

first analogy in the second edition (B225), "that in which succession or simultaneity can be thought only as its determinations."[75]

In the *Metaphysical Foundations*, as explained, the permanent reality in question is a continuously extended distribution of mass or quantity of matter, and substance, accordingly, is material substance as a spatially extended aggregate of movables. In this context, Kant's discussion of the relationship between quantity of matter and quantity of motion or (mechanical) moving force illuminates the transcendental relationship between substance and causality in the first *Critique*. In general, once again, there can be no substance without an exercise of causality, action, and force, and there can be no exercise of these powers, conversely, without a corresponding seat in a substance. In the more specific context of the Mechanics this means that quantity of matter, in accordance with the first proposition, is always estimated or determined by reference to the quantity of motion exchanged in the context of the communication of motion – by the momentum one moving body transfers to another in the course of such interactions. It is only in virtue of this connection with the quantity of motion that the quantity of matter can be determined as a mathematical magnitude. Thus, the total quantity of matter is reciprocally related to the total quantity of motion, and the conservation of the former quantity presupposes the conservation of the latter.[76] The conservation of the quantity of matter, in this way, becomes a principle of balance or equality of reality between cause and effect.[77]

In the pre-critical period Kant typically presents a general conservation principle for the transmission of reality from cause to effect rather than a conservation principle for substance, and he consistently illustrates the

[75] In the schematism discussion simultaneity is specified as the schema of the category of community (A144/B183–84): "The schema of community (interaction [*Wechselwirkung*]), or the mutual [*wechselseitigen*] causality of substances with respect to their accidents, is the simultaneity of the determinations of one with that of the other in accordance with a universal rule."

[76] As pointed out in note 36 above, Newton derives the conservation of momentum from his Third Law of Motion, the equality of action and reaction. For Kant, as we shall see in the sequel, the equality of action and reaction is derived in a "construction" of the communication of motion presented in the fourth proposition of the Mechanics. Nevertheless, although a full understanding of Kant's conception of the conservation of momentum must therefore await a detailed discussion of this proposition, we have already seen enough to know that conservation of mass and conservation of momentum are reciprocally related for Kant.

[77] As explained, Kant expresses this by the formula (539; see the paragraph to which note 28 above is appended): "As the quantity of motion of a body relates to that of another, so also does the magnitude of their action or effect [*Wirkung*]." In particular, then, two initial momenta, considered as causes or (mechanical) moving forces, always stand to one another in the same ratio as their resulting effects (changes of momentum thereby produced).

former by the principle of the conservation of momentum. In the *New Exposition* of 1755, for example, Kant derives a general conservation principle for the transmission of reality from cause to effect as a corollary of his principle of the determining ground or reason [*ratio*] in Proposition x (1, 407): "*There is no more in that which is grounded* [*rationato*] *than is in the ground* [*ratione*]." He illustrates as follows:

The evident character of this rule is easily elucidated in the changes of bodies. If, e.g., body A propels another B by percussion, a certain force, and therefore a reality [*realitas*] is imparted to it. However, an equal quantity of motion is taken away from the impinging body, so that the sum of the forces in the effect is equal in magnitude to the forces of the cause. (1, 407)

Kant further explains that the sum of the forces in question is calculated by "subtracting from one another the motions in contrary directions [*partes contrarias*]," so that "what remains is the motion of the center of gravity, which, as is known from statics, is the same after impact as it was before" (1, 407). Thus, in particular, Kant's discussion of the conservation of momentum here parallels Newton's discussion in Corollaries 3 and 4 to the Laws of Motion.[78]

This parallel is even more noticeable in the essay on *Negative Magnitudes* of 1763. Kant again begins with a general metaphysical principle governing positive and negative "real grounds" or "positings" (2, 194): "*In all natural changes of the world the sum of the positive* [grounds], *in so far as it is estimated in such a way that agreeing (not opposed) positings are added and really opposed are subtracted from one another, is neither increased nor decreased*." And, once again, he illustrates this principle with the conservation of momentum:

I wish to attempt to elucidate this proposition, which seems to me to be important. In the changes of the corporeal world it is firmly established as a mechanical rule already proved since long ago. It is expressed as follows: The quantity of motion, if one adds the forces of bodies made in the same direction and subtracts those which tend in contrary [directions], is not changed by their mutual action (impact, pressure, attraction). (2, 195)

[78] Corollary 3 states (P420): "The quantity of motion, which is determined by adding the motions made in one direction and subtracting the motions made in the contrary direction, is not changed by the action of bodies on one another." Corollary 4 states (P421): "The common center of gravity of two or more bodies does not change its state whether of motion or of rest as a result of the actions of the bodies upon one another; and therefore the common center of gravity of all bodies acting upon one another (excluding external actions and impediments) either is at rest or moves uniformly forward in a straight line." As explained in note 36 above, Newton extends these principles to statics in the Scholium to the Laws of Motion.

Kant's language here recalls Newton's wording in Corollary 3.[79] Moreover, if one notes that Kant formulates a second metaphysical proposition corresponding to the conservation of momentum expressed with respect to the "common center of gravity of all bodies acting upon one another" in Newton's Corollary 4 (note 78 above), the parallel between Kant's metaphysical conservation law and the corresponding Newtonian principles governing the transfer of momentum appears to be very close indeed.[80]

The metaphysical conservation law Kant formulates in the critical period involves the category of substance rather than that of causality, and the corresponding realization or application of this principle in the Mechanics is the conservation of the quantity of matter rather than the conservation of momentum. In the *New Exposition* the central metaphysical principle is the principle of the determining ground or reason [*ratio*], which, as Kant points out (1, 391), is "commonly called the principle of sufficient reason." This principle, as in the Leibnizean tradition more generally, is a purely conceptual, quasi-logical principle derived from the nature of true predication.[81] By contrast, it is a central contention of Kant's critical period that no such principle can be proved analytically or purely conceptually. On the contrary, all such principles, as *synthetic* a priori, essentially involve reference to (A155/B194) a "third [thing] [*ein Drittes*]" (beyond the concepts of subject and predicate) – namely, the synthetic unity of the manifold of intuition in inner sense in accordance with the a priori rules of a possible experience.[82]

[79] Kant states the principle in Latin: "*Quantitas motus, summando vires corporum in easdem partes et subtrahendo eas quae vergunt in contrarias, per mutuam illorum actionem (conflictum, pressionem, attractionem) non mutatur.*" Newton's Corollary 3 (in the original Latin) states (1972, p. 59): "*Quantitas motus quae colligitur capiendo summam motuum factorum ad eandem partem, & differentiam factorum ad contrarium, non mutatur ab actione corporum inter se.*" Kant appears to be paraphrasing Newton while including examples of "impressed forces" from Definition 4 and the discussion in the Scholium to the Laws of Motion.

[80] Kant's "second proposition" reads (2, 197): "*All real grounds in the universe, if one adds those which are in agreement and subtracts those which are opposed to one another, yield a result that is equal to zero.*"

[81] Kant begins in Proposition IV by asserting (1, 391) that "[t]o *determine* is to posit a predicate while excluding its opposite," where "[t]hat which determines a subject with respect of any of its predicates is called the *ground or reason* [*ratio*]." Proposition V, which asserts (1, 393) that "[n]othing is true without a determining ground or reason," then follows from these definitions and the premise that "[e]very true proposition indicates that the subject is determined with respect to a predicate."

[82] For the whole passage see the discussion of "the highest principle of all synthetic judgements," where Kant begins by emphasizing that a "third [thing]" is always necessary in such cases (A155/B194): "What is now this third [thing], as the medium of all synthetic judgements? There is only one totality, wherein all our representations are contained, namely inner sense and its a priori

An especially striking instance of this contention, which concerns the principle of sufficient reason in particular, occurs in Kant's summary discussion of the three analogies of experience as a whole (A215–16/B262–63). Kant concludes this discussion with a methodological consideration of "the mode of proof of which we have made use for these transcendental laws of nature" (A216/B263). Since they are *synthetic* a priori, no "dogmatic" proof "through mere concepts" is possible (A216–17/B263–64): only "[t]he possibility of experience, as a cognition in which all objects must ultimately be given, if their representation is to have objective reality for us" can provide the necessary ground of proof. Kant continues:

> In this third [thing], whose essential form consists in the synthetic unity of apperception of all appearances, we found a priori conditions of thoroughgoing and necessary time determination of all existence in the appearance, without which even empirical time determination would be impossible; and we found rules of a priori synthetic unity by means of which we could anticipate experience. In [the context of a] lack of this method, and under the delusion of pretending dogmatically to prove synthetic propositions which the empirical use of the understanding recommends as its principles, it has then come about that a proof of the principle of sufficient reason has so often been sought, but always in vain. No one has [even] thought of the other two analogies, although they have always been tacitly used, because the guiding thread of the categories was missing, which alone can discover and make noticeable every gap in the understanding, with respect to both concepts and principles. (A217–18/B264–65)

In the absence of the true method for proving "transcendental laws of nature," then, the principle of sufficient reason has illegitimately obscured the importance of the other two analogies – and it has thereby been misunderstood in itself. For the proof of this principle, just like those of the other two analogies, can only be based on its role as an a priori necessary condition of all empirical time determination.[83]

form, time." In order for experience to arise, however, inner sense must be unified by the necessary rules provided by the categories (A156–57/B196):

> Thus experience has a priori principles of its form lying at the basis, namely universal rules of unity in the synthesis of appearances, whose objective reality can always be shown in experience, as [its] necessary conditions – even of its very possibility. Aside from this relation, however, synthetic a priori propositions are completely impossible, because they have no third [thing], namely no object, in which the synthetic unity of their concepts could be verified.

[83] Compare the related discussion in the transcendental doctrine of method (A736–37/B764–65):

> Now the whole of pure reason in its merely speculative use does not contain a single directly synthetic judgement from concepts … [T]hrough pure concepts of the understanding, to be sure, it erects secure principles, but not at all directly from concepts, but rather always only indirectly though the relation of these concepts to something completely contingent, namely *possible experience*; since, if the latter (something as object of possible experience) is presupposed, they

In the *Metaphysical Foundations* Kant is committed to both a synthetic a priori conservation principle for the total quantity of matter corresponding to the category of substance and a synthetic a priori conservation principle for the total quantity of momentum corresponding to the category of causality. But I am now in a position to explain why only the former is formulated explicitly – as the First Law of Mechanics in the second proposition – and, more generally, why the conservation of material substance has priority here. All such a priori laws of nature, in the critical period, are ultimately based on the necessary conditions of empirical time determination, and what Kant calls the substratum of all time determination is an instantiation of the schematized category of substance. The very first requirement for time determination, therefore, is a permanent distribution of some intensive magnitude or reality functioning as "that in which succession or simultaneity can be thought only as its determinations" (B225; see the paragraph to which note 75 above is appended). It is for precisely this reason that the schematized category of causality (as well as the schematized category of community) presupposes the schematized category of substance and, accordingly, that the most fundamental conservation principle, on Kant's critical conception, must be the conservation of substance. Indeed, Kant explicitly asserts this in the first analogy after noting that all philosophers (as well as "the common understanding") have always presupposed the permanence of substance (A184/B227, emphasis added): "But I have never met with even the attempt at a proof of this so synthetic proposition; indeed, it is only seldom found, where it nevertheless belongs, *at the pinnacle of the laws of nature that subsist a priori*." For the critical Kant, therefore, it is the permanence of the quantity of substance rather than the principle of sufficient reason that is first among the "transcendental laws of nature."

With these considerations in mind, let us return to Kant's important discussion of substance, action, and force towards the end of the second analogy (A204–05/B249–50; see note 22 above). He begins (A204/B249) "I cannot leave untouched [here] the empirical criterion of substance, in so far as it appears to manifest itself, not through the permanence of

are then indeed apodictically certain, but in themselves (directly) they cannot even be cognized a priori at all. Thus no one can rigorously comprehend the proposition that everything that happens has its cause from these given concepts. Therefore it is not a dogma, even if it can well and apodictically be proved from another point of view, namely that of the single field of its possible use, i.e., experience. But it is called a *principle* [*Grundsatz*] and not a *theorem* [*Lehrsatz*], even though it must be proved, precisely because it has the special property that it first makes possible its own ground of proof, namely experience, and must always be presupposed in the latter.

the appearance, but better and more easily through action." The passage continues:

> Where there is action, and therefore activity and force, there is also substance, and in the latter alone must the seat of the former fruitful source of appearances be sought. That is well said; but, if one is supposed to explain what one understands by substance, and one wants to avoid a vicious circle, this is not so easy to answer. How is one to infer from the action *to the permanence* of that which acts, which is nevertheless such an essential and peculiar characteristic of substance (*phaenomenon*)? However, in accordance with our foregoing discussion, the solution to this question has no such difficulty, even though it would be completely unresolvable in the common procedure (to proceed merely analytically with its concepts). Action already signifies the relation of the subject of causality to the effect. Now, because all action consists in that which happens, and thus in the changeable, which designates time with respect to succession; then the ultimate subject [of the changeable] is *the permanent* as the substratum of all that is changing, i.e., substance. For, according to the principle of causality, actions are always the first ground of all change in the appearances, and they cannot therefore lie in a subject that itself changes, since otherwise another subject, which determines this change, would be required. Because of this, action, as a sufficient empirical criterion, manifests [*beweiset*] substantiality, without my having to seek for this first by the comparison of perceptions – which could also not proceed, in this way, with the fullness of detail [*Ausfürlichkeit*] required for the magnitude and strict universality of the concept. (A204–5/B250–51)

The crucial transition is thus from substance as "the substratum of all that is changing" – i.e., the substratum of all time determination – to "the [ultimate] subject of causality." The latter must provide us with the "empirical criterion" of the former. For otherwise the substratum of all time determination, independently of causality, could only be found "by the comparison of perceptions" – which would not correspond to the full rigor and strictness demanded by a pure concept of the understanding.[84]

[84] For the distinction between strict and comparative universality see the discussion of the rules associated with the principles of pure understanding (here the principle of causality) in the first introductory section to the transcendental deduction (A91/B124): "Appearances certainly provide us with cases on the basis of which a rule is possible whereby something customarily happens, but never that the result is *necessary* … The strict universality of the rule is in no way a property of empirical rules, which, through induction, can acquire nothing but comparative universality, i.e., extended utility." See also the parallel discussion in the second edition Introduction (B3–4):

> Experience never gives true or strict *universality* to its judgements, but only assumed and comparative [universality] (through induction) – so that, properly speaking, one must say: as far as we have observed so far no exception has been found for this or that rule. Thus, if a judgement is thought with strict universality, i.e., so that no exception at all is admitted as possible, then it is not derived from experience, but is rather valid absolutely a priori … Necessity and strict universality are therefore secure criteria of an a priori cognition, and also belong inseparably to one another.

Kant's discussion of the relationship between quantity of matter as the aggregate of movables and quantity of motion (momentum) in the remark to the first proposition of the Mechanics illuminates the discussion in the second analogy. As explained in section 24 above, since quantity of motion is given by the product of quantity of matter and velocity, while quantity of matter can only be estimated, in turn, by quantity of motion (at a given velocity), Kant is faced with an apparent circularity. He responds:

> This alleged circle would be an actual one, if it were the reciprocal derivation of two identical concepts from one another. However, it contains only the explication of the concept, on the one hand, and that of its application to experience, on the other. The quantity of the movable in space is the quantity of matter; but this quantity of matter (the aggregate of the movable) *manifests itself* [*beweist sich*] in experience only by the quantity of motion at the same velocity. (540; see the paragraph following the one to which note 37 above is appended)

Whereas the concept of quantity of matter is prior to that of quantity of motion in the order of definition, the latter is prior to the former in the order of empirical application. But, according to this same remark, quantity of matter as the aggregate of movables is also the quantity of substance, and "the moving force a matter has *in its inherent* motion alone manifests [*beweise*] the quantity of *substance*" (541; see the paragraph following the one to which note 60 above is appended). Hence, it is precisely the quantity of motion, as the measure of the action exerted by one matter on another in the context of the communication of motion, that here serves as the empirical criterion of substance.

Indeed, the necessity of going beyond the characterization of quantity of substance as the aggregate of movables and appealing to quantity of motion as its empirical criterion has already been emphasized in the second explication and immediately following first proposition. According to the former, quantity of matter can only be subsumed under the concept of *mass* "in so far as all its parts in their motion are considered as acting (moving) simultaneously" (537), so that "a matter *acts in mass*, when all its parts, moved in the same direction, *simultaneously* exert their moving force externally." According to the latter, the relevant aggregate of movables (acting in mass) can then be estimated or determined as a magnitude in general only by appealing to the quantity of motion (at a given velocity). Thus, as explained, the proof of the second proposition shows that only the representation of matter as a continuous and extended aggregate of movables in space enables us to demonstrate the permanence of matter as a realization of the schematized category of substance. But it is also true that this proof, by itself, does not explain how matter so defined acquires a mathematical (measurable) structure in such a way that a quantitative conservation law results. Just as the quantity of matter can only be determined as a magnitude by means of the quantity of motion, a precise mathematical conservation law only results for the quantity of substance by means of the conservation of momentum.

I described in section 24 above how this works in practice. The crucial point is that conservation of momentum allows us to institute a general method for estimating quantity of matter, which begins with the practice of weighing in the terrestrial realm and then proceeds to extend this practice into the celestial realm in the context of universal gravitation.[85] Quantity of substance first becomes determined as a mathematical magnitude, therefore, in the practice of terrestrial weighing, and it is then determined in all cases of material substance in general by the universal extension of this practice effected by the theory of gravitation. Kant appears to have precisely this extension in mind, moreover, in the important remark to the refutation of idealism discussed in section 3 above (B277–78; see the passage to which note 46 of my chapter on the Phoronomy is appended). After suggesting that we begin all time determination from our initial perspective on the surface of the earth and then extend this procedure into the heavens, Kant observes (B278): "[W]e also have nothing at all permanent, which could underlie the concept of a substance, as intuition, except merely *matter*, and even this permanence is not derived from outer experience, but is rather presupposed a priori as necessary condition of all time determination." Thus here, in particular, Kant suggests a connection between his Copernican conception of space and motion, on the one side, and his principle of the permanence of substance, on the other.

More generally, as explained in the remainder of section 3, Kant is here especially concerned with how time itself becomes a mathematical magnitude – and with how the category of substance is involved with our determination of "the magnitude of existence, i.e., duration" (A215/B262). This determination takes place, following Newton, by applying the theory of universal gravitation to the observed motions in the heavens so as to refine and correct their merely apparent temporal uniformity in light of the ideal standard of perfect uniformity implicit in the mechanical laws

[85] For the central role of conservation of momentum in this method see notes 36, 46, and 49 above, together with the paragraphs to which they are appended – and compare the paragraph to which note 76 above is appended.

of motion. So we obtain, in particular, a sequence of ever better approximations to ideal temporal uniformity that is completely parallel, in this context, to the sequence of ever better approximations to what Kant calls "absolute space" in his Copernican conception of space and motion. Both procedures, moreover, equally depend on the conservation of momentum and thus, from Kant's point of view, on his principle of the permanence of the quantity of substance. It is no wonder, then, that Kant revised his statement of the first analogy so as explicitly to emphasize its character as a quantitative conservation principle in the second edition of the *Critique* (note 61 above).[86]

I am now in a position, finally, to answer the questions posed towards the end of section 24 above. How, exactly, does Kant negotiate the transition between the statical concept of weight and the dynamical (for Kant "mechanical") concept of mass, and how are both concepts, in turn, supposed to be connected with the traditional concept of quantity of matter as the "bulk" or "amount" of matter in a given space (i.e., volume)? How does Kant's way of establishing these connections compare with Newton's – which depends, as explained, on the "very accurate experiments with pendulums" (P404) to which Newton refers in his explanation of Definition 1?[87] One way of focussing these questions, as suggested at the beginning of the present section, is to inquire into the relationship between the terrestrially limited or relational notion of weight and the two universal or intrinsic (but conceptually quite different) notions of quantity of matter and mass.

[86] I here complete my argument, in accordance with note 65 above, that a quantitative conservation law for substance is indeed necessary for time determination. Moreover, although Kant only makes this completely explicit in the second edition, I believe it is already implicit in Kant's discussion of the empirical criterion of substance in the first edition (A204–5/B250–51). My argument, in fact, is that Kant's thought undergoes a natural development in this respect from the first edition, through the *Metaphysical Foundations*, to the second edition. Note also that the proof of the first analogy added to the second edition places particular emphasis on the concept of existence (B225):

> But the substrate of everything real, i.e., [what] belongs to the existence of things, is *substance*, in which all that belongs to existence can be thought only as determination. Therefore the permanent, in relation to which all relations of time in the appearances can alone be determined, is substance in the appearance, i.e., the real [in the appearance], which as substrate of everything changing remains always the same. Since this cannot change, therefore, in existence, its quantum in nature can also neither be increased nor diminished.

This argument thereby involves an appeal to the idea of "time itself as a magnitude (the magnitude of *existence*, i.e., duration)" (A215/B262; emphasis added).

[87] See again note 54 above, together with the paragraph to which it is appended and the remainder of section 24. As explained, the point of Newton's pendulum experiments is to delineate an intricate system of empirical connections among the three concepts of quantity of matter, weight, and mass.

Kant begins by explicating the concept of quantity or "amount" of matter as the aggregate of the movable in a given space. He thereby connects this concept with his critical version of a dynamical theory of matter, which is developed in explicit contrast with two other theories: first, what Kant takes to be the standard view – represented by "[a]lmost all investigators of nature" (A173/B25) – that different kinds of matter can differ in their quantities of matter at the same volume only by containing different proportions of a single uniform type of matter and empty space within this volume; second, his own earlier dynamical theory in the *Physical Monadology*, which represents material substances as unextended simple points surrounded by a purely intensive sphere of activity. By contrast with both of these theories, Kant's critical version of the dynamical theory of matter is now connected with the notions of mass and quantity of motion in the Mechanics by the idea that the quantity of matter, so understood, can only manifest itself in experience by "the quantity of motion at a given velocity" (537). Moreover, it is precisely this quantity, in matter, that represents the empirical criterion of substantiality, so that the quantity of substance, in turn, can only be determined by means of the quantity of motion. The principle of the conservation of the total quantity of substance (as Kant reformulates the first analogy in the second edition of the *Critique*) is thereby empirically realized by the conservation of the total quantity of momentum in all ("mechanical") interactions. And the ground of this principle, in the end, rests on the indispensable conditions for all empirical time determination – including the determination of "time itself as a magnitude" (A215/B262).

Kant thus negotiates the connections among the concepts of quantity of matter, mass, and weight by embedding all three within his metaphysical (properly transcendental) discussion of the pure a priori concepts of substance, action, and force. This contrasts sharply with Newton, who eschews such metaphysical considerations in favor of empirical measurement procedures grounded in both traditional statics (including hydrostatics) and his own Second Law of Motion. From Kant's perspective, however, as explained in sections 3 and 6 above, Newton's procedure poses its own metaphysical dangers – chiefly in connection with the concepts of "absolute" space, time, and motion.[88] Kant hopes, among other things, to ward off these dangers by adopting a less global and more "constructive" approach to the application of mathematics to the empirical concepts of

[88] See especially note 82 of my chapter on the Phoronomy, together with the paragraph to which it is appended.

physics. In particular, whereas Newton simply assumes that the fundamental concepts to which he appeals in his initial Definitions are already mathematically well defined independently of the Laws of Motion, Kant intends rather to elucidate their empirical conditions of application step by step – beginning with the concept of velocity in the Phoronomy and now arriving at the concept of mass or quantity of matter in the Mechanics.

Yet the case of mass or quantity of matter turns out to be considerably more complicated than the case of velocity. For the former essentially involves "principles for the determination of the *existence* of appearances in time" (A215/B262, emphasis added), and therefore the analogies of experience. The characteristically metaphysical concepts of substance, causality, and community must thus play an indispensable role in this case – which they do not and cannot play in the case of velocity.[89] In particular, as explained in section 4 above, Kant, unlike Newton, takes the parallelogram construction of the composition of velocities to be a purely mathematical rather than a mechanical construction. For, according to Kant, we must determine velocity as a mathematical magnitude completely independently of any mechanical laws of motion, including Newton's Second Law. In the case of mass or quantity of matter Kant also holds that we must be able to determine it as a mathematical magnitude independently of the Second Law, and, as I have explained, he does not even bother to formulate this law among his own Laws of Mechanics. Nevertheless, as I have also explained, the concept of mass or quantity of matter, unlike that of velocity, cannot be constructed purely mathematically, and, for precisely this reason, Kant's account of its mathematization instead appeals to the metaphysical concepts of substance, action, and force. The result, so far, is a reciprocal relationship between the conservation of the total quantity of matter, on the one side, and the conservation of the total quantity of motion, on the other.

I shall return to the comparison of Newton and Kant on these questions below, after I have discussed Kant's Second and Third Laws of Mechanics. For now, however, I shall confine myself to a remark on the relation of Kant's First Law to the Leibnizean tradition. The circumstance that Kant formulates a conservation principle as his first mechanical law locates him squarely within this tradition – as does, more generally, his overriding focus on the metaphysical concept of substance. It is important

[89] The importance of the analogies of experience in this respect is emphasized in the Introduction: see the paragraph to which note 35 thereof is appended, together with the following paragraph. The remainder of the Introduction focusses on Kant's conception of the application of mathematics to empirical concepts, including the concept of mass or quantity of matter.

to emphasize, however, that Kant's proof of his First Law also involves a radical break with the Leibnizean tradition, in so far as it entirely depends on the idea that material substance (*substantia phaenomenon*) is necessarily infinitely divisible and therefore in no way simple. Moreover, Kant explicitly argues, at the same time, that a purely mental or mind-like being cannot possibly be a candidate for the permanence of substance in his sense.[90] Finally, as explained, Kant is here using Leibnizean metaphysical resources (as is typical with him) to provide a metaphysical foundation for specifically Newtonian mathematical physics – in this case, a metaphysical foundation for the characteristically Newtonian connections among the three conceptually quite different notions of quantity of matter, weight, and mass.

26 INERTIA

The third proposition of the Mechanics is Kant's "*Second Law of Mechanics*" (543): "All changes of matter have an external cause." Immediately after stating this law Kant inserts in parenthesis: "Every body persists [*beharrt*] in its state of rest or motion, in the same direction and with the same speed, if it is not compelled by an external cause to leave this state" (543). He thus makes it clear that he intends a close connection between his own Second Law of Mechanics and what is commonly called the law of inertia – which Newton formulates as his First Law of Motion (P416): "Every body perseveres in its state of being at rest or moving uniformly in a straight line, except insofar as it is compelled to change its state by impressed forces." Kant continues to emphasize this connection in the conclusion of the following proof, which again inserts a parenthetical addition (543): "Hence every change of a matter is based on external causes (i.e., a body persists, etc.)." When we look at the proof itself, however, we find something quite different from Newton's First Law, at least with respect to the overriding emphasis of Kant's discussion:

(From general metaphysics we take as basis the proposition that every change has a *cause*, and here it is only to be proved of matter that its change must always

[90] Watkins (1998, §3) rightly argues that Kant's formulation of a conservation principle as his First Law of Mechanics locates him squarely within the Leibnizean tradition, and Watkins adds illuminating details concerning Kant's place within this tradition. However, although he also rightly stresses that the proof of Kant's conservation principle depends on the infinite divisibility of material substance, Watkins does not sufficiently emphasize, in my view, that precisely this aspect of Kant's proof entails a radical break from the fundamental Leibnizean commitment to substantial simplicity.

have an *external cause*.) Matter, as mere object of the outer senses, has no other determinations except those of external relations in space, and [it] therefore undergoes no change except by motion. With respect to the latter, as change of one motion into another, or of a motion into rest, or conversely, a cause must be found (by the principle of metaphysics). But this cause cannot be internal, for matter has no absolutely internal determinations or grounds of determination. Hence every change in a matter is based on external causes (that is, a body persists, etc.). (543)

Kant, unlike Newton, focusses on the metaphysical notion of cause rather than the mathematical notion of impressed force. Moreover, Kant does not explicitly argue for what we take to be most essential to the law of inertia: that the state preserved by a body under the influence of no external causes is one of either rest or uniform rectilinear motion. Kant instead emphasizes that all determinations of matter are external rather than internal, so that any *change* of such determinations must have an external rather than an internal ground.

The way in which Kant begins this proof – by distinguishing between general metaphysics (transcendental philosophy) and special metaphysics as the pure doctrine of corporeal nature – suggests that the former considers changes and causes in general, internal as well as external, whereas the latter is restricted to external causes ("grounds of determination").[91] The way in which Kant further explains his proof in the following remark confirms this idea:

The inertia of matter is, and means, nothing else than its *lifelessness*, as matter in itself. *Life* is the capacity of a *substance* to determine itself to act from an *internal principle*, of a *finite substance* to change, and of a *material substance* [to determine itself] to motion or rest, as change of its state. Now we are acquainted with no other internal principle in a substance for changing its state except *desiring*, and no other internal activity in general except *thinking*, together with that which depends on it, the *feeling* of pleasure or displeasure, and *desire* or willing. But these actions and grounds of determination in no way belong to the representations of the outer senses, and so neither [do they belong] to the determinations of matter as matter. Hence all matter, as such, is *lifeless*. The principle of inertia says this, and nothing more. (544)

It would appear, then, that general metaphysics or transcendental philosophy considers all substances in general, living as well as non-living, whereas the special metaphysics of corporeal nature considers only non-living material substances – that is, matter as such ("as mere object of

[91] Kant first draws the distinction between general and special metaphysics in the Preface (469–70): see the passage to which note 36 of the Introduction is appended.

the outer senses"). It is for this reason that the general principle of causality must be restricted to specifically external causes here.

An important passage from §5 of the published Introduction to the *Critique of the Power of Judgement* makes the same distinction between causes in general, on the one side, and a further restriction to specifically external causes, on the other – this time in terms of a contrast between "transcendental" and "metaphysical" principles:

A transcendental principle is that through which is represented a priori the universal condition under which alone things can be objects of our cognition in general. By contrast, a principle is called metaphysical if it represents a priori the condition under which alone objects, whose concept must be empirically given, can be further determined a priori. Thus, the principle of the cognition of bodies as substances and as changeable substances is transcendental, if it is thereby asserted that their changes must have a cause; it is metaphysical, however, if it is thereby asserted that their changes must have an *external* cause. This is because in the first case bodies may be thought only by ontological predicates (pure concepts of the understanding), e.g., as substance, in order to cognize the proposition a priori; but in the second case the empirical concept of a body (as a movable thing in space) must be taken as the basis of the proposition – however, as soon as this is done, that the latter predicate (motion only by external causes) belongs to body can be comprehended completely a priori. (5, 181)

So Kant is here alluding to the present discussion in the Mechanics of the *Metaphysical Foundations*, and he is explaining how his Second Law of Mechanics, in particular, restricts the general transcendental principle of the second analogy to the action of specifically external causes.[92]

Both the proof of the third proposition and its accompanying remark are wholly devoted to this restriction – to the idea (544) that "all matter, as

[92] In the terminology of the *Metaphysical Foundations* (469–70), therefore, what Kant calls a "transcendental" principle in the *Critique of the Power of Judgement* belongs to general metaphysics ("the *transcendental* part of the metaphysics of nature"), whereas a "metaphysical" principle belongs to special metaphysics. The restriction Kant emphasizes in this connection points to an important asymmetry between the way in which his First Law of Mechanics realizes the first analogy and the way in which his Second Law realizes the second. In the former case the "essential" step in the proof – the idea that (542) material substance "is possible only in space and in accordance with its conditions, and thus possible only as object of the *outer* senses" – introduces no new restriction in comparison with general metaphysics, for Kant apparently thinks that the general principle of the first analogy is subject to a parallel restriction: see note 62 above, together with the passage to which it is appended, along with note 63 above. It would seem, then, that *all* phenomenal substances are in this sense spatial or material. The reason that an additional restriction is needed in the case of the second analogy, it appears, is that we can still distinguish *living* from *non-living* material substances within the phenomenal world of general metaphysics. Although purely mental substances are of course still thinkable (by "ontological predicates") within transcendental philosophy, they form no part of the phenomenal world at all. I shall return to this issue in the Conclusion.

such, is [essentially] *lifeless*," so that no internal grounds of determination are permitted. The remark concludes by insisting that "[t]he principle of inertia says this, and nothing more" (544). By contrast, Kant nowhere explains how the positive part of the law of inertia – that a body acted upon by no external causes will move uniformly (possibly with zero velocity) and rectilinearly – is itself to be proved. We know from the second explication of the Phoronomy, for example, that Kant defines the "motion of a thing" as "the change of its *external relations* to a given space" (482; see note 11 of my chapter on the Phoronomy). But the proof of the third proposition of the Mechanics simply assumes without argument that such a "change" does not count as a genuine change of *state* if the motion in question is uniform and rectilinear, so that, in this case, no "external cause" is required.[93] It is only if such a "change of external relations" itself changes (i.e., the body undergoes an acceleration) that must we look for an external cause.[94] So the first question we need to ask ourselves, before we examine Kant's conception of the essential "lifelessness" of matter, concerns what entitles him to make this assumption here.

Kant's justification for this assumption – in so far as he has one at this point – rests on the Copernican conception of space and motion introduced in the first explication of the Phoronomy. Here, as explained, Kant characterizes matter as the movable in space and thereby refers back to the fundamental contention in the Preface that "[t]he basic determination of something that is to be an object of the outer senses must be motion" (476). Kant then indicates, in the first remark to the first explication of the Phoronomy, that speed and direction are the two basic properties of a motion – or, as he puts it in the third remark to the second explication (483), "[i]n every motion direction and speed are the two moments for considering it." The characterization of matter as the movable in space therefore implies that motion is a determination or accident of a body, and that both speed and direction serve to individuate such determinations.

[93] Kant touches on this point in a footnote to the second analogy (A207/B252): "One should note that I do not speak of the change of certain relations in general, but rather of a change of state. Therefore, when a body moves uniformly it does not change its state (of motion) at all; but [it does so] when its motion increases or decreases."

[94] Kant clarifies this point in his application of the law of inertia to circular (or curvilinear) motion in the second proposition of the Phenomenology. Since rectilinear motion is "a continuous change of relation with respect to the external space, circular motion is a change of the change of these external relations in space, and therefore a continuous arising of new motions" (557). Therefore, since "according to the law of inertia, a motion, in so far as it arises, must have an external cause … every body in circular motion proves [*beweiset*] by its motion a moving force" (557). I shall discuss this passage in detail in my chapter on the Phenomenology.

So a body moving (changing its relations to an external space) without changing its speed and direction does not change its determination (of motion), whereas a body that changes either one is changing its determination. Kant alludes to these considerations in the proof of the third proposition of the Mechanics when he characterizes the relevant "change [in a body] by motion" as a "change of one motion into another, or of a motion into rest, or conversely" (543). When a body moves uniformly and rectilinearly, then, one motion is not changed into another, or, as Kant puts it elsewhere, no *new* motion arises; the body does not change its state.[95]

A closely related point is that Kant's Copernican conception of space and motion culminates, in the Phoronomy, in an explicit principle of relativity (487): "Every motion, as object of a possible experience, can be viewed arbitrarily as motion of the body in a space at rest, or else as rest of the body, and, instead, as motion of the space in the opposite direction with the same speed." Accordingly, as discussed in section 5 above, the concepts "motion of the body in a space at rest" and "rest of the body and … motion of the space in the opposite direction with the same speed" can be termed really or empirically equivalent on Kant's view (488; see the paragraph to which note 63 of my chapter on the Phoronomy is appended). Motion and rest are thus equivalent or interchangeable determinations, and it is precisely this equivalence that lies at the heart of the modern conception of inertial motion – according to which the "natural" state of a body (acted upon by no external forces) is not a state of rest but one of *either* rest *or* uniform rectilinear motion.

As I also explained in section 5, however, Kant's principle of the relativity of motion articulated in the Phoronomy suffers from a serious lack of specificity with respect to precisely what we now understand as the law of inertia. For, on the one hand, Kant explicitly restricts his principle to rectilinear motions without yet explaining why circular (or curvilinear)

[95] Compare again notes 93 and 94 above. Compare also the related discussion in the first analogy (A187/B230–31):

> Now a correction of the concept of *alteration* [*Veränderung*] is based on this permanence. Arising and perishing are not alterations of that which arises or perishes. Alteration is a manner of existing which follows upon another manner of existing of precisely the same object. Therefore, everything that undergoes an alteration is *enduring* [*bleibend*], and only its *state changes* [*wechselt*]. Since this change [*Wechsel*] thus only concerns the determinations, which can cease to be or begin to be, we can then say, in a somewhat paradoxical seeming expression: only the permanent (substance) is altered, the changeable suffers no alteration, but rather a *change*, where some determinations cease to be and others begin to be.

This passage, however, considers only changes of state in general and is silent on the question of *motion* as a state.

motions are not similarly relative. And, on the other hand, in thus restricting his principle to rectilinear motions, Kant does not also restrict it to *uniform* rectilinear motions. Hence, Kant's principle of the relativity of motion is not identical to what we now call Galilean relativity – according to which all inertial reference frames (moving uniformly and rectilinearly relative to one another) are equivalent with respect to the laws of motion. Kant's lack of specificity in the Phoronomy is a direct consequence of the fact that he deliberately abstracts from all consideration of laws of motion at this stage and explicitly postpones such consideration until the Mechanics. As a result, the Phoronomy leaves us with no well-defined frame of reference (or class of such frames) relative to which the law of inertia is supposed to hold. In sum, Kant has not yet given a clear and unambiguous sense to the idea that a motion – a change of external relations to a *given* space – is, or is not, rectilinear and uniform.[96]

It is helpful to contrast Kant's procedure with Newton's formulation of the law of inertia in the *Principia*. For, in the first place, Newton begins by characterizing the motions he considers with respect to absolute space and absolute time in the Scholium to the Definitions, so that, in particular, to say that a body moves uniformly and rectilinearly is to say that it does so relative to absolute space and absolute time.[97] And, in the second place, Newton's principle of the relativity of motion – Corollary 5 to the Laws of Motion – is precisely what we now call the principle of Galilean relativity (P423): "When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly in a straight line without circular motion." Newton

[96] I shall return to Kant's later reconsideration of the relativity of motion, together with his restriction to rectilinear (but not *uniform* rectilinear) motion, in my chapter on the Phenomenology.

[97] Recall the discussion in the Scholium (P408): "Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration … Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable." Newton then characterizes absolute motion similarly (P409):

> Absolute motion is the change of position of a body from one absolute place to another; relative motion is change of position from one relative place to another. Thus, in a ship under sail, the relative place of a body is that region of the ship in which the body happens to be or that part of the whole interior of the ship which the body fills and which accordingly moves along with the ship, and relative rest is the continuance of the body in that same region of the ship or same part of its interior. But true rest is the continuance of a body in the same part of that unmoving space in which the ship itself, along with its interior and all its contents is moving.

On the following page Newton makes it clear that the exact uniformity (or lack thereof) of a motion depends on the underlying (true) uniformity of time itself (P410): "It is possible that there is no uniform motion by which time may have an exact measure. All motions can be accelerated or retarded, but the flow of absolute time cannot be changed."

derives this principle from his Laws of Motion, and so the law of inertia, in particular, holds in all such relative spaces or frames of reference.

Kant, by contrast, rejects absolute space and time, and he also deliberately abstracts in the Phoronomy from all consideration of mechanical laws of motion. He explicitly postpones such consideration to the present Mechanics, where it turns out, as explained in section 13 above, that the "proper place" for doing this is the discussion of Kant's Third Law of Mechanics in the important footnote to the proof of the fourth proposition (547; see note 30 above, together with the paragraph to which it is appended). Now, in the context of his *third* proposition (Kant's Second Law of Mechanics), this crucial consideration is not yet in place, and it is for this reason, above all, that Kant is not yet in a position to give a clear and unequivocal meaning to the notion of uniform rectilinear motion. Kant now devotes all of his energy to showing that the relevant cause of any "changes of matter" must be external, and he confines what I have called the positive part of the law of inertia – the explanation of how matter will behave in the absence of an external cause – to a parenthetical aside.

The overriding emphasis of Kant's proof is on the *lifelessness* of all matter, "as matter in itself" (544). Since life denotes an "*internal principle*" of action, and no such internal principle can belong to "the determinations of matter as matter," "all matter, as such, is [essentially] *lifeless*" (544). Kant concludes:

> From this very same concept of inertia, as mere *lifelessness*, it follows at once that it does not mean a *positive striving* to preserve its state [*ein positives Bestreben seinen Zustand zu erhalten*]. Only living beings are called inert [*träg*] in this latter sense, because they have a representation of another state, which they abhor, and against which they exert their force. (544)

This conception of inertia as the essential lifelessness of matter thus initiates a sustained polemic against the notion of a "force of inertia (*vis inertiae*)" in the Mechanics. One of Kant's main complaints is that this notion undermines the true basis for the phenomenon of the communication of motion, as presented or "constructed" in his own Third Law of Mechanics. Whereas the communication of motion essentially involves the action of one body on another (and vice versa), the inertia of matter involves no *activity* at all but only a purely passive "mere inability to move itself from itself" (551).[98]

[98] Kant alludes to this point at the beginning of his remark to the proof of the Second Law of Mechanics (544): "This mechanical law must alone be called the law of *inertia* (*lex inertiae*); the law of an equal and opposite reaction for every action cannot bear this name. For the latter says

This complaint applies, in particular, to Kant's earlier conception of inertia in the *Physical Monadology*. Following Newton's famous discussion of "inertial force [*vis inertiae*]" or "innate force [*vis insita*]," Kant there introduces this force as follows (2, 485): "A body in motion colliding with another would have no efficacy, and would be brought by an infinitely small obstacle into a state of rest, if it did not have a force of inertia [*vis inertiae*], by which it strives to persevere in its state of motion [*qua in status movendi perseverare annititur*]."[99] Even more striking, however, is the way in which Kant characterizes this same "force of inertia" in the contemporaneous *New Exposition*. Here, following Newton, Kant uses the terminology of "innate force [*vis insita*]":

> [T]he innate force [*vis insita*], modified by the impact of an external body, resists the collision, from an internal principle of activity [*ex interno efficaciae principio*], with just as much force as it acquires in the direction in which it is impelled. Therefore, all the reality of forces in the phenomenon of motion is equal to that which is already innate [*insitum*] in the body at rest, although the internal power [*interna potestas*], which was indeterminate with respect to direction at rest, is merely directed by the external impulse. (1, 408)[100]

Just as in the *Physical Monadology*, then, Kant here introduces the "force of inertia" directly in connection with the communication of motion, and he also conceives it as a fundamentally active power or principle. Unlike in the *Physical Monadology*, however, Kant also explicitly characterizes it as an "*internal* power" or "*internal* active principle" (emphasis added).

This is important because it is a central feature of the monadic conception of substance articulated in both pre-critical works that one must sharply distinguish between two essentially different types of determinations, internal [*determinationes internae*] and external [*determinationes externae*]. The former belong to the ultimate simple substances solely as a consequence of their individual existence, entirely independently of any other simple substances that may or may not exist. The latter, by contrast, depend on the *co-presence* or *coexistence* of a number of simple substances in a common world, and they are therefore necessarily relational. The two fundamental forces of attraction and repulsion are paradigmatic of (the phenomenal manifestation of) such external determinations, and space

what matter does, but the former only what it does not do, which is more appropriate to the term *inertia* [that is, inactivity – MF]."

99 For my earlier discussion of this passage see note 129 of my chapter on the Dynamics, together with the paragraph to which it is appended.

100 This passage occurs in an elucidation of "the conservation of reality" from cause to effect discussed in the previous section: see the paragraph to which note 78 above is appended.

itself, in its guise as phenomenal co-presence, is constituted by them. Indeed, as explained in section 13 above, it is in virtue of precisely this distinction between internal and external determinations that the *Physical Monadology* attempts to show that the infinite divisibility of space is compatible with the absolute simplicity of the underlying monads.[101] What Kant is saying in the passage from the *New Exposition* quoted above (1, 408), therefore, is that the "force of inertia," unlike the two fundamental forces of attraction and repulsion, belongs to the internal determinations of material substances – those which already inhere in such a substance solely as a consequence of its individual existence independently of any other simple substances that may or may not exist.[102]

In the *New Exposition*, moreover, Kant formulates a complementary principle to "The Principle of Coexistence" (Proposition XIII; note 101), "The Principle of Succession" (Proposition XII), which specifies how simple substances will behave when they are abstracted from their external relations with one other (1, 410): "No change can happen to substances, except in so far as they are connected with others; their reciprocal dependence on one another determines their mutual changes of state." Kant concludes: "Hence a simple substance exempt from every external connection, and thus left entirely alone, is completely immutable in itself" (1, 410). Thus we here have a generalized version of the law of inertia, according to which a simple substance considered entirely on its own can do nothing but preserve unchanging whatever state it happens to be in.

The role of the distinction between internal and external determinations becomes clear in the following proof:

Suppose that some simple substance were to exist alone, dissolved from the connection with others; I say that it could undergo no change of its internal state. Since the internal determinations that already belong to the substance are posited through internal grounds with the exclusion of the opposite, if you want

[101] See note 76 of my chapter on the Dynamics, together with the paragraph to which it is appended. I there refer to the discussion of "The Principle of Coexistence" (Proposition XIII) in the *New Exposition*. Compare also the discussion of the identity of indiscernibles in number 2 of Proposition XI of the *New Exposition* (1, 409–10).

[102] In the *Physical Monadology* Kant calls the two fundamental forces of attraction and repulsion, as well as the force of inertia, "innate." So "innate" is certainly not synonymous with "internal." It here means that a force properly belongs to the material substance itself – and is not, in the case of the force of attraction, for example, explained away by mechanical causes (e.g., by external pressure). Thus the introductory section characterizes universal gravitation as a force that is "innate in bodies at rest and active at a distance" (1, 476; see again the paragraph to which note 17 of the Introduction is appended). Yet the fundamental force of attraction is paradigmatic of an *external* determination of the material substance to which it nonetheless properly ("innately") belongs.

> another determination to succeed, you must also posit another ground. But since the opposite of this ground is internal to the substance, and no external ground is added, by supposition, it is obvious that [the new determination] cannot be introduced into the being. (1, 410)

It follows from the principle of sufficient reason, then, that a simple substance existing on its own has only internal determinations, which, as such, can never admit the opposite of the determinations already (internally) grounded.[103] Any simple substance, in this sense, strives solely to persevere in its own internal state.

In the elucidation of this proposition, accordingly, Kant rejects the Leibnizean–Wolffian conception of force as the ground of *change*:

> This truth depends on an easily understood and infallible chain of grounds. Nonetheless, those who give the Wolffian philosophy its renown, have paid so little attention to this truth that they maintain, on the contrary, that a simple substance is subject to continual changes from an internal principle of activity [*principio activitatis interno*]. Although I for my part am thoroughly familiar with their arguments, I am, nonetheless, convinced of their sterility. For once they have constructed an arbitrary definition of force so that it means that which contains the ground of *changes*, when one ought to declare that it contains the ground of *determinations*, they were bound to fall headlong into error. (1, 411)[104]

There can be very little doubt, therefore, that what Kant calls "innate force [*vis insita*]" in the *New Exposition* and "force of inertia [*vis inertiae*]" in the *Physical Monadology* is precisely (the phenomenal manifestation of) an internal ground of (unchanging) internal determinations in this sense. It is (the phenomenal manifestation of) an "internal principle of activity" (1, 408) by which every simple substance or monad, considered independently and on its own, "strives to persevere" (2, 485) in whatever internal state it finds itself.[105]

[103] Thus the principle of succession, as Kant says, is a consequence of the principle of the determining ground (or reason): see note 81 above, together with the paragraph to which it is appended.

[104] The editor of Kant (1992, p. 421) cites both Wolff and Baumgarten in an explanatory note to this passage.

[105] Space and motion in the pre-critical period exist only as the phenomenal manifestations of the external relations among such substances in a common world. Kant here adds (1, 410): "[I]n a world that was free from all motion (for motion is the phenomenon of a changed [external] connection), no succession at all would be found even in the internal states of substances. Hence, if the [external] connection of substances were abolished, succession and time would likewise disappear." It appears, then, that just as *vis viva*, on the Leibnizean–Wolffian conception, is the phenomenal manifestation of the fundamental internal active force of simple substances by which they determine the *changes* of their internal state, Newtonian *vis inertiae*, for the pre-critical Kant, is the phenomenal manifestation of the fundamental internal active force of simple substances by which they rather determine the *preservation* of their internal state.

From the point of view of Kant's critical conception of material substance and force, therefore, the monadological conception defended in the *New Exposition* and *Physical Monadology* contradicts precisely the principle of the essential *lifelessness* of all matter that Kant now takes to be central to the law of inertia. This becomes especially clear when we recall Kant's fundamental criticism of the "Leibnizean monadology" in the transcendental amphiboly:

> Only that is internal in an object of pure understanding which has no relation at all (with respect to its existence) to anything different from itself. By contrast, the internal determinations of a *substantia phaenomenon* in space are nothing but relations, and it itself is nothing but a totality of mere relations. We are only acquainted with substance in space through forces that are active in space, either driving others into [this space] (attraction) or stopping their penetration into it (repulsion and impenetrability). We are acquainted with no other properties constituting the concept of a substance which appears in space and which we call matter. As object of the pure understanding, on the other hand, every substance must have internal determinations and powers, which pertain to [its] internal reality. However, what can I entertain as internal accidents except those which my inner sense presents to me – namely, that which is either itself a *thought* or is analogous to it? Therefore, Leibniz, after he had taken away everything that may signify an external relation, and therefore also *composition*, made of all substances, because he represented them as noumena, even the constituents of matter, simple substances with powers of representation – in a word, **monads**. (A265–66/B321–22)[106]

For the critical Kant, therefore, any essentially monadic conception of substance, including his own earlier conception in the *New Exposition* and *Physical Monadology*, is committed to the view that the purely internal determinations and powers of such substances must be conceived on the

[106] This is the entire passage from which the first four sentences are quoted in note 77 of my chapter on the Dynamics. Compare the parallel passage (A274/B330) also quoted there, together with a similar later passage (A282–83/B338–39):

> According to mere concepts the internal is the substratum of all relations or external determinations. Therefore, if I abstract from all conditions of intuition, and remain simply with the concept of a thing in general, I can abstract from all external relations and I must nevertheless be left with a concept of that which signifies no relation at all, but merely internal determinations. And it now appears as if it followed that in every thing (substance) there is something which is absolutely internal and precedes all external determinations, in so far as it makes them first possible, and therefore [that] this substratum is something that contains no further external relations in itself (for corporeal things are always only relations, at least of parts external to one another). Moreover, because we are acquainted with no absolutely internal determinations except those by means of our inner sense, this substratum is not only simple, but also (in accordance with the analogy with our inner sense) determined by *representations* – i.e., all things would actually be *monads* or simple beings endowed with representations.

model of "*thinking*, together with that which depends on it, the *feeling* of pleasure, and *desire* or willing" (544). Such a conception is committed, therefore, to just those purely internal determinations, which, according to Kant's present version of the law of inertia, can "in no way belong to the representations of the outer senses, and so neither [do they belong] to the determinations of matter as matter" (544). Indeed, Kant's critical proof of this law entirely rests on the idea that matter, as such, "has no absolutely internal determinations or grounds of determination" (543) but only what the amphiboly calls *comparatively* internal determinations.[107]

It follows that there is a close connection, in this respect, between Kant's critical version of the law of inertia and his principle of the permanence of substance. For, to say that the properties of matter are only comparatively internal is to say that they always depend on the parts of matter (external to one another) – and thus on the parts of these parts and so on *ad infinitum*. It is to say that matter consists of a continuous spatially extended aggregate of movables in just the sense required by Kant's proof of the permanence of material substance. So Kant's rejection of the absolute simplicity of substance (and thus of the point-like physical monads of the pre-critical period) goes hand in hand with a parallel rejection of the idea that material substance, as material, can be in any way mind-like or representational. And this implies, in accordance with Kant's terminology in the remark to his second proposition, that material substances, as material, must be essentially lifeless as well. Kant's First and Second Laws of Mechanics therefore ultimately converge on the same point: a decisive rejection of the "hylozoism" Kant now sees as unavoidable on any monadic conception of material substance (544): "The possibility of a proper natural science rests wholly and completely on the law of inertia (together with that of the permanence of substance). The opposite of the former, and thus the death of all natural philosophy, would be *hylozoism*."[108]

[107] See the discussion in the amphiboly (A277–78/B333–34) quoted in the paragraph to which note 255 of my chapter on the Dynamics is appended.

[108] The more general point, as Kant explains in the second remark to the fourth proposition of the Dynamics, is that the monadological conception of substance, properly understood, "has nothing at all to do with the explanation of natural appearances, but is rather an intrinsically correct *platonic* concept of the world devised by *Leibniz*, in so far as it is considered, not at all as object of the senses, but as thing in itself, and is merely an object of the understanding" (507; see the paragraph following the one to which note 3 of my chapter on the Dynamics is appended). By contrast, if we attempt to combine the monadological conception of substance with natural philosophy (as Kant himself had done in the *New Exposition* and *Physical Monadology*), we necessarily commit a "transcendental amphiboly" that upsets all sound use of the understanding in the investigation of natural phenomena.

In the case of both his First and Second Laws of Mechanics, therefore, Kant focusses primarily on metaphysical considerations deriving from the Leibnizean tradition: the metaphysical concept of substance, together with the distinction between material (composite) and immaterial (simple) substances, in the former; the metaphysical concept of cause or ground of determination, together with the distinction between internal and external grounds of determination, in the latter. In both cases, moreover, Kant holds that a radical transformation of this metaphysical tradition is required in order to give a proper foundation for the elements of Newtonian mathematical physics: the Newtonian concept of mass or quantity of matter, in the former; the Newtonian understanding of the mathematical law of inertia (as involving uniform rectilinear motion in a privileged relative space or reference frame), in the latter. In both cases, finally, Kant maintains that such a metaphysical underpinning is unavoidably required in order to give a properly "constructive" account of the application of mathematics to the empirical concepts of physics. I began to explain how this works in the case of mass or quantity of matter at the end of the last section. I shall explain how Kant's metaphysical foundation for the law of inertia is intimately bound up with the determination of a privileged frame of reference – in which motion is "reduced to absolute space" (545) – in the next.

27 ACTION AND REACTION

The fourth proposition of the Mechanics is a principle of the equality of action and reaction, which Kant calls his "*Third mechanical law*" (544): "In all communication of motion action and reaction are always equal to one another." Of all Kant's three Laws of Mechanics this one is closest, in its explicit formulation, to a Newtonian Law of Motion, in this case the Third Law (P417): "To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and opposite in direction." Although, as explained below, there are significant differences between Kant's and Newton's versions of this law, it is clear that Kant has Newton's Third Law firmly in mind. For, following upon the proof of the proposition, Kant begins his first remark as follows (549): "This, then, is the construction of the communication of motion, which, at the same time, brings with it, as its necessary condition, the law of the equality of action and reaction – which [law] Newton by no means dared to prove a priori, but rather therefore appealed to *experience*."[109] So we can reasonably infer, first, that Kant takes himself to be proving a priori essentially the same law that Newton formulates as the Third Law of Motion, and, second, that an important element of this proof is a mathematical construction, which Kant calls "the construction of the communication of motion." Kant's language also suggests that the present proposition is central and fundamental to the Mechanics as a whole, whose subject in general is precisely the communication of motion.

Kant introduces his proof, as he does for the first two Laws of Mechanics, by a parenthetical appeal to the corresponding principle of pure understanding, i.e., the third analogy of experience:

> (From general metaphysics we must borrow the proposition that all external action in the world is *interaction*. Here, in order to stay within the bounds of mechanics, it is only to be shown that this interaction (*actio mutua*) is at the same time *reaction* (*reactio*); but I can still not wholly leave aside this metaphysical law of community here, without detracting from the completeness of the insight.) (544–45)[110]

This parenthetical remark suggests that the relationship between the present principle of special metaphysics and the corresponding principle of general metaphysics is weaker than in the previous two cases. For in these cases Kant says that the corresponding transcendental principle is

[109] Newton's experimental arguments for the Third Law of Motion occur in the Scholium to the Laws of Motion (compare notes 36, 42, 76, 78, and 79 above). Newton's comments on this law highlight the case of impact (P417): "If some body impinging on another body changes the motion [i.e., momentum] of that body in any way by its own force, then, by the force of the other body (because of the equality of their mutual pressure), it also will undergo the same change in its own motion in the opposite direction." He then adds at the end: "This law is valid also for attractions, as will be proved in the next Scholium" (P417). In the Scholium Newton first describes careful experiments on impact using pendulums (compare note 42 above, together with the paragraph to which it is appended) and remarks (P426):

> On making a test in this way with ten-foot pendulums, using unequal as well as equal bodies, and making the bodies come together from very large distances apart, say of eight or twelve or sixteen feet, I always found – within an error of less than three inches in the measurements – that when the bodies met each other directly, the changes of motions made in the bodies in opposite directions were equal, and consequently that the action and reaction were always equal.

The discussion of attractions follows (P427–28).

[110] In the first edition of the *Critique* the third analogy is entitled the "principle of community" and states (A211): "All substances, in so far as they are simultaneous, stand in thoroughgoing community (i.e., interaction among one another)." In the second edition it is entitled the "principle of simultaneity, in accordance with the law of interaction, or community," and states (B256): "All substances, in so far as they can be perceived as simultaneous in space, are in thoroughgoing interaction."

"taken as basis" (541, 543), whereas he here says only that it is "borrowed." More importantly, he suggests that the corresponding "metaphysical law of community" does not play a central role in the proof itself – one can almost, but "not wholly," leave it aside.[111]

When we examine the following proof this suggestion is amply confirmed, for Kant nowhere appeals to the "metaphysical law of community" (the third analogy) in the proof itself. Rather, the considerations to which he appeals belong entirely to the special metaphysics of corporeal nature or pure doctrine of body. These considerations primarily depend, in fact, on the definition of matter as the movable in space initiating the Phoronomy, together with the principle of the relativity of motion articulated there:

> [B]ecause all change of [matter] is motion, we cannot think any motion of a body in relation to another *absolutely at rest*, which is thereby also to be set in motion; rather, the latter must be represented as only *relatively at rest* with respect to the space that we relate it to, but as moved, together with this space, in the opposite direction with precisely the same quantity of motion in absolute space as the moved body there has towards it. For the change of relation (and thus the motion) between the two is completely mutual [*wechselseitig*]; as much as the one body approaches every part of the other, by so much does the other approach every part of the first. And, since it is here a question, not of the empirical space surrounding the two bodies, but only of the line lying between them (in that they are considered simply in relation to one another, in accordance with the influence that the motion of the one can have on the change of state of the other, abstracting from all relation to [this] empirical space), their motion is therefore considered as determinable merely in absolute space, in which each of the two must have an equal share in the motion that is ascribed to one of them in relative space, in that there is no reason to ascribe more of this motion to one than to the other. (545)

So the point, in general, is that the phenomenon of the communication of motion results in a new perspective on the problem of the relativity of space and of motion that was first introduced in the Phoronomy.

In any case of the communication of motion we are considering one body acting on another by means of its motion so as to transfer or

[111] By contrast, in the first two cases the corresponding metaphysical principle plays a central role in the respective proofs. The principle that no substance either arises or perishes is crucial for Kant's proof of the conservation of the total quantity of matter (see note 61 above, together with the paragraph to which it is appended), and the general principle of causality also figures explicitly in the proof of what Kant calls the law of inertia (see the paragraph preceding the one to which note 91 above is appended). As explained in note 92, however, there is already an asymmetry between these two cases in so far as the proof of Kant's law of inertia depends on an essential *restriction* of the corresponding principle of general metaphysics.

communicate this motion to the other body. The body to which motion is to be communicated is thereby initially conceived to be at rest. But we know from the relativity principle of the Phoronomy that there is no such thing as absolute rest and, more specifically, that it is all the same or equivalent [*einerlei*] "whether I say that a body moves in relation to [a] given space, in such and such direction with such and such speed, or I wish to think the body as at rest, and to ascribe all this, but in the opposite direction, to the space" (488). Moreover, with respect to any given motion of a body relative to a space at rest, I can also redistribute a *part* of this motion to the space instead:

> I can give a part of the given speed to the body, and the other to the space, but in the opposite direction, and the whole possible experience, with respect to the consequences of these two combined motions, is completely identical [*einerlei*] with that experience in which I think the body as alone moved with the whole speed, or the body as at rest and the space as moved with the same speed in the opposite direction. (488)

What Kant is now adding in the present proof is that, from the point of view of the Mechanics, there is actually a privileged way of doing this after all – and thus a privileged relative space or reference frame for considering the motions of both bodies involved in any case of the communication of motion.[112] This yields the point of view of what Kant calls "absolute space," relative to which both bodies have "an *equal* share in the motion" (545, emphasis added).

Kant clarifies the precise meaning of the concepts of absolute space and of having an *equal* share in the motion in the following remarks leading up to his "construction" (545–46):

> On this basis the motion of a body A with respect to another body B at rest, in regard to which it can thereby be moving, is reduced to absolute space; that is, as a relation of acting causes merely related to one another, [this motion] is so considered that both have an equal share in the motion which, in the appearance, is ascribed to body A alone – which can only happen in such a way that the speed ascribed in relative space to body A alone is apportioned between A and B in inverse ratio to their masses: to A alone its [speed] in absolute space, and to B, *together with the relative space* in which it is at rest, [its speed] in the opposite direction. The same appearance of motion is thereby perfectly maintained, but the action in the community of the two bodies is constructed in the following way.

[112] In other words, Kant is here articulating the necessary qualification of his relativity principle arising in Mechanics: see the paragraph to which note 64 of my chapter on the Phoronomy is appended, together with the following paragraph; and compare the paragraph to which note 79 of my chapter on the Dynamics is appended, together with the following paragraph.

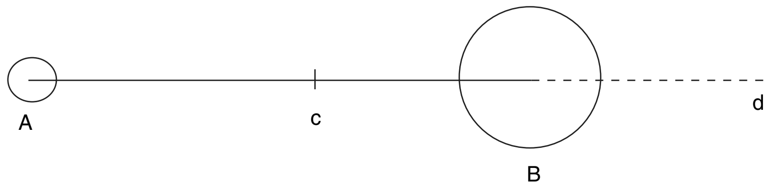

Figure 2

This construction is then illustrated by the communication of motion by impact, where A approaches B initially at rest with the velocity AB. To reduce this to absolute space means to divide the velocity AB into two parts Ac and Bc, such that Ac and Bc are inversely as the masses of A and B respectively: it is to consider the impact relative to the *center of mass* c of the two bodies. Relative to this frame of reference, Kant concludes, the two bodies are at rest after the impact, so that the velocity cA = –Ac is added to Ac and cB = –Bc is added to Bc. (Relative to the frame of reference in which B is initially at rest, by contrast, both A and B move with velocity cB = Bd after the impact.) But Ac and Bc represent equal and opposite momenta; so the momenta thereby transferred or communicated are also equal and opposite. (The same thing happens in the frame of reference in which B is initially at rest; here B acquires the momentum Bd and A the equal and opposite momentum Bc = cA.)

I shall return to the case of the communication of motion by impact below. We already know, however, that the communication of motion in general is by no means confined to impact and, in particular, that it also takes place in cases of attraction (section 22 above). It is no wonder, then, that Kant concludes his proof by generalizing the argument to all cases of the communication of motion as such:

> [Since] the communication of motion by *impact* is different from that by *traction* [*Zug*] only in the direction, in accordance with which the matters resist one another in their motions, it follows that *in all communication of motion* action and reaction are always equal to one another (that any impact can communicate the motion of one body to another only by means of an equal contrary impact, any pressure only by means of an equal contrary pressure, and similarly any traction only by means of an equal contrary traction). (547)[113]

[113] That *traction* (*Zug*) results from attractive force (*Anziehungskraft*) follows from the wording of the second explication of the Dynamics, where Kant also characterizes the force in question as "*drawing* [*ziehende*] force" (498; compare note 12 above). In the note to this explication (which I shall discuss in more detail below), Kant explains that attraction and repulsion differ only in

Thus, for example, if we view the action of an attraction between A and B relative to their center of mass, the resulting change of momentum of A is equal and opposite to the corresponding change of momentum of B – so that the total momentum of the two bodies is unchanged. What Kant purports to have proved a priori is that the total quantity of momentum is similarly conserved in all cases of the communication of motion whatsoever.

It is illuminating to compare Kant's reasoning with Newton's discussion in Corollaries 3 and 4 to the Laws of Motion. Newton derives the conservation of the total quantity of motion in Corollary 3, where he infers it from both the Second and Third Laws of Motion. In the following Corollary 4 he goes on to assert (P421) that "the common center of gravity of all bodies acting upon one another (excluding external actions and impediments) either is at rest or moves uniformly forward in a straight line."[114] In support of this last conclusion Newton begins by pointing out (P422) that "in a system of two bodies acting on each other, since the distances of their centers from the common center of gravity are inversely as the bodies, the relative motions of these bodies, whether of approaching that center or of receding from it, will be equal" (P422). Indeed, the momenta of any two bodies whatsoever along a given straight line – *whether they are interacting or not* – are always equal in magnitude relative to their center of mass along this line.[115] In the following sentence, however, Newton considers changes of momentum in accordance with the Third Law of Motion: "Accordingly, as a result of equal changes in opposite directions in the motions of these bodies, and consequently as a result of the actions of the bodies on each other, that center is neither accelerated nor retarded nor does it undergo any change in its state of motion or rest" (P422). The point is that two bodies *act* on one another, for Newton, when they exert impressed forces on one another. These forces are equal and opposite by the Third Law, and therefore the changes

the direction of action along a given line – just as he does here and in the analogous passage in the remark to the first explication of the Mechanics (see the paragraph to which note 4 above is appended).

114 See note 78 above, together with the paragraph to which it is appended and the following paragraph. The first sentence of the proof of Corollary 3 makes the dependence on the Second and Third Laws of Motion explicit (P420): "For an action and the reaction opposite to it are equal by Law 3, and by Law 2 the changes which they produce in motions are equal and in opposite directions."

115 The distances from the center of mass are always inversely as the masses, by definition, so the velocities relative to this center – i.e., the changes of distance from the center in a given time – must also be inversely as the masses.

in momenta of the two bodies are also equal and opposite by the Second Law. The total momentum of the two bodies – and hence the motion of their center of gravity – is necessarily unchanged.

One important difference between Newton's and Kant's versions of the equality of action and reaction, therefore, is that Newton's principle governs impressed forces in the sense of Definition 4 of the *Principia*, and it is then applied to changes of motion (momentum) by means of the Second Law (compare note 13 above).[116] For Kant, by contrast, the principle is formulated directly in terms of changes of motion rather than impressed forces, and, as explained, Kant does not even state the Second Law in any case (see note 15 above). Moreover, the principle of the equality of action and reaction, for Kant, is essentially involved in a procedure for "constructing" the communication of motion between any system of interacting bodies by constructing a kind of surrogate, as it were, for Newtonian absolute space. In the proof of the fourth proposition of the Mechanics, in particular, the construction depends on the contention (545) that "[a]ll *active* relations of matters *in space*, and all changes of these relations, in so far as they may be *causes* of certain action or effects [*Wirkungen*], must always be represented as mutual." It is precisely here, at the beginning of his argument, that considerations going beyond the merely phoronomical consideration of relative motions essentially enter into Kant's proof.

The importance of these extra-phoronomical considerations is emphasized even more explicitly in the long footnote appended to Kant's proof of the fourth proposition.[117] He there contrasts the cases of Phoronomy and Mechanics by asserting that in Mechanics "another concept of the quantity of motion now comes into play, namely, not that which is thought merely with respect to space, and consists only in the velocity,

[116] Definition 4, together with its explanatory comment, reads as follows (P405):

> *Impressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward*. This force consists solely in the action and does not remain in a body after the action has ceased. For a body perseveres in any new state solely by the force of inertia. Moreover, there are various sources of impressed force, such as percussion, pressure, or centripetal force.

Thus the "actions" referred to in the statement of the Third Law (P417) are precisely impressed forces in this sense. In his comment to *this* statement Newton adds: "By means of these actions, equal changes occur in the motions, not in the velocities – that is, of course, if the bodies are not impeded by anything else" (P417). An editorial footnote explains: "By 'body' Newton means quantity of matter or mass (def. 1) and by 'motion' he means quantity of motion (def. 2) or momentum" (P417).

[117] See again the paragraph to which note 79 of my chapter on the Dynamics is appended, and compare note 30 above, together with the paragraph to which it is appended.

but rather that whereby the quantity of substance (as moving cause) must be brought into the calculation at the same time; and here it is no longer arbitrary, but rather *necessary*, to assume each of the two bodies as moved, and, indeed, with equal quantity of motion in the opposite direction" (547). Kant continues:

> For one [body] cannot act on the other through its own inherent motion, except either in approach by means of repulsive force, or in withdrawal by means of attraction. Since both forces always act mutually and equally in opposite directions, no body can act by means of them on another body through its motion, without just as much reaction from the other with the same quantity of motion. Hence no body can impart motion to an *absolutely resting* body through its motion; rather, the latter must be moved precisely with the same quantity of motion (together with the space) in the opposite direction as that which it is supposed to receive through the motion of the first in the same direction. (547)

It is noteworthy, in particular, that Kant here finally brings to bear the central pure concepts of the understanding employed in the third analogy that do not occur in the proof itself: namely, the concepts of substance and (mutual) interaction.

The third analogy governs the behavior of substances existing external to one another (in space) and states (in the second edition) that "[a]ll substances, in so far as they can be perceived as simultaneous in space, are in thoroughgoing interaction [*Wechselwirkung*]" (B256; see note 110 above). It goes on (also in the second edition) to characterize the notion of interaction (B257–58): "But now the relation of substances, in which the one contains determinations whose ground is in the determinations of the other, is the relation of influence [*Einfluss*], and, if the former contains the ground of the determinations of the other mutually [*wechselseitig*], [it is] the relation of community or interaction." A relation of interaction, therefore, occurs when two substances exert influences or external forces on one another *mutually*.[118] The just-quoted passage from the footnote to the fourth proposition of the Mechanics emphasizes this same condition by explaining that the two bodies in question (now explicitly characterized as substances) not only experience equal and opposite changes of momentum relative to one another but also exert equal and opposite

[118] Thus an influence is an inter-substantial force or relation of causal efficacy that one substance exerts on another substance external to it, and all changes in matter, according to Kant's version of the law of inertia, result from external causal relations in this sense. Forces and causal relations in general, by contrast, can act intra-substantially as well as inter-substantially. But the forces or causal relations considered in the third analogy are all inter-substantial. Compare note 253 of my chapter on the Dynamics.

forces (of attraction and repulsion) on one another. These (dynamical) forces, like the corresponding (mechanical) motions, "always act mutually and equally in opposite directions" (547).

It is important to see, however, that Kant's claim about equal and opposite forces (of attraction and repulsion) does not follow from the proof of the fourth proposition itself – which yields only the equal and opposite changes of momentum necessarily arising in a center of mass frame. Indeed, as explained in section 18 above, Kant already makes the relevant distinction in the remark to the seventh proposition of the Dynamics. He there considers the "law of the equality of interaction" in connection with attractive forces and says that it is "only a principle of mechanics, but not of dynamics" – in particular, "it is a law of the *motions* that follow from attracting forces, not of the proportion of the *attractive forces* themselves" (514–15). Kant illustrates the point with the magnetic attraction between two equally heavy magnets. In this case, as explained, adding a wooden box to one of the magnets moves the center of mass of the system closer to the now heavier box-plus-magnet, and it thereby changes the relative accelerations in this system accordingly, even though the corresponding magnetic *forces* are unchanged. Moreover, a parallel point can be made by considering the attraction of a magnet for a *non-magnetized* piece of iron. Here, according to the Third Law of Motion, both the magnet and the piece of iron experience changes of momentum resulting from the attraction that are necessarily equal and opposite, but the non-magnetized piece of iron exerts no magnetic force on the magnet.[119]

These considerations resurface in the second note to the fourth proposition of the Mechanics. After recapitulating the just proved "*mechanical law* of the equality of action and reaction" (548), Kant goes on to describe a corresponding *dynamical* law:

> But there is still another law of the equality of action and reaction among matters, namely a *dynamical* [one] – not in so far as one matter *communicates* its motion to another, but rather [in so far] as it originally *imparts* this motion to it and, at the same time, produces the same in itself through the latter's resistance. This can easily be shown in a similar way. For, if matter A exerts traction [*zieht*] on matter B, then it *compels* the latter to *approach* it, or, which is the same [*einerlei*], it *resists* the force with which the latter might strive to *remove* itself. But since it is all the same [*einerlei*] whether B removes itself from A, or A from B, this resistance is, at the same time, a resistance exerted by body B against A, in

[119] See notes 174 and 176 of my chapter on the Dynamics, together with the paragraphs to which they are appended. See also Tanona (2000, §§5, 6) for a detailed and helpful discussion of this point.

so far as the latter may be striving to remove itself from the former; and so traction and counter-traction are equal to one another. (548–49)

Note that Kant here says that the equal and opposite contrary motion A experiences from the mechanical resistance of B is an effect of the force "originally" exerted by A itself. A "originally *imparts* [a] motion" to B and thereby "produces the same [equal and opposite motion] in itself through the latter's resistance." Kant does not say that B also "originally imparts" a motion to A. So this "dynamical law" of the equality of action and reaction holds equally well in the case of a magnet and a non-magnetized piece of iron, where only the former actually exerts a magnetic force. This "dynamical law," in other words, really says nothing more than what has already been proved in the corresponding "mechanical law." It does not say, in particular, that there are equal and opposite "originally moving" dynamical forces exerted by *both* bodies.[120]

But precisely this is asserted in the footnote to the proof of the fourth proposition quoted above, where Kant introduces the two fundamental (dynamical) forces of attraction and repulsion into the argument and explicitly states (547) that "both forces always act mutually and equally in opposite directions."[121] These forces, as fundamental or "original," are

[120] The contrast between "mechanical" and "dynamical" is not quite the same here as it is in the passage from the Dynamics (514–15) cited in the previous paragraph. There Kant is contrasting the universally valid "mechanical" law of the equality of action and reaction with the special "dynamical" property of gravitational force that it is always proportional to the mass of the body that exerts this force. In the passage from the remark to the first proposition of the Mechanics discussed in section 24 above (see notes 44 and 45 above, together with the paragraph to which they are appended and the following paragraph) Kant returns to this latter contrast, and he explains that, although the estimation of mass using universal gravitation does indeed depend on the special "dynamical" property of gravitational force just noted, it is still "in fact mechanical, although only indirectly so" (541). It counts as "mechanical," in particular, because "the attracting body also thereby imparts to itself a velocity of its own inherent motion (by the resistance of the attracted body), which, in like external circumstances, is exactly proportional to the aggregate of its parts" (541). Thus, what makes this estimation "mechanical" is precisely the circumstance that what Kant, in the second note to the fourth proposition, calls the "dynamical" law of the equality of action and reaction still governs this particular interaction – as it indeed governs all dynamical "actions" whatsoever, even the case of magnetic force where the special "dynamical" property of gravitational force certainly does not hold.

[121] That Kant intends to invoke the two "original" forces of attraction and repulsion at this point is indicated by the circumstance that his wording mirrors that in the note to the second explication of the Dynamics cited in note 113 above. There Kant characterizes the forces of repulsion and attraction (respectively) as "the one through which the two points [on a line] *remove* themselves from one another, the second through which they *approach* one another," and concludes that "only these two kinds of forces can be thought, **as forces to which all moving forces in material nature must be reduced**" (498–99, bold emphasis added). For the demand that all other forces must be reduced to fundamental or "original" forces see the passage from the end of the general note to dynamics (534) quoted in the paragraph following the one to which note 203 of my chapter on the Dynamics is appended.

universal properties of all matter as such, and so we *can* now conclude that all the bodies involved necessarily exert them mutually on one another.[122] Therefore, if one body acts on another by means of either of the two fundamental forces, we here have a true *inter*-action in the sense of the third analogy, that is, a genuine "dynamical community" of mutual influence:

> [E]ach [of two simultaneous] substance[s] (since it can only be a consequence [in time] with respect to its determinations) must contain in itself the causality of certain determinations in the other and, at the same time, the effects of the causality of the other; that is, they must stand in dynamical community (immediately or mediately), if [their] simultaneity in any possible experience is to be cognized. But now everything is necessary with respect to the objects of experience without which experience of these objects would itself be impossible. Therefore, it is necessary for all substances in the appearance, in so far as they are simultaneous, to stand in thoroughgoing community of interaction among one another. (A212–13/B259–60)

In genuine dynamical community, then, each of the two substances involved functions as the seat of the causality of certain determinations in the other. So what we have just seen is that this last condition is in fact always fulfilled in the cases of the two fundamental forces of attraction and repulsion: each body involved is the seat of a causal action or influence determining the changes in the motion (momentum) of the other.[123]

I observed at the beginning of this section that the role of the third analogy in the argument of Kant's Third Law of Mechanics is quite different from the roles of the first two analogies in his arguments for the previous two laws. For Kant does not (and need not) appeal to the third analogy explicitly in his "construction of the communication of motion" (549). Nevertheless, Kant does say (544) that his proof "must borrow the proposition that all external action in the world is *interaction*" and

[122] By contrast, magnetism, for Kant, is a derivative force, which ultimately derives from the action of a "magnetic matter" (532) or aether: compare note 56 of my chapter on the Dynamics, and also the discussion of the "magnetic matter penetrating all bodies" acting on "attracted iron filings" in the first *Critique* (A226/B273). A magnet, on this view, causes the surrounding aether to be put into a certain state, which then causes the attracted iron filings to move – presumably by some kind of pressure.

[123] Again, in the case of the magnet and the non-magnetized piece of iron the latter does not exert a dynamical action on the former. Rather, the inertia (mass) of the latter contributes (mechanically) to the determination of the center of mass of the two bodies – relative to which their changes in momentum are then necessarily equal and opposite. Moreover, inertia, as explained in section 26 above, is not a genuine dynamical force or source of action, for Kant, but rather a purely passive "mere inability to move itself from itself" (551; see the paragraph to which note 98 above is appended). I shall return to this last passage (551) below in the context of further discussion of the precise role of inertia in the communication of motion.

(544–45) that "[h]ere, in order to stay within the bounds of mechanics, it is only to be shown that this interaction (*actio mutua*) is at the same time *reaction* (*reactio*)." I am now in a position to elucidate this situation. It is true that Kant's proof does not depend on the assumption of genuine dynamical community or interaction; it depends only on the construction of the center of mass of the two bodies and thus holds equally (as in the case of magnetism) where no true dynamical community (*mutual* influence) need be involved. Nevertheless, as Kant indicates in the footnote to his proof, its primary application is to the two fundamental forces of attraction and repulsion, where genuine dynamical community does in fact hold. It is precisely here, therefore, that Kant's "construction of the communication of motion" makes necessary contact with the argument of the third analogy – and it follows (545) that "[we] can still not wholly leave aside this metaphysical law of community here, without detracting from the completeness of the insight."

So how exactly is the relevant insight to be completed? In the passage from the third analogy just quoted (A212–13/B259–60) Kant says that any two simultaneous substances stand in a relation of genuine dynamical community. But he also asserts that, since all substances (in the phenomenal) world are simultaneous or coexistent with one another (in space), each such substance stands in dynamical community with all others (A213/B260): "[I]t is necessary for all substances in the appearance, in so far as they are simultaneous, to stand in thoroughgoing community of interaction among one another." Moreover, as observed in section 17 above, Kant illustrates the sense in which all substances stand in community with one another by appealing to the role of both light and universal gravitation in our perception of the heavenly bodies. He first appeals (in the second edition) to the mutual interaction between the earth and the moon as the ground for our perception of their simultaneity (B257–58), then remarks (in the passage just quoted) that such dynamical community can be either immediate or mediate, and finally illustrates the case of mediate community by the propagation of light. In particular (A213/B260): "[T]he light, which plays between our eyes and the heavenly bodies, effects a mediate community between us and them and thereby proves their simultaneity," so that "we cannot empirically change our place (perceive this change) without matter everywhere making it possible for us to perceive our position, and only this [matter], by means of its mutual influence, can verify their [the heavenly bodies'] simultaneity and therefore, all the way to the most distant objects, their coexistence." Thus, whereas universal gravitational attraction between the heavenly bodies (including the earth and the moon) effects an immediate dynamical community among them, it is the mediate community effected by light propagated through the aether that puts us into perceptual contact with the heavenly bodies in the first place.

Kant's primary example of dynamical community therefore involves a situation of thoroughgoing mutual interaction throughout the entire cosmos – including an interaction between the earth and the other heavenly bodies by means of which our own relative position and motion on the earth within the cosmos is determined. We here have the first step in Kant's Copernican conception of space and motion, whereby we realize (with Copernicus) that the earth (along with everything on it) is actually in a state of motion among the other heavenly bodies. Moreover, given the central importance of Newton's moon test in the argument of the *Metaphysical Foundations* so far, it is not implausible to suppose that the example of an interaction between the earth and the moon added to the second edition of the *Critique* is also an allusion to this crucial part of Newton's argument. But the moon test is also the key step in depicting how an initial frame of reference fixed at the center of the earth is to be projected, as it were, outwards into the heavens. For, as explained in section 26 above, it is precisely at this point that the terrestrial concept of weight can be extended to the universal concept of mass or quantity of matter – which then plays a crucial role in the construction of an appropriate (celestial) center of mass frame (see the paragraphs to which notes 42 and 43 above are appended). The upshot, it appears, is that Kant's "construction of the communication of motion" articulated in the proof of the fourth proposition of the Mechanics must be embedded within the extended context provided by Kant's Copernican conception of space and motion in order for the relevant insight to be completed.

This conception is finally fully articulated in the Phenomenology, where Kant describes a procedure in which "all motion and rest must be reduced to absolute space" (560) modeled on Newton's argument of Book 3 of the *Principia*.[124] So when Kant speaks of a motion being "reduced to absolute space" in his proof of the fourth proposition of the Mechanics (545), he is also anticipating the discussion in the Phenomenology here. Kant's

[124] See my discussion in the Introduction in the paragraphs to which notes 28 and 31 thereof are appended. As explained in my chapter on the Phoronomy, the discussion of absolute space in the Phenomenology is already implicit in the first explication of Kant's treatise: see note 2 of my chapter on the Phoronomy, together with the paragraph to which it is appended, and compare (the references in) note 70 of the same chapter, together with the paragraph to which it is appended and the following paragraph.

point, on my reading, is that a proper understanding of the significance and role of Newton's Third Law of Motion requires precisely this larger context, for it is only in this context that we can ultimately grasp the true significance and role of Newtonian "absolute space." And to do this, from Kant's perspective, we also require a metaphysical investigation of the necessary conditions for thoroughgoing mutual interaction in a whole of existing (in this case *co*existing) substances. Such an investigation – like Kant's previous investigations of the physical concepts of quantity of matter and inertia – must therefore appeal to the fundamental metaphysical concepts of substance, causality, and community derived from the Leibnizean tradition.

With these considerations in mind, let us turn to the first remark to Kant's fourth proposition, which begins as follows:

> This, then, is the construction of the communication of motion, which, at the same time, brings with it, as its necessary condition, the law of the equality of action and reaction – which [law] Newton by no means dared to prove a priori, but rather therefore appealed to *experience*, [and] which others, for its sake, introduced into natural science a special force of matter, under the name, first introduced by Kepler, of a *force of inertia* (*vis inertiae*), and thus they also derived it in principle from experience. (549)

In the case of his critical remark addressed to Newton, we know that Kant has in mind Newton's experimental arguments for the Third Law from the Scholium to the Laws of Motion (note 109 above). The idea here is that it is misleading to appeal to experience on behalf of this law, because, according to precisely the argument of the Phenomenology, we unavoidably need to *presuppose* it in order to make motion itself a proper object of experience in the Kantian sense. But who are the unnamed "others" that Kant critically addresses after his remark about Newton, and why does Kant single out them (rather than Newton) for having wrongly introduced a force of inertia (*vis inertiae*) into nature?[125]

The answer to this question has been convincingly documented in an important paper on the Leibnizean purview of Kant's Third Law of Mechanics by Marius Stan (see note 15 above). Stan (in press, §11) traces in detail a tradition for treating the relation between action and reaction in the impact of two bodies that begins with Leibniz, continues

[125] This last question is especially pressing because, although Newton himself introduces the force of inertia in Definition 3 (see again note 129 of my chapter on the Dynamics), Kant does not criticize him for this reason here (see again note 109 above, together with the paragraph to which it is appended).

with his protégé and disciple Jacob Hermann, and is then taken up by a number of Leibniz's followers including Christian Wolff. The basic idea of all these treatments is that we begin by considering a case of (one-dimensional) impact between a moving body and a body initially at rest, and we try to explain the resulting changes in motion by appealing to a balance or equilibrium between the (active) moving force or impetus (*vis motrix*) of the impacting body and the (passive) *force of resistance* of the body at rest due to its own inertia (*vis inertiae*). On this picture, therefore, we take inertia, following Kepler, to be manifested primarily in a body's resistance to being set into motion in the first place, and we view the equality of action and reaction, in this case, as applying to two distinct Leibnizean forces – one active and the other passive. Stan (in press, §III) then describes the continuities between this tradition and Kant's own treatments of impact – both early, in the *New System of Motion and Rest* (1758), and later, in the Mechanics of the *Metaphysical Foundations* – and argues that Kant is criticizing the traditional picture from within rather than simply rejecting it out of hand on behalf of Newton's Third Law.[126]

Thus when, in the first remark to the fourth proposition of the Mechanics, Kant rebukes unnamed "others" for appealing to a force of inertia – as "first introduced by Kepler" (549) – to explain the communication of motion from an impacting body to an initially resting body, there can be very little doubt that he has principally in mind the treatment in the tradition of Leibniz, Hermann, and Wolff. Indeed, Leibniz's enthusiastic embrace of Kepler and the force of inertia was well known at the time from a number of his more general works – including the *Theodicy* (1710), the *Correspondence with Clarke* (1717), and the *New Essays*

[126] As representative works of this tradition Stan cites, among others, Leibniz's *Specimen Dynamicum* (1695), Hermann's *Phoronomia* (1716), and Wolff's *Cosmologia Generalis* (1731). Stan summarizes his view of the place of Kant's early (1758) treatment to this tradition as follows (in press, §III.1):

> At mid-century, the Hermann–Wolff model of action and reaction held sway over German rationalist dynamics. But it was open to censure from two sides. One is external: it takes Newton's side and wrecks the model with just two blows. Another is criticism from within, as it were: it finds internal tension and relieves it by tweaking the model, instead of replacing it altogether. Kant takes the latter route, in the 1758 *New Doctrine of Motion and Rest*.

In the following section on the *Metaphysical Foundations* Stan details the significant overlap between Kant's treatment of impact in the Third Law of Mechanics and his earlier treatment in 1758 and concludes (in press, §III.2): "Arresting continuities between his early and Critical concepts of action and reaction explain the un-Newtonian traits of Kant's third law in *MAN*."

on Human Understanding (1765).[127] As explained in section 26 above, however, Kant had earlier appealed to the force of inertia as efficacious in precisely the impact of a moving body with one at rest in the *Physical Monadology* (2, 485; see the passage to which note 99 above is appended). His discussion of inertia in the second proposition of the Mechanics then initiates a sustained polemic against such a force – for the reason, among other things, that one should carefully guard against confusing the law of inertia with the law of the equality of action and reaction (544; see note 98 above, together with the paragraph to which it is appended).

The crucial point is that mechanical resistance, for the critical Kant, is not an "originally moving" dynamical force at all, and it cannot be manifested, in particular, in a state of rest. Indeed, there can now be no communication of motion at all, for Kant, unless *both* bodies are conceived as moving – as explained at the beginning of his second note to the fourth proposition of the Mechanics:

> This, then, is the *mechanical law* of the equality of action and reaction, which rests on the fact that no *communication* of motion takes place, except in so far as we presuppose a *community* of these motions, and thus on the fact that no body impacts another that is at rest *relative to it*; rather, the second body is at rest relative to space, only in so far as it moves, *together with this space*, in the same amount, but in the opposite direction, with that motion which then falls to the first as its relative share, and together would originally yield the quantity of motion that we would ascribe to the first in absolute space. For no *motion* that is to be *moving* with respect to another body, can be *absolute*; but if it is relative with respect to the latter, then there is no relation in space that would not be mutual and equal. (548)

In all cases of the communication of motion, in other words, there is a privileged frame of reference (representing absolute space) for considering the interaction determined by the center of mass of the two bodies. In this frame both the impacting body A and the impacted body B "have an equal share in the motion which, in the appearance, is ascribed to body A

[127] The discussion in the *New Essays* (which was certainly well known to Kant) explicitly invokes the Leibnizean model of impact. In particular, when discussing the "sources for a body's resistance to another body which tries to make it give way," Leibniz (1765, p. 123) continues:

> Within the body itself there are two – one passive and constant, and the other active and changing. The former is what I follow Kepler and Descartes in calling *inertia*. This renders matter resistant to motion, so that force must be expended to move a body, independently of its having weight or being bonded to other things. Thus a body that seeks to drive another along must encounter such resistance as a result. The other cause – the active and changing one – consists in the body's own impetus: the body will not yield without resistance at a time when its own impetus is carrying it to a given place.

alone – which can only happen in such a way that the speed ascribed in relative space to body A alone is apportioned between A and B in inverse ratio to their masses – to A alone its speed in absolute space, and to B, *together with the relative space* in which it rests, its speed in the opposite direction" (545).

Hence the appearance of A impacting on B in a state of rest is just that – a merely apparent motion occurring in a second frame of reference in which B is initially at rest. In the first (privileged) frame of reference representing absolute space the two bodies, after impact, set one another mutually into a state of rest (by a mutual cancellation of their two equal and opposite momenta or mechanical moving forces). In the second frame of reference, however, all the motion (momentum) is initially possessed by A, and then, after impact, the two bodies move off together in (merely) relative space with the same velocity (but in the opposite direction) initially possessed by B in absolute space. The appearance of mechanical resistance in a state of rest results simply from translating the communication of motion in absolute space into a second frame of reference viewed as a (mere) relative space. In reality – in absolute space – B is not at rest after all but is moving towards A in the opposite direction with precisely the same momentum that A has towards B. The mechanical moving force that B thereby exerts on A is thus of precisely the same kind as that which A exerts on B (impetus or *vis motrix*), and there is no need at all to appeal to another force (inertia or *vis inertiae*) exerted also (or only) in a state of rest.

In his discussion of the third Definition initiating the *Principia* Newton identifies "innate force (*vis insita*)" or "inertial force (*vis inertiae*)" (P404; see again note 129 of my chapter on the Dynamics) with "mass (*masse*)" – that is, with what we now call *inertial* mass. He says that "a body exerts this force only during a change of its state, caused by another force impressed upon it, and this exercise of force is, depending on the point of view, both resistance and impetus" (P404). He concludes: "Resistance is commonly attributed to resting bodies and impetus to moving bodies; but motion and rest, in the popular sense of the terms, are distinguished from each other only by point of view, and bodies commonly regarded as being at rest are not always truly at rest" (P404–5). Despite his use of the term *vis inertiae*, then, Newton's conception is actually rather close to that defended by Kant: impetus and resistance are manifestations of the very same phenomenon, depending only on the frame of reference in which one views the two interacting bodies. Moreover, mass or inertial force, in Newton's view, is definitely not a dynamical force in Kant's sense – active and inherent in bodies independently of their state of motion. For, "a body exerts this force only during a change of its state, caused by another force impressed upon it" (P404). Newton is also perfectly clear, finally, that the state in question is one "*either* of resting *or* moving" (P404, emphasis added). So it is for all of these reasons, it appears, that Kant does not single out Newton in the polemic against the force of inertia in the Mechanics.

By contrast, the way in which the Leibniz–Hermann–Wolff tradition appeals to Kepler, and privileges the resistance of matter in a state of *rest*, can easily lead to a fundamental misunderstanding of the law of inertia – according to which a body only thereby strives to remain in a state of rest.[128] Moreover, even if we avoid this misunderstanding, and view *vis inertiae* as maintaining either a state of rest *or* a state of uniform rectilinear motion, we can still easily be led to view *vis inertiae* as an innate or inherent force of bodies possessed entirely independently of their state of motion. For a body's mass is certainly innate or inherent in precisely this sense. And if we do not sufficiently appreciate, following Newton, that "this exercise of [inertial] force is, depending on the point of view, *both* resistance *and* impetus" (P404, emphasis added), we can thereby be led to assimilate *vis inertiae* to a fundamental dynamical force of matter in the sense of the critical Kant.[129] But the laws governing fundamental dynamical forces, for the critical Kant, can only be inferred "from data of experience" (534; see the paragraph following the one to which note 23 of my chapter on the Dynamics is appended). It is for this reason, it appears, that Kant, in the first remark to the fourth proposition of the Mechanics, says that the "others" who mistakenly introduced a force of inertia into nature "also derived it [the law of the equality of action and reaction] in principle from experience" (549; see again the paragraph to which note 125 above is appended).

Kant sums up his case against *vis inertiae* in the second remark to the fourth proposition:

[128] Kepler took his *vis inertiae* to be just such a striving, and Clarke accuses Leibniz of the same fundamental misunderstanding in the *Leibniz–Clarke Correspondence*. See Leibniz (1717, 5, first note to Clarke's reply to §99; 1956, pp. 111–12): "[Inertia is] that passive force, not by which (as Mr. Leibnitz from Kepler understands it) matter resists motion, but by which it equally resists any change from the state 'tis in, either of rest or motion." Given that Leibniz does take inertia, following Kepler, to be a resistance to *motion*, and, moreover, explicitly distinguishes inertia (*vis inertiae*) from impetus (*vis motrix*), Clarke's charge is certainly understandable.

[129] Again, it appears that Kant himself came very close to this misunderstanding in the *Physical Monadology*: see the paragraph to which note 130 of my chapter on the Dynamics is appended.

Regardless of the famous name of its creator [i.e., Kepler – MF], the terminology of inertial force (*vis inertiae*) must therefore be entirely banished from natural science, not only because it carries with it a contradiction in terms, nor even because the law of inertia (lifelessness) might thereby easily be confused with the law of reaction in every communicated motion, but primarily because the mistaken idea of those who are not properly acquainted with the mechanical laws is thereby maintained and strengthened – according to which the reaction of bodies discussed under the name of inertial force would amount to a draining off, diminution, or eradication of the motion in the world; but the mere communication of motion would not be effected thereby, because the moving body would have to apply a part of its motion solely in overcoming the inertia of the one at rest (which would then be a pure loss), and could only set the latter in motion with the remaining part; but if none were left over, it would completely fail to move the latter by its impact, because of its great mass. (550–51)

Kant's first complaint is the standard (and rather trivial) one that the very wording of "force of inertia" – that is, the activity (*vis*) of inactivity (*inertiae*) – is self-contradictory. Kant's second complaint is the one on which we have been concentrating so far, and it is directed, as observed, against the Leibniz–Hermann–Wolff tradition of explaining impact by a law of the equality of action and reaction balancing the impetus or *vis motrix* of the impacting body with the inertia or *vis inertiae* of the impacted body. But who does Kant have in mind by "those who are not properly acquainted with the mechanical laws" and maintain that "the reaction of bodies discussed under the name of inertial force would amount to a draining off, diminution, or eradication of the motion in the world"?

Stan (in press) suggests that Christian Wolff is likely one of Kant's targets here – specifically, in the first note to the proof of the Third Law of Mechanics, which asserts (548) that "any body, however great its mass, must be *movable* by the impact of any other body, however small its mass or speed."[130] What is even more striking, however, is that Kant himself had defended the "draining off" of mechanical motion via the resistance exerted by the force of inertia in his pre-critical *New Exposition* (1755). There, as explained in section 26 above, Kant introduces the force of inertia under the rubric of "innate force [*vis insita*]" and characterizes its role in impact as follows (1, 408; see the paragraph to which note 100 above

[130] See Stan (in press, §III.2n. 124), which cites Wolff (1731, §§388–89) in this connection. The proposition demonstrated in §389 is especially striking (1731/1737, p. 282): "*If the inertial force* [*vis inertiae*] *of the resting body exceeds the force of the incoming body no motion follows & the motion of a non-elastic incoming body is extinguished*." Here the Leibniz–Hermann–Wolff model of a conflict between the inertial force of the resting body and the motive or impulsive force of the incoming body appears to be particularly evident.

is appended): "[A]ll the reality of forces in the phenomenon of motion is equal to that which is already innate [*insitum*] in the body at rest, although the internal power [*interna potestas*], which was indeterminate with respect to direction at rest, is merely directed by the external impulse." The notion of "reality" here is subject to a metaphysical conservation principle – which, as explained in section 25 above, Kant associates with the conservation of (vector) momentum (see the paragraph to which note 78 above is appended).

Yet, as Kant argues on the previous page, this conservation principle is in no way incompatible with "the complete destruction of motion by the resistance of matter" (1, 407):

> With regard to the complete destruction of motion by the resistance of matter, it does not so much cancel this rule as elevate it and establish it more strongly. For the force which arises from rest by a consensus of causes will be again reduced to rest by expending as much in resisting impediments as it has acquired, and the situation remains the same as before. Therefore, the inexhaustible duration of mechanical motion is impossible; for, since it always expends a part of its force in resistance, it would be contrary to both this rule and sound reason [to suppose that] an undiminished power to renew itself should nonetheless persist. (1, 407)

What Kant appears to be supposing is that the total (scalar) momentum expended by bodies in their impacts with one another is latent in these same bodies in a state of rest. It is initially stored in these bodies in the form of an innate or inherent force of inertia, then redistributed among the same bodies in a variety of mechanical motions (scalar momenta), and eventually returned to the innate force of inertia in a state of rest by the consequent destruction of their mechanical motions by impact. In all cases, however, the total *vector* momentum is conserved, and it is always constantly zero in the center of mass frame.[131]

[131] This conception appears to be confused. For, in the first place, it is necessary to distinguish elastic from inelastic impact. In the latter scalar momentum necessarily diminishes to zero, while in the former it does not: *vis viva* (mv^2) is conserved as well. Yet the difference between these two cases has nothing to do with the inertia (mass) common to all bodies but rather depends, in the second place, on particular distinguishing properties of bodies such as hardness, softness, or elasticity. As a result, the mechanical resistance due to the inertia (mass) common to all bodies must be sharply distinguished from the specifically *inelastic* resistance of (supposed) perfectly hard bodies. A lack of clarity about these distinctions was very widespread at the time, and it is evident, for example, in the debate concerning hard-body atomism in the *Leibniz–Clarke Correspondence*: see Scott (1970) for further discussion. Kant attempts to clarify some of these questions in the Mechanics of the *Metaphysical Foundations* – notably, in the footnote on the relationship between absolutely hard and elastic bodies appended to his discussion of the "*transfusionists* of motion" in the first remark to the fourth proposition (549–50). In particular, *elastic* resistance is characterized by a special (dynamical) force of matter (compare the discussion of "spring-force" (529) in the third number of the general remark to dynamics), whereas

Three years later, however, in the *New System of Motion and Rest* (1758), Kant decisively rejects the force of inertia, together with the role it was supposed to play in impact according to the *New Exposition*:

Perhaps it would never have occurred to anyone to assert that a body is completely at rest so long as a body approaching it does not yet touch [it], or, if one likes, is in an equilibrium of forces [in this state], and it is nevertheless supposed to assume suddenly of itself a motion [directed] against the impacting body at the moment of impact, or is supposed to shift itself into a disequilibrium, in order to cancel an oppositely directed force in the latter, if it did not emerge from experience that in a state which everyone takes to be a state of rest, the body reacts on [*entgegen wirkte*] every such acting [*handelnden*] body with the same degree [of force]. But I have shown that what one falsely takes to be a state of rest with respect to the impacting body is in fact [a state of] motion relative to it. So it is obvious that this force of inertia is needlessly invented; and in every impact a motion of the body with respect to another moved against it in the same degree is to be found – which explains the equality of action and reaction very easily and comprehensibly, without allowing the invention of a special type of force of nature. (1, 19–20)[132]

So Kant rejects the force of inertia here for essentially the same reasons he does in 1786. *Both* bodies must be assumed to be in motion before the impact, so that what one falsely took to be a special force of mechanical resistance exerted by the impacted body at rest – against the impetus or momentum of the impacting body – is really an equal and oppositely directed impetus or momentum of the impacted body.[133] There is no room, accordingly, for an inherent force of inertia that is supposed to be characteristically efficacious in a state of rest.[134] Thus, as soon as Kant first introduces the Copernican conception of space and motion, and, accordingly, explains the equality of action and reaction in impact by a construction

inelastic resistance is characterized by the absence of this force. But I shall not be able to pursue these questions further here.

[132] Stan (in press, §III.1) rightly emphasizes the importance of the rejection of inertial force in the *New System* and quotes parts of this passage as well; he does not consider its relation to the earlier *New Exposition*.

[133] It is noteworthy, too, that the following section of the *New System*, "on the law of continuity, in so far as it is inseparable from the concept of inertial force," then considers and rejects what Kant later calls the "transfusionist" conception of a gradual and continuous transfer of motion from the impacting to the impacted body (see note 131 above). Moreover, although Kant's discussion here is certainly not the same as in 1786, the two still have considerable overlap.

[134] In 1758 Kant explains that, although he can admit (1, 20) that "all bodies have a force of inertia with respect to those moved against them, i.e., a force to react [*entgegen zu wirken*] to the action [*Handlung*] in the same degree," it is still true, nevertheless, that "they only appear to have this in [a state of] complete rest as an internal force in themselves; for they in fact have it merely because they are in actual and equal motion [directed] against the approaching [body], and they never have such [a force] in so far as they are found to be at rest relative to it" (1, 20).

of the communication of motion in the center of mass frame, he completely eliminates the innate or inherent force of inertia.[135]

I explained in section 26 that a rejection of the force of inertia represents a crucial step, for Kant, in moving beyond the monadological conception of substance dominant in the pre-critical *New Exposition* and *Physical Monadology*. In particular, the force of inertia, in these works, represents the purely internal determinations of a simple substance that it possesses intrinsically and on its own independently of any other such substances. The two fundamental (dynamical) forces of attraction and repulsion, by contrast, are essentially relational, and they express the co-presence or coexistence of a number of simple substances in a common world. This conception, as explained, therefore contradicts Kant's present conception of the essential lifelessness of all matter – and thus his present conception of the law of inertia. What I am now in a position to add is that Kant's pre-critical view of the role of the force of inertia within a monadological conception of substance also stands in fundamental opposition to the construction of the communication of motion, and accompanying Copernican conception of space and motion, that Kant had already formulated in 1758. And since, as explained, the relevant insight at the basis of Kant's critical construction of the communication of motion in the fourth proposition of the Mechanics can only be completed in the context of the third analogy of the first *Critique* (see the two paragraphs following the one to which note 123 above is appended), Kant's critical understanding of the category of community must also replace his pre-critical conception of substantial coexistence.

It turns out, moreover, that the primary application of the category of community in the critical period is to the argument for universal gravitation in Book 3 of Newton's *Principia*, and this argument, as explained, is the focus of Kant's procedure by which all motion and rest must be reduced to absolute space articulated in the following Phenomenology. Kant thus seeks a metaphysical foundation for Newtonian physics employing the fundamental concepts of substance, causality, and community derived from the Leibnizean tradition. In the case of each of these concepts, however, Kant needs radically to revise the Leibnizean understanding. Material substance (*substantia phaenomenon*)

[135] The most important difference between the *New System* and the *Metaphysical Foundations*, in this respect, is that the latter explicitly extends the construction of the communication of motion to attractive force, and thus to universal gravitation as well, whereas the former considers only the communication of motion by impact. This decisive advance, as suggested, is made in the Phenomenology, which I shall consider in detail below.

is necessarily infinitely divisible and therefore in no way simple. Causality in matter cannot be internal at all, but necessarily involves the external influence of another (material) substance. The interaction of material substances, finally, is not the result of a divinely instituted pre-established harmony among otherwise independently evolving simple substances but a necessary condition for our perception of the coexistence of such substances at the same time. Thus, although, according to the critical Kant, Newtonian physics does require a broadly Leibnizean metaphysical foundation, the Leibnizean tradition must also be radically revised in the process. A metaphysical "first philosophy" based on a reality lying beneath or behind the observed phenomena of nature must ultimately give way to the critical procedure of time determination characterizing the conditions of the possibility of the objects of our (human) experience.[136]

28 MOVING FORCE, QUANTITY OF MATTER, AND THE LAWS OF MECHANICS

As we have seen, a sharp distinction between mechanical and dynamical moving forces is central to the Mechanics. Kant emphasizes that dynamical moving forces (attraction and repulsion) are active also at rest, so that their exercise is entirely independent of the state of motion of the body exerting the force. Mechanical moving forces, by contrast, are exerted only in the context of the communication of motion, whereby both bodies, for Kant, must be necessarily conceived as moving. 'Mechanical moving force', in this context, is simply another name for the *momentum* that is transferred from one body to another in the context of such communication. Kant makes it equally clear, however, that there is nonetheless an intimate connection between the two kinds of forces. As he explains at

[136] At the end of my discussion of Kant's First Law of Mechanics (section 25) I suggested that, although Watkins (1998) rightly argues that Kant's formulation of a conservation principle here locates him squarely within the Leibnizean tradition, Watkins does not sufficiently emphasize that Kant's critical proof of this principle entails a radical break from the fundamental Leibnizean commitment to substantial simplicity (see note 90 above). Similarly, although Stan (in press) rightly argues for the Leibnizean provenance of Kant's Third Law, he does not, in my opinion, sufficiently emphasize the radical transformation of the Leibnizean tradition entailed by Kant's proof of this law. In particular, although Stan is clear that the force of inertia has now been rejected once and for all, he appears not sufficiently to appreciate how close Kant's position on rest and the force of inertia has thereby become to Newton's. He appears also not sufficiently to appreciate the circumstance that Kant not only extends his law from impact to universal gravitation (conceived as a fundamental force acting immediately at a distance) but also considers its application to the latter context to be precisely where his underlying insight (the Copernican conception of space and motion) is to be completed.

the outset of the Mechanics, mechanical moving forces *presuppose* the corresponding dynamical ones (536–37). Kant reiterates the same point at the beginning of the general remark to mechanics that concludes the chapter (551): "The communication of motion takes place only by means of such moving forces that also inhere in a matter in [a state of] rest (impenetrability and attraction)."

Neither mechanical nor dynamical moving force, in Kant's sense, is an "impressed force" in the technical sense of Definition 4 of Newton's *Principia*. The latter is more like a dynamical than a mechanical moving force, however, in so far as it is "*the action exerted on a body to change its state either of resting or of moving uniformly straight forward*" (P405; see note 116 above). The action is exerted *on* a body to effect a change in its motion; it is not exerted *by* the body in virtue of its motion.[137] So, in this respect, we can view Newton's Second Law of Motion (which is not formulated by Kant) as establishing a correlation between dynamical and mechanical moving force in Kant's sense – whereby the latter (change of momentum) provide a quantitative measure of the former (see note 13 above, together with the paragraph to which it is appended). Yet, because of fundamental differences between Kantian dynamical forces and Newtonian impressed forces, we have to proceed very carefully in articulating this correlation.

The most fundamental of these differences is that the Kantian notion of a dynamical moving force is physical (or metaphysical) rather than mathematical. As introduced in the proof of the first proposition of the Dynamics, it is precisely the notion of a *causal action* exerted by one body to change the state of motion of another.[138] For Newton, by contrast, while an impressed force is indeed an "action" exerted on a body, Definition 4 does not specify what other body (if any) thereby counts as the cause of the change of motion. Instead, Newton takes pains to abstract from all questions concerning the cause of the action in favor of a purely mathematical consideration of the action by itself. He does list,

[137] Newton drives the point home in the first sentence of the following comment (P405): "This force consists solely in the action and does not remain in a body after the action has ceased." The impressed force remains "in" a body only while it is changing its state, so that an inertially moving body "has" no such force at all – no matter how fast it is moving.

[138] See again the discussion in the proof of the first proposition (497): "[T]he resistance that a matter offers in the space that it fills to every penetration by other matters is a cause of the motion of the latter in the opposite direction. But the cause of a motion is called a moving force." Note that "motion" here signifies velocity rather than momentum, for the notion of mass has not yet been introduced. Indeed, one of the main tasks of the Mechanics is to explain how mass – and therefore momentum – first enters the picture for Kant.

by way of example, three "sources" of impressed force (P405): "percussion, pressure, or centripetal force." In the case of this last and most important example, however, Newton goes out of his way to express what we might call a mathematical agnosticism concerning the true causes or seats of the action:

> I use interchangeably and indiscriminately words signifying attraction, impulse, or any sort of propensity toward a center, considering these forces not from a physical but only from a mathematical point of view. Therefore, let the reader beware of thinking that by words of this kind I am anywhere defining a species or mode of action or a physical cause or reason, or that I am attributing forces in a true and physical sense to centers (which are mathematical points) if I happen to say that centers attract or that centers have forces. (P408)

Newton thus makes it clear, in particular, that his notion of an "attractive" impressed force directed towards a center does not, by itself, imply any claim at all concerning the causal efficacy of this center.[139]

Newton's mathematical agnosticism is evident in the first proposition of Book 1, which shows that the trajectory generated by any centripetal force satisfies Kepler's area law with respect to the center S. The idea is to represent the action of the force by a series of instantaneous

[139] It seems clear, in addition, that one of the motivations for Newton's agnosticism is to forestall objections to his notion of gravitational attraction by proponents of the mechanical philosophy. The comments to Definition 5 (the definition of centripetal force) added to the second edition give a brief précis of the argument for universal gravitation of Book 3 (P405–6), and the agnosticism stated in the above-quoted passage (P408) follows the last definition relevant to centripetal force (Definition 8, P407). In the introduction to §11 of Book 1, which points towards the transition from mathematical to physical considerations completed in Book 3, Newton still takes pains to emphasize that his "attractions" are purely mathematical (P561):

> I now go on to set forth the motion of bodies that attract one another, considering centripetal forces as attractions, although perhaps – if we speak the language of physics – they might more truly be called impulses. For here we are concerned with mathematics; and therefore, putting aside any debates concerning physics, we are using familiar language so as to be more easily understood by mathematical readers.

Finally, the Scholium to this section reiterates Newton's mathematical agnosticism in even more explicit terms – and makes it clear, in particular, that the "impulsive" action of an external aether may indeed be the true physical cause (P588):

> I use the word "attraction" here in a general sense for any endeavor whatever of bodies to approach one another, whether that endeavor occurs as a result of the action of the bodies either drawn toward one another or acting on one another by means of spirits emitted or whether it arises from the action of aether or air or any medium whatsoever – whether corporeal or incorporeal – in any way impelling [*impellentis*] toward one another the bodies floating therein. I use the word "impulse [*impulsus*]" in the same general sense, considering in this treatise not the species of forces and their physical qualities but their quantities and mathematical proportions, as I have explained in the definitions.

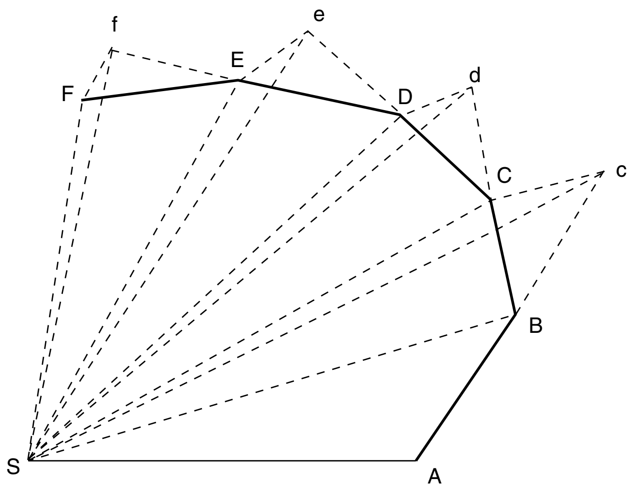

Figure 3

"impulses" towards S (acting at A, B, C, and so on) separated by equal times, with the result that (by the law of inertia) the areas of the corresponding triangles (SAB, SBC, SCD, and so on) are also equal (see Figure 3). Thus, let a body first move uniformly from A to B in accordance with the law of inertia. In a second and equal interval of time the body would, if nothing hindered it, proceed in a straight line to c, where Bc is equal to AB and the triangular area SBc is equal to the corresponding area SAB. But now (P44): "When the body comes to B, let a centripetal force act with a single but great impulse and make the body deviate from the straight line Bc and proceed in the straight line BC." By the parallelogram construction of Corollary 1 to the Laws of Motion, the body will now be found at C at the end of the second interval of time. But triangles SBC and SBc are equal, and thus the triangular area swept out by the motion – SBC – is also equal to SAB. Kepler's area law, in this way, is a consequence of the law of inertia, and it thereby provides a precise standard of temporal equality for (non-inertial) centripetal motions.[140]

[140] To complete this argument Newton needs to extend his reasoning to the case of a continuously acting centripetal force successively approximated by an increasing number of discrete impulses – letting "the number of triangles be increased and their width decreased indefinitely" (P445). This procedure involves substantial mathematical difficulties; for the clearest and most up-to-date discussion see Pourciau (2003).

What is important, in the present context, is that Newton's reasoning is indeed purely mathematical. He speaks of instantaneous impulses directing the moving body at A, B, C, and so on towards the center of S. But he has already cautioned the reader not to infer that the cause of these impulses is a series of successive impacts (exerted by an external aetherial medium, for example) driving the body towards S.[141] Nor is there any reason to infer that the cause is a true (physical) attraction for the body exerted by S itself (which, after all, is a mere mathematical point). All that matters are the lines of direction of the "action" – *whatever* its true cause might be – and the circumstance that the changes of direction experienced by the body (at A, B, C, and so on) are separated by equal times. Newton has thereby represented the impressed force or action in a purely mathematical (rather than physical) fashion – as what we could now call a vector directed towards S – and shown that this suffices to derive further mathematical properties of the resulting motion: the important property, in this case, that it satisfies Kepler's area law.

That Kant's notion of dynamical moving force is explicitly causal or physical does not imply, however, that it lacks the mathematical structure of a Newtonian impressed force. For it is precisely a cause of a change of motion (addition of velocities) in the sense of the Phoronomy. Moreover, the entire point of the Phoronomy, as discussed in sections 3 and 4 above, is to explain how *motion* is possible as a mathematical magnitude with respect to both speed and direction. The first proposition of the Dynamics then introduces the notion of a dynamical force into Kant's treatise, and the note to the second explication refers back to the Phoronomy (implicitly) in arguing that only two possible kinds of dynamical moving force – attraction and repulsion – "can be thought" (498): "For all motion that one matter can impress [*eindrücken*] on another, since in this regard each of them is considered only as a point, must always be viewed as imparted in the straight line between the two points."[142] The action of a dynamical

[141] Newton makes it clear later that an external aetherial medium *may* be the physical cause (P588; see note 139 above), but it also may not be. At this point we simply do not know one way or another.

[142] For the full passage (498–99) and my earlier discussion of it see the paragraph to which note 11 above is appended. As I observed in this discussion, Kant still needs to explain how the pointwise (so far merely phoronomical) approach can be applied to bodies consisting of a continuous aggregate of acting points: see notes 14 and 15 above, together with the paragraphs to which they are appended. I shall return to this issue below. But I meanwhile observe that Newton responds to essentially the same problem in §11 of Book 1 (compare note 139 above), where he anticipates the later application of his purely mathematical approach to forces in physics in Book 3 (P561):

Up to this point, I have been setting forth the motions of bodies attracted toward an immovable center, such as, however, hardly exists in the natural world. For attractions are always directed force in Kant's sense – that is, the motion imparted by this force – thereby acquires the vectorial structure of a Newtonian impressed force. And, in particular, arguments based on the addition or composition of such vectors (such as the argument of Proposition 1 of Book 1) can be thus carried over into Kant's framework.[143]

To fully characterize a vectorial quantity, however, one needs both its direction and its "length" or magnitude. Kant gives the former via the straight line connecting the two interacting bodies in question – here considered as mere mathematical points. Newton's Second Law of Motion further characterizes the magnitude of an impressed force, in effect, by the change of motion or momentum (mass times velocity) thereby generated (P416; see note 36 of my chapter on the Phoronomy): "*A change of motion is proportional to the motive force impressed and takes place along the straight line in which the force is impressed.*" The second explication of the Dynamics gives us a change of velocity in the straight line connecting the two bodies, so the problem is now to explain how mass or quantity of matter itself becomes possible as a mathematical magnitude. As suggested at the end of section 25 above, however, Kant aims for a more "constructive" explanation of this quantity than Newton provides in the

toward bodies, and – by the third law – the actions of attracting and attracted bodies are always mutual and equal; so that if there are two bodies, neither the attracting nor the attracted body can be at rest, but both (by corol. 4 of the laws) revolve about a common center of gravity as if by a mutual attraction.

The following Scholium to Section 11 begins (P588):

By these propositions we are directed toward the analogy between centripetal forces and the central bodies toward which those forces tend. For it is reasonable that forces directed toward bodies depend on the nature and the quantity of matter of such bodies, as happens in the case of magnetic bodies. And whenever cases of this sort occur, the attractions of the bodies must be reckoned by assigning proper forces to their individual particles and then taking the sums of these forces.

[143] Stan (in press, §III.3) considers the "limits of Kant's a priori mechanics" and asks, in particular, how – in the absence of the Newtonian concept of impressed force – Kant can deal with cases where one needs to distinguish the line or direction of motion from the line or direction of the action of such a force. In the cases that Stan considers (oblique impact and orbital motion) we have an inertial component of uniform motion that needs to be combined or composed with the action of a force (impulsive or gravitational) in a different direction. Stan suggests that the primarily metaphysical rather than mathematical character of Kant's discussion of action and reaction in the fourth proposition of the Mechanics (as ultimately derived from the Leibnizean tradition) may create serious obstacles for Kant in such cases. But this suggestion, on my reading, is too hasty, for it underestimates both the intimate relationship between Kantian dynamical and mechanical moving forces and the intimate connection, in turn, between the former forces and the mathematical treatment of the composition of motions (velocities) in the Phoronomy. Nevertheless, despite my disagreements with Stan on these points, I am indebted to his discussion for raising this important question – to which I shall return below.

Definitions leading up to the Second Law. Hence, in attempting better to understand the relationship between the Kantian Laws of Mechanics and the Newtonian Laws of Motion, it is precisely to Kant's explanations of mass and quantity of matter that we should now turn.

The second explication of the Mechanics, once again, characterizes *quantity of matter* as "the aggregate of the movable in a determinate space" (537). Kant thereby links this concept to his dynamical theory of matter articulated in the previous chapter, according to which matter fills the space it occupies as a continuum with no empty interstices. So there is no such thing as an isolated point-mass, and it follows that in the Mechanics we must necessarily go beyond the Phoronomy, where matter is considered as a mere moving point. But Kant also indicates, in the same explication, that such an aggregate of movables only counts as a *mass* [*Maße*] "in so far as all its parts in their motion are considered as acting (moving) simultaneously" (537): "[O]ne says that a matter *acts in mass*, when all its parts, moved in the same direction, *simultaneously* exert their moving force externally." In order to attach a well-defined mathematical magnitude or quantity of matter in the sense of (inertial) mass to an infinite and continuous aggregate of movables, therefore, one must consider them as all moving in the same direction and simultaneously exerting their moving force externally on some other matter.

Indeed, according to the immediately following first proposition, it is precisely because such an aggregate is continuous and infinitely divisible that "the quantity of matter, in comparison with *every* other matter, can be estimated only by the quantity of motion at a given velocity" (537; compare the first paragraph of section 24 above). In the following remark to this proposition Kant indicates that he has in mind primarily the phenomenon of *weight*, whereby one matter or aggregate of movables stands in (static) equilibrium with another through the equal momenta with which they press on the equal arms of a balance (540): "The quantity of the movable in space is the quantity of matter; but this quantity of matter (the aggregate of the movable) *manifests itself* in experience only by the quantity of motion at the same velocity (for example, by equilibrium [*Gleichgewicht*])."[144] Finally, Kant indicates in the same remark that he also has in mind Newton's extension of the terrestrial concept of weight to the universal concept of mass or quantity of matter in Book 3 of the *Principia*, whereby the *dynamical* measure of quantity of matter given by

[144] See the two paragraphs following the one to which note 37 above is appended for the full passage and my earlier discussion of it in section 24.

universal gravitation is also necessarily connected to its *mechanical* measure (541): "[*O*]*riginal attraction*, as the cause of universal gravitation, can still yield a measure of the quantity of matter, and of its substance (as actually happens in the comparison of matters by weighing), even though a dynamical measure – namely attractive force – seems here to be the basis, rather than the attracting matter's own inherent motion."[145]

Towards the end of section 24 I discussed how both Newton and Kant view the argument of Book 3 as involving a complex system of relationships between three conceptually quite different notions: quantity or "amount" of matter as the product of volume and density, weight as manifested in static equilibrium, and (inertial) mass as operative in mechanical resistance and the communication of motion.[146] I suggested that Newton and Kant negotiate these relationships in rather different ways, and (as just noted) I suggested at the end of section 25 that Kant thereby aims for a more "constructive" account than Newton's of how quantity of matter becomes possible as a mathematical magnitude. We are now in a position to put the pieces together so as better to appreciate precisely what this last suggestion amounts to.

What is needed, at this point, is a more explicit discussion of how Kant's account of quantity of matter in the Mechanics is intertwined with the dynamical theory of matter developed in the previous chapter. For the balancing argument of the Dynamics chapter, as explained in section 16 above, already involves a system of relationships between three different concepts of quantity of matter: (i) a dynamical concept related to the fundamental force of repulsion through a notion of density linked to the possibility of compression; (ii) a dynamical concept related to the fundamental force of attraction by the circumstance that the accelerations produced by this force are directly proportional (at a given distance) to the attracting body's mass; (iii) a mechanical concept related to the communication of motion and therefore to the concepts of impetus and inertia.[147] We have just reminded ourselves that the second concept (involving attraction) is linked to the mechanical (inertial) concept in the remark to the first proposition of the Mechanics. And the first concept (involving repulsion and resistance to compression) is also linked to the mechanical

[145] See the paragraph preceding the one to which note 46 above is appended, together with the surrounding paragraphs, for the full passage and my earlier discussion of it in section 24.

[146] See again note 54 above, together with the paragraph to which it is appended and the remainder of section 24.

[147] See again note 132 of my chapter on the Dynamics, together with the paragraph to which it is appended and the following paragraphs, for my earlier discussion in section 16.

(inertial) concept by the relationship between dynamical and mechanical resistance – by the claim, more specifically, that "no matter would impress proportionate motion on another matter lying in the straight line *ahead of it* in its way, if both did not possess original laws of repulsion" (536; see again the paragraph to which note 4 above is appended).

The two dynamical concepts are linked to the second explication of the Mechanics (quantity of matter as the aggregate of the movable in a given space) by the balancing argument. For, on the one hand, matter fills a space by the repulsive force exerted by *all* the points in the given space – in explicit contrast with the pre-critical dynamical theory of the *Physical Monadology*. And, on the other hand, the original attraction is a penetrating force exerted from *each* point in the space filled by one matter on *all* the points in the space filled by another – "and for this reason alone [it] is always proportional to the quantity of matter" (516).[148] In other words, it is precisely because gravitational attraction is a penetrating force, for Kant, that the gravitational acceleration thereby effected by one body on another (e.g., the acceleration of the moon towards the earth) is always proportional, at a given distance, to the quantity of matter of the attracting body (in this case the earth). Moreover, it is precisely this property of gravitational attraction to which Newton crucially appeals in his determination of the masses of the primary bodies in the solar system via the accelerations experienced by their satellites in the corollaries to Proposition 8 of Book 3.[149]

Proposition 8 treats the planets (including the earth) as spherical bodies with spherically symmetric distributions of mass around their centers, and, on this assumption, it shows that the gravitational forces of bodies towards different planets (of falling bodies towards the earth and the moons of Jupiter towards Jupiter, for example) are given in terms of the inverse-square gravitational force acting between the *center* of the planet and that of the body. It thereby justifies the idealization of these bodies as isolated point-masses, and, at the same time, it provides important support for the idea that terrestrial gravity (or weight) is in fact identical to the (centripetal) force of celestial attraction responsible for the motions

[148] See note 156 of my chapter on the Dynamics, together with the paragraph to which it is appended, for the full passage. This is a note to the seventh explication – and is a consequence, for Kant, of the preceding seventh proposition stating that (512) the original attraction "is an immediate action at a distance of one matter on other matter through empty space."

[149] See the paragraph to which note 119 of my chapter on the Dynamics is appended for my first discussion of these corollaries. It appears in section 15 above, where I consider Kant's remark to the fifth proposition of the Dynamics (the beginning of the balancing argument).

of the heavenly bodies in their orbits. It also supports the moon test of Proposition 4 showing that the (inverse-square) acceleration of the moon towards the center of the earth coincides with the (constant) terrestrial acceleration g of gravity as the moon is imagined to descend to the surface of the earth.[150] In the context of Newton's Second Law of Motion, therefore, it thus provides support for the identification of inertial mass with what we now call passive gravitational mass – and it allows us to determine both via the terrestrial quantity of weight (= mg).[151] The argument concludes, finally, with the identification of inertial mass with what we now call active gravitational mass, so that we can indeed (in accordance with the corollaries to Proposition 8) take the *accelerations* produced on satellites by their primary bodies (the moon towards the earth, the moons of Jupiter towards Jupiter, and so on) as measures of the relevant primary body's inertial mass.[152]

The key to these identifications, from Kant's point of view, is the mechanical concept of quantity of matter characterized in the second explication of the Mechanics. For this is his version of the concept of quantity or "amount" of matter – the product of density and volume – where density, for Kant, is a function of the specific balance of repulsive and attractive force governing a specific type of matter (together with its state of compression). Precisely because specifically different types of matter have specifically different intrinsic densities, however, the first proposition of the Mechanics asserts that quantity of matter in this sense can only be mathematically estimated mechanically, via the communication of motion, and thus in terms of (inertial) mass. Moreover, the concept of quantity of matter is also essentially linked to what we now call active gravitational mass by the circumstance – essential to Kant's dynamical theory – that the fundamental force of attraction is a penetrating force.

150 See note 113 of my chapter on the Dynamics, together with the paragraph to which it is appended. Proposition 8 supports this argument by showing that, although the universal attraction on any body near the surface of the earth is actually compounded of very many attractive forces, most of which are not directed towards the center, it may still be treated exactly as if it were solely directed towards the center.

151 This property of specifically gravitational force is demonstrated in Proposition 6 of Book 3 (which depends on Proposition 4). See note 181 of my chapter on the Dynamics, together with the paragraph to which it is appended. The Second Law is used in concluding that the passive gravitational mass of the attracted body is precisely cancelled by its inertial mass, leaving only its *acceleration* towards the attracting body in the inverse-square law.

152 This crucial identification is demonstrated in Proposition 7 of Book 3. It depends on the previous Proposition 6, the Third Law of Motion, and (from Kant's point of view) both the immediacy and universality of gravitational interactions. See notes 192–95 of my chapter on the Dynamics, together with the paragraphs to which they are appended.

It is essential to Kant's dynamical theory, in other words, that quantity of matter is identical to active gravitational mass, which, by the first proposition of the Mechanics, is therefore identical to inertial mass as well. According to this same dynamical theory, finally, the static concept of weight signifies "the striving [of a matter] to move in the direction of greater gravitation," understood as the action of the universal and original force of attraction.[153] Hence the static concept of weight provides a measure of what we now call passive gravitational mass, and, for Kant, it thereby provides a measure of inertial mass as well.[154]

Section 25 above explains how Kant's consideration of material substance and quantity of matter in the Mechanics depends on his dynamical theory of matter developed in the preceding chapter – namely, on the infinite divisibility of material substance established in the fourth proposition of the Dynamics and the rejection of his pre-critical version of a dynamical theory in the first remark to this proposition.[155] And so we are now in a position to see, more generally, that the purpose of Kant's critical version of a dynamical theory of matter, in this context, is to elucidate

[153] See the second note to the eighth proposition of the Dynamics (518): "The *action* of universal attraction, which all matter immediately exerts on all [other matter] and at all distances, is called *gravitation*; the striving to move in the direction of greater gravitation is *weight* [*Schwere*]." The eighth proposition asserts (516) that the original attraction "extends immediately to infinity throughout the universe, from every part of matter to every other part." The preceding paragraph of the second note begins (518): "The original attraction is proportional to the quantity of matter and extends to infinity."

[154] As explained, Newton derives the relationship between weight (mg) and the passive gravitational mass of any body attracted via the inverse-square law by the argument of the moon test. By the Second Law of Motion, Proposition 6 of Book 3 then yields the identification of inertial mass (m) with passive gravitational mass (see again note 151 above). It is not entirely clear how Kant incorporates this identification into his dynamical theory, but it appears to be a necessary condition for his claim that the *dynamical* determination of quantity of matter (active gravitational mass) via universal attraction is, in the end, *mechanical*. See the passage (541) to which note 145 above is appended, together with the reference in the note back to my earlier discussion in section 24. I there illustrate, in particular, how, in the case of two gravitationally interacting bodies, the acceleration of the second towards the first is proportional (at a given distance) to the first body's active gravitational mass, and, at the same time, the change of momentum of the first body towards the second due to the (inertial) resistance of the latter is also proportional (at the same distance) to the first body's inertial mass by the equality of action and reaction. But this depends on the fact that all gravitational forces are independent of the passive gravitational masses of the bodies acted upon and thus, in the end, on the equality of inertial mass with passive gravitational mass in all such cases. In other words, it is only in virtue of precisely this fundamental property of gravitational force – that bodies fall with the same acceleration in any gravitational field – that Kant's first proposition of the Mechanics can be extended to the procedure of determining the active gravitational masses of attracting bodies by the accelerations they produce on attracted bodies.

[155] See the paragraph to which note 60 above is appended, together with the following paragraph, for my earlier consideration of this point in section 25.

those structural features of the concept of matter that make it suitable for the universal mathematization of the concept of quantity of matter effected by the Newtonian theory of universal gravitation. For it is this theory alone that has successfully extended the static concept of weight into a universal measure (mass) for all matter in the universe wherever it may be found, and, at the same time, has given a precise (and similarly universal) mathematical meaning to the traditional concept of "bulk" or "amount" of matter as the product of volume and density. Newton's mathematization thereby involves an intricate system of relationships between these three conceptually quite different properties of matter. Yet, as explained in sections 24 and 25 above, although Newton supports this system of relationships with a correspondingly intricate system of empirical findings, the conceptual connections underlying them remain comparatively unclear.[156]

The purpose of Kant's dynamical theory of matter, by contrast, is to extract just those features of Newton's theory that make his concept of the quantity of matter (as the product of volume and density) into a mathematically precise measure applicable to all matter in the universe – and then to build just these features into the concept of matter itself. For, according to Kant, we must, at least in principle, already be in possession of such a mathematically precise concept in order properly to establish the empirical laws that are supposed to govern it. Thus, for example, while Definition 1 of the *Principia* characterizes the quantity of matter as the product of density and volume, we can only measure specific densities, at this point, by the Archimedean procedure based on terrestrial hydrostatics. But when Newton turns, much later, to considering the relative densities of the primary bodies in the solar system in Corollaries 3 and 4 to Proposition 8 of Book 3, he determines their relative quantities of matter by their active gravitational masses (the accelerations produced in their satellites) and then divides by their volumes. There is no empirical or mathematical problem, from Kant's point of view, with this Newtonian argument. On the contrary, it is precisely this argument for which Kant is now attempting to provide a metaphysical foundation. But the metaphysical foundation for the argument (as opposed to the argument itself) has the task of explaining how such a truly universal mathematical concept of quantity of matter is possible in the first place. And this task,

[156] For Newton's intricate system of empirical findings see my earlier discussion in section 24 beginning with the paragraph to which note 54 above is appended and continuing to the end of that section. For the remaining conceptual tensions see the first three paragraphs of section 25. This last discussion culminates in the consideration cited in note 155 above.

in Kant's eyes, can only be accomplished by isolating just those conceptual interconnections among quantity of matter, weight, and mass that he has attempted to elucidate in the Dynamics and Mechanics of the *Metaphysical Foundations*. It is in precisely this sense, in the end, that Kant's procedure, as I have suggested, is indeed more "constructive" than Newton's.

I argued in section 20 that Kant's proposed metaphysical-dynamical program for the empirical investigation of the structure of matter in the general remark to dynamics should not be understood as one more speculative or hypothetical attempt among others to anticipate the inner structure of matter. In particular, it does not compete, in this respect, with the hard-body atomism of either the mechanical natural philosophy or Newton – or, for that matter, with the force-center atomism of Boscovich and the early Kant.[157] The present considerations both confirm and deepen this earlier point. For we now see that the heart of Kant's (critical) dynamical theory of matter, as developed in the balancing argument articulated in the last four propositions of the Dynamics, is not a contribution to hypothetical or speculative matter theory at all – whether physical or metaphysical. Instead of trying to delineate in advance matter's true inner structure, it aims rather to elucidate those features of the empirical concept of matter that explain the possibility of Newton's successful mathematization of the concept of quantity of matter in the argument of Book 3 of the *Principia*. Kant's critical version of the dynamical theory of matter, in this sense, is more like what we might conceive as an inquiry into conceptual presuppositions than a piece of hypothetical or speculative matter theory. Kant's "metaphysical" foundation for the Newtonian concept of quantity of matter, in his own terms, is thus an elucidation of precisely the conditions of the possibility of our experience – our experience, in this case, of the objects of Kant's dynamical concept of matter.[158]

This reading of the relationship between the concept of matter elucidated in the Dynamics and the treatment of quantity of matter in the Mechanics illuminates the important divergences between Kant's three Laws of Mechanics and the Newtonian Laws of Motion. For, in the first

[157] See note 72 above, together with the paragraph to which it is appended, for my earlier discussion of this point in section 25.

[158] Kant distinguishes his *dynamical* concept of the filling of space (with relative impenetrability) from the *mathematical* concept of the filling of space (with absolute impenetrability) in the fourth explication of the Dynamics and its accompanying remarks (501–2). I discuss this distinction in relation to the first proposition of the Dynamics in section 10.

place, it makes good sense of the circumstance that Kant's First Law of Mechanics is a principle of the conservation of matter, which Newton does not formulate at all. The primary purpose of Kant's dynamical theory of matter is to explain the possibility of Newton's mathematization of the *quantity* of matter, and so an account of this quantity (in the first proposition of the Mechanics) and a law of its total conservation (in the second proposition) are appropriately placed by Kant "at the pinnacle of the laws of nature that subsist a priori" (A184/B227; see the paragraph following the one to which note 83 above is appended). This reading, in the second place, also illuminates the way in which Kantian dynamical moving forces acquire the mathematical structure of a Newtonian impressed force without explicit use of the Second Law of Motion. As explained, dynamical moving force (the cause of a change of motion) acquires the vectorial structure of line and direction in the second explication of the Dynamics in virtue of the (vectorial) composition of velocities demonstrated in the Phoronomy (see the paragraph to which notes 142 and 143 above are appended). Kant's remaining problem is then to account for the magnitude of such a vector by reference to the (inertial) mass of the body acted upon by the force in question (see the paragraph following the one last mentioned), and precisely this is accomplished by the lengthy argument that I have just reviewed.[159]

[159] As indicated in note 142 above, Kant needs to go beyond points to bodies (continuous aggregates of points) in order properly to incorporate mass into the magnitude of the force-vector. In the case of attraction in general he follows Newton in considering the sum of all such point-forces originating in the body in question – where, given a spherically symmetrical distribution of mass, it thereby follows that the body may be considered as if all its mass were concentrated in its central point. Once mass is thus properly incorporated, moreover, the orbital motions Newton considers in Book 3 result from applying the composition of motions already demonstrated in the Phoronomy to the tangential and centripetal components of the motion. In the other case considered by Stan (in press, §III.3; see note 143 above) – oblique impact – the two bodies cannot be considered as mere point-centers, but Kant can still construct their motions after the impact by composing their initial inertial motions with the action of impact in the standard way. The fundamental force of repulsion is a contact or surface force rather than a penetrating force (516), where "[c]ontact in the physical sense is the immediate action and reaction of *impenetrability*" (511). Moreover, such physical contact presupposes mathematical contact (512), where, in the case of two spherical bodies, the (initial) surface of contact is the point of tangency. In order to determine the line of direction of the action of impenetrability, accordingly, we take the normal to the plane tangent to both spheres at this point. We then compose the two initial inertial motions, considered in the center of mass frame, with the two (similarly equal and opposite) components of momentum directed along the normal at the point of tangency. Although Kant does not explicitly mention this construction in the *Metaphysical Foundations*, he does observe that representing "light matter as an agglomeration of little spheres … would yield a lateral motion of light in accordance with their varying obliquity to the direction of impact" (520; compare note 64 of my chapter on the Dynamics, together with the passage from the general remark to Dynamics to which it is appended). I am here indebted to discussion with Daniel Warren.

Kant's "constructive" account of the mathematization of quantity of matter, dynamical moving force, and quantity of motion culminates in his version of the equality of action and reaction, which establishes an intimate relationship between the fundamental dynamical forces involved in any interaction and the quantities of motion thereby exchanged. Kant thus begins to explain, at the same time, how, and in what sense, the changes of motion produced by dynamical moving forces can be determined as "true" as opposed to merely "apparent" such changes. For we only have a true (as opposed to merely apparent) *force* when we have a true change of motion as well. As explained in section 27 above, however, Kant has only begun this important part of the explanation in the discussion of his Third Law of Mechanics. So this explanation, once again, is completed only in the following Phenomenology, where the argument of Book 3 of the *Principia* is more systematically reconstructed from Kant's point of view.

29 CONTINUITY, TIME DETERMINATION, AND THE CATEGORIES OF RELATION

After re-emphasizing, as we have seen, the dependence of mechanical forces on dynamical forces, Kant begins the general remark to mechanics as follows:

> The action [*Wirkung*] of a moving force on a body in an instant is the *solicitation* [*Sollizitation*] of the body; the velocity effected [*gewirkte*] in the latter through solicitation, in so far as it can increase in equal proportion to the time, is the *moment* of acceleration. (The moment of acceleration must therefore contain only an infinitely small velocity, because otherwise the body would thereby attain an infinite velocity in a given time, which is impossible; moreover, the possibility of *acceleration* in general, by means of a continued moment thereof, rests on the law of inertia.) (551)

The action of a (dynamical) moving force, therefore, is instantaneous, continuous, and cumulative. At any given instant the force produces an instantaneous velocity in the body affected (which, in general, is infinitesimal or infinitely small), and, as it continues to act throughout a given interval of time, the instantaneous velocities thereby produced add up or accumulate so as to result, at the end of the interval in question, in a single total (finite) effect – a single total (finite) change of velocity.[160]

[160] The Latinate term *Sollizitation* appears to derive from Leibniz's *Specimen Dynamicum* (1695, p. 149; 1969, p. 438), which introduces the term solicitation for the action of a moving

Kant describes the action of such continuous and incremental causality in more general terms in an important passage towards the end of the second analogy:

[E]very alteration has a cause, which manifests [*beweist*] its causality in the entire time in which the former proceeds. Therefore, this cause does not bring its effect about suddenly (at once or in an instant), but rather in a time, in such a way that, as the time increases from the initial moment a until its completion b, the magnitude of the reality (b – a) is also generated through all smaller degrees contained between the first and last. All alteration is therefore only possible through a continuous action [*kontinuierlicher Handlung*] of causality, which, in so far as it is uniform, is called a moment. The alteration does not consist of these moments, but is generated from them as their effect [*Wirkung*]. (A208–9/B253–54)

It is striking, in particular, that Kant also uses the notion of a *moment* in this passage – again characterized in terms of uniform change (change in equal proportion to the time). Kant uses the same notion (but without an explicit restriction to uniform change) when mentioning the action of instantaneous and continuously acting causality ("in passing") in the anticipations of perception (A168–69/B210): "If one considers this reality [an intensive magnitude in the appearance – MF] as cause (whether of a sensation or another reality in the appearance, e.g., an alteration), then one calls the degree of the reality as cause a moment, e.g., the moment of gravity, because the degree designates only those magnitudes whose apprehension is not successive, but instantaneous."

Kant's example of the moment of gravity allows me to illustrate our initial passage from the Mechanics (551) concretely. Consider a falling body

force – possessed by a body *in virtue of its own motion* (see note 5 above, together with the paragraph to which it is appended) – arising from an infinitely small rather than actual (finite) motion of this same body. Such a solicitation, according to Leibniz, is paradigmatic of the action of "dead force," where that of "living force [*vis viva*]" arises by "an infinite number of continuous impressions of dead force." Since Kant, however, has decisively rejected the distinction between living and dead force in favor of the (Newtonian) concept of force as possessed by a body as *cause of motion in another body* (see again note 5 above, together with the paragraph to which it is appended and notes 7, 9, and 28 above), he here intends something quite different: namely, that a continuously acting *dynamical* force (in Kant's sense) is the cause of an infinitely small change of motion (mdv) in the body on which it acts in an instant – which, continued over a finite time, then yields a finite change of motion (mv) in the latter. Euler's *Mechanica* (1736) uses the notion of solicitation (forms of the verb *sollicitare*) with the same (Newtonian) meaning. So it appears that Kant may be following Euler in his transformation of the Leibnizean terminology. This terminological (and conceptual) development deserves further investigation, but I am meanwhile indebted to Marius Stan for emphasizing the importance of *Specimen Dynamicum* here. Compare also Pollok (2001, pp. 461–65).

dropped from rest continuously accelerating under the influence of gravity. According to Galileo's law of fall, the velocity it acquires at the end of any finite time t increases linearly as this time: using modern notation, $v = gt$, where g represents the constant acceleration of gravity. At each succeeding instant of time, moreover, the body acquires a new infinitesimal or infinitely small velocity dv, which is then added to the finite velocity already acquired to produce a new total velocity $v + dv$ (so $dv = gdt$ represents the moment of acceleration = the moment of gravity here). The law of inertia figures essentially in the incremental generation of a total (finite) velocity, because the previously acquired (finite) velocities must be conserved in all succeeding instants of time when the newly produced infinitesimal velocities (moments of acceleration) are successively added. It is also clear that the moment of acceleration can only contain an infinitesimal velocity dv, for otherwise the successive addition of an infinite number of finite – and equal – velocities would necessarily produce an infinite velocity.

Newton employs an infinitesimal version of Galilean uniform acceleration in describing a continuously acting (and in general variable) force suitable for representing orbital (curvilinear) motions. Although the acceleration of gravity, as described by the inverse-square law, for example, is now variable rather than constant, it is an essential part of Newton's procedure that the infinitesimal velocity generated at any single instant can still be treated as a linear function of time. In accordance with Galileo's law, therefore, the infinitesimal distance so produced can still be expressed as proportional to the square of the time.[161] Perhaps the simplest example of this procedure is the derivation of the law for centripetal acceleration (or, equivalently, centrifugal force) in the case of uniform circular motion.

Consider a body moving uniformly from A to D along the circumference of the circle with center C; let AB be tangent to the circle at A; and let the line BE intersect the circle at D and E through the center C (so that DE is therefore a diameter of the circle):

[161] Newton formulates the general principle in question as Lemma 10 of §1 of Book 1 in the *Principia* (P437–38): "The spaces which a body describes when urged by any finite force, whether that force is determinate and immutable or is continually increased or continually decreased, are at the very beginning of the motion in the squared ratio of the times." The clearest exposition of this Lemma (together with the other initial lemmas of §1) is found in Pourciau (1998). It depends only on the continuity of the force in question: more precisely, on the (sufficient) smoothness of the resulting trajectory as a function of time (as Pourciau explains, it expresses, in modern terms, the existence of the second derivative).

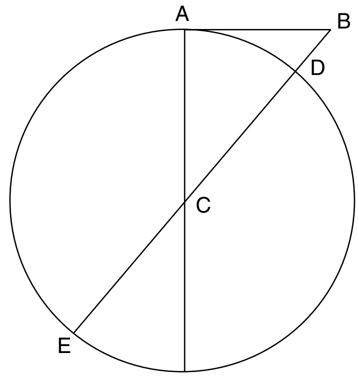

Figure 4

By Euclid III.36, BE × BD = BA^2. And, if we consider the desired centripetal acceleration a_c as we approach the (arbitrarily chosen) initial instant of the motion, D approaches A, BD becomes the infinitesimal distance "fallen" during this initial moment, and thus, in the limit, BD = $\frac{1}{2}a_c t^2$. Moreover, AB approaches AD, and BE approaches the diameter DE. In general, the (linear) velocity v is given by πDE/T, where T is the period of uniform circular motion, and t/T = AD/πDE. Therefore, in the limit, $t^2 = AB^2/v^2 = (DE \times BD)/v^2$ (by Euclid III.36). It follows that the desired centripetal acceleration a_c is given by v^2/r, where r is the radius of the circle = ½DE.[162]

The essence of this procedure is to represent a circular (and, more generally, curvilinear) motion as the combination of a rectilinear inertial motion tangent to the trajectory in question with a (rectilinear) centripetal acceleration towards a fixed central point. The centripetal acceleration satisfies Galileo's law of fall infinitesimally, or in the limit, and we can then use the spatial geometry of the resulting figure precisely to describe this acceleration – and therefore the continuously acting force that produces it. The above derivation of the law for centripetal acceleration in uniform circular motion, using this method and the Euclidean geometry of the circle, originates with Huygens, although Newton discovered it independently. Newton's further contribution was to extend it to a treatment of (continuous) centripetal accelerations in general, including analogous

[162] I have simplified (and modernized) the argument using equalities instead of proportionalities (and also taking DE to be a diameter, which is not strictly necessary). For a detailed exposition of the original argument see Brackenridge (1995, pp. 58–63).

derivations for more general figures such as the ellipse – where, of course, we obtain the inverse-square law as a function of the distance from one focus of the ellipse to an arbitrary point of this curve.[163]

It is clear that Kant was familiar with both Huygens's first application of this procedure to uniform circular motion and Newton's later extension to orbital motion in general and the inverse-square law. For, in §38 of the *Prolegomena*, when explicitly discussing the inverse-square law, Kant refers to both the geometrical property of the circle employed in the first derivation (Euclid III.36) and the generalization of this property to conic sections employed in the second.[164] Moreover, as I shall begin to explain below, Kant's developing treatment of circular (and, more generally, curvilinear) motion in the *Metaphysical Foundations* explicitly invokes the general features of this same Newtonian procedure. In our initial passage from the general remark to mechanics concerning the motions produced by a continuously acting force, therefore, Kant very likely has in mind not only the Galilean description of motion under the uniform acceleration of terrestrial gravity but also the Newtonian extension of this description to orbital motions in the solar system governed by non-uniform celestial gravitation in accordance with the inverse-square law.[165]

One of Kant's main concerns in the general remark to mechanics is decisively to reject the model of instantaneous discontinuous action traditionally associated with the phenomenon of impact. This kind of action, from Kant's point of view, could only arise from the impact of perfectly rigid or absolutely hard bodies, which would result in a finite change of momentum (of both bodies) at a single instant. Kant here argues that such bodies (and thus such instantaneous but finite changes) are impossible:

Hence, an absolutely hard body, that is, one that would, on impact, *instantaneously* oppose a body moved at finite speed, with a resistance equal to the total force of that body, is impossible. Consequently, by means of its impenetrability

[163] Newton first derives a generalized form of Kepler's area law for *any* centripetal force from the law of inertia in Proposition 1 of Book 1 (see the paragraph to which note 140 above is appended). He then uses a generalized form of Euclid III.36 applied to the ellipse in Proposition 11. The generalized form valid for all (central) conics was well known from Apollonius, and Newton presents it as such in Book 1, Lemma 12. For a detailed discussion of this generalized derivation (for central conics) see Brackenridge (1995, pp. 102–18).

[164] See *Prolegomena* §36 (4, 320–21). See also Friedman (1992b, chapter 4, pp. 186–94) for a detailed discussion of this passage. For Kant's familiarity, more generally, with the relative contributions of Huygens and Newton to the theory of universal gravitation see Friedman (1992b, chapter 5, §1).

[165] Compare note 160 above. From the perspective of universal gravitation, once again, even terrestrial gravity is in fact variable rather than constant, and Galileo's law of fall holds only approximately: compare note 43 above.

or cohesion, a matter attains instantaneously only an infinitely small resistance to the force of a body in finite motion. And from this there now follows the mechanical law of continuity (*lex continui mechanica*): namely, that in no body is the state of rest or motion, or the speed or direction of the latter, changed by impact instantaneously, but only in a certain time, through an infinite series of intermediate states, whose difference from one another is less than that between the first state and the last. A moved body that impacts on a matter is thus not brought into a state of rest by the latter's resistance all at once, but only through a continuous retardation; and one that was at rest is put into motion only through a continuous acceleration; and it is changed from one degree of speed to another only in accordance with the same rule. (552)

For Kant, therefore, *all* actions of a (dynamical) moving force – whether in impact due to repulsion or in orbital motion due to attraction – are necessarily continuous.

Kant's argument against the possibility of absolutely hard bodies begins by considering hydrostatic equilibrium – the case of "a compressed [volume of – MF] air that bears a weight" (551). The expansive force or pressure of the air is what Kant calls a surface force, which acts only with respect to an infinitely thin surface of contact and thus entirely independently of the quantity of matter found "behind" this surface.[166] In such a case the mechanical moving force (i.e., momentum) coordinated with this exercise of causality is "the motion of an infinitely small quantity of matter, which must therefore take place with finite velocity in order to be equal to the motion of a body of finite mass with infinitely small velocity (a weight)" (551). In other words, expansive force or pressure is here in equilibrium with weight, and weight is the motion of a finite mass with infinitely small velocity (see note 41 above, together with the paragraph to which it is appended). Equilibrium requires, therefore, that the counterbalancing velocity associated with pressure be finite. By contrast, the fundamental force of attraction is a *penetrating* force, and so, in this case, "a finite quantum of matter exerts moving force on a likewise finite quantum of another [matter]" (551). Kant concludes that "the solicitation of attraction must therefore be infinitely small, because it is equal to the moment of acceleration (which must always be infinitely small)" (551–52). No such attraction could occur with finite velocity, therefore, because then "matter would have to *penetrate* itself through its own attractive force" (552).

Kant defines an absolutely hard body as one that can suffer no distortion of its figure by means of any weight, no matter how great (552):

[166] Kant draws the distinction between surface and penetrating forces in the seventh explication of the Dynamics: see note 114 and (especially) note 122 of my chapter on the Dynamics.

"An absolutely hard body would be one whose parts attracted [*zögen*] one another so strongly that they could not be separated by any weight nor changed *in their situation* with respect to one another." We are to imagine, then, an arbitrarily large weight hanging from the body, which, as perfectly hard or rigid, could be thereby neither broken nor stretched.[167] The moment of acceleration associated with such resistance would therefore have to be infinitely greater than the corresponding moment of acceleration of gravity, and this, Kant holds, is impossible:

Now since the parts of the matter of such a body would have to attract [*ziehen*] one another with a moment of acceleration that would be infinite with respect to gravity, but finite with respect to that of the mass that is to be driven thereby, the resistance by means of impenetrability, as expansive force, since it always occurs with an infinitely small quantity of matter, would then have to take place with a more than finite velocity of solicitation, that is, the matter would strive to expand with infinite velocity, which is impossible. (552)[168]

What matters for my reading are not the details of Kant's argument against absolutely hard bodies but the connection it makes between the dynamical theory of matter and the phenomenon of impact – the connection, in particular, between the notions of dynamical and mechanical resistance.

The former, as we already know, is a resistance to compression acting entirely independently of the state of motion of the body in question, whereas the latter acts only in the communication of motion by impact. But we now know, in accordance with Kant's argument, that no body can be absolutely hard or rigid and, therefore, that all bodies must suffer

[167] Compare the discussion of elasticity (spring-force) in the third number of the general remark to dynamics (529): "An iron wire, stretched by a hanging weight, springs back into its [original] volume when the band is cut." Here we are to imagine the "attractive elasticity" of the wire resisting the action of the weight to some extent and then producing a corresponding rebound when this action ceases. Compare also the second number of this remark, where Kant distinguishes *cohesion* or resistance to separation from *friction* or resistance to displacement. Fluid matters exemplify the former but only solid bodies exemplify the latter (527): "A **solid** – or better a **rigid** – body (*corpus rigidum*) *is one whose parts cannot be* [mutually] *displaced by every force* – and therefore resist displacement with a certain degree of force." The supposed absolutely hard bodies are thus absolutely rigid in this sense: they resist displacement to an infinite degree. A few pages later Kant characterizes the absolutely impenetrable matter assumed by the mechanical natural philosophy as exhibiting (533) an "*absolute insurmountability* of the cohesion of matter" in the fundamental (absolutely hard) particles postulated in this philosophy.

[168] Why exactly is this impossible? The second proposition of the Dynamics states that the expansive force of matter must have a determinate (i.e., *finite*) degree (500; see note 48 of my chapter on the Dynamics, together with the paragraph to which it is appended). The reason for this, in general, is that "beyond any given force a greater force must be thinkable, for that force beyond which no greater is possible would be one whereby an infinite space would be traversed in a finite time (which is impossible)" (499). In other words, an infinite velocity as such is simply impossible.

some compression or deformation on impact. It is in this sense, therefore, that the operative force in the communication of motion by impact is the fundamental force of repulsion responsible for the original expansive elasticity of all matter. For it is in virtue of precisely such original elasticity that all matter as such is compressible by the action of any external force but resists such compression all the more the more it is actually compressed.[169] So we can now conclude, in addition, that all changes of momentum involved in the communication of motion by impact – and thus all exercises of the mechanical moving force of resistance – must satisfy the mechanical law of continuity.[170]

As explained in section 10 above, Kant introduces the concept of dynamical resistance, together with the contrasting concept of mechanical resistance, in the first proposition of the Dynamics. The conclusion of this proposition is that the dynamical resistance in question must be due to a moving force: the fundamental force of repulsion responsible for the original expansive elasticity of all matter. The argument hinges on an implicit contrast between the spatio-temporally continuous motion resulting from such a moving force and the discontinuous motion that would result on the assumption of perfectly hard or absolutely impenetrable matter. More precisely, the spatio-temporal trajectory resulting from the latter assumption would lack a well-defined tangent (or derivative) at the very moment of attempted penetration, whereas the merely relative impenetrability resulting from the former assumption entails the existence of a spatio-temporal tangent (or derivative) everywhere – including at the moment of attempted penetration. The argument thereby points back to Kant's remarks to the third explication of the Phoronomy, where the case of perfect uniform reflection at an instantaneous turn-around point is contrasted with continuously decelerating and accelerating motion under the influence of Galilean (terrestrial) gravity. In the present passage from the general remark to mechanics (552) Kant has now come full circle. For the apparently discontinuous transfer of motion in impact in cases

[169] This is the content of the third proposition of the Dynamics (501) as explained in the following remark (501): see the paragraph to which note 61 of my chapter on the Dynamics is appended.

[170] It appears from the footnote on the relationship between absolutely hard and elastic bodies appended to the first remark to the fourth proposition (549–50; see note 131 above) that such continuity holds, for Kant, whether the impact in question is elastic or inelastic. The only way in which a genuine discontinuity could arise, Kant suggests, is by considering absolutely hard (absolutely non-deformable) bodies as paradigmatic of *elastic* impact. Kant further suggests that his construction of the communication of motion is equally valid whether the impact is elastic or inelastic, in so far as he is only considering what happens up to the point that two impacting bodies set one another mutually at rest – and independently, therefore, of all consideration of a subsequent rebound. But, once again, I shall have to leave these questions aside here.

of mechanical resistance is now explicitly assimilated to the model of a continuously acting moving force in accordance (infinitesimally) with Galileo's law of fall.[171]

At the end of his statement of the mechanical law of continuity Kant inserts a parenthetical reference to an extension of this law to the case of attraction. After remarking that in no body is either the speed or the direction of its motion "changed by impact instantaneously, but only in a certain time, through an infinite series of intermediate states" (552), Kant first applies this principle to changes of speed and only then to changes of direction as well (552–53): "In the same way, the direction of its motion is changed into one that makes an angle with this [direction] in no other way than by means of all possible intermediate directions, that is, by means of motion in a curved line (which law can also be extended to the change of state of a body through attraction on similar grounds)." So here, for the first time in the Mechanics, Kant explicitly considers curvilinear rather than merely rectilinear exercises of the communication of motion. He also explicitly draws a connection between the communication of motion by impact and the communication of motion by attraction (i.e., gravitation). This passage therefore reminds us of Proposition 1 of Book 1 of the *Principia*, where Newton establishes a parallel connection.

In this Proposition, as explained, Newton represents the action of a centripetal force by a series of instantaneous and discontinuous "impulses" directed towards a common center, and he expresses his mathematical agnosticism by deliberately leaving it open whether these impulses do (or do not) result from a series of instantaneous impacts by the particles of an external aether.[172] As explained in section 18 above, however, Kant explicitly rejects Newton's mathematical agnosticism in the second remark to the seventh proposition of the Dynamics, which argues that such agnosticism concerning the true physical cause of gravitational attractions in

[171] See note 21 of my chapter on the Dynamics, together with the paragraph to which it is appended and the preceding paragraph. Kant's argument is therefore ultimately circular, for the proponent of absolute hardness or impenetrability (*mathematical* as opposed to *dynamical* impenetrability) would reject the argument of the first proposition of the Dynamics – and therefore the entire dynamical theory of matter erected on this basis. Nevertheless, as we shall see, the circularity in question presents no obstacle to our appreciation of the depth and subtlety of Kant's response to the idea of absolute hardness, which ultimately involves the continuity of time itself.

[172] See note 139 above, together with the paragraph to which it is appended and the following paragraph. As explained, Newton thereby (among other things) attempts to mollify criticisms of the notion of attraction (at a distance) by proponents of the mechanical philosophy. Newton is very careful, however, to be completely neutral on this question. The impulses may be due to external impacts but they need not be; at this point we simply do not know: see note 141 above, together with the paragraph to which it is appended.

fact sets Newton "at variance with himself" (515). Kant, more generally, has no problem at all conceiving gravitational attraction as a true and immediate action at a distance – and so he has no reason to attempt to mollify proponents of the mechanical philosophy by deriving the model of a continuously acting centripetal force from the supposedly more physically transparent model of instantaneous perfect uniform reflection on impact.[173] Indeed, the present general remark to mechanics begins with the model of continuous action (apparently derived from the Newtonian theory of gravitation) and only then goes on to argue that even the apparently discontinuous communication of motion by impact must be conceived on the model of genuinely continuous action as well.

Why, then, does Kant derive the mechanical law of continuity applied to the curvilinear motions produced by gravitational attraction as an extension of this same law applied to collisions (thereby suggesting Proposition 1 of Book 1)? And why, more generally, does he persistently give pride of place to the phenomenon of impact in discussing the laws of the communication of motion throughout the Mechanics, where the later application to the phenomenon of attraction is typically mentioned as an afterthought? The answer, I believe, is that the principle of the conservation of momentum – together with the closely related principle of the equality of action and reaction – first arose, both historically and conceptually, in the context of the phenomenon of impact. Indeed, the connection between these fundamental mechanical principles and the phenomenon of impact was so close that it took a very considerable (and very controversial) effort to apply them subsequently to the phenomenon of gravitation. One of Newton's greatest achievements was to put this extended application into practice, where the equality of action and reaction, in particular, is applied directly to the attractions between two gravitating bodies independently of any supposed aether lying in between them. Yet Newton himself repeatedly insisted that he is not committed to genuine action at a distance and explicitly left it open that the

[173] Although Newton does attempt to mollify such proponents in his discussion of Proposition 1, his motivations are perhaps better seen as largely mathematical, in so far as his appeal to discrete impulses allows a particularly clear and simple demonstration of the all-important connection between the area law and the law of inertia (where, as observed in note 140 above, the extension to a continuous motion by a limiting argument is not simple at all). Brackenridge (1995, Part 1) suggests, more generally, that Newton's mathematical dynamics is a fruitful synthesis of the "polygonal approximation" used in Proposition 1 and the "parabolic approximation" involving the infinitesimal version of Galileo's law applied directly to continuously acting forces. It is clear, in the end, that the latter – genuinely continuous – model is primary for Newton.

gravitational attraction in question could in fact be merely apparent.[174] Kant, on my reading, is suggesting a rather different perspective on Proposition 1 of Book 1 in his statement of the mechanical law of continuity. Whereas Newton's own discussion is consistent with a direct balancing of (discontinuous) changes of momentum between the orbiting body and the impulsive mechanical moving forces exerted by an external aether, Kant appeals instead to a direct balancing of (continuous) changes of momentum between the orbiting body and the (immediately) attracting body now placed at the (initially) fixed center of centripetal force.[175]

I began by quoting the passage at the outset of the general remark to mechanics where Kant depicts the action of a (dynamical) moving force as instantaneous, continuous, and incremental (551; see the paragraph to which note 160 above is appended). I also quoted a passage from the general discussion of time determination at the end of the second analogy where Kant describes continuously and incrementally acting causality in strikingly similar terms (A208–9/B253–54). This latter discussion begins by asking how change is possible in general:

> But how something may be changed in general – how it is possible that from one state at one point of time a contrary [state] at another may follow – of this we have not the slightest concept a priori. For this an acquaintance with actual forces is required, which can only be empirically given – e.g., [an acquaintance with] moving forces, or, what is the same thing, [with] certain successive appearances, which (as motions) indicate such forces. (A206–7/B252)

The procedure of the *Metaphysical Foundations* agrees with this discussion. For Kant first introduces a purely instantaneous characterization of motion (i.e., velocity) as a mathematical magnitude in the Phoronomy and then introduces two empirically given fundamental forces of attraction and repulsion in the Dynamics.

As explained in section 13 above, the concept of causality (and hence the concept of force) operative in the Dynamics is the pure or unschematized category, which therefore omits reference to "the succession of the manifold, in so far as it is subject to a rule" (A144/B183; see note

[174] This is the issue that first surfaces in Newton's correspondence with Roger Cotes: see notes 195 and 196 of my chapter on the Dynamics, together with the paragraph to which they are appended and the preceding paragraph.

[175] One central consequence of this substitution is that the initially fixed center can no longer be taken to be truly fixed. As observed, Newton explicitly emphasizes this point in his introductory remarks to Book 1, §11 by stating that "neither the attracting nor the attracted body can be at rest, but both (by corol. 4 of the laws) revolve about a common center of gravity *as if by a mutual attraction*" (561, emphasis added; see note 142 above). In the next sentence, however, Newton re-emphasizes his mathematical agnosticism (see note 139 above).

75 of my chapter on the Dynamics chapter, together with the paragraph to which it is appended). The corresponding notion of force is thus also purely instantaneous, and so time determination as such, which essentially involves the determination of "certain *successive* appearances, which (as motions) indicate such forces" (A207/B252; emphasis added), has still not been fully put into effect. Indeed, it is only in the Mechanics that the schematized concepts of causality and force come fully into play. In particular, our passage from the beginning of the general remark introducing the "*moment* of acceleration" (551) concludes by stating that "the possibility of *acceleration* in general, by means of a continued moment thereof, rests on the law of inertia" (551). Galilean free fall produced by the earth's gravity, for example, depends on conserving earlier finite velocities as new infinitesimal velocities are instantaneously added (as moments of acceleration). And, even more obviously, the action of (universal) gravitation producing the curvilinear orbital motions in the heavens depends on the inertial tendency of these same bodies to proceed rectilinearly along a tangent. The main point of the general remark, finally, is that the action of the fundamental force of repulsion is to be assimilated, in this respect, to the (continuous and incremental) action of (universal) gravitation.

In the context of the Mechanics, therefore, it is the law of inertia that first makes time determination possible. It does so, in general, by telling us where a moving body would end up in space and time if no dynamical forces acted on it after (or before) a given instant. The effect over time of a continuously acting dynamical force can then be derived from its instantaneous actions (solicitations resulting in moments of acceleration) together with the law of inertia. In this sense, the law of inertia binds together different and otherwise independent moments of time by specifying the naturally persisting state of motion of a body on the basis of which *changes* of state – due to the actions of external forces – can then be determinately ordered.[176] Kant suggests just this kind of role for the law of inertia, moreover, in the continuation of the passage from the second analogy (A206–7/B252) quoted above. He states (A207/B252) that "the form of any change" can indeed be "considered a priori" while implying that "the content [of this change], i.e., the state that is changed" cannot, and he adds a footnote specifying that only accelerated (but not uniform)

[176] In contemporary mathematical terms the law of inertia thereby binds together different and otherwise independent tangent spaces at different points of the space-time manifold by inducing an *affine structure* on this manifold – where the inertial trajectories in Newtonian space-time, for example, define just such a [flat] affine structure (see notes 42 and 52 of my chapter on the Phoronomy).

motion counts as a genuine change of state (see notes 93 and 94 above, together with the paragraph to which they are appended).

But there is no pre-existing absolute space given prior to and independently of the actual empirically given motions, for Kant, and he instead employs his three Laws of Mechanics to regulate an indefinitely extended corrective procedure by which we construct better and better empirical approximations to such a privileged space. By the same token, there is no pre-existing absolute time but only a parallel and complementary indefinitely extended corrective procedure by which we construct better and better empirical approximations to true temporal uniformity. The notion of uniform rectilinear motion central to the law of inertia only receives fully determinate empirical meaning at the very end of this procedure – which, moreover, can never be actually attained. Or, to put it less abstractly, this notion, like the Laws of Mechanics themselves, functions solely as a rule or standard for correcting the lack of true temporal uniformity in any actually given empirical motions.[177]

For a concrete example of how this procedure is supposed to work, imagine that it has been extended as far as the solar system, where, in particular, the center of gravity of this system is now determined as our (provisionally) fixed point of reference representing absolute space. The orbital motions in our system, under the action of universal gravitation, are also thereby determined, with the result that all the primary bodies in the solar system, including the sun, execute orbital motions around this (provisionally) fixed point. In this frame of reference there is a rectilinear inertial component of motion tangent to the curvilinear path in question and a rectilinear accelerated component of motion directed towards the (provisionally) fixed central point generating each such case of orbital motion. That is, these two components, acting together, determine the succeeding (and preceding) instantaneous states of motion in accordance with the law of universal gravitation. Relations of simultaneity between these same instantaneous states of motion are similarly determined, since gravitational forces act immediately and instantaneously across arbitrarily large spatial distances. And, by taking account of all the gravitational

[177] See the paragraph to which note 52 of my chapter on the Phoronomy is appended, together with the following paragraph. In the contemporary terms of the preceding note, therefore, the point is that, just as there is no pre-existing absolute space or time for Kant, there is also no pre-existing affine structure given by a family of idealized inertial trajectories. The entire mathematical structure of what we now call Newtonian space-time, including especially its affine structure, is to be constructed from our experience of actual bodies on the basis of a priori principles ultimately derived from the categories.

interactions between all the bodies in the solar system, we are then able to correct our common astronomical standards of temporal uniformity. For example, we can (at least in principle) correct the (actually non-uniform) sidereal rotation of the earth by reference to the tidal friction created by the attraction of the sun and the moon, and so on.[178] The three fundamental modes of time – duration, succession, and simultaneity – can thus be empirically constructed (again, at least in principle) from the observable given motions.

This same concrete realization of time determination in the Mechanics also provides us with a concrete realization of the more general conception of continuously acting causality Kant presents at the end of the second analogy. Kant expresses the result as a similarly general "law of the continuity of all changes" (A209/B254), according to which "every transition in perception to something following in time is a determination of time through the generation of this perception, and since the latter [i.e., time] is always and in all of its parts a magnitude, [every such transition is] the generation of a perception as a magnitude through all degrees, of which none is the smallest, from zero up to its determinate degree" (A210/B255).[179] It is all the more surprising, then, that, at the end of his discussion of the "mechanical law of continuity" in the general remark to mechanics, Kant takes pains to distance this discussion from the more general "metaphysical law of continuity" in particularly strong and explicit terms:

This [mechanical] *lex continui* is based on the law of the inertia of matter. Since, by contrast, the *metaphysical* law of continuity would have to be extended to all changes in general (inner as well as outer), and would thus have to be based on the mere *concept of a change in general*, as magnitude, and on its generation (which would necessarily proceed continuously in a certain time, as does time itself), [this metaphysical law] therefore finds no place here. (553)

[178] See note 50 of my chapter on the Phoronomy, together with the paragraph to which it is appended and the two preceding paragraphs.

[179] Several pages earlier Kant makes it clear that this law of continuity (like all pure principles of time determination) arises from the need to represent the properties of absolute time (which is not an object of perception) in the empirically given appearances themselves. See again the passage quoted in note 45 of my chapter on the Phoronomy, according to which a universal rule grounded in the understanding generates "precisely the same order and *continuous connection* in the series of possible perceptions … as is found a priori in the form of inner intuition (time), wherein all perceptions must have their place" (A200/B245, my emphasis). At the end of the previous paragraph Kant states (A199/B244, emphasis in the original) that "*only in the appearances can we empirically cognize this continuity in the interconnection* [*Zusammenhange*] *of times*."

It is not surprising that Kant distinguishes the two laws, for Kant makes a parallel distinction in the case of each of his three Laws of Mechanics.[180] What is peculiar is the assertion that the "metaphysical law" has "*no* place here" (emphasis added) – something that Kant certainly does not say in the other cases. Such an assertion seems entirely inappropriate, more generally, in any case where a more specific law of corporeal nature is in fact an instance or realization of a more general law of pure understanding.[181]

A full discussion of this situation requires a more general exploration of the relationship between the transcendental principles of the first *Critique* and the special metaphysics of corporeal nature expounded in the *Metaphysical Foundations*, and I shall undertake such an exploration in the Conclusion. But I can now begin to clarify a number of relevant points, on the basis of which it will then be possible to illuminate the precise role of the principle of continuity in the argument of the Mechanics as a whole. The first point is already implicit in the text of the second analogy where Kant formulates the metaphysical (i.e., *general* metaphysical) law of continuity in question. For, as we have seen, Kant introduces his argument (A206–7/B252) by explicitly excluding empirically given "moving forces" from the present discussion because such "actual forces" cannot be considered purely a priori. What can be considered purely a priori is rather the form of a change in general (A207/B252): "But the form of any change, the condition under which it, as the arising of another state, can alone proceed (the content of the change, i.e., the state that is changed, may be what it will), and thus the succession of the states itself (that which happens), can still be considered a priori in accordance with the law of causality and the conditions of time.*"[182]

[180] For the first case see note 61 above, together with the paragraph to which it is appended; for the second see notes 91 and 92 above, together with the paragraphs to which they are appended and the preceding paragraph; for the third see note 111 above, together with the paragraph to which it is appended. In the first two cases Kant calls the more general transcendental principle from the first *Critique* a proposition of "general metaphysics," but in the third case it is also called simply a "metaphysical law." That general metaphysics or transcendental philosophy considers all objects of our senses in general, inner as well as outer, whereas the special metaphysics of corporeal nature is limited to objects of specifically outer sense, emerges from the paragraph from the Preface (469–70) cited in note 91 above.

[181] In the case of his third "mechanical law" (see again the paragraph to which note 111 above is appended) Kant does not actually *derive* it from the corresponding "metaphysical law," but he still insists (544) that "we must borrow" the corresponding principle of pure understanding at this point and that (545) we "can still not wholly leave aside this metaphysical law of community here, without detracting from the completeness of the insight."

[182] I observed in the paragraph to which note 176 above is appended that Kant suggests in his footnote that the law of inertia specifies the content of the change in the case of moving forces in particular.

By contrast, the argument for the mechanical law of continuity in the general remark to mechanics depends on taking the Newtonian mathematical treatment of orbital (and, more generally, curvilinear) motion as the paradigm for a continuously acting force. Beginning with this Newtonian model, and then appealing to the balancing of the two fundamental forces central to his dynamical theory of matter, the import of Kant's argument for the mechanical law of continuity is to assimilate the apparently discontinuous transfer of motion on impact to the Newtonian model of gravitational force. So this argument, in particular, depends on the actual empirical givenness of the two fundamental dynamical moving forces of attraction and repulsion – which are explicitly excluded from the discussion of the (general) metaphysical law of continuity in the second analogy. These forces are precisely what must be abstracted from in establishing a purely a priori argument for the continuity of change based solely on the "*form* of any change … in accordance with the law of causality and the conditions of time" (A207/B252; emphasis added). The proof of the mechanical law of continuity depends on the de facto continuity of the action of the two fundamental forces, while the proof of the metaphysical law depends only on the relevant a priori principle of pure understanding (the principle of causality) and the equally a priori pure intuition of time.

A second and even more important point emerges if we ask exactly how the metaphysical law of continuity derived in the second analogy is supposed to be proved there. Kant emphasizes the importance of this question at the very end of his discussion. After remarking (A209/B254) that "[w]hatever uses this proposition may have in the investigation of nature does not concern us here," he continues (A209/B254–25): "But how such a proposition, which appears to extend our cognition of nature so much, may be possible completely a priori, very much requires our examination, even though it is obviously actual and correct, and one would therefore like to believe that the question of how it is possible is presumptuous." He then presents his answer:

All increase of empirical cognition, and every advance of perception, is nothing but an extension of the determination of inner sense – that is, a progress in time – whatever the objects may be, appearances or pure intuitions. This progress in time determines everything and, in itself, is determined by nothing further: i.e., the parts of this [progress] are only given in time, and through the synthesis of [time], they are not given prior to it. For the sake of this [progress], every transition in perception to something following in time is a determination of time through the generation of this perception, and since the latter [time] is always and in all of its parts a magnitude, [every such transition is] the generation of a perception as a magnitude through all degrees, of which none is the smallest, from zero up to its determinate degree. From this the possibility of cognizing a priori a law of changes, in accordance with their form, is now clear. We anticipate only our own apprehension, whose formal condition, since it dwells in us prior to all given appearances themselves, certainly must be able to be cognized a priori. (A210/B255–56)

In other words, the "form" or "formal condition" of any change is given by time as the form of inner sense, in so far as it is synthesized or determined by the a priori activities of the understanding – a claim that Kant makes explicitly in the following concluding paragraph.[183]

The relevant process of apprehension that we are here in a position to anticipate is what Kant calls (A98) the "synthesis of apprehension in intuition" in the first edition version of the transcendental deduction – the first of the three syntheses discussed there. Kant begins his discussion of this synthesis with a general remark about time:

Wherever our representations may arise from, whether they are effected through the influence of outer things or through inner causes, whether they arise a priori or empirically, as appearances, they nevertheless belong to inner sense as modifications of the mind, and, as such, all of our cognitions are ultimately subject to the formal condition of inner sense, namely time, as that in which they all must be ordered, connected, and brought into relation. This is a general remark, which must be taken as the basis of all that follows. (A98–99)

This general remark, in turn, echoes Kant's earlier discussion in the transcendental aesthetic where he is concerned to explain the sense in which time is the form of *inner* sense:

Time is the formal a priori condition of all appearances in general. Space, as the pure form of all outer intuition is limited, as an a priori condition, merely to outer appearances. By contrast, because all representations, whether they have outer things as object or not, still belong in themselves, as determinations of the mind, to [its] inner state, and this inner state belongs under the formal conditions of inner intuition, and therefore to time, [it follows that] time is an a priori condition of all appearances in general – and, in fact, [it is] the immediate

[183] This paragraph reads in full (A210–11/B55):

Therefore, just as time contains the sensible a priori condition for the possibility of a continuous progress of an existent to the following [one], the understanding, by means of the unity of apperception, is the a priori condition of the possibility of a continuous determination of all positions of the appearances in time, through the series of causes and effects – where the former unavoidably draw the existence of the latter after themselves, and thereby make the empirical cognition of temporal relations valid for every time (universally) and thus objectively [valid].

condition of inner [appearances] (of our souls) and precisely thereby also the mediate condition of the outer appearances. If I can say a priori [that] all outer appearances are in space, and are determined a priori in accordance with spatial relations; I can say completely generally, on the basis of the principle of inner sense, [that] all appearances in general, that is, all objects of the senses, are in time, and necessarily stand in temporal relations. (A34/B50–51)

Time as the form of inner sense is therefore a much more general a priori condition of appearances than space as the form of outer sense. The synthesis of apprehension whose form Kant anticipates in his proof of the metaphysical law of continuity in the second analogy thereby shares in this greater generality.

It is in precisely this way that the metaphysical law of continuity proved in the second analogy achieves its much greater generality vis-à-vis the mechanical law of continuity proved in the general remark to mechanics. It is in precisely this way, in the words of the general remark, that the metaphysical law, unlike the mechanical law, is "extended to all changes in general (inner as well as outer)" (553; see the paragraph to which note 180 above is appended). The proof of the mechanical law, by contrast, explicitly depends, like everything else in the *Metaphysical Foundations*, on a consideration of space as the form of outer intuition. And so it is thereby limited, like everything else in the *Metaphysical Foundations*, to objects of specifically outer intuition. Kant marks this contrast in the general remark by insisting (553) that the metaphysical law is "based on the law of the inertia of matter." Moreover, Kant is not merely referring here to the principle of rectilinear inertia but, above all, to his own statement of the Second Law of Mechanics – whose primary import, as explained in section 26 above, is to assert that all matter as such is essentially lifeless and thus by no means characterized by *internal* principles. On the contrary, all determinations of matter as such are essentially spatial, and any cause of a change in matter (i.e., a motion) must be found in another matter to which it is spatially external.[184]

Indeed, the proof of the mechanical law of continuity depends on all three of Kant's Laws of Mechanics: in particular, on the principle of the conservation of momentum closely associated with the first and the equality of action and reaction formulated in the third. And these laws,

[184] Compare once again the paragraph to which note 91 above is appended, together with the two following paragraphs. As I emphasize in the paragraph to which note 98 above is appended, together with the following paragraphs, this marks the beginning of a sustained polemic against the idea of a "force of inertia," especially as expressed in Kant's earlier pre-critical writings. I shall return to the latter below.

therefore, necessarily involve a consideration of space as the form of outer intuition. The first considers the quantity of matter as an "aggregate of … movables [spatially] external to one another" (541–42; see the paragraph to which note 61 above is appended, together with the following paragraph); the third explicitly involves a procedure by which motions are "reduced to absolute space" (545; see the paragraph to which note 112 above is appended, together with the following paragraph). More generally, as explained above, the proof of the mechanical law of continuity depends on taking the Newtonian mathematical treatment of orbital (and, more generally, curvilinear) motion as the paradigm for a continuously acting force, and the essence of this treatment is to exploit features of the spatial geometry of the curvilinear figures in question to derive corresponding features of the relevant forces. The mechanical law of continuity thereby essentially depends on a consideration of space and its geometry, whereas the whole point of the corresponding metaphysical law is deliberately to abstract from just this consideration.

In the Preface to the *Metaphysical Foundations* Kant explains that only the doctrine of body, but not the doctrine of the soul, can be a science in the strict or "proper" sense, because "a doctrine of nature will contain only as much proper science as there is mathematics applicable there" (470). The doctrine of the soul cannot be a "proper" natural science because mathematics is not applicable there:

> [B]ecause mathematics is not applicable to the phenomena of inner sense and their laws, the only option one would have would be to take the *law of continuity* in the flux of inner changes into account – which, however, would be an extension of cognition standing to that which mathematics provides for the doctrine of body approximately as the doctrine of the properties of the straight line stands to the whole of geometry. For the pure intuition in which the appearances of the soul are supposed to be constructed is *time*, which has only one dimension. (471)[185]

It follows, then, that the metaphysical law of continuity, unlike its mechanical counterpart, cannot be a mathematical law. For the former considers the changing appearances in question only as "modifications of the mind" (A99), and the time in which all these appearances "must be ordered, connected, and brought into relation" (A99) is itself considered, first and foremost, as "the immediate condition of inner [appearances] (of our soul)" (A34/B50).

[185] See the paragraph to which note 8 of the Introduction is appended for the full passage.

So Kant's point in the general remark to mechanics is not to deny that the mechanical law of continuity is an instantiation or realization of the metaphysical law. In particular, there is no reason that the continuously acting dynamical forces of attraction and repulsion conforming to the specifically Newtonian model of a continuously acting force cannot instantiate the more general conception of continuously acting causality articulated in the second analogy.[186] Kant's point is rather that the proof of the metaphysical law formulated in the second analogy does not lead by any natural or direct continuation to a corresponding proof of the mechanical law. For example, we cannot simply restrict the metaphysical law to specifically spatial appearances in order thereby to derive the corresponding mechanical law.[187] On the contrary, the latter essentially involves a fundamental change of perspective, whereby space and its geometry, the three Laws of Mechanics, and the empirically given dynamical forces of attraction and repulsion are all explicitly bought into play. This change of perspective, for Kant, implies that the very meaning of the metaphysical law of continuity, together with the precise way in which it serves to "extend our cognition of nature" (A209/B254), must also be fundamentally rethought.

The source of the metaphysical law of continuity in the tradition within which Kant is writing is Leibniz. Kant makes this clear in §14 of the *Inaugural Dissertation* (1770), which focusses on the idea that "*time is a continuous magnitude* and the principle of the laws of the continuous [*lex continui*] in the changes of the universe":

Now the metaphysical law of *continuity* [*lex continuitatis metaphysica*] is this: *All changes are continuous* or flow: i.e., opposed states follow one another only through an intermediate series of different states. For, because two opposed states are in different moments of time, [and] between two moments of time

[186] As explained, there is no doubt that the former does provide Kant with his primary instantiation of the latter in the corporeal realm: see the paragraph to which note 179 above is appended, together with the earlier discussion in the first two paragraphs of the present section. In the terms of notes 176 and 177 above, we thereby obtain a one-dimensional ordering of three-dimensional instantaneous spaces or "planes of absolute simultaneity" in Newtonian space-time.

[187] As explained (in my earlier discussions cited in note 180 above), both the first and second mechanical laws are directly derived from the corresponding transcendental principle by explicitly restricting it to spatial objects. In the case of the Third Law, it is true, the mechanical principle is not literally derived from the transcendental principle at all, but, nevertheless, the former is still intimately connected with the latter (see note 181 above). Indeed, this connection is so important that Kant modifies his statement of the third analogy in the second edition to say that the co-existent substances in question "can be perceived as simultaneous *in space*" (B256, my emphasis; see note 110 above). I shall return to this point below.

there is always some intervening time, in the infinite series of moments of which the substance is neither in the one given state, nor in the other, nor in no state [at all], [it follows that] it will be in different states, and so on to infinity. (2, 399)

This proof of the metaphysical law of continuity is therefore quite similar in its basic idea to the analogous proof in the second analogy.[188] And it is explicitly characterized in the next sentence of the *Inaugural Dissertation* as "this law of Leibniz [*hanc Leibnizii legem*]" (2, 399).[189] So it is very likely that Kant regards the metaphysical law of continuity proved in the second analogy as also of Leibnizean origin.[190]

Although, in the *Inaugural Dissertation*, Kant portrays himself as a proponent and defender of this Leibnizean law, he goes on in the following discussion sharply to attack the Leibnizean conception of time. Opposed to Kant's own view that time is a "*pure intuition* [*intuitus purus*]" are both the opinion of certain "English philosophers" that it is "some kind of continuous flux within existence, but even without any existing thing" and the view that it is "something real abstracted from the succession of internal states – the view maintained by Leibniz and his followers" (2, 400). Thus here, just as in the following discussion of space in §15, Kant intends to stake out a middle ground between the Newtonians and the Leibnizeans.[191] But what is of most interest in the context of my present discussion is that Kant criticizes the Leibnizean view for "completely neglecting *simultaneity*,* the most important consequence of time" (2, 401) and adds the following striking remarks in the footnote:

[S]imultaneous [things] are joined together at the same moment of time in just the same way that successive [things] are at different moments. Therefore, although time has only one dimension, still the *ubiquity* [*ubiquitas*] of time (to speak with Newton), whereby *all* [things] sensitively thinkable are at *some time*,

[188] The essence of this proof (A208–9/B253–54) is quoted in the second paragraph of the prersent section.

[189] Here Kant responds to a challenge raised by Abraham Kästner to the defenders of this "law of Leibniz," namely, to show that the continuous motion of a point along all the sides of a triangle is impossible. Kant's response is closely related to the considerations he presents in the first proposition of the Dynamics, where he rejects an instantaneous turn-around point at the initial moment of penetration (see again note 171 above, together with the paragraph to which it is appended). The discussion in the *Inaugural Dissertation* concludes (2, 400): "Therefore, in accordance with the teachings of Leibniz, a body changes its direction in continuous motion only in accordance with a line none of whose parts are straight, i.e., a curved [line]."

[190] The main difference between the two is that the proof in the *Inaugural Dissertation* makes no mention of the synthesizing activities of the understanding as it *determines* the pure intuition of time. I shall return to this point below.

[191] See note 15 of the Introduction, together with the paragraph to which it is appended, for my earlier and more extensive discussion of this matter.

adds another dimension to the magnitude of actual [things], in so far as they hang, as it were, from the same point of time. For if one designates time by a straight line proceeding to infinity, and that which is simultaneous at any point of time by ordinates to this line, then the surface thereby generated represents the *phenomenal world*, with respect to both substance and accidents. (2, 401)[192]

Kant's point is that one cannot simply define simultaneous events as those that are not successive, for the relation of simultaneity requires just as much of a positive (causal) relation between events as the relation of succession. When one takes this point seriously into account, moreover, one sees that the Leibnizean conception of the sense in which time has only one dimension can in fact be seriously misleading.

These remarks appear even more striking if we juxtapose them with the passage from the Preface to the *Metaphysical Foundations* where Kant explains that "mathematics is not applicable to the phenomena of inner sense and their laws" (471). True, we could apply the law of continuity here, but, precisely because time "has only one dimension" (471), this "would be an extension of cognition standing to that which mathematics provides for the doctrine of body approximately as the doctrine of the straight line stands to the whole of geometry." The remarks concerning simultaneity in §14 of the *Inaugural Dissertation* appear to be making essentially the same point against the Leibnizean view of time. Since time, on this view, is "abstracted from the succession of internal states" (2, 400), it leaves out the fundamental *connection* between time and space necessarily involved in the relation of simultaneity between spatially distant events in the phenomenal world. We therefore lose the possibility of using the geometry of space in order further to develop the application of mathematics to time, and, in the end, a proper science of time would then be impossible.

The reference to Newton in these remarks is also striking. For, as we know, one of the main points of natural philosophy on which Kant strongly differs from Leibniz is that Kant enthusiastically embraces universal gravitation as an immediate action at a distance. Indeed, Kant's main concrete instantiation or realization of the procedure of time determination (the determination of duration, succession, and simultaneity) is provided by

[192] See note 33 of the Introduction (where the entire footnote is quoted), together with the paragraph to which it is appended. The reference to Newton is to the General Scholium added to the second (1713) edition of the *Principia*, according to which "each and every particle of space is *always* [*semper*], each and every indivisible moment of duration is *everywhere* [*ubique*]" (P941). The context is an argument that God must similarly exist "*always* and *everywhere* [*semper & ubique*]" (P941).

the Newtonian theory of universal gravitation (see the paragraph to which note 178 above is appended), and the relation of (distant) simultaneity, in particular, is determined by instantaneous gravitational interactions. We can therefore say, more generally, that, since the Leibnizean monadology recognizes only *intra*-substantial causality (see note 118 above, together with the paragraph to which it is appended), the only temporal relations that it can comprehend are relations of succession within a single substance.[193] What Kant wants to put in its place, therefore, is precisely the Newtonian procedure for progressively mathematizing time, on the basis of all of the *geometrical* resources (including instantaneous action at a distance) provided by the theory of universal gravitation.[194]

We can deepen our appreciation of these points by asking what Leibnizean text might have been the background for Kant's discussions of the metaphysical law of continuity – both in the *Inaugural Dissertation* and the corresponding passage in the second analogy (together with the passage from the general remark to mechanics with which we began). One likely source, it seems, is Leibniz's treatment of the law of continuity in the Preface to the *New Essays on Human Understanding*, which appeared in 1765 and produced a strong impression on contemporary German philosophy. The context is Leibniz's defense of innate ideas, based on the "minute perceptions [*petit perceptions*]" of which we are only *potentially* consciously aware. Such minute perceptions are intimately connected with the law of continuity, because, in waking from sleep to full conscious awareness, for example, it is necessary that there be intermediate states of less than fully conscious awareness in between.

Leibniz concludes this discussion as follows:

In a word, *insensible perceptions* are of just as great use in pneumatology [*Pneumatique*] as insensible corpuscles are in physics [*Physique*], and it is just as

[193] More precisely, the Leibnizean conception uses pre-established harmony between substances to define a kind of surrogate for inter-substantial causality and explains simultaneity (or spatial co-existence) by the absence of this kind of causality. Kant's objection in the *Inaugural Dissertation* is that the *absence* of a causal relation does not, by itself, give rise to any kind of (positive) efficacious causality.

[194] The crucial point, in the contemporary mathematical terms of note 186 above, is that we thereby represent time as a one-dimensional ordering of three-dimensional instantaneous spaces or "planes of absolute simultaneity" in Newtonian space-time – where, in particular, two events are placed in the same plane of absolute simultaneity just in case they (as instantaneous changes of momentum) are connected by instantaneous gravitational forces in accordance with the Third Law of Motion. Moreover, as Stein (1977, p. 13) suggests, in view of Kant's own image in the *Inaugural Dissertation* of the phenomenal world as a (two-dimensional) surface generated by ordinate lines representing simultaneous events perpendicular to the (one-dimensional) flow of time, this contemporary mathematical representation is not quite so anachronistic as it may first appear.

unreasonable to reject the one as the other on the pretext that they are beyond the reach of our senses. Nothing takes place suddenly, and it is one of my great and best confirmed maxims that *nature never makes leaps* – which I have called *the law of continuity* … and the use of this law is very considerable in physics. It implies that any change from small to large, and vice versa, passes through what is intermediate, both with respect to degrees and with respect to parts, and that no motion ever arises immediately from rest or is reduced to [rest] except through a lesser movement [*un movement plus petit*], just as one could never traverse a certain line or distance without traversing a shorter one [*une ligne plus petite*]; nevertheless, those who have propounded the laws of motion have not observed this law, believing that a body can receive in an instant a motion contrary to the preceding one. All of this supports the judgement that *noticeable perceptions* [*perceptions remarquables*] arise by degrees from those that are too minute to be noticed. (1765, pp. 56–57)[195]

It appears likely, as I say, that Kant had just this discussion in mind in both the *Inaugural Dissertation* and the later parallel treatments of the metaphysical law of continuity in the second analogy and general remark to mechanics. For, on the one hand, we know that Kant was intensively occupied with the *New Essays* in the years just prior to the publication of the *Inaugural Dissertation*.[196] And, on the other hand, the passage from the second analogy says that Kant will here leave aside "[w]hatever uses this proposition [i.e., the metaphysical law of continuity] may have in the investigation of nature [*Naturforschung*]" (A209/B254; see again the paragraph to which note 183 above is appended). Finally, at the same time, it is clear from the corresponding discussion of the metaphysical law in the general remark to mechanics that a principle of continuous transfer of motion on impact is the primary application in physics that Kant has in mind.[197]

Moreover, it is noteworthy that, in the Preface to the *New Essays*, Leibniz envisions two parallel and complementary sub-parts of knowledge or science in general: "pneumatology" or the pure doctrine of the

[195] For Leibniz, the fundamental law of motion is the conservation of *vis viva* in perfectly elastic impact. Moreover, no motion can ever be lost during impact (elastic or inelastic), since any apparent loss (in inelastic impact) is only transferred to the (essentially elastic) insensible parts of the bodies. As a result, impacting bodies always suffer some deformation on impact (whether or not they rebound), and the change of motion is always continuous. In any case, however, the restriction of all interaction to impact implies that Leibniz has no real account of (distant) simultaneity – which, for Kant, depends on precisely a true and immediate gravitational attraction through empty space.

[196] See, e.g., Cassirer (1918, pp. 97–98).

[197] See again the passage (552) quoted in the paragraph following the one to which note 165 above is appended. It is clear from note 189 above that this is the primary application that Kant has in mind in the *Inaugural Dissertation* as well.

soul and "physics" or the pure doctrine of body. For this is precisely what Kant later does as well in the Preface to the *Metaphysical Foundations*. The fundamental difference, however, is that Kant decisively rejects the idea that there can be a strict or "proper" science of the soul – precisely because "mathematics is not applicable to the phenomena of inner sense and their laws" (471). Indeed, the difference between Leibniz and Kant on this point goes deeper still, for the central idea of the Leibnizean monadology is that pneumatology or the pure doctrine of the soul provides a metaphysical foundation for physics or the pure doctrine of body. The ultimate constituents of reality are monads, unextended mind-like simple substances modeled on our own inner experience of the soul, and the empirically given reality of bodies interacting mechanically in space is a derivative well-founded phenomenon of this underlying monadic reality.[198] For Kant, by contrast, a genuine metaphysical foundation for physics cannot possibly proceed in this way. And it is for precisely this reason that the idea of such a monadology "has nothing at all to do with the explanation of natural appearances, but is rather an intrinsically correct *platonic* concept of the world devised by Leibniz, in so far as it is considered, not at all as object of the senses, but as thing in itself, and is merely an object of the understanding" (507; see the paragraph following the one to which note 3 of my chapter on the Dynamics is appended). More generally, according to the critical Kant of the *Metaphysical Foundations* and the first *Critique*, the idea of such a monadological foundation for physics entirely misconstrues the route from self-reflection by means of the pure understanding to our empirical knowledge of nature.[199]

[198] Leibniz only alludes to this monadological metaphysics in the Preface to the *New Essays* – where, for example, he mentions his "marvelous pre-established harmony between the soul and the body, and indeed among all the monads or simple substances" and then refers to a "sound pneumatology, comprising knowledge of God, souls, and simple substances in general" (1765, pp. 55, 57; and compare p. 375 for a discussion of the sensible world as a "linking together of phenomena"). Nevertheless, this metaphysics emerges naturally here when read in conjunction with the *Monadology* (1714) – another work certainly known to Kant (see the following note). More generally, there is no doubt that Kant himself ascribes such a view to Leibniz, as can be seen from the passage from the amphiboly quoted in section 21 above, where Kant says that "in the pure understanding matter precedes form, and *Leibniz* consequently first assumed things (monads), together with an inner power of representation, in order afterwards to ground their external relations and the community of their states (namely, their representations) on this" (A267/B323; see the paragraph to which note 77 of my chapter on the Dynamics is appended).

[199] See the paragraph to which note 255 of my chapter on the Dynamics is appended. Again, what Leibniz appears to suppose is that we have knowledge of our own mind, by the pure understanding, as it is in itself, and we then project this knowledge, also by the pure understanding, onto the ultimate monadic constituents of matter. Leibniz suggests such a procedure in the Preface to the *New Essays*, where he defends innate (and a priori) knowledge

In grounding our empirical knowledge of nature (and thereby providing a metaphysical foundation for physics), the understanding does in fact begin by linking intellectual self-reflection (pure apperception) to the pure intuition of time. So time, as Kant says, is indeed "the immediate condition of inner [appearances] (of our soul)" (A34/B50; see the paragraph preceding the one to which note 184 above is appended). He calls this procedure "figurative synthesis," or the "transcendental synthesis of the imagination," in §24 of the second edition transcendental deduction and says that it is "an action of the understanding on sensibility and its *first* application to objects of an intuition possible for us (and at the same time the ground of all other applications)" (B151–52, emphasis added). Kant then considers an apparent paradox in his notion of inner sense, namely, that it "presents ourselves to consciousness only as we appear to ourselves, not as we are in ourselves, because we intuit ourselves only as we are internally *affected*, which seems to be contradictory, in that we would have to relate to ourselves as passive" (B152–53).

Here is the solution:

> That which determines inner sense is the understanding and its original capacity to combine the manifold of intuition, i.e., to bring it under an apperception (as that on which its own possibility rests) … [The understanding] therefore exerts this act [of combination], under the title of a *transcendental synthesis of the imagination*, on the *passive* subject whose *faculty* it is – so that we can correctly say that inner sense is thereby affected. Apperception and its synthetic unity is so far from being identical with inner sense that the former, as the source of all combination, rather extends to the manifold of an *intuition in general* [and] under the name of the categories, prior to all sensible intuition, to objects in general. By contrast, inner sense contains the mere *form* of intuition, but without combination of the manifold therein, and thus not yet any *determinate* intuition at all – which is only possible through the consciousness of the determination of [the manifold] by the transcendental action of the imagination (synthetic

by asserting that "we are innate to ourselves, as it were, and … we include Being, Unity, Substance, Duration, Change, Action, Perception, Pleasure, and hosts of other objects of our intellectual ideas" (1765, p. 51). Compare also §30 of the *Monadology* (1878–90, vol. VI, p. 612; 1969, p. 646):

> It is also through the knowledge of necessary truths and through their abstractions that we rise to *reflective acts*, which enable us to think of that which is called "*I*" and enable us to consider that this or that is in *us*. And thus, in thinking of ourselves, we think of being, of substance, of the simple and of the composite, of the immaterial and of God himself, by conceiving that that which is limited in us is limitless in him. And these reflective acts furnish the principal objects of our reasonings.

influence of the understanding on inner sense) which I have called figurative synthesis. (B153–54)[200]

The first stage of the determination of objects of experience by the pure understanding, through which alone empirical knowledge is possible, is thus a determination of the pure intuition of time (wherein all of our representations, whatever their objects may be, must have their place). We can in this sense "correctly say that inner sense is thereby affected" (B153–54) by the self-reflection of the understanding and that our soul thereby appears to ourselves.

The crucial point, however, is that we do not, for Kant, obtain any knowledge of the soul as an object (e.g., as a simple substance) in this way. Accordingly, Kant explains the resulting illusions of "rational psychology" in great detail in the paralogisms of pure reason.[201] It follows that "pneumatology," in Leibniz's sense, is similarly impossible and, *a fortiori*, so is the monadology. We do not know an object – a monadological simple substance – through our own self-reflection, and so we cannot possibly project this "knowledge" onto nature in delimiting a further realm of analogous simple substances underlying the empirical phenomena. On the contrary, the only way in which we can obtain genuine knowledge of nature is by further extending the procedure of transcendental determination begun by the transcendental synthesis of the imagination. We must (spatio-)temporally schematize the pure understanding with respect to all of its pure logical forms of judgement, thereby obtaining the schematized categories. The relational categories, in particular, are essential to all time determination, for these three categories (substance, causality, and community) correspond (respectively) to the three modes of time (duration, succession, and simultaneity). And these categories, when schematized, can in no way represent a monadological realm of unextended simple substances in which all changes are effected by *intra*-substantial causal relations operating within the monads in question. Material substance or *substantia phaenomenon* is necessarily spatially extended, and a community of such substances necessarily depends on *inter*-substantial causal

[200] A manifold of intuition in general is some given "matter" or other that is to be (actively) synthesized by the pure forms of the understanding (the pure categories) but is not yet specified in terms of any particular (passive) forms of sensibility or receptivity – in particular, it is not yet specified as *spatio-temporal*. The idea of a synthesis of a manifold of intuition in general thereby indicates an earlier stage of considering the pure understanding – on its own, as it were, independently of its application to our spatio-temporal forms of sensibility. For further discussion see Friedman (2003), together with the literature cited there.

[201] I shall consider the relevant discussion in the paralogisms and its connection with Kant's divergence from Leibniz in the Conclusion.

relations operating between different substances and across spatial distances.[202] It also follows, according to the refutation of idealism, that our empirical self-knowledge of our own internal (psychological) existence in time is entirely parasitic on the external time determination already established, by all three relational categories, among spatially extended material substances existing in both space and time.[203]

The final stage of the procedure of transcendental time determination is reached in the Mechanics of the *Metaphysical Foundations*, where Kant presents his own version of a metaphysical foundation for physics via his three Laws of Mechanics – each of which, in one way or another, is intimately connected with the corresponding transcendental principle of pure understanding articulated in the analogies of experience. These principles of pure understanding are thereby implicated with the mathematical determination of time as a magnitude exemplified in the Newtonian theory of universal gravitation. As Kant understands this theory, in particular, the conservation of momentum, the equality of action and reaction, and the continuously evolving accelerations effected by instantaneous action-at-a-distance forces provide us with our paradigmatic concrete instantiation of time determination in general. It is in precisely this way, in the end, that Kant, in the critical period, achieves a radically new form of middle ground or synthesis between Newtonian physics and Leibnizean metaphysics. Pure metaphysical concepts or categories derived from the Leibnizean tradition – the concepts of substance, causality, and community – are the a priori foundation for all empirical time determination, and they thereby take over the role, in this procedure, of Newtonian absolute time. By the same token, however, these same metaphysical concepts or categories must now be fundamentally reconceived, so that they no longer represent a monadological realm existing behind the empirical phenomena but rather articulate the "form" of the empirical phenomena themselves – in accordance with the way in which these phenomena, in particular, are described by Newtonian physics.

[202] Compare the discussion of community in the general remark to the system of principles where Kant presents his own conception as an alternative to precisely the Leibnizean pre-established harmony (B293; see the paragraph to which note 33 of the Introduction is appended): "But we can make the possibility of community (of substances as appearances) conceivable very well, if we represent them to ourselves in space, and therefore in outer intuition. For the latter already contains within itself a priori formal outer relations as conditions of the possibility of real [relations] (in action and reaction, and thus community)."

[203] Compare my remarks on the refutation of idealism in the paragraph to which note 9 of the Introduction is appended, together with the preceding and following paragraphs. Again, I shall return to this topic in the Conclusion.

The pre-critical system of the *New Exposition* and *Physical Monadology* had attempted an earlier such middle ground or synthesis, but this system, too, is now seen as fundamentally misconceived. Kant had then sought to reconcile Newton and Leibniz by retaining a monadological metaphysics of ultimate simple substances lying behind the empirical phenomena, while, at the same time, rejecting the Leibnizean principle that only *intra*-substantial causality is possible. What binds together the ultimate simple substances into a common world, in this view, are the external relations of (real) interaction between them – relations that manifest themselves phenomenally as Newtonian attractive and repulsive forces (acting, respectively, at long-range and short-range distances). Nevertheless, as ultimate simple substances or monads, the metaphysical constituents of this common world must still have a purely internal nature, and this internal nature is itself phenomenally manifested as the force of inertia (*vis inertiae*) by which bodies exert resistance to motion. But it is precisely here, from Kant's critical point of view, that we now find a fundamental mistake. For, as explained, one of the main burdens of the overall argument of the Mechanics is explicitly and definitively to reject this force. What is actually operative in the communication of motion is the law of the equality of action and reaction, where this law necessarily governs the *external* relations of community of material substances in space. The idea of a purely internal force of inertia operative also in a state of rest represents nothing but a mere appearance of the true situation. For it arises precisely from viewing the interaction in question in a frame of reference in which one of the bodies is at rest rather than from the uniquely privileged perspective determined by their center of mass.

It is in this context, then, at the conclusion of his polemic against the force of inertia in the second remark to the fourth proposition, that Kant introduces the correspondence between his three Laws of Mechanics and the relational categories:

> Nothing can resist a motion except the contrary motion of another [body], but never its [state of] rest. Thus here the inertia of matter, that is, the mere inability to move itself from itself, is not the cause of a resistance. A special entirely peculiar force merely to resist, without being able to move a body, under the name of an inertial force, would be a word without any meaning. The three laws of general mechanics could therefore more appropriately be named the laws of *self subsistence*, *inertia*, and *reaction of matters* (*lex subsistentiae, inertiae, et antagonismi*) *in all of their changes*. That these laws, and thus all propositions of the present science, precisely answer to the categories of *substance*, *causality*, and *community*, in so far as these concepts are applied to matter, needs no further discussion. (551)

It is more appropriate, in particular, to assimilate the *resistance* formerly attributed to inertial force to the *reaction* arising in Kant's construction of the communication of motion. It is similarly more appropriate, by the same token, to conceive the three Laws of Mechanics (like the three relational categories they instantiate) as necessarily operating in unison to regulate the mutually effected changes of material substances standing in thoroughgoing (external) dynamical community within a common spatio-temporal world. And, although Kant may have here drastically overstated the extent to which his new critical conception of the correspondence between metaphysical categories and mechanical principles "needs no further discussion," the extraordinary richness and subtlety of this conception can only excite our admiration and wonder.

CHAPTER FOUR

Phenomenology

30 THE MOVABLE AS AN OBJECT OF EXPERIENCE

Beginning with the first section of my chapter on the Phoronomy, I have placed what I call Kant's Copernican conception of the relativity of space and motion at the center of the argument of the *Metaphysical Foundations*. I began by calling attention to the relationship between Kant's conception of "absolute" and "relative" space and that of Newton's Scholium on space, time, and motion, and I observed that Kant returns to this question in the Phenomenology. Here, in particular, Kant explains that absolute space is necessary "as an idea, which is to serve as the rule for considering all motion and rest therein merely as relative" (560), and he also refers several times to Newton's Scholium explicitly.[1] I also observed that the Preface to the second edition of the *Critique* takes Newton's argument for universal gravitation – "the invisible binding force of the universe" – definitively to establish what Copernicus had himself put forward only as an hypothesis (Bxxii).[2] The main purpose of the present chapter is to provide a detailed reading of Kant's very compressed discussion in the Phenomenology of exactly how "true" (as opposed to merely "apparent") motions are to be determined. I aim thereby to show that Kant's engagement in this text with the argument for universal gravitation in Book 3 of the *Principia* is extraordinarily rich and subtle, much more so, in fact, than previous commentators – including myself – have suspected.

Like the other chapters of the *Metaphysical Foundations*, the Phenomenology begins with an explication – the only one in this chapter (554): "Matter is the movable in so far as it, as such a thing, can be an object of experience." As explained in the second note of my chapter

[1] See note 2 of my chapter on the Phoronomy, together with the paragraph to which it is appended and the preceding paragraph.

[2] See note 10 of my chapter on the Phoronomy, together with the paragraph to which it is appended (including the following note).

on the Mechanics, Kant's wording here is precisely parallel to that of the first explication of the Mechanics (536): "Matter is the movable in so far as it, as such a thing, has moving force." In both cases we are concerned with a characterization of matter *as* something movable. In sharp contrast to the first explication of the Mechanics, however, the explication in the Phenomenology does not add any further distinguishing predicate or (partial) concept (*Teilbegriff*) to the list of predicates definitive of the concept of matter: being an object of experience, unlike having moving force, is *not* a further such predicate. Indeed, the fourth or Phenomenology chapter is the only one in the *Metaphysical Foundations* that does not add a further distinguishing predicate in this way. For Kant begins by characterizing matter as the movable in space in the Phoronomy, goes on further to characterize it in terms of the filling of space in the Dynamics, and then adds the concept of (mechanical) moving force in the Mechanics.

The discussion of the categories of modality in the corresponding chapter in the first *Critique* on the postulates of empirical thought makes a parallel but more general point from the perspective of transcendental philosophy:

> The categories of modality have the following peculiarity: that, as determination of the object, they do not in the least augment the concept to which they are ascribed as predicate, but they only express the relation [of this concept] to the faculty of cognition. If the concept of a thing is already entirely complete, I can still ask of this object whether it is merely possible, or also actual, or, if so, whether it is [also] even necessary. No further determinations are thereby thought in the object itself, but the question is only how [this object] (together with all of its determinations) relates to the understanding and its empirical employment, to the empirical power of judgement, and to reason (in its application to experience). (A219/B266)

So the categories of modality, unlike the other categories, do not add a further predicate (or conceptual "determination") to any object. They do not, for example, characterize an empirically given object as either a magnitude (in accordance with the categories of quantity), a reality (in accordance with the categories of quality), or a substance (in accordance with the categories of relation). Instead, they describe how the object in question is related to one (and eventually all) of our three intellectual cognitive faculties: understanding, the power of judgement, and reason.

I shall discuss this important passage and its relationship with the argument of the Phenomenology in the final section of the present chapter (section 36). For now, however, I return to the first explication characterizing matter, in so far as it is movable, as an object of experience. Kant's remark to this explication explains the sense of "experience [*Erfahrung*]" at issue by distinguishing it from "appearance [*Erscheinung*]":

Motion, like everything that is represented through the senses, is given only as appearance. For its representation to become experience, we require, in addition, that something be thought through the understanding – namely, besides the mode in which the representation inheres in the *subject*, also the determination of an *object* thereby. Hence the movable, as such a thing, becomes an object of experience, when a certain *object* (here a material thing) is thought as *determined* with respect to the *predicate* of motion. (554)

This notion of appearance, therefore, is associated with the senses rather than the understanding, whereas only the understanding can then give rise to experience – as opposed to mere appearance. The present distinction between appearance and experience diverges from Kant's more familiar usage, where appearances are rather distinguished from things in themselves and are accordingly *equated* with objects of experience (that is, empirical objects).[3]

In the same remark Kant clarifies his meaning by drawing a further distinction between "appearance [*Erscheinung*]" and "semblance [*Schein*]":

[I]f the movable, *as such a thing*, namely, with respect to its motion, is to be thought of as determined, i.e., for the sake of a possible experience, it is necessary to indicate the conditions under which the object (matter) must be determined in one way or another by the predicate of motion. At issue here is not the transformation of semblance into truth, but of appearance into experience; for, in the case of semblance, the understanding with its object-determining judgements is always in play, although it is in danger of taking the subjective for the objective; in the appearance, however, no judgement of the understanding is to be met with at all – which needs to be noted, not merely here, but in the whole of philosophy, because otherwise, when appearances are in question, and this term is taken to have the same meaning as semblance, one is always poorly understood. (555)

In the case of *semblance*, then, the understanding is already involved, and it is liable, in particular, to be seduced into a false judgement. In the case of what Kant here calls *appearance*, by contrast, there is (as yet) no judgement of the understanding at all: we have a mere presentation to the senses that is completely neutral as far as the understanding is concerned. Kant takes special pains, therefore, to insist that what he here calls appearance

[3] See, for example, the chapter on phenomena and noumena (A238/B298): "The transcendental use of a concept in any principle is when it is referred to *things in general* and *in themselves*; the empirical use, however, is when it is referred merely to *appearances*, i.e., objects of a possible *experience*."

(as something that is about to be transformed into experience) has no connotation of falsity or illusion.[4]

A semblance, by contrast, can easily give rise to a false judgement, and, as such, it cannot be transformed into a truth. The third remark to the first part of the *Prolegomena* provides a striking illustration of this point:

> The senses represent to us the motion of the planets as now progressive, now retrogressive; and herein is neither falsehood nor truth, because, so long as one acquiesces for the moment in this being only appearance [*Erscheinung*], one does not yet judge in any way concerning the objective character of the motion. However, because a false judgement can easily arise if the understanding has not taken sufficient care to prevent the subjective mode of representation from being taken for objective, one says that they present a semblance [*sie scheinen*] of retrogression. Yet the semblance [*Schein*] here is not to be charged to the senses but to the understanding, whose province alone it is to make an objective judgement from the appearances. (4, 291)

In this passage, as in the Phenomenology, the distinction between *appearance* and *experience* arises in the context of determining the "true" motions of the heavenly bodies from their "apparent" motions. It thereby underscores the importance of Kant's Copernican conception of motion and its relativity in this same context. In particular, the apparent motions – which are initially presented or given to our senses – are just those described from our parochial perspective here on earth.[5] There is absolutely no falsity or illusion involved so far, because the motions in question are indeed correctly described *relative to this perspective*. Nevertheless, it is the distinctive task of the understanding to seek for successively more adequate and comprehensive perspectives (frames of

[4] Kant uses "appearance" in this sense when he first introduces the term at the beginning of the transcendental aesthetic (A19–20/B34): "The action of an object on the faculty of representation, in so far as we are affected by it, is *sensation*. That intuition which is related to an object through sensation is *empirical*. The undetermined object of an empirical intuition is *appearance*." Kant distinguishes between sensibility and understanding right before this (A19/B33): "Thus objects are *given* to us by sensibility, and it alone provides us with *intuitions*; but they are **thought** by the understanding, and from it arise *concepts*." Kant begins the transcendental aesthetic, therefore, by considering objects only in so far as they are given to us (i.e., in so far as they affect us), and thus they are so far mere objects of *sensation* (empirical intuition). They only become objects of *experience*, as opposed to mere *appearances* in this sense, in so far as the understanding is then able to "determine" them (one way or another) by means of concepts.

[5] For the connection between the notion of a mere object of sensation – the "undetermined object of an empirical intuition" (A20/B34) in the sense of the preceding note – and the Copernican conception of motion and its relativity see note 14 of my chapter on the Phoronomy, together with the paragraph to which it is appended.

reference) as it undertakes a process of progressively transforming appearances into experience.

Kant appeals to the different possible perspectives of an observer in the proof of the following first proposition of the Phenomenology:

> Whether a body is said to be moved in a relative space, and the latter at rest, or whether, conversely, the latter shall be said to be moved, with the same speed in the opposite direction, while the former is at rest, is not a dispute about what pertains to the object, but only about its relation to the subject, and belongs therefore to appearance and not experience. For if the observer [*Zuschauer*] locates himself in that same space as at rest, the body counts as moved for him; if he locates himself (at least in thought) in another space comprehending the first, relative to which the body is likewise at rest, then that [first] relative space counts as moved. (555)

Thus, from the point of view of appearance, as opposed to experience, all the relative spaces in which a subject might locate itself are so far on a completely equal footing, and motion, accordingly, is so far entirely relative. To speak of the motion of a body, at this point, is simply to indicate its relation to one or another possible observer located in one or another frame of reference. Moreover, although we ourselves take our starting point from a perspective here on the surface of the earth, we can easily transport ourselves ("at least in thought") into another perspective or frame of reference in which the earth itself now appears to be in motion. Only the understanding, for Kant, can then intervene to adjudicate such thoroughgoing relativity, by determining which relative space (if either) is, in fact, more adequate.

It is instructive, at this point, to consider Kant's well-known remarks in the Preface to the second edition of the *Critique* where he compares his own (proposed) revolution in metaphysics to the Copernican revolution in astronomy. Metaphysics – the attempt to derive substantive a priori knowledge about objects from concepts alone – has not yet been able to attain "the secure path of science" (Bxiv).[6] Following the examples of mathematics and natural science, however, we might be able to do better by a sudden revolution in our "manner of thinking" – which, Kant

[6] At greater length (Bxiv):

> *Metaphysics*, a completely isolated speculative cognition of reason, which elevates itself completely above [all] instruction from experience and, indeed, through mere concepts (not like mathematics through the application of [concepts] to intuition) – [and] where reason itself is therefore supposed to be its own student – has until now not been so favored by fate as to have been able to enter upon the secure path of a science.

proposes, involves supposing that "objects must conform to our cognition" rather than the other way around (Bxvi).[7] He then introduces the famous Copernican analogy:

It is precisely the same here as with the first thoughts of Copernicus, who, after he was not able to progress well in the explanation of the motions of the heavens when he assumed that the entire starry host rotates around the observer [*Zuschauer*], attempted [to see] whether it might not succeed better if he allowed the observer to rotate and, on the contrary, the stars [to remain] at rest. One can now make a similar attempt in metaphysics with respect to the *intuition* of objects. (Bxvi–xvii)

Kant here (in 1787) invokes a shift of perspective analogous to the one he contemplates in the proof of the first proposition of the Phenomenology.[8]

Moreover, the way that Kant immediately proceeds to spell out his proposed analogy is even more striking:

If the intuition had to conform to the constitution of the objects, then I do not comprehend how one could know [*wissen*] something about them a priori; but if the object (as object of the senses) conforms to the constitution of our faculty of intuition, then I can very well represent this possibility to myself. However, because I cannot remain with these intuitions, if they are to become cognition, but [I] must refer them, as representations, to something as object and determine the former through the latter; then, I can either assume [that] the *concepts*, by which I effect this determination, also conform to the objects (and then I am back in the same dilemma concerning the manner in which I could know something about them a priori), or else I assume that the objects – or, which is the same thing, the *experience*, in which alone they (as given objects) are cognized – conform to these same concepts. [Under this last assumption] I then immediately comprehend an easier way out, since experience itself is a mode of cognition

[7] At greater length (Bxvi):

Until now one assumed that all of our cognition must conform to the objects; but all attempts to decide something about them a priori through concepts, whereby our cognition would be expanded, came (under this presupposition) to nothing. One should therefore once attempt [to see] whether we might make better progress in metaphysics by assuming that the objects must conform to our cognition – which already agrees much better with the required possibility of a cognition of [objects] that is supposed to establish something about objects before they are given to us.

[8] It is important to emphasize, however, that the first proposition of the Phenomenology is explicitly limited to cases of *rectilinear* motion – where an important kind of thoroughgoing relativity indeed turns out to hold. Kant considers the case of the (true) rotation of the earth with respect to the (truly non-rotating) starry heavens in the following second proposition. This reflects the qualification to the principle of (thoroughgoing) relativity of motion that he states in the Phoronomy – which, as explained in section 5 above, already points forward, in turn, to the case of the true rotation of the earth as discussed in the Phenomenology. I shall return to these issues in detail below.

that requires the understanding, and I must presuppose its [the understanding's] rule in myself – before any objects are given to me and therefore a priori – which [rule] is expressed in a priori concepts, in accordance with which, therefore, all objects of experience must necessarily conform and agree. (Bxvii–xviii)

Although Kant does not use the terminological contrast between "experience" and "appearance" here, the main point of his proposed Copernican solution to the problem of how we can have a priori knowledge of objects involves the equivalent contrast between mere objects of the senses or intuition and full-blown objects of experience. The latter must be determined a priori by the faculty of understanding, and it is in precisely this sense that these objects – objects of experience – must necessarily conform to our cognition.[9]

Kant's Copernican solution is a bit more subtle, and considerably more difficult, than it may first appear. It is relatively easy to understand how we can have a priori knowledge of mere objects of the senses for Kant. Since these are necessarily given to us in our a priori forms of intuition (space and time), it is clear that such objects – objects of an empirical intuition – must necessarily conform to the a priori structure of these forms. Objects of experience, however, are necessarily *further determined* (conceptually) by the a priori categories of the understanding. And, since the latter have their origin in the pure understanding alone, entirely independently of our forms of sensibility, it is so far unclear why not yet determined objects given to our senses (empirical intuitions) must necessarily conform to the a priori structure of the categories as well. For an object of experience is not merely an object thought by the pure understanding but is rather an empirically given object of intuition that, in addition, is necessarily determined by the understanding. Showing that all objects that can be given to our senses are *also* necessarily subject to the categories (and thus to the understanding) is precisely the task of the transcendental deduction.

Indeed, this is how Kant explicitly describes the task of the transcendental deduction in his revised presentation in the second edition. In §26 of this version, entitled "transcendental deduction of the universally possible use in experience of the pure concepts of the understanding," Kant explains what still needs to be established:

What is now to be explained is the possibility of knowing a priori, by means of *categories*, whatever objects *may present themselves to our senses* – not, indeed,

[9] A mere object of the senses or intuition, it seems, is just "the *undetermined* object of an empirical intuition" (A20/B34, emphasis added) in the sense of notes 4 and 5 above.

with respect to the form of their intuition, but with respect to the laws of their combination – and thus [how] they prescribe laws to nature, as it were, and even make nature possible. For without this it would not be clear how everything that may merely be presented to our senses must stand under laws that arise a priori from the understanding alone. (B159–60)

Although here is not the place to go into the details of Kant's extraordinarily compressed and notoriously difficult argument for this conclusion, I do want to emphasize, at this point, that the conclusion involves the contrast between mere objects of the senses or intuition presented in our forms of sensibility and full-blown objects of experience resulting from a determination of the former objects by the understanding.

But this is the same contrast, as I have explained, as the one that Kant first clearly articulates in the Phenomenology of the *Metaphysical Foundations* as a distinction between "appearance" and "experience."[10] He then reiterates this contrast in the Copernican revolution passage in the Preface to the second edition using different but equivalent terminology. The central point, in all three cases (Phenomenology of the *Metaphysical Foundations*, Preface and §26 of the second edition of the *Critique*), is that cognition begins with mere presentations given to our sensibility that already have a distinctively spatio-temporal form, and it then proceeds to *determine* such empirical intuitions by means of concepts of the understanding so as finally to arrive at what can alone be properly called experience. It is in precisely this way, in particular, that the understanding now underwrites the very possibility of experience. So Kant's appeal to the Copernican revolution in astronomy in explaining his own proposed revolution in metaphysics is no mere rhetorical flourish. On the contrary, there is an essential and fundamental connection between Kant's understanding of the former revolution in astronomy (as completed by Newton) in the Phenomenology and the crux of his own revolutionary theory of experience in the first *Critique*.[11]

[10] I observed that Kant uses the term "appearance" in the transcendental aesthetic for the "undetermined object of an empirical intuition" in both editions of the *Critique* (A20/B34; see note 4 above). But Kant develops no contrasting account of "experience" there – and no account, in particular, of the "determination" of appearances in this sense by the understanding.

[11] The main point of the revolution in metaphysics effected in the first *Critique* is not the idea that space and time are pure forms of our sensible intuition. This idea was already very prominent in the *Inaugural Dissertation* in 1770, but Kant was still left wondering – in his well-known letter to Marcus Herz of February 21, 1772 – how the *pure intellect* could possibly represent its objects a priori. The problem was that the pure intellect in the *Inaugural Dissertation* is not yet *schematized* in terms of the forms of intuition, and so Kant did not yet have an account of how the purely intellectual activities of the understanding could synthesize and thereby determine the objects of sensibility.

31 KANT, LAMBERT, AND PHENOMENOLOGY

I explained in section 11 that the relationship between Kant and Lambert was an especially important one, in so far as the two correspondents saw themselves as embarked on a common project for reforming the Leibnizean–Wolffian metaphysical tradition that they inherited in light of recent advances in the mathematical exact sciences.[12] The topic of section 11 was the remark to the first proposition of the Dynamics, where Lambert is named explicitly (along with some unspecified "others") as a representative of that concept of solidity or absolute impenetrability to which Kant's own dynamical theory of matter is opposed – and it was especially instructive, therefore, to compare Kant's conception of solidity or impenetrability with Lambert's. Although Kant does not explicitly refer to Lambert in the present context, it is equally instructive to compare Kant's conception of the task of a "phenomenology" with Lambert's. For, as it turns out, Lambert was very likely the first philosopher to employ the term, and it is virtually certain that Kant adopted this at the time unusual terminology from Lambert.

The first of his philosophical works that Lambert mentions in his initial letter to Kant of November 13, 1765 (see again note 12) is the recently published *New Organon, or Thoughts on the Discovery and Designation of the True and its Differentiation from Error and Semblance* [*Schein*] (1764). In this work, as Lambert explains in the Preface, he has chosen to borrow Greek terms for its four main parts: *Dianoiologie* or the doctrine of the laws of the understanding, *Alethiologie* or the doctrine of truth (as distinct from error), *Semiotik* or the doctrine of the designation of thoughts and things by means of signs, and finally *Phänomenologie* or the doctrine of semblance (*Lehre von dem Schein*). The aim of the latter is "to acquaint [us] with semblance and to give [us] the means for avoiding it and penetrating to the truth" (1764, vol. 1, p. iv).[13] In his letter to Lambert of

[12] See notes 27–31 of my chapter on the Dynamics, together with the paragraph to which they are appended and the following paragraph. In particular, both men saw their own work on the nebular structure of the Milky Way and the universe as a whole as a generalization or extension of the modern tradition in mathematical astronomy beginning with Copernicus and Kepler. Moreover, they both hoped to use the model of Euclid's geometry as a crucial part of their respective projects to reform Leibnizean–Wolffian metaphysics (see note 32 of my chapter on the Dynamics, together with the paragraph to which it is appended). I shall turn to a more detailed discussion of their two very different approaches to the nebular cosmological conception that they shared below.

[13] The editors of the most recent (1990) edition of Lambert's *Neues Organon* state, in their explanatory notes to the *Phänomenologie*, that Lambert was the first to use this concept. Moreover it

September 2, 1770 accompanying the *Inaugural Dissertation* Kant then uses the Latin term *phaenomologia generalis* to describe a "merely negative science … in which the validity and limitations of the principles of sensibility are determined, so that they do not confuse judgements about objects of pure reason" (10, 98).[14] He also uses the term, now in a Germanized form (*phaenomologie überhaupt*), in the famous letter to Herz of February 21, 1772 (10, 129; compare note 11 above). And he uses it once again, finally, now in the same Germanized form used by Lambert, in the title of the fourth chapter of the *Metaphysical Foundations*. Yet, as we have seen, Kant takes particular pains to distinguish his sense of phenomenology (as the "doctrine of appearance [*Erscheinung*]") from Lambert's (as the "doctrine of semblance").[15] Kant's problem is not to explain how we move from "semblance" to "truth," but rather to describe how we transform "appearance" into "experience." It is precisely here, on my reading, that Kant's most important divergence from Lambert can best be appreciated.

Lambert begins his *Phänomenologie* (§1) by explaining the relationship between "semblance" and "truth" in naively realistic terms. Aside from possible confusions arising from the designation of things by signs (as explained in the previous *Semiotik*), there is another source of confusion that often makes it difficult for us to secure the "correctness and agreement of concepts with the things themselves" (1764, vol. 11, p. 217). This is because our process of cognition involves not only truth and falsity but also an element that makes us liable to confuse them (pp. 217–18): "[W]e find in our cognition between these two [truth and falsity] an intermediate thing, called *semblance* [*Schein*], which brings it about that we very often represent the thing under another form [*Gestalt*], and makes it easy for us to take that which it *seems* [*scheinen*] to be for that which it *actually is*, or, in turn, to confuse the latter with the former." *Phänomenologie* (the doctrine of semblance) gives us the means for avoiding this deception – for "penetrating through the semblance to the truth" and thereby "seeking to recognize what is true in itself [*das Wahre an sich zu erkennen*]" (p. 218).

appears from these same notes (1990, pp. 1004–7) that Kant was the second. The page numbers of the original (1764) edition appear in the margins of this (1990) edition.

14 Towards the end of his reply to this letter on October 13, 1770, Lambert refers explicitly to several sections of his own "*Phaenomenol*" (10, 110).

15 Kant equates the doctrine of appearance [*Erscheinungslehre*] with phenomenology in the first footnote to the general remark to Phenomenology (560): "In the doctrine of appearance … In precisely the same phenomenology …" I shall return to this passage below.

This does not mean, however, that Lambert is proposing a discipline for penetrating beyond the sensible world to knowledge of noumena or things in themselves in the Kantian sense. On the contrary, he immediately proceeds to illustrate his meaning in the following sections (§§2–4) by familiar examples of visual or optical semblance (that a circle can appear oblong when viewed from an angular perspective, and so on). In these cases the sciences of optics and perspective – to which Lambert has recently made significant contributions – give us precisely the means that we need for correcting the semblance of a thing (an apparent oblong shape) and thereby penetrating to the truth (the true circular shape).[16] What Lambert is proposing is thus a generalization or extension of these sciences to what he calls a "transcendent optics" and "transcendent perspective," whereby we "can extend these concepts, and with them the concept of semblance, to their true generality" (1764, vol. II, p. 220). This generalized science has the task of determining for every type of semblance (not just optical or even merely sensory semblance) how semblance and truth are systematically related.[17] Kant, however, takes special pains sharply to *distinguish* the "transcendental" sense of what he calls "appearance [*Erscheinung*]" from its ordinary or empirical sense – so that Lambert's realism, from Kant's point of view, naively assimilates the empirical object to what he calls a thing in itself.[18]

[16] Lambert's scientific contributions at the time included his *Photometria* (1760) and *Freie Perspektive* (1759). We know that Kant owned a copy of the latter in his personal library, along with the *Cosmological Letters* (1761). The *Photometria* discusses the inverse-square law for degree of illumination to which Kant refers in the first remark to the eighth proposition of the Dynamics: see note 203 of my chapter on the Dynamics, together with the paragraph to which it is appended.

[17] Lambert summarizes the idea in §266 (1764, vol. II, p. 421):

> Phenomenology occupies itself in general with determining what is real and true in every type of semblance, and to this end it develops the particular causes and circumstances that bring about and alter a semblance, so that one can infer from the semblance to that which is real and true. We have already remarked in the first chapter (§§2ff.) that optics has given us a doctrine of *visual* semblance since antiquity, and that phenomenology can be called a transcendent optics in its most general extent, in so far as it determines the semblance from the true in general and, in turn, determines the true from the semblance. Optics does this with reference to the eye.

[18] See the discussion of this point in the transcendental aesthetic (A45/B62–63):

> We normally distinguish, among the appearances, that which attaches essentially to their intuition, and is valid for every human sense in general, from that which pertains to them only accidentally, in so far as it is not valid in relation to sensibility in general but only in relation to the particular placement or organization of this or that sense. And we here call the former cognition one that represents the object in itself, the latter only its appearance. But this distinction is only empirical. If one remains at this point (as commonly happens), and one does not view every empirical intuition in turn (as it should happen) as mere appearance (so that one can find nothing at all in it that pertains to a thing in itself), then our transcendental distinction is lost – and

We can gain a deeper understanding of what is missing from Lambert's realism, from Kant's point of view, if we compare their two different applications of "phenomenology" to a particularly important case: the inference from the "apparent" structure of the heavens (as seen from the perspective of the earth) to their "true" structure. Lambert introduces this problem at the very beginning of his *Phänomenologie* (§2), as the task of "inferring the true arrangement of the cosmic structure from the apparent shape of the heavens [*aus der scheinbaren Gestalt des Himmels auf die wahre Einrichtung des Weltbaues zu schließen*]" (1764, vol. II, p. 218), and he then recurs to it throughout the remaining text. In §4, for example, he redescribes the astronomical problem in question as resting on the difference between "spherical" and "theoretical" astronomy (p. 220). And in §51, accordingly, Lambert illustrates how we can penetrate from semblance to truth (p. 247): "In this way, theoretical astronomy is engendered from at first merely spherical astronomy, where [in the former] the cosmic structure is represented completely differently from that according to the judgement of the senses." Similarly, the whole problem of Kant's Phenomenology (not merely an illustration of this point, as it was for Lambert) is determining the true structure of the heavens from their apparent structure and, in particular, the true motions of the heavenly bodies from their merely apparent motions. So what is the precise relationship, we now need to ask, between Kant's understanding of the problem of determining the true structure of the heavens and Lambert's?

What Lambert has primarily in mind is not the problem of inferring true from apparent motions but the problem of inferring the nebular or galactic distribution of the stars from their apparent spherical distribution around the earth. This is precisely the problem addressed in his

> we then in fact believe that we cognize things in themselves, even though everywhere (in the sensible world), even up to the deepest investigation of its objects, we are involved with nothing else but appearances.

Kant illustrates his point with the example of the rainbow, which (according to modern optics since Descartes) is a mere appearance or semblance (produced by refraction) of water droplets illuminated by the sun. The illuminated water droplets, from Lambert's point of view, are paradigmatic of what he means by real and true objects (as opposed to "semblance") – whereas, from Kant's point of view, they are rather paradigmatic of precisely what he means by "appearance" (in the transcendental sense). The terminology can be confusing, however, because Lambert does not have the Kantian distinction between "appearance" and "semblance" – where the latter, unlike the former, involves a false (or at least misleading) judgement of the understanding. This, as we shall see, is because Lambert's entire discussion of the transformation of semblance into truth, from Kant's point of view, is missing the crucial idea that *experience* results from determining the *appearances* presented to the senses by the laws of the understanding.

Cosmological Letters (1761), whose full title is "Cosmological Letters concerning the Arrangement of the Cosmic Structure [*Cosmologische Briefe über die Einrichtung des Weltbaues*]." Lambert's principal conclusion, based on a careful consideration of the visual appearance of the Milky Way, is that the true distribution of the stars is not spherical but flat or disk-shaped:

For since I assume that the stars in this band [i.e., the Milky Way – MF] are as widely separated from one another as any of the closest fixed stars are from our sun, I must therefore necessarily put them in inconceivably long rows behind one another, and I thereby conclude that the whole *system* of the fixed stars visible to us is not *spherical* but flat, approximately like a disk whose diameter is many times longer than its thickness. (1761, pp. 127–28)

Lambert's argument for this conclusion is optical, based on his own previous work in the *Photometria* (1760).[19]

So it is by no means surprising that Lambert puts exclusive emphasis on the science of optics when first introducing the problem in §2 of his *Phänomenologie*:

Moreover, *optics* or *the art of vision* was so unavoidable for astronomers – who had to infer the true arrangement of the cosmic structure from the apparent shape of the heavens – that they long since needed to search for and apply the more difficult optical propositions. In modern times, telescopes and magnifying glasses have yielded new material for expanding the optical sciences, and thus motivation, diligence, and care in this part of phenomenology have not

[19] See Lambert (1761, p. 138):

The main question here seems to me to depend on whether the stars that we see through telescopes in the Milky Way are at least as far from one another as the nearest fixed stars are to our sun. For, if this is so, then it is soon shown that they must lie behind one another in indescribably long rows. I take, e.g., two similar stars from the Milky Way that appear to be separated from one another by only one second. If I posit that they are equally distant from us, I then have an isosceles triangle whose two longer sides make an angle of one second [of arc], but whose shorter side is the distance between the two. *Trigonometry* yields [the result] that each of the longer [sides] must be 206,265 times greater than this [shorter side]. But the latter is at least 500,000 times greater than the distance of the earth from the sun. Therefore, such stars must be 200,000 times 500,000, or 100,000,000,000 (i.e., a hundred thousand million) times further from us than the sun. Since I cannot imagine that we should still be able to see them, I prefer to conclude that they must either lie closer to one another [than do the nearest fixed stars to the sun – MF], or they must lie in long rows behind one another.

Thus, from the inverse-square law governing degree of illumination (note 16 above), it follows that a star of the same intrinsic brightness as the sun, at a distance n times further from us than the sun, will appear only $1/n^2$ as bright. For very large values of n, as in Lambert's example, such stars will therefore not be individually visible. The only way that the Milky Way could then be visible at all – assuming that the distances between its stars are as Lambert suggests – is if a very large number of such stars were then lined up in rows behind one another. For the number 500,000 in the above calculation see the following note.

been lacking – no matter how much the remaining parts have remained behind. (1764, vol. II, pp. 218–19)[20]

By contrast, Lambert devotes virtually no attention to the problem of inferring true from apparent motions. On the contrary, he simply takes it for granted that the basis for a solution to this problem has already been given in the earlier work of Copernicus and Kepler, and, more generally, Lambert takes a naively realistic attitude towards the concepts of (absolute) motion and rest. As explained in section II above, Lambert's axioms for the concept of moving force take the solid to be naturally at rest and include versions of Newton's first two Laws of Motion (limited, in accordance with the mechanical philosophy, to forces due to impact or pressure) governing the (absolute) motions thereby produced.[21] However, in taking the solid to be naturally at rest, Lambert betrays a much stronger commitment to absolute motion than Newton. Lambert's commitment, in fact, appears to be more in line with Kepler's original conception of inertia, according to which a body tends to remain in a state of (absolute) rest and resists any effort to be put into motion. In his famous discussion of *vis insita* or *vis inertiae* in the *Principia*, by contrast, Newton clearly takes inertial motion and rest to be dynamically equivalent.[22]

In the Eleventh of his *Cosmological Letters* Lambert begins from the fundamental conclusion of Copernicus (and Kepler) concerning the true orbital motions in the solar system and then pleads for a cosmological extension of this conclusion (1761, p. 134): "Now I comprehend completely why you, Sir, always said that we have not, for a long time,

[20] The importance of the optics of the telescope in this connection is due to Lambert's reliance on Bradley's results on the aberration of starlight (see note 6 of my chapter on the Phoronomy). Bradley had argued that, due to the combined effect of the finite velocity of light and the orbital motion of the earth, a telescope would need to tilt slightly to receive the light from a star. Such tilting, carried throughout the yearly orbit of the earth, would then yield a shift in the apparent position of the star describing a small circular trajectory, and, from the size of this circle, one could then deduce the distance of the star from the earth (by a form of parallax). For the closest visible stars to the sun, Bradley's observations, as Lambert reports in §1157 of his *Photometria* (1760, p. 504), result in a distance at least 400,000 times that of the sun. That Lambert has Bradley's telescopic results in mind in the *Cosmological Letters* is indicated by his remark in the Tenth Letter that the ancients had "less precise instruments," so that "the refraction of light-rays, the aberration of light, and the *nutation* of the earth were unknown [to them]" (1761, p. 123). Since Bradley discovered nutation by observing minute oscillations in the circumference of the small circular trajectories due to aberration, there is thus no doubt, in particular, that Lambert had a much better understanding of Bradley's results in this respect than Kant (compare note 50 of my chapter on the Phoronomy).

[21] See the paragraph preceding the one to which note 36 of my chapter on the Dynamics is appended.

[22] See note 128 of my chapter on the Mechanics, together with the paragraph preceding the one to which it is appended.

been thinking in a sufficiently Copernican way. It would not be enough to disturb the earth from its rest; rather, not a single body in the entire firmament should remain at rest … No point of the entire cosmic system remains, even for a moment, in an *absolute* [state of] rest." For, as Lambert explains in his Eighteenth Letter, it follows from his conception of nested galactic systems (which is similar to Kant's) that the earth not only orbits in an ellipse around the sun, but, since the sun in turn orbits in an ellipse around the center of the system of stars to which it belongs, the true trajectory of the earth in space is really a cycloid.[23] On Lambert's conception, therefore, the true geometry of the orbital trajectories in space follows from that assumed by Copernicus and Kepler in the context of the more complicated cosmological structure articulated in the *Cosmological Letters*.

In one of his rare discussions of the distinction between true and apparent motions in the *Phänomenologie*, finally, Lambert returns to this same conception:

> The sun seems [*scheint*] to traverse 360 degrees of a circle in 24 hours. This semblance [*Schein*] can be produced either by the motion of the earth or the sun or both. If both the earth and the sun revolve, then the appearance indicates only the sum or difference of the [two] motions: the sum if the directions are opposed, the difference if they go in the same direction. And in both cases it amounts to 360 degrees in 24 hours, without our being able to infer what in this is to be ascribed to the earth or the sun in particular. In a similar way, the Copernican system indicates only the relative motion of the earth around the

[23] See Lambert (1761, p. 260):

> In order to begin with an example that is closest to us, and already known, we assume that the moon moves in an *ellipse* around the earth, and that it returns in [this orbit] in approximately 27 days. This would be so if the earth remained unmoved. But if we assume that the earth orbits in an ellipse around the sun, then the *ellipse* of the moon completely disappears, and now the moon orbits in a *cycloidal*-curve around the sun – and, in fact, somewhat more rapidly than the earth, because it needs to take detours, and nevertheless goes around the sun in the same time. This is again the case so long as we assume that the sun is unmoved. But the sun changes its position as much as any fixed star. If we assume that it orbits in an *ellipse* around the center of the system of fixed stars to which it belongs, then the *ellipse* of the earth and the *cycloidal*-curve of the moon disappear, and they both are transformed into *cycloidal*-curves – the earth moves in one of the first degree, the moon in one of the second. It is obvious that the speed always increases.

Strictly speaking, the curves Lambert is describing are more complicated than cycloids. Whereas a cycloid is traced out by a point on a circle rolling along a straight line, a *hypocycloid* is traced out by a point on a circle rolling along the exterior of the circumference of another circle, and an *epicycloid* by a circle rolling along the interior of such a circumference. Lambert's curves are traced out by rotating circles whose centers lie on the circumference of another rotating circle (as in an epicycle) – or, more precisely still, by points on Keplerian ellipses one of whose foci lie on the circumference of another such ellipse.

sun, and this is elliptic – where the true [motion], by contrast, is cycloidic. (1764, vol. II, p. 249)

What Lambert means by the true motions in the heavens, therefore, are simply those that result from the extrapolation of the Copernican–Keplerian description of the solar system into the extended context of his own cosmological theory.

Lambert's conception, then, is essentially geometrical – or, more precisely, purely kinematical. For, in sharp contrast to Newton, Lambert does not discuss the *forces* that produce the true motions in question. He instead simply assumes (i) that there is an absolute distinction between true motion and rest, and (ii) that the Copernican–Keplerian description of the orbital motions in the solar system can be taken as a reliable guide for implementing this distinction within the cosmos as a whole. Lambert thus entirely ignores the problem – fundamental to the argument of Book 3 of the *Principia* – of empirically deciding between the Copernican–Keplerian system, on the one side, and the Tychonic system, on the other. In particular, whereas Galileo's telescopic observations of the phases of Venus rule out the Ptolemaic system in its original form, they still leave the Tychonic system (where the planets orbit the sun rather than the earth) completely open. More generally, there were no purely optical or kinematical arguments at the time that could empirically decide between Kepler and Tycho. And it was precisely this problem that Newton solved in Book 3 of the *Principia* by determining the center of mass of the solar system. Newton's implementation of the distinction between true and apparent motion, therefore, does not rely on optical or kinematical considerations but on precisely his Laws of Motion governing the fundamental dynamical concepts of mass, force, and interaction (action and reaction).[24]

We already know, however, that Kant's perspective on the problem of determining the true from the apparent motions has just this Newtonian procedure in view. In the Mechanics of the *Metaphysical Foundations* Kant formulates his own three Laws of Mechanics governing mass,

[24] See note 118 of my chapter on the Dynamics, together with the paragraph to which it is appended. I suggest in the note that, assuming the fixed stars to be at rest, telescopic observations can in principle decide between the Keplerian and Tychonic systems via stellar parallax. Since the nearest stars are so very far away, however, no such parallax could be observed at the time – and it was not in fact observed until the late nineteenth century. Moreover, Lambert provides no empirical justification for the assumption that the fixed stars are at rest in any case. By contrast, as I shall explain below, Newton's dynamical conception of true motion is eventually capable (in principle) of showing that the fixed stars are indeed at rest (or moving inertially) to a very high degree of approximation.

force (as an external cause of change of motion), and interaction. In the fourth proposition, in particular, he takes the center of mass of any interacting system of bodies to define a privileged frame of reference for "reduc[ing]" the observable motions within this system "to absolute space" (545; see the paragraph following the one to which note 112 of my chapter on the Mechanics is appended). In the Phenomenology Kant returns to a fuller discussion of this procedure, whereby "all motion and rest must be reduced to absolute space" (560) – and, as I shall argue in detail in what follows, he thereby engages with the argument of Book 3 of the *Principia* in an extraordinarily insightful fashion. If I am not mistaken, therefore, there is no doubt that Kant's perspective on determining the true from the apparent motions is, like Newton's, fundamentally dynamical.

Unlike Lambert's *Cosmological Letters* (and the *Phänomenologie* of his *Architectonic*), Kant's *Metaphysical Foundations* is dominated by the problem of determining the true motions of the heavenly bodies from their apparent motions – and, indeed, the problem of providing a dynamical *definition* of the notion of "true motion" in the first place. Far from taking a naively realistic conception of absolute motion and rest for granted, Kant begins in the Phoronomy by explicitly rejecting such a conception in favor of his Copernican conception of motion and its relativity. This conception, as I have also explained (section 1 above), is first presented in the *New System of Motion and Rest* of 1758, which, in turn, echoes the hierarchical picture of a nested sequence of rotating galactic structures developed in the *Theory of the Heavens* of 1755. Thus, both Kant's problem of determining the true motions of the heavenly bodies from their apparent motions and Lambert's problem of determining the true arrangement of the cosmic system from its apparent structure as seen from the earth can be traced back to their common formulation of a nebular or galactic structure for the universe in the years 1755–61.[25] Yet now, in the *Metaphysical Foundations*, Kant takes a crucial step that Lambert never does: he conceives his own Laws of Mechanics as the indispensable means

[25] Although Kant and Lambert agree on the disk-shaped structure of the Milky Way, and they also agree that the Milky Way is only one galactic system in a nested sequence of such systems, their conceptions are not in all respects the same. First, Kant takes the solar system and the visible stars closest to the sun to be part of the Milky Way galaxy while Lambert does not. Second, and more importantly, Kant has an evolutionary conception of the origin of these systems from an initial cloud of vaporous material while Lambert does not. In particular, Kant gives an evolutionary explanation of the galactic constitution of the cosmos as a whole while Lambert gives a teleological explanation (in terms of the need for empty spaces so that the galactic motions can occur without collisions); and, in this sense, it is indeed more appropriate to designate what we now know as the nebular hypothesis as the "*Kant*–Laplace hypothesis."

for determining the true motions from the apparent and thereby implementing a (dynamical) procedure for reducing all motion and rest to absolute space that is finally completed in the Phenomenology. Moreover, these Law of Mechanics, in turn, are more specific realizations or instantiations in the special metaphysics of corporeal nature of the three analogies of experience articulated in the first *Critique*. It is in precisely this sense that the movable is now to be determined as what Kant calls an "object of *experience*" (554, emphasis added).

From Kant's point of view, therefore, what is missing from Lambert's naively realistic conception of the relationship between semblance and truth is precisely the characteristically Kantian notion of experience. Empirical truth, for Kant, is determined by fundamentally causal or dynamical principles, which are articulated at the most general level by the three analogies of experience. For Lambert, by contrast, his procedure for inferring the true structure of the heavens from its apparent structure relies exclusively on geometrical and optical propositions – which, from Kant's point of view, count as merely "mathematical" as opposed to "dynamical" principles.[26] Indeed, experience, in Kant's technical sense, begins from what he calls "perceptions" (which already conform, therefore, to the anticipations of perception and thus to the mathematical principles) and then subjects these same perceptions to an essentially dynamical order of necessary connections via the analogies of experience.[27] Lambert's naive realism, by contrast, has no room at all for *this* notion of experience – which, of course, is absolutely central to Kant's distinctively transcendental form of idealism (see again notes 11 and 18 above, together with the paragraphs to which they are

[26] For Kant's important distinction between mathematical and dynamical principles see the paragraph to which note 24 of my chapter on the Dynamics is appended. Compare also note 203 of my chapter on the Dynamics, where one of the main optical contributions of Lambert's *Photometria* (note 16 above) is explicitly subsumed under the *mathematical* principles.

[27] The principle of the analogies in the second edition reads (B218): "Experience is possible only through the representation of a necessary connection of perceptions." The following proof (also added in the second edition) begins (B218–19):

> Experience is an empirical cognition, that is, a cognition that determines an object through perceptions. It is therefore a syntheses of perceptions, which is not itself contained in perceptions, but rather contains the synthetic unity of the manifold of perceptions in a consciousness – which constitutes what is essential in a cognition of the *object* of the senses, that is, experience (not mere intuition or sensation of the senses).

Compare also the following discussion of the distinction between mathematical and dynamical principles (A178–80/B220–23), where, in particular, Kant describes the postulates of empirical thought as concerning "the synthesis of mere intuition (the form of appearance), of perception (the matter of appearance), and of experience (the relation of these perceptions)" (A180/B223).

appended).[28] So what is most remarkable, in the present context, is that this crucial divergence between Lambert and Kant is already flagged at the beginning of the Phenomenology of the *Metaphysical Foundations* by Kant's substitution of his own contrast between appearance and experience for Lambert's "phenomenological" contrast between semblance and truth.[29]

32 RECONSIDERING THE RELATIVITY OF MOTION

As explained in section 5 above, the Phoronomy centrally involves a principle of the relativity of motion (487): "Every motion, as object of a possible experience, can be viewed arbitrarily as motion of the body in a space at rest, or else as rest of the body, and, instead, as motion of the space in the opposite direction with the same speed." Kant's first proposition of the Phenomenology appears to be very similar (555): "The rectilinear motion of a matter with respect to an empirical space, as distinct from the opposite motion of the space, is a merely *possible* predicate." Moreover, the parallel with the relativity principle of the Phoronomy becomes even clearer in the first part of the following proof: "Thus in experience (a cognition that determines the object validly for all appearances) there is no

[28] Lambert develops a notion of "experience" in his *Dianoiologie*, but this has the common-sense meaning of that which is established by ordinary or refined *perception* as opposed to any inferences that may be drawn therefrom. Thus, for example, Lambert distinguishes (§557) between common experience in astronomy (that the sun rises and sets, and so on) from the more refined experience that he calls "observation," which "requires more attention and longer time" – such as that "the moon always turns the same side to us, the times from one new moon to another are not equal but have regular deviations, that the planets change their position among the stars, etc." (1764, vol. I, p. 352). Lambert then remarks in the *Phänomenologie* (§91) that astronomy has long used the language of "appearance [*Schein*]" so as not to "mix what one has actually experienced or observed with the inferences drawn therefrom" (1764, vol. II, p. 273). Hence Lambert would count as "experience" what Kant counts as mere appearance as opposed to experience – as, for example, in the case of the observed retrograde motions of the planets discussed in the passage from the *Prolegomena* (4, 291) quoted in the paragraph to which note 5 above is appended.

[29] In light of the terminology Kant finally articulates in the second edition of the *Critique* (compare note 27 above), the former distinction is perhaps better expressed as a contrast between "perception" and experience. Thus, whereas the notion of appearance at the beginning of the Phenomenology is formulated in such a way that absolutely no judgements of the understanding are yet involved, the notion of perception implies only that no judgements governed by the specifically *dynamical* categories are yet involved. The terminological instability visible here is related to the distinction between "judgements of perception" and "judgements of experience" formulated in the *Prolegomena*, which then appears to give way in the second edition of the *Critique* in favor of the idea that all judgements whatsoever necessarily involve the categories (§19 of the transcendental deduction). In any case, however, I do not need to go further into this difficult area here, because the crucial difference between Lambert and Kant with which I am now concerned involves the central – or, as Kant says, "essential" (B218) – role of the specifically dynamical categories in constituting *experience*.

difference at all between the motion of the body in the relative space, and the body being at rest in absolute space, together with an equal and opposition motion of the relative space" (555).[30] Hence, given the strong similarity between this statement and the relativity principle in the Phoronomy, it is by no means surprising that Kant goes on in the immediately following remark to assert that the first proposition of the Phenomenology "determines the modality of motion with respect to *phoronomy*" (556).

Just as Kant qualifies his principle of relativity in the Phoronomy by the statement that (488) he "*assume*[*s*] *all motions to be rectilinear*," he here introduces the same qualification into both the statement of the proposition to be proved and the conclusion of the following proof (note 30).[31] He also explains in the Phoronomy that curvilinear motions (as in the case of the daily rotation of the earth, for example) constitute an exception to his relativity principle and asserts that later, in the Mechanics, it will be shown that not even (mutual) rectilinear motions can be apportioned completely arbitrarily. Similarly, Kant follows his first proposition of the Phenomenology with a second proposition wholly excluding circular motions (including the axial rotation of the earth) from such relativity and then adds a third proposition (based on his Third Law of Mechanics) according to which (558): "In every motion of a body, whereby it is moving relative to another, an opposite and equal motion of the latter is *necessary*."[32]

[30] This is the conclusion of the reasoning quoted in the paragraph following the one to which note 5 above is appended. The connection between the conclusion Kant draws here and the statement of the first proposition emerges in the immediately following reasoning, which explicitly adds the restriction to *rectilinear* motion (555–56):

> Now the representation of an object through one of two predicates, which are equally valid [*gleichgeltend*] with respect to the object, and differ from one another only in regard to the subject and its mode of representation, is not a determination in accordance with a *disjunctive judgement*, but merely a choice [*Wahl*] in accordance with an *alternative judgement* … This means that through the concept of motion, as object of experience, it is in itself undetermined, and therefore equivalent [*gleichgeltend*], whether a body is represented as moved in the relative space, or the latter with respect to the former. Now that which is in itself undetermined with respect to two opposed predicates is to that extent *merely possible*. Hence, the rectilinear motion of a matter in an empirical space, as distinct from the equal [and] opposite motion of the space, is a merely possible predicate in experience.

I shall return below to the precise import of this argument – which crucially depends on the parenthetical sentence in the ellipsis and the footnote appended thereto.

[31] See the paragraph to which note 65 of my chapter on the Phoronomy is appended, together with the following paragraphs, for a full statement and discussion of the qualifications to his relativity principle that Kant introduces in the Phoronomy.

[32] Kant explicitly refers to the fourth proposition of the Mechanics in this connection in his proof of the third proposition of the Phenomenology. As I have emphasized, it is in precisely the fourth proposition of the Mechanics (and, in particular, the long footnote appended to the proof of this proposition) that Kant explains the qualification to the relativity principle of the

Despite these parallels and similarities, however, the first proposition of the Phenomenology has a significantly different meaning and import from the relativity principle stated in the Phoronomy. First of all, whereas Kant states the relativity principle of the Phoronomy in a completely unrestricted form and only subsequently introduces qualifications limiting it to specifically rectilinear motions, the statement of the corresponding proposition of the Phenomenology already builds in this limitation from the beginning. Secondly and more importantly, whereas the relativity principle of the Phoronomy is supposed to be entirely independent of the mechanical laws of motion, Kant's discussion in the Phenomenology depends on precisely these laws. The case of circular motion, as Kant says (557), explicitly depends on the law of inertia, and the case of (mutual) rectilinear motions, as Kant also says (558), explicitly depends on the equality of action and reaction. Thirdly, and more generally, whereas the entire discussion in the Phoronomy falls under the categories of quantity, so that neither the relational nor the modal categories are yet at issue, the discussion in the Phenomenology falls under the modal categories. And, since the relevant notion of possibility expressed in the first category of modality concerns precisely the possibility of *experience*, the relational categories (governed by the analogies of experience) are therefore presupposed here as well.[33]

Indeed, it is only if we keep this situation firmly in mind that the argument Kant presents for the first proposition of the Phenomenology is comprehensible at all. In my exposition so far I have presented various parts of the argument separately (compare note 30 above), and it is now time to put the pieces together. I begin with the first part:

Whether a body is said to be moved in a relative space, and the latter at rest, or whether, conversely, the latter shall be said to be moved, with the same speed in the opposite direction, while the former is at rest, is not a dispute about what

Phoronomy according to which not even (mutual) rectilinear motions can be apportioned completely arbitrarily.

[33] The first of the three postulates of empirical thought states (A218/B265): "That which agrees with the formal conditions of experience (in accordance with intuition and concepts) is *possible*." The relevant "formal conditions of experience," therefore, comprise *all* pure concepts of the understanding – including, especially, the categories of relation. Similarly, when then articulating the concepts of possibility, actuality, and necessity in the following passage, Kant explains (A219/B266, emphasis added; see the paragraph preceding the one to which note 3 above is appended): "No further determinations are thereby thought in the object itself, but the question is only how [this object] (together with all of its determinations) relates to *the understanding and its empirical employment*, to the empirical power of judgement, and to reason (in its application to experience)." Thus, once again, it follows that all pure concepts of the understanding (including, of course, the relational categories) are involved in the (real) possibility of an object.

pertains to the object, but only about its relation to the subject, and belongs therefore to appearance and not experience. For if the observer locates himself in that same space as at rest, the body counts as moved for him; if he locates himself (at least in thought) in another space comprehending the first, relative to which the body is likewise at rest, then that [first] relative space counts as moved. Thus in experience (a cognition that determines the object validly for all appearances) there is no difference at all between the motion of the body in the relative space, and the body being at rest in absolute space, together with an equal and opposition motion of the relative space. (555)

This argument is at first sight very puzzling. From the point of view of mere *appearance* – as opposed to *experience* – it would appear that all motion whatsoever is entirely relative. In particular, a perceiving subject, from this point of view, could equally well take itself to be at rest in a *rotating* frame of reference (on the rotating earth for example), and this would flatly contradict Kant's argument in the following second proposition. So it is only if we extend our considerations beyond mere appearance to experience that the restriction to rectilinear motion in the first proposition is intelligible. Kant signals this extension, moreover, in the concluding sentence of the argument by explicitly invoking the notion of experience – as "a cognition that determines the object validly for *all* appearances" (555, emphasis added). In particular, Kant is here looking forward to the applications of both the law of inertia in the second proposition and the equality of action and reaction in the third. He is presupposing, in other words, that the mechanical laws of motion – and, accordingly, the analogies of experience – are already in play. For there is otherwise nothing in the text of the proof of the first proposition that can explain the restriction to specifically rectilinear motions.

If this is correct, however, we are now faced with two further difficulties. In the first place, the statement of the principle of relativity in the Phoronomy also considers the motion in question as "object of a possible *experience*" (487, emphasis added; compare the first paragraph of this section). And further such references to "experience" occur throughout the following argument on behalf of this principle.[34] How, then, can the reference to "experience" in the proof of the first proposition of the Phenomenology possibly indicate a difference, as I have claimed, in the meaning and import of the two statements? In the second place, and

[34] Thus, for example, the argument begins (487): "To make the motion of a body into an experience it is required that not only the body, but also the space in which it moves, be an object of outer experience and thus material. An absolute motion – that is, a motion in relation to a non-material space – is capable of no experience at all and hence is nothing for us." The entire argument is quoted in section 5 above.

even more seriously, how, on this reading, is the proof of the first proposition of the Phenomenology supposed to work? How, in particular, are we to explain the still puzzling circumstance that Kant restricts his statement to rectilinear but not *uniformly* rectilinear motions? In my discussion of the corresponding puzzle in the Phoronomy I appealed to the idea that Kant does not yet have the mechanical laws of motion available. But, as I have just argued, Kant certainly has these laws available in the Phenomenology – including, in the first instance, the law of inertia to which he explicitly appeals in the proof of the following second proposition. So why does Kant not formulate his first proposition as what we now call the principle of Galilean relativity, which holds only for frames of reference moving *both* rectilinearly *and* uniformly?

In order to address the first difficulty, I begin with Kant's second remark to the first explication of the Phoronomy. This remark, which contains Kant's initial argument for the relativity of motion, begins by distinguishing space as the mere form of outer sensible intuition from the matter sensibly given to us within this form (481): "*Matter*, as opposed to *form*, would be that in the outer intuition which is an object of sensation [*Empfindung*]." Further: "In all experience something must be sensed [*empfunden*], and that is the real of sensible intuition [*das Reale der sinnlichen Anschauung*]" (481). Thus it follows, in particular, that "the space in which we are to arrange our experience of motion must also be sensible [*empfindbar*], that is, it must be designated through what can be sensed" (481). This space (a relative, material, or empirical space) is therefore movable in turn, and so on *ad infinitum*.[35] So Kant is here invoking the notion of experience primarily to indicate that the matter of outer intuition (as opposed to the mere form of this intuition) is something that must be given to us as an object of *sensation*. It is precisely because the space (or frame of reference) "in which we are to arrange our experience of motion" must itself be an object of sensation that this space, too, must be material and therefore movable.

Hence, when Kant here characterizes matter (and its motion) as an object of experience, he is describing it, in the first instance, as precisely an object of sensation or intuition – as that which affects our senses as an "undetermined object of an [outer] empirical intuition" (A20/B34;

[35] More explicitly (481): "But this space, as material, is itself movable; and a movable space, however, if its motion is to be capable of being perceived, presupposes once again an expanded material space in which it is movable; this latter presupposes in precisely the same way yet another; and so on to infinity." For the entire passage see the paragraph to which note 16 of my chapter on the Phoronomy is appended, together with the reference in the note back to an earlier quotation.

compare again notes 4 and 5 above). In the terminology of the first paragraph of the Preface to the *Metaphysical Foundations*, therefore, Kant is considering corporeal or material nature (the object of the doctrine of body) in what he calls its "*material* meaning … as the sum total of things, in so far as they can be *objects of our* [outer] *senses*, and thus also of experience" (467). So Kant's conception of matter (and its motion) as an object of experience in the Phoronomy is actually much closer to what he now calls *appearance* as opposed to *experienc*e in the Phenomenology. It is much closer, that is, to a mere object of sensation or (empirical) intuition – which, as such, has not yet been subject to the full synthesizing or determining activities of the understanding. The circumstance that Kant in the Phoronomy considers motion as an object of "experience" in *this* sense, therefore, is perfectly consistent with the claim that Kant's reference to "experience" in the Phenomenology is intended to signal an important difference between the two.[36]

In the second remark to the first explication of the Phoronomy Kant is concerned with a transition from pure to empirical intuition – from mere empty space considered as the pure form of outer intuition to what may then be presented to us within this form as "*objects of our* [outer] *senses*" (467). He is thereby concerned, as explained in section 6 above, with a parallel transition from mathematical to empirical motion – from the motion of a mere mathematical point in pure intuition to the motion of an empirical spatial object or body that can only be perceived by our (outer) senses. Indeed, Kant already indicates this transition in the first remark to the first explication. He begins by asserting that matter viewed phoronomically can be considered as a mere (mathematical) point but then goes on to say that he will sometimes use the concept of *body* nonetheless – in order "to anticipate to some extent the application of the principles of phoronomy to the more determinate concepts of matter that are still to follow" (480).[37] Moreover, Kant has in some sense completed this

[36] It is significant, in this connection, that the distinction Kant draws between nature considered in its "formal" and "material" meanings in the Preface to the *Metaphysical Foundations* is then echoed in §26 of the second edition transcendental deduction. Here, in particular, Kant begins with "nature, as the totality of all appearances (*natura materialiter spectata*)" (B163) and then moves to the assertion that "all appearances in nature stand under the categories in accordance with their combination – on which [categories] nature (merely considered as nature in general) depends, as the original grounds of its necessary lawfulness (as *natura formaliter spectata*)" (B165). Here, once again, Kant's aim is to show that the totality of (mere) appearances or empirical intuitions in space and time *also* necessarily stand under the pure concepts of the understanding: compare the paragraph preceding the one to which note 10 above is appended.

[37] See note 72 of my chapter on the Phoronomy (including the reference there to the earlier note 34), together with the paragraph to which it is appended.

transition by the end of the second remark, where he asserts that the movability of an [empirical] *object* in space involves a concept that can only be "given through experience" (482).[38] Finally, the purpose of the transition from pure to empirical intuition (from mathematical to empirical motion) that Kant undertakes in the Phoronomy is precisely to introduce his characteristically Copernican conception of the relativity of space and motion: the idea, namely, that we start with a frame of reference determined by our own bodily position in space and then extend this perspective throughout a sequence of ever more extended frames of reference (material and thus movable spaces) to infinity.[39]

In the Phenomenology, by contrast, Kant now has fully available the more determinate concept of matter or body articulated in the Dynamics and Mechanics: the concept of the movable as that which fills a space and, in turn, exerts moving force through its motion on (and thereby exchanges momentum with) other such movables. It is on precisely this basis that Kant now proposes fully to articulate his Copernican conception of space and motion – to describe how, and by what principles, we move from our parochial perspective here on earth to the center of mass of the solar system, the center of mass of the Milky Way galaxy, and so on. Kant's three Laws of Mechanics (and therefore the three analogies of experience) play a leading role here, and it is in precisely this sense that Kant is now centrally concerned with the transition from appearance to *experience*.[40] In sharp contrast with the Phoronomy, therefore, Kant is raising no question at this point about the transition from pure to empirical intuition, from the pure form of outer intuition to the matter or objects of (outer) sensation. Rather, Kant begins with the notion of what he here calls appearance – a so far undetermined object (in space) of intuition or sensation – and moves to the much richer notion of a now conceptually determinate object of cognition or experience.

These considerations are confirmed and also further clarified by Kant's initial discussion of his procedure for reducing all motion and rest to absolute space in the general remark to phenomenology. He begins by explaining that in phenomenology (558–59) there "appear three concepts, whose use in general science is unavoidable, and whose precise determination is therefore necessary, although not that easy or comprehensible – namely, [first,] the concept of *motion in relative* (movable) *space*, second,

[38] See the paragraph following the one referred to in the previous note.

[39] See note 75 of my chapter on the Phoronomy, together with the paragraph to which it is appended and the four following paragraphs.

[40] See again notes 27 and 29 above, together with the paragraph to which they are appended.

that of *motion in absolute* (immovable) *space*, and third, that of *relative motion* in general, as distinct from absolute motion." These three concepts thereby correspond, respectively, to the first, second, and third propositions of the Phenomenology. Kant immediately adds, however that "[t]he concept of absolute space is the basis for *all* of them" (559, emphasis added). So it follows, in particular, that the concept of absolute space (and therefore Kant's procedure for reducing all motion and rest to absolute space as well) is already presupposed in the first proposition.

Kant now asks what he takes to be the central question (559): "How do we arrive at this peculiar concept [of absolute space], and what underlies the necessity of its use?" His answer begins:

> It cannot be an object of experience, for space without matter is no object of perception, and yet it is a necessary concept of reason, and thus nothing more than a mere *idea*. For in order that motion be given, even merely as appearance, an empirical representation of space is required, with respect to which the movable is to change its relation: but the space that is to be perceived must be material, and thus itself movable, in accordance with the concept of a matter in general. (559)

So Kant is here explicit that the starting point of his conception of motion and its relativity (as initiated, originally, in the Phoronomy) depends on the circumstance that motion is first given to us "merely as *appearance*" (emphasis added). It is only at the conclusion of the long paragraph that begins with the passages already quoted, however, that Kant finally arrives at motion as an object of *experience* (560): "Absolute space is therefore necessary, not as a concept of an actual object, but rather as an idea, which is to serve as a rule for considering all motion therein merely as relative; and all motion and rest must be reduced to absolute space, if the appearance thereof is to be transformed into a determinate concept of experience [*Erfahrungsbegriff*] (which unites all appearances)."[41]

Nevertheless, although Kant thus arrives at motion as an object of experience at the end of the paragraph under consideration, he also makes it clear in the first part of his discussion that the mere concept of motion as an appearance is already sufficient to generate at least the beginnings of his Copernican conception of space and motion (559; immediately following the passages quoted at the beginning of the preceding paragraph):

[41] This passage should be compared with the conclusion of the initial argument of the first proposition, where Kant also explicitly invokes the notion of "experience" in this sense – as "a cognition that determines the object validly for all appearances" (555; see the paragraph following the one to which note 33 above is appended).

Now to think of [this material space] as moved, one may think it only as contained in a space of greater extent, and take the latter to be at rest. But the same can be done with the latter, with respect to a still further extended space, and so on to infinity, without ever arriving by experience at an immovable (immaterial) space, with respect to which either motion or rest might be absolutely [*schlechthin*] attributed to any matter. Rather, the concept of these relational determinations will have to be continually revised, according to the way that we will consider the movable in relation to one or another of these spaces.

The argument that Kant presents here, on the basis of considering the movable "merely as appearance" (559), is essentially the same as what he has already presented in the Phoronomy in connection with the principle of the relativity of motion articulated there.

However, as Kant explains in the immediately following passage, it is precisely on account of this so far completely unstable sequence of continual revisions in our relational determinations (of motion and rest) that the concept of absolute space – as in idea of reason – must at this point most definitely be introduced:

Now since the condition for regarding something as at rest or moved is always conditioned in turn, *ad infinitum*, in relative space, it becomes clear, *first*, that all motion or rest can be relative only and never absolute, that is, that matter can be thought as moved or at rest solely in relation to matter, and never with respect to mere space without matter, so that absolute motion, thought without any relation of one matter to another, is completely impossible; and *second*, for precisely this reason, that no concept of motion or rest valid *for all appearance* is possible in relative space. Rather, one must think a space in which the latter can itself be thought as moved, but which depends for its determination on no further empirical space, and thus is not conditioned in turn – that is, an absolute space to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively-mutual,* but none as absolute motion or rest (where, while one is said to be moved, the other, in relation to which it is moved, is nonetheless represented as absolutely [*schlechthin*] at rest). (559–60)[42]

The concluding sentence of the paragraph already quoted above (560) then follows. It is precisely the *idea* of absolute space, conceived as "a rule for considering all motion therein merely as relative" (560), which finally introduces stability and determinacy into Kant's Copernican conception

[42] I shall return below to Kant's important footnote to this passage explaining the concept of an "alternatively-mutual [*alternativ-wechselseitig*]" determination of motion.

of space and motion – and thereby unites all the possible appearances of so far merely relative motions into a coherent concept of experience.

Of course it is not yet clear what exactly Kant means by this, and I shall not be in a position fully to explain Kant's procedure until we go further into the general remark. My task now, however, is to clarify Kant's reconsideration in the first proposition of the principle of relativity introduced in the Phoronomy, and I can make further progress with this task by looking briefly at how Kant initially sets up the problem in the remark to the (sole) explication. In particular, immediately after the first part of the remark (554; see the paragraph to which note 3 above is appended), Kant continues:

> But now motion is change of relation in space. There are thus always two correlates here, such that either, *first*, the change can be attributed in the appearance to one just as well as to the other, and either the *one* or the *other* can be said to be moved, because the two cases are equivalent [*gleichgültig*]; or, *second*, one must be thought in experience as moved to the exclusion of the other; or, *third*, both must be necessarily represented through reason as equally moved. In the appearance, which contains nothing but the relation in the motion (with respect to its change), none of these determinations are contained. But if the movable, *as such a thing*, namely, with respect to its motion, is to be thought of as determined, i.e., for the sake of a possible experience, it is necessary to indicate the conditions under which the object (matter) must be determined in one way or another by the predicate of motion. (554)[43]

Even without yet understanding what Kant is trying to say, we can immediately note two important features of this passage. On the one hand, motion is defined as "change of relation in space." In terms of the second explication of the Phoronomy, therefore, it is clear that Kant is now explicitly considering the motions of *bodies* as opposed to mere mathematical *points*.[44] On the other hand, however, Kant also makes it clear that in the appearance, as opposed to experience, *none* of the three "determinations" in question is contained – where the task of such determination, more generally, therefore depends essentially on the perspective of experience.[45]

[43] This passage then continues through the conclusion of the remark, as quoted in the paragraph to which note 4 above is appended.

[44] See again note 11 of my chapter on the Phoronomy, where we see that in the second explication of the Phoronomy Kant is intending to characterize the motion of bodies as opposed to mere points – and where, in addition, he gives the daily rotation of the earth as an example of such motion.

[45] This passage is thus in accord with what Kant goes on to say in the proof of the first proposition – which emphasizes that it is precisely from the point of view of "experience (a cognition that determines the object validly for all appearances)" that the rectilinear motion of an object with

But the most important feature of this passage, from our present point of view, is its insistence that motion always involves two "correlates" – a body, on the one side, and a (material) relative space or frame of reference with respect to which the body is either in motion or at rest, on the other. And, since our present objective is to understand Kant's argument in the first proposition (where, in the case of *rectilinear* motion, the motion of an object with respect to a given relative space and the equal and opposite motion of the space are completely equivalent), it is instructive to compare this case with the following second proposition (where, in the case of *circular* motion, the two possible motions in question are definitely not equivalent). For, in the case of the circular motion considered in the latter proposition, we already know from the remark to the explication that, of the two "correlates" in question, "one must be thought in experience as moved to the *exclusion* of the other" (554, emphasis added). In particular, as Kant explains in the general remark, the daily rotation of the earth relative to the starry heavens is by no means equivalent to an equal and opposition rotation of the heavens relative to a (presumptively) fixed earth. On the contrary, a properly formed judgement of experience locates all of the (daily) rotation in the earth itself and positively excludes any contribution from the surrounding heavens.[46]

Indeed, as Kant emphasizes in the remark to the second proposition, it is even possible thereby to determine cases of circular motion in the absence of any surrounding materially given space at all:

> Moreover, *Newton's* Scholium to the Definitions he has prefixed to his *Principia* may be consulted on this subject, towards the end, where it becomes clear that the circular motion of two bodies around a common central point (and thus also

respect to a given relative space and the equal and opposite motion of the space are "equivalent [*gleichgeltend*]" (555–56; see note 30 above, together with the paragraph to which it is appended). Finally, it is interesting to compare the present passage with the passage from the postulates of empirical thought quoted in note 33 above – where Kant describes an analogous progression from "the understanding and its empirical employment, to the empirical power of judgement, and to reason (in its application to experience)" (A219/B266). To make the two passages line up completely, we would need to equate the reference to "experience" in the passage from the first remark with "the empirical power of judgement" in the postulates and, accordingly, to subsume the reference to "the appearance" in the former passage under "the understanding and its empirical employment." I shall return to this situation below.

[46] Compare the discussion of the second proposition in the general remark (560–61):

> [R]elative [circular] motion with respect to the external space (for example, the axial rotation of the earth relative to the stars of the heavens) is an *appearance*, in place of which the opposite motion of this space (of the heavens) in the same time can be posited, as fully equivalent [*gleichgeltend*], but which, according to this [second] proposition, may not be so posited throughout in experience.

the axial rotation of the earth) can still be known by experience even in empty space, and thus without any empirically possible comparison *with an external space*; so that a motion, which is a change of external relations in space, can therefore be empirically given, even though this space is not itself empirically given, and is no object of experience. (557–58)[47]

It is for precisely this reason that, when introducing three concepts of motion at the start of the general remark, Kant characterizes the one corresponding to the second proposition as "*motion in absolute* (immovable) *space*" (559) – and, when he turns to the case of circular motion in detail, Kant begins by saying that this type of motion certainly *seems* to be absolute.[48]

The contrast with rectilinear motion, in this respect, could not be clearer. For Kant emphasizes in the second part of the statement of the first proposition that rectilinear motion, "thought in no relation at all to a matter external to it, i.e., [thought] *as absolute motion*, is *impossible*" (555). In the following proof, after arguing, as the first point to be proved, that a rectilinear motion of a body with respect to an empirically given relative space is equivalent to an equal and opposite motion of the space, Kant proceeds to the second point:

Since, moreover, a relation, and thus also a change thereof, i.e., motion, can be an object of experience only in so far as both correlates are objects of experience, whereas the pure space that is also called absolute space, in contrast to relative (empirical) space, is no object of experience, and in general is nothing, a rectilinear motion without reference to anything empirical, i.e., absolute motion, is completely impossible – which was the second [point to be proved]. (556)

[47] The reference is to the final paragraph of Newton's Scholium (P414):

For example, if two balls, at a given distance from each other with a cord connecting them, were revolving about a common center of gravity, the endeavor of the balls to recede from the axis of motion could be known from the tension of the cord, and thus the quantity of circular motion could be computed … In this way both the quantity and direction of this circular motion could be found in any immense vacuum, where nothing external and sensible existed with which the balls could be compared.

I shall return below to this example and Kant's understanding of it.

[48] Thus, immediately before the passage quoted in note 46 above, Kant begins (560): "Because *circular motion*, according to the second proposition, can be given as actual motion in experience, even without reference to the external empirically given space, it indeed seems [*scheint*] to be absolute motion." Immediately after the same quoted passage he adds (561): "Hence, that rotation is not to be represented as externally relative, which sounds as if this kind of motion is to be taken to be absolute." In the passage from the remark to the second proposition quoted in the text, Kant takes the circumstance that a circular motion can thus "be empirically given, even though this space is not itself empirically given, and is no object of experience," to be "a paradox that deserves to be solved" (558). Kant's solution, explained below, is then presented in the corresponding part of the general remark.

This sharp asymmetry between circular and rectilinear motion therefore provides us with a welcome opportunity to clarify Kant's intentions in the first proposition as well – and, in particular, more carefully to consider his restriction to specifically rectilinear motions there.

It might seem, at first sight, that we could capture Kant's intentions by inserting an additional restriction to *uniformly* rectilinear motions here and, accordingly, by applying Newton's Corollary 5 to his Laws of Motion (i.e., what we now call the principle of Galilean relativity). Kant's view, on this reading, would be that no uniform rectilinear motion (i.e., no inertial motion) of a body (or system of bodies) in an otherwise empty (absolute) space could be empirically distinguished from a state of rest, while the circular (rotational) motion of a body (or system of bodies) can be empirically distinguished from a state of rest (even in an otherwise completely empty space). The argument of the concluding paragraph of Newton's Scholium to the Definitions entails the latter; Corollary 5 to the Laws of Motion implies the former. It thereby follows that (at least some) states of *non-uniform* (accelerated) rectilinear motion should be also in principle empirically distinguishable from rest, and so Kant's first proposition of the Phenomenology would then need explicitly to exclude them as well – and to subsume them, instead, under the second proposition governing circular motion.[49]

The problem with this suggestion, however, is not just that Kant himself never inserts such a restriction to *uniform* rectilinear motion.[50] In addition, the way in which he later discusses the question of the rectilinear motion of the system of all matter in the general remark is completely at odds with it:

Only the rectilinear motion of the *cosmos* [*Weltganzen*], that is, of the system of all matter, would be such [an absolute motion]. For if, external to a matter, there were any other matter at all, even separated from it by empty space, then the motion would already be relative. For this reason, any proof of a law of motion, which amounts to showing that its opposite would have to result in a rectilinear motion of the entire cosmic system [*Weltgebäude*], is an apodictic proof of its truth, simply because absolute motion would then result, which is completely impossible. Of such a kind is the law of *antagonism* in all community of matter

[49] Corollary 6 to the Laws of Motion introduces an important complication, in so far as it states that Galilean relativity can be extended to *accelerated* motion as well, provided that all bodies in the system under consideration are equally accelerated along parallel lines. I shall return to this issue in the technical note at the end of section 34 below.

[50] Kant is perfectly clear about the distinction between uniform and non-uniform rectilinear motion: see again note 65 of my chapter on the Phoronomy, together with the paragraph to which it is appended.

through motion. For any deviation from it would shift the common center of gravity of all matter, and thus the entire cosmic system, from its place – which would not happen, by contrast, if one wanted to imagine this system as rotating on its axis. Hence it would always be possible to think such a motion, although to assume it, so far as one can foresee, would be entirely without any conceivable use. (562–63)

Here, once again, Kant is contrasting two different cases: a rotational motion of the entire system of all matter (which would therefore not be rotating relative to any other matter external to it) and a rectilinear motion of this same system (which would similarly not be moving rectilinearly relative to any other matter external to it). The latter, Kant says, is completely impossible, while the former, on the contrary is always possible (or at least it is always possible "to think such a motion").[51]

The important point, however, is that Kant does *not* describe the (presumptive) case of a rectilinear motion of the entire cosmic system as we would describe it following Newton. Corollary 4 to the Laws of Motion (which is closely related to the following Corollary 5) states that the common center of gravity of all interacting bodies is either at rest or "moves uniformly forward in a straight line" (P421; see the paragraph to which note 114 of my chapter on the Mechanics is appended). In particular, in accordance with Corollary 5 and Galilean relativity, Newton thereby asserts that the center of gravity of the system of all matter is *either* at rest *or* undergoes uniform rectilinear motion. In accordance with this same principle, moreover, it is then impossible empirically to determine which of the two is actually the case. For Kant, by contrast, only the former is a genuine real possibility, since the latter would clearly "shift the common center of gravity of all matter, and thus the entire cosmic system, from its [absolute – MF] place" (563). For Kant, in other words, the common center of gravity of all matter can only be at rest (although the cosmos could still possibly undergo a rotation around this point), because he is taking the construction of a (non-rotating) frame of reference centered on this point to *define* the distinction between true motion and rest in the first place. Kant is not, as we would expect, constructing a particular example of an inertial frame and then observing, in accordance with

[51] Kant is here presupposing his solution to the "paradox" of true circular motion relative to no external empirically given space (compare note 48 above, together with the paragraph to which it is appended), which he has just presented in the previous pages. I shall turn to the details of this solution below, but the basic idea is that (just as in Newton's example) there are *internal* effects of the rotation (produced, e.g., by centrifugal forces) that allow its truth or "actuality" to be determined empirically.

Galilean relativity, that any other inertial frame (moving uniformly and rectilinearly relative to the first) is equally good. He is rather constructing an empirically definable surrogate for Newtonian absolute space, and, in this sense, his conception is quite incompatible with our modern understanding of Galilean relativity.[52]

How, then, does Kant understand the rectilinear (but not necessarily uniform) motions he considers both here and in the first proposition? An important clue is provided by his reference to "the law of *antagonism* in all community of matter through motion" (563). This, as we know from the Mechanics, is just Kant's Third Law of Mechanics – his construction of the equality of action and reaction (section 27 above).[53] Kant applies this law, as we also know, to all cases of the communication of motion (i.e., momentum), whether by impact or by attraction.[54] In one case we consider the equal and opposite uniform rectilinear motions (defined relative to their common center of mass) of two bodies approaching one another in impact, in the other the equal and opposite non-uniform (accelerating) rectilinear motions (defined relative to their common center of mass) of two bodies attracting one another via universal gravitation. In both cases, by the equality of action and reaction, neither body can be properly considered to be at rest. Instead, they both possess an exactly equal share of the total motion (momentum or change of momentum) involved in the interaction in question.[55]

[52] Compare note 28 of the Introduction. On our modern understanding of Galilean relativity, which crystallized in the late nineteenth century, we replace Newtonian absolute space with the class of inertial frames of reference – all of which are moving uniformly and rectilinearly relative to one another. Although Newton himself does not have this modern understanding, his commitment to what we now call Galilean relativity is nonetheless very clear and explicit in his corollaries to the Laws of Motion (especially Corollaries 4 and 5). By contrast, Kant (to the best of my knowledge) never comes close to formulating or even acknowledging Galilean relativity in any of his writings.

[53] Kant calls this the "law of antagonism [*lex antagonismi*]" when he sets up the correspondence between his three Laws of Mechanics and the categories of relation immediately before the general remark (551; see the final paragraph of my chapter on the Mechanics).

[54] See note 113 of my chapter on the Mechanics, together with the paragraph to which it is appended.

[55] As explained in note 112 of my chapter on the Mechanics, it is precisely here that Kant articulates the necessary qualification to his statement of the thoroughgoing relativity of rectilinear motion in the Phoronomy. As observed in notes 143 and 159 of my chapter on the Mechanics, in connection with Stan (in press), we actually need to distinguish between the line of motion and the line of interaction in both cases. From that point of view, moreover, the cases of impact and (gravitational) attraction are exactly parallel, in so far as both involve an inertial component of uniform rectilinear motion composed with an interactive component involving changes of momentum and thus accelerations. Since the Phenomenology, as we shall see, is concerned almost exclusively with universal gravitation, the rectilinear motions at issue in the first proposition turn out to be mutual gravitational accelerations along the line of interaction.

From this point of view, therefore, the contrast between rectilinear and circular motion in the first two propositions of the Phenomenology can be understood in the following way. Rectilinear motion, in this context, always involves a mutual interaction (resulting in a mutual change of momentum) between two different bodies. Circular or rotational motion, by contrast, can perfectly well characterize a single body (or system of bodies) alone, with no need at all to consider any further relations, in turn, between this body (or system of bodies) and external matter located outside it. Thus, in the case of the two correlates involved in circular motion (the body and a possible material relative space external to it), the entire motion (in experience) can be assigned to one to the exclusion of the other. But the two correlates involved in rectilinear motion (the two interacting bodies) must *both* be in motion for Kant – and, indeed, in such a way that the two motions in question are always exactly equal and opposite. So when, in the proof of the first proposition of the Phenomenology, Kant says that "through the concept of motion, as object of experience, it is in itself undetermined, and therefore equivalent, whether a body be represented as moved [rectilinearly] in the relative space, or the latter with respect to the former" (556), what he means, I suggest, is that, from the perspective of the privileged (non-rotating) center of mass frame he calls "absolute space" (545), *neither* can be taken to be moved to the exclusion of the other.[56] It is in precisely this sense, as Kant puts it in the statement of the proposition (555), that "[t]he rectilinear motion of a matter with respect to an empirical space, as distinct from the opposite motion of the space, is a merely *possible* predicate."[57]

I am suggesting, more specifically, that in Kant's progressive procedure for reducing all motion and rest to absolute space we can begin by considering rectilinear motions arising in the interactions between any two bodies. We can do this in such a way, in particular, that we provisionally

[56] For the relevant notion of "absolute space" in the construction of the communication of motion in the Mechanics see again the paragraph following the one to which note 111 of my chapter on the Mechanics is appended.

[57] Compare the longer extract from the conclusion of the proof quoted in note 30 above. In the ellipsis of that quotation Kant has (556): "(In the former [disjunctive judgement], of two *objectively* opposed predicates, one is assumed to the exclusion of the other for the determination of the object; in the latter [alternative judgement], of two judgements objectively equivalent, yet subjectively opposed to one another, one is assumed for the determination of the object without excluding the opposite – and thus by mere choice.)*" In the footnote Kant indicates that he will further explain the distinction between "disjunctive and alternative opposition" in the general remark. He does this in the footnote there to the phrase "alternatively-mutual" (559; see note 42 above, together with the paragraph to which it is appended), to which I shall return below.

choose to consider one of the bodies as at rest and the other in motion, without in any way excluding a corresponding (equal and opposite) rectilinear motion of the first body. We can do this so long as we recognize, at the same time, that the choice is indeed provisional and, accordingly, that eventually assigning the corresponding (equal and opposite) motion to the first body is a problem reserved for a later stage. This, I suggest, is what Kant has in mind in the proof of the first proposition – according to which, in the case of two bodies moving rectilinearly relative to one another, an "observer" has a "choice" as to where (that is, on which body) to "locate himself" (555–56).[58] When an observer makes such a choice, moreover, the sense in which the resulting description of rectilinear motions is then "merely possible" is that it is *provisional* in precisely this way. In the third proposition, by contrast, Kant goes on to assert that, in all cases of (rectilinear) motion of one body relative to another, "an opposite and equal motion of the latter is *necessary*" (558; see the paragraph to which note 32 above is appended). But here, I suggest, he is referring to a later stage of the same procedure, when we are finally in a position to assign the precise (equal and opposite) shares of the total (relative) motion to both bodies – and thereby to discharge the provisional assumption (the arbitrary choice of one of the two bodies to be at rest) with which we began.[59]

A centrally important illustration of this procedure with which Kant was certainly familiar is provided by Newton's argument in Book 3 of the *Principia*. Newton begins with the "phenomena" encoded in Kepler's laws. Taking the fixed stars to be at rest, then, relative to the primary bodies in the solar system (the sun, the earth, Mercury, Venus, Mars, Jupiter, and Saturn), the satellites of these bodies obey Kepler's laws (in cases where there

[58] Compare again note 30 above, together with the paragraph following the one to which note 33 above is appended.

[59] I am here indebted to the following objection from Martin Carrier: if, in accordance with the equality of action and reaction, an equal and opposite motion of both bodies is *necessary*, how can an assignment of motion to just one body (on my reading of the first proposition) be *possible*? The answer I am now proposing is that the two different attributions (of possibility and necessity respectively) are made at two different stages of a single procedure. So "possible" in the first proposition means "provisionally acceptable." As such, it is to be contrasted both with rectilinear motion "in *no* relation at all to a matter external to it" (555, emphasis added) – which, according to the second part of the first proposition, is "*impossible*" (555) – and, in the case of (actual) circular motion of a body (such as the daily rotation of the earth), with an "opposite motion of a relative space [e.g., the starry heavens – MF], *assumed instead of the motion of the body*" (557, emphasis added). The latter, according to the second proposition, is "mere semblance" (557). I shall return to a consideration of the second contrast (between rectilinear and circular motion) below.

are satellites, i.e., for the sun, the earth, Jupiter, and Saturn).[60] However, in the pivotal contested case of the earth–sun orbit Newton deliberately leaves it open whether the earth is at rest with the sun orbiting around it (in accordance with Kepler's laws) or vice versa.[61] The point, for Newton, is that all of these motions are generated by inverse-square forces – and thus accelerations – directed towards the (presumed) center of the orbit in question.[62] In particular, the earth experiences an inverse-square acceleration directed towards the sun (relative to a [non-rotating] frame of reference fixed at the sun's center), and the sun experiences an inverse-square acceleration directed towards the earth (relative to a [non-rotating] frame of reference fixed at the earth's center).[63] So we can begin by describing all of

[60] Newton states this explicitly for the satellites of Jupiter and Saturn – with respect to Kepler's second or area law and third or harmonic law – in Phenomena 1 and 2. For example, Phenomenon 1 reads (P797): "The circumjovial planets [or satellites of Jupiter], by radii drawn to the center of Jupiter, describe areas proportional to the times, and their periodic times – the fixed stars being at rest – are as the 3/2 powers of their distances from that center." (Phenomenon 2 is completely parallel.)

[61] See Newton's statement of Phenomenon 4 (P800): "The periodic times of the five primary planets and of either the sun about the earth or the earth about the sun – the fixed stars being at rest – are as the 3/2 power of their mean distance from the sun." This is a statement of Kepler's third or harmonic law now applied *both* to the orbits of the five primary planets (Mercury, Venus, Mars, Jupiter, and Saturn) and the earth relative to the sun *and* to the solar orbit with the earth as center.

[62] Compare Proposition 2 of Book 3 (P802): "The forces by which the primary planets are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the sun and are inversely as the squares of their distances from its center." This is followed by Proposition 3 (P802): "The force by which the moon is maintained in its orbit is directed toward the earth and is inversely as the square of the distance of its places from the center of the earth." Thus, Newton has here derived inverse-square "acceleration fields" (see the paragraph to which note 181 of my chapter on the Dynamics is appended) around both the sun and the earth. Note, however, that he does not here mention an inverse-square acceleration of the sun directed towards the earth: see the following note.

[63] As Howard Stein first pointed out to me, however, there is a subtle difficulty concerning the earth–sun orbit. If we ascribe an inverse-square acceleration to the sun directed towards the earth, this acceleration does not belong to the same acceleration field as that governing the orbit of the moon. Similarly, while Kepler's third law holds across all the orbits of the planets (including the earth) relative to the sun, it does not hold across the orbits of both the moon and the sun from the perspective of a fixed earth. It appears to be for this reason, in fact, that Newton does not appeal to Kepler's third law in inferring either an inverse-square acceleration of the moon towards the sun or an inverse-square acceleration of the sun towards the earth. He infers the former from the (relative) stability of the lunar orbit in Proposition 3 and arrives at the latter only by applying the Third Law of Motion from the point of the view of the center of mass of the earth–sun system in the Corollary to Proposition 12 (see the following note). Moreover, while Newton infers the acceleration-field property for the sun's gravitational attraction from Kepler's third law applied across the orbits of all the planets, he infers the corresponding property of the earth's gravitational attraction in the context of the moon test: see again the paragraph to which note 181 of my chapter on the Dynamics is appended. (Here I am indebted to George E. Smith.) Newton *could* have applied the 3/2-power law to the earth–sun orbit to derive an inverse-square acceleration of the sun directed towards the earth – but only within the solar orbit itself (not comprising both the sun and the moon). This asymmetry between considering the earth–sun

these phenomena from the point of view of the earth – resulting in a version of the Tychonic system (see note 24 above). And it is then only later, in Propositions 11 and 12 of Book 3, that we can finally argue, appealing to Newton's measurement of the masses of the (main) primary bodies in the solar system (the earth, the sun, Jupiter, and Saturn), that the center of mass of the entire system is actually very close to the center of the sun – resulting in a close approximation to the Keplerian–Copernican system.[64] In his description of the initial phenomena, however, Newton is allowing us to proceed from *either* the Tychonic *or* the Keplerian–Copernican description of the relative motions. He is thereby presenting us, on my reading, with what Kant in the Phenomenology calls an "alternative judgement" (compare note 57 above).

A simpler – but closely related – illustration of this procedure is provided by the behavior of falling bodies near the earth. We begin by noting that this behavior is governed (to a close approximation) by Galileo's law of fall, according to which there is a uniform (rectilinear) acceleration constant for all bodies directed towards the center of the earth. We also note that the same uniform acceleration (= g) underlies the terrestrial determination of quantity of matter in the form of weight.[65] Newton's moon test then allows us to connect the (approximately uniform) acceleration of terrestrial gravity with the (non-uniform) inverse-square accelerations of universal gravitation, so that we can now extend the terrestrial determination of quantity of matter via weight to a celestial determination of this same quantity in the form of mass.[66]

orbit from the perspective of the sun or the earth, respectively, already suggests, therefore, that the sun, not the earth, is (much closer to) the true center of motion.

[64] Compare the paragraph to which note 24 above is appended, together with the reference in that note to the Dynamics chapter. For Newton's own conclusion see the Corollary to Proposition 12 (P817):

> Hence the common center of gravity of the earth, the sun, and all the planets can be considered the center of the universe. For since the earth, sun, and all the planets gravitate toward one another and therefore, in proportion to the force of the gravity of each of them, are constantly put in motion according to the laws of motion, it is clear that their mobile centers cannot be considered the center of the universe, which is at rest. If that body toward which all bodies gravitate most had to be placed in the center (as is the commonly held opinion), that privilege would have to be conceded to the sun. But since the sun itself moves, an immobile point will have to be chosen for that center from which the center of the sun moves away as little as possible and from which it would move still less, supposing that the sun were denser and larger [i.e., more massive – MF], in which case it would move less.

[65] See the paragraph to which note 41 of my chapter on the Mechanics is appended. In the case of two weights in equilibrium, therefore, we have $m_1a_1 = m_2a_2$, where $a_1 = g = a_2$; so equality of weights implies equality of masses.

[66] See again the paragraph to which note 41 of my chapter on the Mechanics is appended, together with the three following paragraphs. For the connection, by way of an approximation, between (apparently) uniform terrestrial gravity and (non-uniform) celestial gravity see note 43 of this same chapter.

In the context of universal gravitation, however, it turns out that the earth undergoes corresponding (rectilinear) accelerations of its own. In accordance with the equality of action and reaction, it accelerates both towards all falling bodies and towards the moon – where, in both cases, the center of mass of the interaction in question now replaces the (provisionally immobile) center of the earth. Thus, once again, although we begin by considering the earth to be at rest in both cases, it then turns out, at a later stage, that the earth is in motion as well – and, indeed, necessarily so (compare note 59 above, together with the paragraph to which it is appended).

Kant's description of the beginnings of his procedure for reducing all motion and rest to absolute space in the general remark to phenomenology is entirely in accord with this reading. For, in the first place, Kant explains his procedure in the initial case of rectilinear motion (corresponding to the first proposition) as follows:

> Thus the rectilinear motion of a body in relative space is reduced to absolute space, when I think the body as in itself at rest, but this space as moved in the opposite direction in absolute space (which is not apprehended by the senses), and when I think this representation as that which yields precisely the same appearance, whereby all possible appearances of rectilinear motions that a body may have at the same time are reduced to the concept of experience which unites them all, namely, that of merely relative motion and rest. (560)

The rectilinear motion of a body is reduced to absolute space, then, when this body is (provisionally) assumed to be at rest, and, accordingly, all cases in which it moves rectilinearly relative to other bodies are described from this point of view. In all such cases, that is, the (in fact) mutually relative rectilinear motions are (provisionally) ascribed to precisely the other bodies in question. We begin, for example, by considering the earth to be at rest, and by ascribing all the relative rectilinear accelerations in question – of falling bodies towards the earth, the moon towards the earth, and (even) the sun towards the earth – to bodies other than the earth. It is only at a later stage that we appeal to the equality of action and reaction (in the context of universal gravitation) so as to conclude that the earth accelerates towards all of these other bodies in turn – and, once again, necessarily so. My preceding discussion makes it clear, moreover, that Kant was quite familiar with this example.

In the second place, however, the way in which Kant goes on to describe the determination of the true rotation of the earth (in accordance with his second proposition) confirms that this is what he has in mind. For he begins by assuming a rectilinear acceleration directed towards the center of the earth in conformity with Galileo's law of fall and then appeals to observed *deviations* from this rectilinear fall (due to what we now call Coriolis forces) to infer the true rotation of the earth.[67] Kant begins, therefore, by provisionally considering the earth to be at rest and recording the (approximately uniform) rectilinear acceleration of fall from this point of view. At a later stage, when we go on to observe small deviations from rectilinear fall, we do not conclude that we were initially mistaken but rather that these deviations can be explained by the true rotation of the earth. To be sure, this (partial) revision of our initial description is essentially different from those that we have been considering so far (in accordance with Kant's third proposition), since we are now concerned with establishing true states of rotation while positively *excluding* the opposite motion of the relative space in question (the starry heavens). We are therefore not applying the equality of action and reaction at this stage. Yet we still begin by taking the earth to be (provisionally) at rest and describing the rectilinear motions of other bodies towards the earth from this point of view.[68]

It is also very significant, in the third place, that Kant concludes the introductory paragraph where he first introduces his procedure for reducing all motion and rest to absolute space (immediately before the paragraph we have just considered corresponding to the first proposition) by characterizing "absolute space" as follows:

[67] The crucial passage reads (561):

> [I]f I represent to myself a deep hole descending to the center of the earth, and I let a stone fall into it, I find, however, that the falling stone deviates from its perpendicular direction continuously, and, in fact, from west to east, even though gravity, at all distances from the center of the earth, is always directed towards [this center], and I conclude, therefore, that the earth is rotating on its axis from west to east.

I shall return to a more detailed consideration of this passage below.

[68] The relationship between Kant's second and third propositions in his procedure for reducing all motion and rest to absolute space turns out to be quite complicated and subtle, and I shall only be in a position to unravel it completely in my further discussion of this procedure in the next two sections. The basic idea, however, is that we can put all the pieces coherently together only from the point of view of the finished theory of universal gravitation – where the rectilinear acceleration of fall, the (true) orbital motions in the solar system, and the (true) axial rotation of the earth (and of other celestial bodies) are all integrated into a single description. From this point of view, in particular, the gravitational acceleration of fall is indeed rectilinear (because it follows from the law of universal gravitation), the earth is indeed truly rotating (since it manifests centrifugal forces counterbalancing gravitational forces), and, as already explained, the earth is also accelerating rectilinearly towards all other bodies under the action of the same force of universal gravitation (in accordance with the equality of action and reaction). Although I can only put the whole story together over the course of the next two sections, I shall return to the crucial difference between rectilinear and rotational motion at the end of the present section.

[It is a space] to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively-mutual,* but none as absolute motion or rest (where, while one is said to be moved, the other, in relation to which it is moved, is nonetheless represented as absolutely [*schlechthin*] at rest). (559–60)[69]

The phrase "alternatively-mutual [*alternativ-wechselseitig*]," as Kant indicates in the footnote, is then worthy of further consideration:

* In logic the *either–or* always signifies a *disjunctive* judgement, where, if the one is true, the other must be false. For example, a body is *either* moved *or* not moved, that is, at rest. For here [in logic] one speaks solely of the relation of the cognition to the object. In the doctrine of appearance [*Erscheinungslehre*], where it is a matter of the relation to the subject, so as to determine therefrom the relation to the object, the situation is different. For here the proposition that the body is either moved and the space at rest, or conversely, is not a disjunctive proposition in an objective relation, but only in a subjective one, and the two judgements contained therein are valid *alternatively*. In precisely the same phenomenology, where the motion is not merely considered phoronomically, but rather dynamically, the disjunctive proposition is instead to be taken in an *objective* meaning; i.e., in the place of the rotation of a body I cannot assume a state of rest of the latter and the opposite motion of the space instead. But wherever the motion is considered *mechanically* (as when a body approaches another seemingly [*dem Scheine nach*] at rest), then even the judgement that is disjunctive in form is to be used *distributively* in relation to the object, so that the motion must not be attributed *either* to the one *or* to the other, but rather an equal share of it to each. (559–60)

This footnote thereby refers back to an earlier passage in the proof of the first proposition, where Kant first introduced the distinction between *alternative* and *disjunctive* judgements: "(In the former [disjunctive judgement], of two *objectively* opposed predicates, one is assumed to the exclusion of the other for the determination of the object; in the latter [alternative judgement], of two judgements objectively equivalent, yet subjectively opposed to one another, one is assumed for the determination of the object without excluding the opposite – and thus by mere choice.)*" (556)[70]

It is clear, therefore, that an alternative attribution of the predicate of motion corresponds to the first proposition, a disjunctive attribution to the second, and a distributive attribution to the third. But what, then, is an

[69] For the full passage see again the paragraph to which note 42 above is appended.

[70] This passage was already quoted in note 57 above, where I also referred back to the surrounding context in the proof of the first proposition. The footnote to the passage just quoted (556) then refers us forward to the present discussion in the general remark.

"alternatively-mutual" attribution? It is an attribution, I suggest, that has one foot, as it were, in the first proposition (as "alternative") and another in the third (as "mutual"). For, on the one hand, the concept of "mutual [*wechselseitig*]" attributions of motion is characteristic of Kant's interpretation of the equality of action and reaction in the fourth proposition of the Mechanics.[71] And, on the other hand, Kant uses what appears to be the same concept later in the general remark in describing "[e]mpty space in the *phoronomical* sense, which is also called absolute space" (562).[72] The phrase "alternatively-mutual" thereby corresponds to what I have called a *provisional* attribution of rectilinear motion. Such a judgement attributes (rectilinear) motion to one of two interacting bodies (without excluding a complementary motion of the other), and, however, we deliberately leave it open for further reconsideration in light of the equality of action and reaction (where an equal and opposite motion of the other body is then necessary). It is not, therefore, what Kant calls a distributive judgement, for there an equal and opposite attribution of two motions is already in place. Rather, we begin from a number of alternative possibilities (either the earth or the sun as the center of the earth–sun orbit, for example), each of which provides an equally good starting point for the application of Kant's procedure. It is precisely this situation, on my reading, that Kant is attempting to capture in the reconsideration of the relativity of (rectilinear) motion that he presents in the first proposition.[73]

[71] See again the passage from the beginning of the proof of the fourth proposition of the Mechanics where Kant characterizes the "absolute space" for considering the motion of two interacting bodies as one in which "the change of relation (and thus the motion) between the two is completely mutual [*wechselseitig*]; as much as the one body approaches every part of the other, by so much does the other approach every part of the first" (545; see the paragraph following the one to which note 111 of my chapter on the Mechanics is appended, and compare note 56 above).

[72] Kant there says (562) that "absolute space" is necessary in order "to think motion, not merely in a one-sided fashion [*einseitig*] as absolute, but always mutually [*wechselseitig*], as a merely relative predicate." Earlier, when discussing the third proposition, Kant had considered the "mutually opposed and equal [*wechselseitig-entgegengesetzten und gleichen*] motion of the two bodies" (562) involved in cases of action and reaction.

[73] I suggest that this is what Kant intends in the proof of the first proposition by taking *alternative* judgements to be such that, "of two judgements objectively equivalent, yet subjectively opposed to one another, one is assumed for the determination of the object without excluding the opposite – and thus by mere choice" (556). Thus, continuing our example, we could begin Kant's determination of the true motions in the heavens (following Newton's argument in Book 3) by initially taking any body in the solar system to be at rest (or, at least, any body that has satellites of its own). All such starting points, in this sense, are "objectively equivalent, yet subjectively opposed to another." Since they are "objectively equivalent," however, we deliberately take care, at this point, that we do not positively exclude any of them but merely choose a given alternative in order to get started somewhere. We thereby keep the final stage of the procedure continually (and prospectively) in view – where all attributions of rectilinear motion must eventually be fully mutual.

We can deepen our appreciation of this point and also connect it with Kant's threefold distinction between "appearance," "semblance," and "experience" at the beginning of the Phenomenology (554–55; section 30 above) if we now consider more carefully Kant's contrast between rectilinear motion (the subject of the first proposition) and circular motion (the subject of the second). For it may seem, on my reading, that there is no longer an asymmetry between the two cases after all. In particular, we begin Kant's procedure for reducing all motion and rest to absolute space not only by taking the earth to be at rest at the center of the earth–sun orbit (in accordance with the first proposition) but also by taking the earth to be non-rotating as well (in apparent violation, therefore, of the second proposition). Thus, if an alternative judgement, on my reading, is simply one from which we can begin Kant's procedure for determining the true motions while simultaneously holding it open for further revisions, then it would appear that this description fits both cases equally well – contrary to Kant's explicit intentions in the two propositions.

One place where Kant appears to begin his procedure by starting from a non-rotating earth is in the discussion of time determination in the second remark to the refutation of idealism, where Kant says that "we can undertake [*vornehmen*] all time determination only by the change of external relations (motion) in relation to the permanent in space (e.g., motion of the sun with respect to objects on the earth)" (B277–78).[74] By the argument of the second proposition of the Phenomenology, however, it turns out that all of the motion in question (the daily rotation of the earth relative to all of the celestial bodies, including the sun) is to be ascribed to the earth and none ascribed to the heavenly bodies (again including the sun). Our question, therefore, concerns how this initial starting point for Kant's procedure, on my reading, is also consistent with the argument of the first proposition, where all initial attributions of *rectilinear* motion are taken to be "alternatively-mutual." Since we here begin by taking the earth to be at rest with regard to *circular* motion as well, why is this attribution (just like an initial attribution of rest with regard to rectilinear motion) not also alternatively-mutual?

An important clue to the asymmetry Kant sees here emerges in the statement of the second proposition itself (556–57): "The circular motion of a matter, as distinct from the opposite motion of the space, is an *actual*

[74] Compare the paragraph to which note 46 of my chapter on the Phoronomy is appended. The note itself suggests a close relationship between this passage from the first *Critique* and the argument of the *Metaphysical Foundations*.

predicate of this matter; by contrast, the opposite motion of a relative space, assumed instead of the motion of the body, is no actual motion of the latter, but, if taken to be such, is a mere semblance [*Schein*]."[75] The first proposition says that there is no difference between the (rectilinear) motion of the body in a relative space and the body being at rest with the space in motion – so that the two, as Kant puts it, are objectively "equivalent [*gleichgeltend*]" (556). By contrast, in cases of *actual* (rotational) motion of a body, according to the second proposition, the latter scenario (with the space in motion instead) is a mere *semblance*. Indeed, the conclusion of the following proof then restates this idea even more strongly, now in terms of the threefold distinction between appearance, semblance, and experience:

> Now the motion of the space, as distinct from that of the body, is merely *phoronomic*, and thus has no moving force. Therefore, the judgement that here either the body is moved, or the space is moved in the opposite direction, is a *disjunctive* judgement, whereby, if one of the terms (namely the motion of the body) is posited, the other (namely that of the space) is excluded. Thus the circular motion of a body, as distinct from that of the space, is an *actual* motion, [and] therefore the latter, even though it agrees with the former according to the appearance, nevertheless contradicts it in the context of all appearances, i.e., [the context] of a possible experience, and thus [is] nothing but mere semblance. (557)

In this case, therefore, although the two descriptions in question (rotation of a body and opposite equal rotation of the surrounding space) are perfectly in agreement in the appearance, they cannot be combined with one another in the context of a possible experience. And, for precisely this reason, the latter description must be counted as a mere semblance.

Hence, when we begin Kant's procedure from a non-rotating earth (as in the example from the refutation of idealism), we are in this case beginning from a mere *appearance* (*Erscheinung*) rather than from an alternative *judgement*. We are not, as in the first proposition, making a judgement – from the point of view of "a cognition that determines the object validly for all appearances" (555; compare note 30 above, together with the paragraph to which it is appended) – that two different descriptions of the same relative motion (either the earth or the sun as center of the earth–sun orbit, for example) represent equally good provisional starting points. Indeed, it follows from the argument of the second proposition (which I

[75] The second of the two sentences reads (556–57): "[*D*]*agegen ist die entgegengesetzte Bewegung eines relativen Raums, statt der Bewegung des Körpers genommen, keine wirkliche Bewegung des letzteren, sondern, wenn sie dafür gehalten wird, ein bloßer Schein*." It seems that *des letzteren* here should refer back to *eines relativen Raums* rather than *des Körpers*.

shall examine in detail in the next section), that – in conformity with the law of inertia – the two different descriptions of the relative motion in this case (of the earth or the starry heavens) cannot possibly represent equally good starting points. In particular, if we now take one of the two to be *actual* (as opposed to merely possible), the other is positively *excluded*. We cannot later apply the equality of action and reaction to this case so as eventually to strike a balance – "in the context of all appearances" (557) – between the two disjuncts of an alternative judgement. In this sense, the law of inertia (as actually applied in the second proposition) functions quite differently in Kant's procedure from the equality of action and reaction (as prospectively applied in the first proposition).[76]

Further light is shed on this issue by the related example Kant considers in the *Prolegomena* of the retrograde motion of the planets as seen from the perspective of the earth. He says that there is here "neither falsehood nor truth" considered as "only appearance," where "one does not yet judge in any way concerning the objective character of the motion" (4, 291; see the paragraph following the one to which note 4 above is appended). Nevertheless, "because a false judgement can easily arise," one also speaks of a "semblance" of retrogression: "Yet the semblance here is not to be charged to the senses but to the understanding, whose province alone it is to make an objective judgement from the appearances" (4, 291). In this case, too, we begin from the perspective of an immobile earth, and we describe the motions of the planets, in accordance with the Ptolemaic system, as exhibiting retrograde motions (moving on epicycles fixed on deferent circles uniformly rotating relative to the earth). And, as Kant says, there is nothing wrong with this description if one takes it simply as an appearance (of merely relative motion) with no judgement of the understanding (concerning true as opposed to apparent motion) yet in

[76] Compare note 73 above, together with the paragraph to which it is appended. The idea that a later application of the equality of action and reaction is kept *prospectively* in view in the case of an alternative judgement represents the heart of my reading of the phrase "alternatively-mutual." By contrast, if we begin from a non-rotating earth, although this is (and must be) perfectly legitimate when taken as a mere appearance, there is here no judgement at all concerning the true motions – not even a provisional alternative judgement that is self-consciously held open for future revisions. To introduce a judgement at all, for Kant, is to introduce the point of view of experience (as opposed to appearance). So, by the argument of the second proposition, if we take either the earth or the starry heavens to be (truly) rotating in the context of experience, the other is positively excluded and thus relegated to the status of a mere semblance. As Kant puts it in the remark to the (sole) explication of the Phenomenology, "in the case of semblance, the understanding with its object-determining judgements is always in play, although it is in danger of taking the subjective for the objective; in the appearance, however, no judgement of the understanding is to be met with at all" (555; see the paragraph to which note 4 above is appended).

question. By contrast, however, as soon as such a judgement is in question (from the point of view of experience as opposed to appearance), the retrograde motion of the planets must now be taken as a mere semblance.

This, however, is not a case of axial (rotational) motion, but rather an effect (from the Copernican point of view) of the orbital motion of the earth relative to the sun. And did I not just say, in the case of the earth–sun orbit, that we then had a choice (in accordance with an alternative judgement) of whether to take the earth or the sun to be at (orbital) rest? Certainly; yet this choice, in the Newtonian argument that I believe Kant takes as his model, is not between Copernicus and Ptolemy but between the Keplerian–Copernican and Tychonic systems – where, according to the latter, the sun orbits around the earth, and the remaining primary planets all have orbits centered on the sun.[77] In particular, we must now take the orbits of the remaining primary planets to be centered on the sun in accordance with Newton's Phenomenon 5 (P801): "The primary planets, by radii drawn to the earth, describe areas in no way proportional to the times but, by radii drawn to the sun, traverse areas proportional to the times."[78] Newton is here appealing to Kepler's second or area law, and he is observing that this law holds for the primary planets with respect to the sun (taken as one of the foci of a Keplerian ellipse) but does not hold at all for the five primary planets with respect to the earth. This decides the issue, for Newton, between the Copernican and Ptolemaic systems, but it leaves the choice of the Tychonic system still open. And it is precisely this last choice, for Newton, which is then finally settled in the context of the equality of action and reaction applied from the point of view of the center of mass of the solar system.

According to Propositions 1 and 2 of Book 1 of the *Principia*, however, Kepler's area law is a direct result of the law of inertia. It is in virtue of this law, in particular, that deviations from rectilinear inertial motion due

[77] Compare note 61 above, together with the paragraph to which it is appended. It is in Newton's Phenomenon 4, in particular, that the choice between an earth-centered and sun-centered earth–sun orbit is explicitly left open. On the Tychonic system, the planets still retrogress (on epicycles centered on the sun) relative to the (fixed) earth, but they no longer retrogress relative to the true center of their orbits (the sun): compare note 24 above, including the reference there to my chapter on the Dynamics.

[78] Compare Newton's comment on this Phenomenon (P801):

> For with respect to the earth they sometimes have a progressive motion, they sometimes are stationary, and sometimes they even have a retrograde motion; but with respect to the sun they move always forward, and they do so with a motion that is almost uniform – but, nevertheless, a little more swiftly in their perihelia and more slowly in their aphelia, in such a way that the description of areas is uniform.

to centripetal forces are equivalent to areas swept out by radii directed towards the center that are directly proportional to the times.[79] Hence, Kant's application of the law of inertia in the argument of the second proposition of the Phenomenology (as I shall explain in more detail below) allows one to infer a true rectilinear acceleration towards the sun for all of the primary planets in the solar system.[80] Yet, since the earth–sun orbit remains Keplerian whether one takes the earth or the sun to be the center, it is also possible to infer a true rectilinear acceleration of the sun towards the earth.[81] At this point, therefore the choice between the Keplerian–Copernican and Tychonic systems still remains open. And it is just this ambiguity, as already explained, which is then settled by an application of the equality of action and reaction – in accordance with which we can show, in particular, that the center of mass of the earth–sun system is actually very close to the center of the sun. In this case, therefore, all *three* propositions of the Phenomenology are in play. We can begin, in accordance with the first proposition, by taking either the earth or the sun as the center of the earth–sun orbit. We can then infer, in accordance with the second proposition, that there are rectilinear accelerations of all the primary planets (including the earth) directed towards the sun and of the sun towards earth. We can finally conclude, in accordance with the third proposition, that both the earth and the

[79] Proposition 1 reads (P444): "The areas which bodies made to move in orbits described by radii drawn to an unmoving center of forces lie in unmoving planes and are proportional to the times." Proposition 2 reads (P446): "Every body that moves in some curved line described in a plane and, by a radius drawn to a point, either unmoving or moving uniformly forward with a rectilinear motion, describe areas around that point proportional to the times, is urged by a centripetal force tending toward that same point." As explained in the paragraph to which note 140 of my chapter on the Mechanics is appended, the crux of Newton's argument involves viewing the motions in question as approximated by successive inertial motions forming the sides of successive triangles whose areas (by the law of inertia and Euclidean geometry) are equal. In this way, the equality of *areas* prescribed by Kepler is shown to be a direct reflection of the equality of *distances* prescribed by the law of inertia.

[80] There is no doubt that Kant is completely familiar with this part of Newton's argument. See the discussion at the beginning of the First Part of the *Theory of the Heavens* (2, 244):

> If the orbits of the heavenly bodies were precisely circles, then the simplest possible analysis of the composition of curvilinear motions would show that a restraining impulse towards the center is required; however, although in all planets, and also comets, they are ellipses in whose common focus is the sun, nevertheless, the higher geometry, with the help of the Keplerian analogy (according to which the radius vector, or the line drawn from the planet to the sun, always cuts off such areas [*Räume*] from the elliptical path that are proportional to the times), still shows with indubitable certainty that a force must unceasingly impel the planet towards the center of the sun throughout the entire orbit.

[81] For the qualifications that are necessary in the case of Kepler's third (harmonic) law see note 63 above.

sun accelerate towards one another, although the earth accelerates much more than does the sun.[82]

Before leaving this example, there is a further important feature of Newton's discussion of the choice between the Keplerian–Copernican and Tychonic systems that is well worth noting. I explained in section 31 above that Newton's way of making this choice does not rely on optical or kinematical considerations (such as attempted observations of stellar parallax) but is thoroughly dynamical – relying on the forces that give rise to the orbits in question and the Laws of Motion governing such forces.[83] I am now in a position to add, however, that the same is true of Newton's argument based on Phenomenon 5 for the choice of the Tychonic over the Ptolemaic system as the only genuine alternative to the Keplerian–Copernican system. For Newton does not appeal in Phenomenon 5 to Galileo's telescopic observations of the phases of Venus.[84] Instead, as just explained, he appeals to his own initial propositions of Book 1, according to which the centripetal force producing the planetary orbits must be directed towards the sun rather than the earth. Thus, because this result is a direct consequence of the law of inertia, Newton's demonstration of it (in Proposition 2 of Book 3) is just as essentially dynamical (rather than optical or kinematical) as his demonstration of the (approximate) truth of the Keplerian–Copernican system (in Propositions 11 and 12). And since, as I have just argued (and will continue to argue), Kant is closely following the *Principia* throughout his discussion of true and apparent motions, we can conclude that his conception of the proper treatment of the (apparent) retrograde motions of the planets is precisely as dynamical as Newton's.[85]

[82] See again note 68 above. This example allows us to anticipate some of the subtleties and complexities involved in applying both the first and second propositions in Kant's procedure. The crucial point, in this case, is that when we begin from an alternative judgement (in accordance with the first proposition), we also have in mind an application of the law of inertia (in accordance with the second proposition) that positively *excludes* rectilinear accelerations of the remaining primary planets directed towards the earth. Once again, however, the full story can only emerge in the sequel – where, in particular, I shall explore the intimate connection Kant makes between states of true rectilinear acceleration of the *parts* of a rotating system and states of true axial rotation of the system itself.

[83] See again note 118 of my chapter on the Dynamics, together with the paragraph to which it is appended.

[84] He does allude to Galileo's observations in Phenomenon 3, however, and also extends this optical argument to Mercury, Mars, Jupiter, and Saturn as well. Phenomenon 3 states (P799): "The orbits of the five primary planets – Mercury, Venus, Mars, Jupiter, and Saturn – encircle the sun."

[85] See the paragraph to which note 25 above is appended, together with the preceding paragraph. The situation is intimately connected, as explained in this earlier discussion, with Kant's conception of what it means to determine the motion of the movable as an object of *experience*.

33 DETERMINING TRUE CIRCULAR MOTION

According to the second proposition of the Phenomenology, the circular motion of a matter or body is "an *actual* predicate of this matter," while "the opposite motion of a relative space, assumed instead of the motion of the body ... is a mere semblance" (556–57; see the paragraph to which note 75 above is appended). It is now time to examine Kant's proof of this proposition:

> Circular motion (like all curvilinear motion) is a continuous change of rectilinear motion, and, since the latter is itself a continuous change of relation with respect to the external space, circular motion is a change of a change in these external relations in space, and is thus a continuous arising of new motions. Now since, according to the law of inertia, a motion, in so far as it arises, must have an external cause, while the body, at every point on this circle (according to precisely the same law), is striving, for its own part, to proceed in the straight line tangent to the circle, which motion acts in opposition to this external cause, it follows that every body in circular motion manifests [*beweiset*] a moving force by its motion. [But] the motion of the space, as distinct from that of the body, is merely *phoronomic*, and has no moving force. (557)[86]

This argument appears, at first sight, to be completely straightforward. Suppose that a body is moving in a circular orbit. At every point of its orbit there must be a force (directed towards the center) responsible for the body's deviation from rectilinear (tangential) motion. Hence, the presence of this force dynamically distinguishes the orbit in question from a merely "apparent" motion.

On closer consideration, however, the argument appears more problematic. First of all, the law of inertia applies equally to cases of *rectilinear accelerated* motion: in such cases, as well, there must be a force responsible for the body's deviation from inertial (constant velocity) motion.[87] But we cannot infer, in such cases, that we have a true or actual motion that positively excludes – in accordance with a disjunctive judgement – an opposite motion of the relative space. Indeed, the whole point of Kant's first proposition is that rectilinear motions (including rectilinear accelerated motions) differ fundamentally from circular motions in just this respect,

[86] The remainder of Kant's argument, which appeals to the notion of a disjunctive judgement and concludes that the opposite motion of the space is a mere semblance, is quoted in the paragraph to which note 75 above is appended.

[87] See again the footnote from the second analogy (A207/B252) quoted in note 93 of my chapter on the Mechanics (and compare also the paragraph to which that note is appended). It is clear, in particular, that *any* deviation from inertial motion (whether rectilinear or curvilinear) "manifests" an external moving force for Kant.

since, in the case of rectilinear motions, we are faced with an alternative rather than a disjunctive judgement.[88] Secondly, the point of the second proposition is to allow us to *determine* true or actual circular motions to begin with. So we cannot simply assume such a motion at the start, and then infer the existence of a force as the cause of this motion. Rather, we must somehow be given the force in question independently of the motion to be determined, and we must then use this independently given force to establish the motion as actual.

Both of these issues are clarified by the example that Kant presents in the following remark. There, as already observed, Kant introduces Newton's example of two balls rotating around a common center of gravity connected by a cord, where the tension in the cord then allows us to infer that the rotation in question is in fact a true one (see note 47 above, together with the paragraph to which it is appended). Moreover, Kant himself describes the example in precisely this way in his more detailed presentation in the general remark. In an important footnote, in particular, he quotes (in Latin) the words with which Newton initiates his discussion of the example in the Scholium to the Definitions (562; compare P414): "It is certainly very difficult to find out the true motions of individual bodies and actually to differentiate them from apparent motions, because the parts of that immovable space in which the bodies truly move make no impression on our senses. Nevertheless, the case is not utterly hopeless." Kant continues: "He [Newton] then lets two balls connected by a cord revolve around their common center of gravity in empty space, and shows how the actuality of their motion, together with its direction, can nonetheless be discovered by means of experience" (562). So Kant's primary example of determining true motion, following Newton, is the rotational motion of two bodies around a common center of gravity, and the fundamental idea is that we can determine the truth or actuality of this motion by the centrifugal force thereby produced (here the tension in the cord).[89]

[88] It is especially clear that non-uniform (accelerated) rectilinear motions are subject to the first proposition if we take seriously Kant's use of the term "alternatively-mutual." For we then see that the equality of action and reaction is (prospectively) in play applied to mutually accelerating bodies. Compare note 76 above, together with the paragraph to which it is appended and the preceding paragraph.

[89] A parallel situation arises in the case of Newton's much more frequently discussed example of the rotating bucket presented two paragraphs earlier in the Scholium. In this case the concavity produced in the surface of the rotating water plays the same role as the tension in the cord in Newton's second example. As we shall see, Kant has good reasons in the Phenomenology for focussing on this second example.

In terms of the first issue raised above, then, Kant is imagining a case of mutual orbital motion (around a common center of gravity) of (at least) two bodies, not a single orbit (with respect to a purely mathematical center) of one body. We are thereby considering an *axial rotation* (of the line connecting the two bodies with respect to their common center of gravity), and this is why, in the passages under consideration, Kant consistently assimilates what he calls "circular motion" to axial rotation. In his remark to the second proposition, for example, Kant speaks of determining "the circular motion of two bodies around a common central point (and thus also the axial rotation of the earth)" (557–58). At the conclusion of the footnote just quoted from the general remark he adds (562): "I have attempted to show this also in the case of the earth moved around its axis, in somewhat altered circumstances."[90] Hence the sharp distinction Kant draws between circular and rectilinear motion (including rectilinear accelerated motion) in the first two propositions, is, at bottom, a distinction between axial rotation (whether of the line connecting two bodies rotating around their common center of gravity or of a single body around its axis), on the one side, and rectilinear motion (whether uniform or accelerated), on the other. In the latter case, according to Kant, we are always dealing with a relative motion involving two bodies, but in the former only one body (or system of bodies) need actually be considered. The reason, as explained, is that cases of rectilinear motion (whether of two colliding bodies approaching one another inertially or two attracting bodies accelerating towards one another) are always subject to the equality of action and reaction, whereas cases of axial rotation are not (see note 55 above, together with the paragraph to which it is appended). Hence, rectilinear motion, for Kant, always involves a relative motion between two "correlates" (a body and the surrounding space), whereas axial rotation does not (see the paragraph to which note 57 above is appended).[91]

[90] As we shall see, determining the earth's true axial rotation is in fact the main example of reducing all motion and rest to absolute space that Kant discusses (at length) in the general remark – where, as observed, Kant begins his discussion of true circular motion with precisely this example (see note 46 above, together with the paragraph to which it is appended). Indeed, Kant already used this same example (the rotating earth) to illustrate the way in which the principle of the relativity of motion formulated in the Phoronomy must (later) be qualified in the case of "curvilinear motions" (488, and compare the still earlier use of the example at 482).

[91] The situation is rather subtle, however. In the case of two bodies mutually rotating around a common center, for example, there is of course more than one body, and the two bodies in question experience rectilinear accelerations towards one another that are necessarily equal and opposite. Nevertheless, the axial rotation itself need involve no external correlate of the *system* of the two bodies, and no application of the equality of action and reaction need apply

The second issue is considerably more complex. The true circular motions in question are supposed to be determined by the centrifugal forces thereby produced. Where there is a centrifugal force or endeavor (directed away from the center of motion), however, there must also be a counterbalancing centripetal force (directed towards the center).[92] Otherwise, there would be no circular motion after all, and the body or bodies under consideration would simply fly off (uniformly and rectilinearly) along the tangent. It is precisely this situation that Kant is emphasizing in his proof of the second proposition when he explains that any circularly moving body (or part of a body) "is striving, for its own part, to proceed in the straight line tangent to the circle, which motion acts in opposition to this external cause [i.e., the external 'moving force' that follows from the law of inertia – MF]" (557; see the paragraph to which note 86 above is appended). Thus, in the case of two balls rotating around a common center connected by a cord, the centripetal or attractive force is the elastic force of the cord that keeps the two joined together and thereby resists the counterbalancing centrifugal endeavor arising from rotational motion. The tension in the cord can then be measured, for example, by its stretching (in accordance with Hooke's law) against the elastic force in question.[93] So, in this way, there is a clearly observable difference between two balls connected by a cord experiencing the centrifugal effects of a true

there. Similarly, in the case of the axial rotation of the earth, the *parts* of the earth (located opposite to one another at equal distances along a diameter) also experience equal and opposite rectilinear accelerations towards one another. When Kant speaks of orbiting "bodies" in the proof of the second proposition, therefore, he means orbiting parts of an axially rotating system of bodies – a point, as explained below, that becomes even clearer in the general remark.

[92] I say centrifugal "force or endeavor" to call attention to the circumstance that centrifugal force, properly speaking, is no genuine (dynamical) force at all; it is rather, in modern terminology, a "pseudo force" arising only in non-inertial frames of reference (here a rotating frame of reference). And the criterion for this, again in modern terms, is that such "forces" (accelerations) are not correlated with equal and opposite changes of momentum in accordance with the equality of action and reaction – which, like all mechanical laws of motion, holds only in inertial frames of reference. I shall return to this situation below, but I meanwhile note that Kant himself is clear that the effect of centrifugal force in question occurs "without any dynamical repulsive cause" but "though actual [rotational – MF] motion" instead (562; see the passage to which note 95 below is appended). I am indebted to George E. Smith for stressing the need to clarify the notion of "centrifugal force" at this point.

[93] In terms of the third number of the general remark to dynamics, therefore, the force exerted by the cord is a case of what Kant calls "attractive elasticity," and he illustrates this notion by an "iron wire, stretched by a hanging weight" (529; see note 167 of my chapter on the Mechanics). It was standard at the time to illustrate the centrifugal force produced in a connecting cord by a rotating body with the tension produced in this same cord by a hanging weight – thereby indicating a transition from statics to dynamics.

(mutual) rotation and two balls connected by the same cord experiencing no such rotation at all.[94]

This case, therefore, is relatively straightforward. But what is its analogue in the case of the rotating earth? At the end of his lengthy discussion of determining the true rotation of the earth in the Phenomenology (the discussion to which the above-quoted footnote to Newton's Scholium is attached), Kant explains that the truth or actuality of the earth's rotation "rests on the representation of the mutual and continuous *withdrawal* of any part of the earth (outside the axis) from any other part lying opposite to it on a diameter at the same distance from the center" (561–62) and concludes:

> For this motion is actual in absolute space, in that the reduction of the distance in question, which gravity by itself would produce in the body, is thereby continuously made up, and, in fact, without any dynamical repulsive cause (as can be seen from the example chosen by Newton in the *Principia*, page 10 of the 1714 edition*); hence, it is made up by actual motion, which relates, however, to the space inside the moved matter (namely its center), and not to that outside it. (562)[95]

So Kant is clearly envisioning a counterbalancing of the centrifugal force or endeavor due to rotation by *gravity*. The true or actual rotation of the earth results in a (centrifugal) tendency of the parts of the earth (located opposite to one another at equal distances along a diameter) to withdraw from one another. But gravity induces a (centripetal) tendency of these same parts to approach one another. If gravity acted alone, therefore, without (actual) rotation, no (centrifugal) withdrawal would take place at all.[96]

[94] In the case of Newton's rotating bucket (compare note 89 above), the counterbalancing centripetal force is of course produced by the bucket's walls, which resist the outwardly directed centrifugal pressure and conserve the rotational motion of the water. Centrifugal pressure combined with surface tension then results in the characteristic concave shape.

[95] The attached footnote is just the one that we have been considering (see the paragraphs to which notes 89 and 90 above are appended). Kant is citing a (pirated) 1714 version of the second (1713) edition of the *Principia* that circulated widely in Northern Europe at the time.

[96] There are two fundamentally different – but inter-translatable – ways of representing this centrifugal withdrawal. In a frame of reference in which the earth is at rest (a rotating frame) it appears as a centrifugal *acceleration* acting in the opposite direction to the centripetal acceleration due to gravity. In a frame of reference in which the earth is rotating (an inertial frame), however, it appears as the result of the tangential component of *inertial* motion that is compounded with the centripetal acceleration of gravity to yield a circular (more generally curvilinear) orbital motion of the parts at equal distances from the axis of rotation along a common diameter (see again note 91 above). In this last case, therefore, the relevant endeavor of motion is *perpendicular* to the diameter, and the "reduction of distance" due to gravity is represented (in

What is the observable effect, in this case, analogous to the observed tension (or stretching) of the cord in Newton's example? I believe that Kant is here considering the empirically determinable shape or figure of the earth, in so far as the rotation in question produces an observable flattening at the poles and bulge at the equator.[97] In the next section I shall present evidence that Kant has in mind, in particular, the results of Propositions 18, 19, and 20 of Book 3 of the *Principia*. Here Newton first outlines in general how the oblate spheroidal shape in question follows from a counterbalancing of gravity and centrifugal force, then presents a quantitative analysis of this counterbalancing for the case of the earth, and finally (in the third edition) argues that known measurements of the earth's surface gravity at different latitudes support his analysis. I shall return to some of the details of Newton's analysis when I discuss Kant's argument in the general remark at greater length below. But what is most relevant, at this point, is Proposition 18 of Book 3, where Newton first sketches the general idea for all rotating planets:

If it were not for the daily circular motion of the planets, then, because the gravity of their parts is equal on all sides, they would have to assume a spherical figure. Because of that circular motion it comes about that those parts, by receding from the axis, endeavor to ascend in the region of the equator. And therefore if the matter is fluid, it will increase the diameters at the equator by ascending, and will decrease the axis at the poles by descending. (P821)[98]

the limit) by the line segment BD in the figure appearing in the paragraph to which note 162 of my chapter on the Mechanics is appended. In the modern terms of note 92 above, then, the centrifugal *acceleration* (directed away from the center along the diameter) arising in the first representation is the manifestation of a "pseudo force" and is therefore no true or actual acceleration at all. Only the representation in an inertial frame can properly represent the true or actual motion – which, in this case, results from compounding the true centripetal acceleration due to gravity with the tangential inertial endeavor. Thus, aside from the true centripetal acceleration of gravity, there is also what we now call a true angular velocity. I shall return to this situation below and, in particular, to the relationship between Kant's conception and our modern one. I am meanwhile indebted to Marius Stan for prompting me to consider more explicitly these two different representations of the centrifugal withdrawal in question.

[97] Hilary Putnam first suggested to me that Kant here has in mind the problem of the shape of the earth. In my earlier discussion in Friedman (1986) – repr. in Friedman (1992b) – I had too hastily assimilated this passage to Kant's allusions elsewhere to the moon test. Although, as we shall see below, I still think that Kant is implicitly referring to the moon test in the course of his discussion in the general remark, the language Kant employs in this particular passage – "the mutual and continuous *withdrawal* of any part of the earth (outside the axis) from any other part lying opposite to it on a diameter at the same distance from the center" (561–62) – unmistakably suggests a centrifugal bulging at the equator.

[98] Proposition 18 states (P821): "The axes of the planets are smaller than the diameters that are drawn perpendicularly to those axes."

The earth consists of fluid matter such that each part attracts every other part by the force of universal gravitation. This matter would thereby take an equilibrium spherical shape in the absence of true rotation. Such rotation, however, leads to a new equilibrium, where the distances between the parts are increased perpendicular to the axis of rotation and decreased along the axis itself.[99]

On this reading, therefore, the analogy between the case of the axial rotation of the earth and the case of the two mutually rotating balls connected by a cord is that gravity plays the role of the (attractive) elasticity of the cord, and the observable withdrawal of parts against the resistance of gravity (the observable equatorial bulging of the earth) plays the role of the observable tension or stretching of the cord against the resistance of its (attractive) elasticity.[100] Just as, in the case of (attractive) elasticity, the stretched cord would return to its original configuration if the mutual rotation of the two balls were to cease, now, in the case of gravity, a rotating fluid earth would return to its original spherical shape if its axial rotation were to cease. It is no wonder, then, that Kant's immediately following discussion of the third proposition in the general remark refers back to "the active dynamical influences, given through experience, that are required in the second case [the second proposition – MF] (gravity or a tensed cord)" (562). The example of the (oblate spheroidal) shape produced by the earth's rotation thereby makes clear how the empirical

[99] As I shall explain in detail in the following section, it is clear from the fifth chapter of Part Two of the *Theory of the Heavens* that Kant was familiar with Newton's quantitative analysis, and also with the fundamentally important differences between Newton's analysis, based on universal gravitation between each part of the earth and every other part, and Huygens's opposing analysis, based on uniform (Galilean) gravity directed towards the center. Newton's increasingly detailed discussions of this situation in the second and third editions are due, in part, to Huygens's objections to universal gravitation. The potentially observable differences in the shape of the earth predicted by the two competing analyses then led to the first great test case for universal gravitation in the eighteenth century (which, as noted in the following section, was never conclusively resolved). I reserve a more detailed discussion for the following section, because it is there that the difference between universal gravitation and uniform (Galilean) gravity becomes most directly relevant for my reading of Kant's consideration of the rotation of the earth in the Phenomenology – including, in particular, the relationship between Kant's initial examples involving Coriolis force (see note 67 above, together with the paragraph to which it is appended) and the case now under consideration involving centrifugal force.

[100] Compare note 91 above. Just as, in the case of the balls connected by a cord, the centrifugal endeavor due to rotation are counterbalanced by centripetal (rectilinear) accelerations towards their common center of gravity, here, in the case of the rotating earth, the centrifugal endeavor of diametrically opposed *parts* of the earth at equal distances along a diameter is counterbalanced by corresponding centripetal (rectilinear) accelerations (due to universal gravitation) directed towards the center of the earth.

determination of this rotation is indeed analogous to Newton's example of the two bodies connected by a cord.

If this is correct, however, we have a new question. Since Kant is here occupied primarily with the earth's true rotation, why does he begin with Newton's example of the two balls connected by a cord from the Scholium and then go out of his way to show, rather tortuously, that this case is analogous to the case of the rotation of the earth? Why, in particular, does Kant not directly cite Newton's discussion of the rotation of the earth in Propositions 18, 19, and 20 of Book 3 (with the results of which, as pointed out in note 99 above, Kant was familiar) rather than the discussion in the Scholium? In order to answer these questions properly, it turns out, we need first to ask what the purpose of the discussion in the Scholium is supposed to be.

So let us look at Newton's final paragraph of the Scholium at greater length. It begins as follows:

> It is certainly very difficult to find out the true motions of individual bodies and actually to differentiate them from apparent motions, because the parts of that immovable space in which the bodies truly move make no impression on our senses. Nevertheless, the case is not utterly hopeless. For it is possible to draw evidence partly from apparent motions, which are the differences between the true motions, and partly from the forces that are the causes and effects of the true motions. For example, if two balls, at a given distance from each other with a cord connecting them, were revolving about a common center of gravity, the endeavor of the balls to recede from the axis of motion could be known from the tension of the cord, and thus the quantity of circular motion could be computed. Then, if any equal forces were simultaneously impressed upon the alternate faces of the balls to increase or decrease their circular motion, the increase or decrease of the motion could be known from the increased or decreased tension of the cord, and thus, finally, it could be discovered which faces of the balls the forces would have to be impressed upon for a maximum increase in the motion, that is, which were the posterior faces, or the ones that are in the rear in a circular motion. Further, once the faces that follow and the opposite faces that precede were known, the direction of the motion would be known. In this way both the quantity and direction of this circular motion could be found in any immense vacuum, where nothing external and sensible existed with which the balls could be compared. (P414)

Newton begins by considering the rotating balls in an otherwise empty space, and he explains how, in Kant's language, "the actuality of their motion, together with its direction, can nonetheless be discovered by means of experience" (562; see the paragraph to which note 89 above is appended). So far, therefore, Kant's discussion of this example is completely parallel to Newton's.[101]

The Scholium continues (and concludes) by imagining that a number of other bodies are then added to the space in question – bodies that play the same role in this scenario as the fixed stars do with respect to the observed motions in the solar system:

Now if some distant bodies were set in that space and maintained given positions with respect to one another, as the fixed stars do in the region of the heavens, it could not, of course, be known from the relative changes of position of the balls among the bodies whether the motion was to be attributed to the bodies or to the balls. But if the cord was examined and its tension discovered to be the very one which the motion of the balls required, it would be valid to conclude that the motion belonged to the balls and that the bodies were at rest, and then, finally, from the change of position of the balls among the bodies, to determine the direction of the motions. But in what follows, a fuller explanation will be given of how to determine true motions from their causes, effects, and apparent differences, and, conversely, of how to determine from motions, whether true or apparent, their causes and effects. For this was the purpose for which I composed the following treatise. (P414–15)

Thus, although the mere relative motion of the balls with respect to the newly introduced bodies (playing the role of the fixed stars) does not suffice to show whether the former or the latter are truly in motion, the rotational motion in question could nonetheless be truly ascribed to the balls rather than the surrounding bodies by examining the tension in the cord. Newton appears to be suggesting, therefore, that the example of the two balls connected by a cord can be taken as a model for the argument of Book 3 determining the true motions in the solar system from the merely apparent motions recorded in Newton's initial Phenomena. For these Phenomena, as we have seen, begin by taking (assuming) the fixed stars to be at rest, and the ensuing argument then arrives at a point where we have finally determined the true motions independently of the fixed stars.[102]

[101] Although he is thus clearly aware, in particular, that Newton's discussion involves the determination of both the truth or actuality of the (rotational) motion and its direction, Kant's own discussion focusses exclusively on the former. I shall basically follow Kant here.

[102] For Phenomena 1 and 2 see note 60 above; for Phenomenon 4 see note 61 above. Although Newton does not mention the fixed stars explicitly in Phenomenon 5 (see the paragraph to which note 78 above is appended), the retrograde motions of the planets to which Newton appeals are recorded against the background of the fixed stars (as seen from the earth) as well. Compare also note 24 above, together with the paragraph to which it is appended, on the role of the fixed stars in Newton's essentially dynamical method for determining the true motions.

Now Kant, as observed, explicitly suggests that the Newtonian force of universal gravitation plays the same role in the argument of Book 3 that the (attractive) elasticity of the tensed cord plays in the example from the Scholium (see again note 100 above, together with the paragraph to which it is appended). And a more detailed look at the last paragraph of the Scholium reveals that Newton appears to be making the same suggestion. The crucial question, in both cases, therefore concerns exactly how the argument of Book 3 is supposed to be parallel to the scenario in the Scholium where the cord connecting the two balls is "examined and its tension discovered to be the very one which the motion of the balls required" (P414). Kant's lengthy discussion of the rotation of the earth in the Phenomenology, on my reading, culminates with the example of the oblate spheroidal shape induced by the earth's true rotation. And it is to precisely his discussion of this example that the crucial footnote (562) concerning Newton's Scholium – quoting from (and briefly summarizing) its final paragraph – is attached. Moreover, Newton's own discussion of the shape of the earth occurs in Propositions 18, 19, and 20 of Book 3 – and, in my reading of the text of the *Metaphysical Foundations* so far, I have been repeatedly led to Propositions 11 and 12 of Book 3 where Newton determines the center of mass of the solar system as the true center of orbital motion.[103] So it is reasonable, at this point, to consider the argumentation in Book 3 between Propositions 12 and 18.

Proposition 13 marks a pivotal point in Newton's argument. It appears, at first sight, simply to be stating what we already know from the (Keplerian) initial Phenomena (P817): "The planets move in ellipses that have a focus in the center of the sun, and by radii drawn to that center they describe areas proportional to the times." Newton immediately emphasizes, however, that we have now (after the law of universal gravitation is in place and we know where the true center of motion actually lies) achieved a fundamentally new perspective (P817): "We have already discussed these motions from the phenomena. Now that the principles of motions have been found, we deduce the celestial motions from these principles a priori." He continues:

Since the weights [i.e., gravitational forces – MF] of the planets toward the sun are inversely as the squares of the distances from the center of the sun, it follows … that if the sun were at rest and the remaining planets did not act upon one

[103] See, most recently, note 64 above (including the references there to earlier discussions), together with the paragraph to which it is appended.

another, their orbits would be elliptical, having the sun in their common focus, and they would describe areas proportional to the times. (P817–18)

According to the theory of universal gravitation, therefore, Proposition 13 is not strictly and exactly true but instead describes an idealized situation in which certain small perturbations can be ignored (P818): "The actions of the planets upon one another, however, are so very small that they can be ignored, and they perturb the motions of the planets in ellipses about the mobile sun less … than if those motions were being performed about the sun at rest."[104] It turns out, however, that "the action of Jupiter upon Saturn is not to be ignored entirely" (P818), especially when these two (heaviest) planets are in conjunction. In addition: "The perturbations of the remaining planets are still less by far, *except* that the orbit of earth is sensibly perturbed by the moon" (P818, emphasis added).[105]

The logic of Newton's argument is as striking as it is subtle. We begin with the initial Phenomena described by Kepler's laws, which describe the observed orbital motions of the planets relative to the sun against the background of the fixed stars. Moreover, for planets that themselves have satellites (Jupiter, Saturn, and the earth), we have versions of Kepler's laws governing the observed orbital motions of these satellites relative to their primary bodies as well – also against the background of the fixed stars. By the argument of Propositions 1 through 7, however, we thereby arrive at the law of universal gravitation: every body (and indeed every *part* of every body) in the solar system exerts a force on every other that is inversely proportional to the distance between them and directly proportional to their masses. It then follows from this conclusion that our (Keplerian) descriptions of the Phenomena are not exactly correct after all, for the universality of the gravitational forces in question necessarily results in (relatively small) perturbations of the initial orbits. Nevertheless, to the precise extent that we can then account for each of these perturbations in turn within the evolving theory of universal gravitation, we are

[104] The two ellipses contain references to propositions from Book 1 that justify the relevant claims – namely, to Propositions 1, 11, and 13 (Corollary 1), and to Proposition 66, respectively.

[105] In more detail (P817–18):

> The common center of gravity of the earth and the moon traverses an ellipse about the sun, an ellipse in which the sun is located at one focus, and this center of gravity, by a radius drawn to the sun, describes areas (in the ellipse) proportional to the times; the earth, during this time, revolves around this common center with a monthly motion.

Thus Newton here argues explicitly – for the first time – that the Tychonic system is to be definitively discarded in favor of the Keplerian–Copernican system. Compare Newton's Corollary to the immediately preceding Proposition 12 (P817), quoted in note 64 above.

justified in concluding that the true rotational motions in the solar system (both axial and orbital) can indeed be ascribed to the bodies in this system rather than to the fixed stars.

Proposition 14 provides a graphic illustration of this logic. It deduces from first principles – the area law derived in Proposition 1 of Book 1 and the inverse-square law derived in Proposition 11 – that the aphelia and nodes of the planetary orbits are at rest. (Aphelia are the points where the elliptical orbits are furthest from the sun; nodes are the points where the orbits intersect the plane of the ecliptic.) Yet Newton immediately points out that this conclusion is not exact, due to precisely the perturbations produced by the other gravitational forces in the system that are not directed towards the focus of the orbit in question (P819): "But yet from the actions of the revolving planets and comets upon one another some inequalities will arise, which, however, are so small that they can be ignored here."[106] Nevertheless, he also concludes, in Corollary 1 to this Proposition (P819), that "[t]he fixed stars are at rest, because they maintain given positions with respect to the aphelia and the nodes." When Newton earlier derived the area law and inverse-square laws from the initial Phenomena, however, he was already assuming that the fixed stars are at rest – or, more precisely, that they can be *taken* to be so. And his justification for now drawing this as a conclusion is nothing more nor less than the hopeful expectation that all such perturbations will continue to be accommodated within his evolving theory.[107]

[106] Newton expounds on this at greater length in the following Scholium (P819):

> Since the planets nearer to the sun (namely, Mercury, Venus, the earth, and Mars) act but slightly on one another because of the smallness of their bodies [i.e., because their masses are small – MF]; their aphelia and nodes will be at rest, except insofar as they are disturbed by the forces of Jupiter, Saturn, and any bodies further away. And by the theory of gravity it follows that their aphelia move slightly forward with respect to the fixed stars, and do this as the 3/2 powers of the distances of these planets from the sun. For example, if in a hundred years the aphelion of Mars is carried forward 33′20″ with respect to the fixed stars, then in a hundred years the aphelia of the earth, Venus, and Mercury will be carried forward 17′40″, 10′53″, and 4′16″ respectively. And these motions are ignored in this proposition because they are so small.

Kant, to the best of my knowledge, never comments on these details of the motions of the planetary aphelia. As we shall see below, however, there is good reason to think that he was familiar with the basic idea of the planetary perturbations first announced in the discussion of Proposition 13.

[107] The situation is especially interesting with respect to the inverse-square law. In Proposition 2 of Book 3, for example, Newton adds that the inverse-square law for the planets "is proved from the greatest exactness from the fact that the aphelia are at rest" (P802), for "the slightest departure from the ratio of the square would (by book 1, prop. 45, corol. 1) necessarily result in a noticeable motion of the apsides in a single revolution and an immense such motion in many revolutions" (P802). Similarly, Newton does not appeal to Kepler's third or harmonic law at all in deriving the inverse-square law for the moon in Proposition 3 but only to "the very

I am now in a position to explain more precisely the relationship between this argument and the example of the two balls connected with a cord in the Scholium. In the latter scenario the circumstance that the two balls are rotating around a common center relative to a frame of reference determined by the fixed stars does not, by itself, settle the question whether this rotational motion is a true one. The question can be settled, however, by examining the observable tension in the cord and finding, in particular, that it is "the very one which the motion of the balls required" (P414). In other words, the centrifugal tension or stretching of the cord exactly corresponds to the centripetal acceleration – due to what Kant calls attractive elasticity – governing the presumed rotational motion, and it is for just this reason that the motion now counts as true. In the argument from Phenomena in Book 3 Newton also begins by recording the observable relative motions with respect to a frame of reference determined by the fixed stars. Here, of course, the bodies are not connected by any visible cords whose observable tension or stretching could then allow us to establish the truth of the presumed rotational motions. Rather, the argument of Book 3 results in an inverse-square attractive force acting between all bodies (and all parts of bodies) in the solar system, and it is precisely this force of universal gravitation – which Kant calls "the invisible binding force of the universe" (Bxii; see note 1 above, together with the paragraph to which it is appended) – that is now to be coordinated with the observable relative motions with which Book 3 begins. Unlike the case of the two rotating balls, however, this coordination is by no means direct or straightforward. It rather requires an ongoing process of mutual delicate adjustment in which the initial Phenomena are subject to correction, in turn, by the evolving theory of universal gravitation. To exactly the extent to which all perturbations of the initial (Keplerian) orbits can be successfully calculated within this theory by reference to the universal inverse-square attractive forces in question, we are then justified in concluding that this force is indeed the ground of the truth or actuality

slow motion of the moon's apogee" (P802–3). He again invokes Corollary 1 to Proposition 45 of Book 1 and argues than any (apparent) deviation from the inverse-square law in this case can be fully explained by the action of the sun on the moon (in accordance with an *exact* inverse-square law). Finally, as George E. Smith has emphasized to me, the stability of the orbits in question with respect to the fixed stars was by no means firmly established at the time – and Newton does not list it among his initial Phenomena. When he nonetheless takes the orbits in question to be very nearly stable at the beginning of his argument, all he is really doing is committing himself to the *program* of explaining any (small) deviations from such stability within the theory of universal gravitation. And, to precisely the extent to which this program succeeds, Newton's procedure is thereby justified. For further discussion of this essentially programmatic dimension of Newton's procedure see Smith (2002a, 2002b, and 2012).

of all the rotational motions (both axial and orbital) that have now been determined independently of the fixed stars.[108]

But the most important point, for both Newton and Kant, is that we have thereby made a transition from an essentially optical, kinematical, or phoronomical conception of motion to a fundamentally dynamical conception. The true motions, as opposed to the apparent ones, are determined by the dynamical forces that give rise to them rather than by observed changes in position relative to the (presumably) fixed stars. For the mechanical laws of motion governing such forces determine what we now call the class of inertial frames completely independently of the fixed stars. In the case of the observable relative motions in the solar system, in particular, the force of universal gravitation allows us empirically to determine a unique inertial frame by precisely Newton's argument: it is centered on the center of mass of the solar system, and, moreover, all true accelerations in it are precisely balanced by corresponding contrary accelerations in accordance with the equality of action and reaction. Against the background of his intimate involvement with this Newtonian argument, Kant's conception of how true circular motion is determined is thereby eventually implicated with a rather considerable amount of the theory of universal gravitation.[109] In the following section I shall attempt to put all the pieces together so that the relationship between Kant's main example of the rotating earth and Newton's exposition of this theory stands out even more clearly. For it is only in this way, on my reading,

[108] Newton makes a transition between considering orbital and axial rotations in Proposition 17 of Book 3 (P820): "The daily motions of the planets are uniform, and the libration of the moon arises from its daily motion." This Proposition then sets the stage for considering the resulting oblate spheroidal shape of the planets in Proposition 18: see note 98 above, together with the paragraph to which it is appended. The intervening Propositions (15 and 16) concern two "Problems" (aimed at finding the principal diameters, eccentricities, and aphelia of the planetary orbits) that are not directly relevant here.

[109] Both the fundamentally dynamical character of Kant's conception of true rotation and his reliance on universal gravitation are confirmed by the first sentence of Kant's remark to the second proposition (557): "This proposition determines the modality of motion with respect to *dynamics*; for a motion that cannot take place without the influence of a continuously acting external moving force manifests [*beweiset*], directly or indirectly, originally moving forces of matter, whether of attraction or repulsion." The passage to which note 47 above is appended then immediately follows. So it is clear, in particular, that the most important dynamical force now at issue is precisely what Kant calls the fundamental force of attraction. I shall consider Kant's third proposition in more detail in the following section, but it is already clear that the requirement that all true accelerations be balanced by corresponding contrary accelerations (in accordance with the equality of action and reaction) is an essential constituent of his conception of motion. This requirement, moreover, does in fact suffice to eliminate all "pseudo forces" and restrict us to true accelerations in the modern sense (compare note 96 above). I shall discuss this issue further below, culminating in the technical note to the following section.

that we can fully appreciate what Kant means by the transformation of "appearance" into "experience" – and thus, in the end, what he means by "absolute space."

34 REDUCING ALL MOTION AND REST TO ABSOLUTE SPACE

It is only in the general remark to phenomenology that Kant finally explains the sense in which "absolute space" is indeed a necessary concept of natural science – or, more precisely, a necessary idea of reason (560; see the paragraph to which note 41 above is appended): "Absolute space is therefore necessary, not as a concept of an actual object, but rather as an idea, which is to serve as a rule for considering all motion therein merely as relative." Since, as we know, Kant has already used the concept of motion being "reduced to absolute space" in the fourth proposition of the Mechanics (545; compare note 56 above, together with the paragraph to which it is appended), the fact that the general remark immediately follows the third proposition of the Phenomenology – which explicitly depends on the fourth proposition of the Mechanics – has the effect of emphasizing this earlier usage here.[110]

Nevertheless, there is an important difference between Kant's earlier and later uses of the concept. In the fourth proposition of the Mechanics he illustrates the idea with the communication of motion by impact. One body approaches another initially at rest with a given uniform rectilinear velocity, and this motion is then reduced to absolute space by adopting the center of mass frame of the interaction in which both bodies now have an equal share of the motion (i.e., equal and opposite momenta). The primary example Kant has in mind in the Phenomenology, by contrast, is suggested by the two mutually rotating balls Newton introduces towards the end of the Scholium to the Definitions – which Kant then adapts to the axial rotation of the earth. Here, in particular, we are to consider

[110] The proof of the third proposition of the Phenomenology reads (in its entirety) as follows (558):

> According to the Third Law of Mechanics (Proposition 4), the communication of motion of bodies is possible only by the community of their original moving forces, and the latter only by mutually opposite and equal motion. The motion of both is therefore actual. But since the actuality of this motion does not rest (as in the second proposition) on the influence of external forces, but follows immediately and unavoidably from the concept of the relation of the *moved* in space to anything else *movable* thereby, the motion of the latter is *necessary*.

Kant adds, in the following remark, that this proposition "determines the modality of motion with respect to mechanics" (558).

two bodies (or parts of bodies) interacting with one another via gravitational attraction, whereby they experience equal and opposite changes of momenta relative to their common center of mass. And in the latter case, as explained, we are dealing with non-uniform (accelerated) motions rather than uniform motions.[111]

This point reflects a fundamental difference between the perspectives of the Mechanics and the Phenomenology more generally. As observed in section 22 above, Kant begins the Mechanics by focussing almost exclusively on the phenomenon of impact. Indeed, he makes a special point of excusing himself for concentrating on the communication of motion via repulsive forces at the expense of (gravitational) attraction – and he illustrates the case of attraction only parenthetically by the example of "a comet, of stronger attractive power than the earth," which might "drag the latter in its wake in passing ahead of it" (537; compare the paragraph to which note 10 of my chapter on the Mechanics is appended). In the Phenomenology, by contrast, Kant's focus is squarely on interactions via gravitational attraction, and this provides strong confirmation, I believe, for my reading. Kant's procedure for reducing all motion and rest to absolute space is modeled on the Copernican conception of the relativity of space and motion that he had introduced, for the first time, in the *New System of Motion and Rest* (1758) – which is itself based, in turn, on the cosmological conception that he had already articulated in the *Theory of the Heavens* (1755).[112]

As observed, Kant initiates his discussion of reducing all motion and rest to absolute space by making a transition from his earlier version of the Copernican conception of space and motion developed in the Phoronomy to his new perspective on this same conception developed in the Phenomenology. In particular, he begins with the conditions for representing motion "merely as appearance" (559) and moves to the conditions for representing this appearance via "a determinate concept of experience [*Erfahrungsbegriff*] (which unites all appearances)" (560). Moreover, in thus moving from *appearance* to *experience*, he also moves from a sequence of ever more extensive relative space subject to

[111] See again note 55 above, together with the paragraph to which it is appended.

[112] Compare notes 5–11 of my chapter on the Phoronomy, together with the paragraphs to which they are appended. That the at first sight rather fanciful example of a comet dragging along the earth appears to involve a clear allusion to Lambert's *Cosmological Letters* (see note 4 of my chapter on the Mechanics), also provides confirmation, I believe, for my juxtaposition of the *Theory of the Heavens* and the *Cosmological Letters* in connection with precisely Kant's procedure in the Phenomenology (section 31 above).

an indeterminate process of continual revision lacking all internal stability to a determinate "absolute space to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively-mutual" (559). It is in precisely this absolute space that all *appearances* of motion and rest can finally be transformed into a determinate *experience* that unifies all such appearances in one system.[113]

I have argued, on this basis, that the point of view of absolute space thereby necessarily involves mutually counterbalancing motions, in accordance with the equality of action and reaction, in all cases of interactions corresponding to true motions: uniform (and non-uniform) rectilinear motions arising in interactions via repulsive forces, non-uniform (accelerated) rectilinear motions arising in interactions via attractive forces, and circular or rotating motions – where the latter, in Kant's view, also essentially involve (mutually balancing) attractive (centripetal) forces preventing a (rectilinear) escape along the tangent. Moreover, even in cases of (uniform or non-uniform) rectilinear motions falling under Kant's first proposition, the equality of action and reaction (and thus Kant's third proposition) is still prospectively in play. It is for this reason, on my reading, that even cases falling under the first proposition count as essential parts (initial stages) of Kant's procedure for reducing all motion and rest to absolute space.[114]

Indeed, Kant begins his discussion of this procedure, as explained, with precisely the first proposition. He says that "the rectilinear motion of a body in relative space is reduced to absolute space, when I think the body as in itself at rest" (560; see the paragraph following the one to which note 66 above is appended), and, as I have suggested, the example that is most directly relevant in this context is that of the earth – which is *provisionally* assumed to be at rest at the beginning of Kant's procedure. It will later emerge, in accordance with the third proposition, that the earth cannot be truly at rest after all, for it must necessarily experience counterbalancing rectilinear accelerations towards (for example) falling bodies, the moon, and the sun. Nevertheless, the decisive difference between this case and that of Kant's second proposition is that the provisional assumption that

[113] See my earlier discussion in the paragraphs to which notes 41 and 42 above are appended, together with the intervening paragraph.

[114] The core of my argument for these claims, which depends on a particular reading of the crucial concept of an "alternatively-mutual" attribution of motion, is presented in the paragraphs to which notes 70–73 above are appended

some bodies other than an arbitrarily chosen single body are in rectilinear motion (falling bodies, the moon, and the sun rather than the earth) is made without excluding the opposite assumption that the latter body (the earth) is in fact rectilinearly moving (towards all these other bodies) after all. In the case of circular motion (e.g., the true axial rotation of the earth with respect to the starry heavens), by contrast, we do positively exclude the opposite assumption (that the earth is at rest and the starry heavens are rotating) once and for all. The reason, as explained, is that it is the law of inertia rather than the equality of action and reaction that is most directly relevant at this stage.[115]

Kant next turns to the case of determining true circular motion and, in particular, to the example of determining the true axial rotation of the earth with respect to the starry heavens. Indeed, the discussion of this example, which occupies two full pages out of a total of nine in the entire general remark, is by far the most detailed discussion Kant provides anywhere of determining true as opposed to merely apparent motion. Moreover, he provides two distinct examples of such determination. The first part of the discussion concentrates on determining the true rotation of the earth from what we now call Coriolis forces producing deviations from rectilinear fall towards the center, while the second part focusses on an analogous determination from the centrifugal forces producing deviations from the earth's otherwise spherical shape (compare note 99 above, together with the paragraph to which it is appended and the two preceding paragraphs). In light of the central position of this discussion within the general remark, therefore, we are left with two

[115] My argument occupies the bulk of section 33. As explained, there is a delicate relationship between the two cases of correcting our initial perspective on an earth at rest by means of either the second proposition (via the law of inertia) or the third proposition (via the equality of action and reaction). In the former case we begin from a mere *appearance* and then avoid a false judgement or *semblance* in favor of a disjunctive judgement. In the latter, by contrast, we begin from a (provisionally acceptable) alternative judgement that is then refined into a distributive judgement. Moreover, there is a further important subtlety arising here. If, as I claim, Kant is presupposing the analogies of experience (in the guise of his three Laws of Mechanics) in all three propositions of the Phenomenology, then what exactly is he presupposing in the case of the first proposition? This question can only be fully addressed in what follows. But I can begin by observing that Kant is definitely assuming that the "movable" whose true motions are to be determined is a *body* rather than a mere moving *point* (see the text of the proof of the first proposition quoted in the paragraph following the one to which note 33 above is appended). Otherwise, for example, the question of the motion of the (three-dimensional) earth relative to a surrounding frame of reference defined by the fixed stars could not even arise. So if we can here assume that Kant means a body "in the *mechanical* meaning" (537, emphasis added), it follows that the concepts of quantity of motion, quantity of matter, and (therefore) quantity of substance are implicated as well: compare notes 21 and 22 of my chapter on the Mechanics, together with the paragraph to which they are appended.

especially pressing questions. First, why does Kant take pains to illustrate his point with two different examples rather than a single one? Second, if, as I have claimed, he here means to invoke all the main steps of Newton's argument for determining the true motions from the apparent motions in Book 3 of the *Principia*, then why does Kant devote such disproportionate space to the determination of the earth's true rotation – at the expense, apparently, of such other crucially important steps in the argument as the moon test, the determination of the center of gravity of the solar system, and so on?

As observed, Kant initiates his discussion of the earth's true rotation by indicating that this is now the place to solve the "paradox" he had formulated in his remark to the second proposition: namely, that such circular motion "can be given as actual motion in experience, even without reference to the external empirically given space," so that, in particular, "it indeed seems [*scheint*] to be absolute motion" (560; see note 48 above, together with the paragraph to which it is appended). He continues:

> But it should be noted that it is here a question of true (actual) *motion*, which does not, however, *appear* [*erscheint*] as such, so that, if one wished to evaluate it merely in accordance with empirical relations to space, it could be taken for *rest*; it is a question, that is, of *true motion* as distinct from *semblance* [*Schein*], but not of absolute motion in contrast to relative. Thus, circular motion, even if it exhibits no change of place in the appearance, that is, no phoronomic change in the relations of the moved body *to* (empirical) *space*, nonetheless exhibits a continuous dynamical change, demonstrable through experience, in the relations of matter *within its space*, for example, a continual diminution of attraction in virtue of a striving to escape, as an action or effect [*Wirkung*] of the circular motion, and thereby assuredly indicates its difference from semblance [*Schein*]. (561)

Kant here claims that a dynamical counterbalancing of a centripetal force (of attraction) by a centrifugal endeavor (due to rotation) can "assuredly" indicate the actuality of circular motion. It can do this, moreover, even in the absence of an external relative space:

> For example, one may represent to oneself the earth as rotating on its axis in infinite empty space, and also verify this motion by experience, even though neither the relation of the earth's parts among one another, nor to the space outside it, is changed phoronomically, that is, in the appearance [*Erscheinung*]. For, with respect to the first, as empirical space, nothing changes its position on or within the earth; and, as regards the second, which is completely empty, no externally changed relation, and thus no appearance of a motion, can take place anywhere. (561)

Yet one may wonder, at this point, precisely how one is supposed to perform the required verification.

It is precisely here that Kant inserts his first example, using what we now call a Coriolis force or endeavor to explain small deviations from the centrally directed rectilinear descent of falling bodies:

> But if I represent to myself a deep hole descending to the center of the earth, and I let a stone fall into it, I find, however, that the falling stone deviates from its perpendicular direction continuously, and, in fact, from west to east, even though gravity, at all distances from the center of the earth, is always directed towards [this center], and I conclude, therefore, that the earth is rotating on its axis from west to east. (561; compare note 67 above)

As the stone descends perpendicularly into the hole under the influence of terrestrial gravity, one has also to take into account its horizontal velocity due to the earth's eastward rotation. The corresponding (linear) velocity of the earth's rotation, however, is greater at the surface than it is at any point further down in the hole. Hence, while the stone experiences a perpendicular rectilinear acceleration of fall, it also experiences a constant horizontal velocity equal to the (linear) velocity of the earth's rotation at the surface, and, as a result, the stone continuously deviates from rectilinear fall from west to east.[116]

It is so far unclear, however, how this example is supposed to illustrate the point at issue – that "even if it exhibits no [phoronomic] change of place in the appearance," a circular motion could nonetheless be determined as actual "in infinite empty space" (561). This is not a case, in particular, where "neither the relation of the earth's parts among one

[116] This continuous eastward acceleration during fall (due to the continuously changing relationship between successively lower tangential velocities of rotation within the earth) is an effect, as remarked, of what we now call a Coriolis force. Like centrifugal force, therefore, it is a manifestation of the tangential endeavor (a "striving to escape") due to rotational motion in accordance with the law of inertia – and it is thus also a "pseudo force" in the sense of note 92 above. Unlike centrifugal force, however, it is directed along the tangent rather than perpendicular to it – and, accordingly, it only affects bodies (like the falling stone) that also have some component of (rectilinear) motion perpendicular to the rotation. Kant appeals to such forces to explain the directional tendency of the trade winds in his *Theory of the Winds* of 1756 (see note 150 of my chapter on the Dynamics, together with the paragraph to which it is appended). A wind, according to Kant, arises from temperature differences in a given layer of equal pressure at a given height above the earth's surface. Hence, if we consider such a wind directed along a meridian away from the equator, for example, it will also acquire a continuous eastward acceleration due to the decrease in (tangential) velocities of rotation as we move away from the equator. In this case, therefore, we are concerned with a deviation of a motion (of the wind) from its otherwise horizontal trajectory (at a given distance from the surface of the earth) along a meridian, not a deviation from a vertical (downward) trajectory due to gravity. Nevertheless, gravity is involved in this example as well, since the concentric layers of air at constant pressures are maintained in equilibrium by precisely this force.

another, nor to the space outside it, is changed phoronomically, that is, in the appearance," and it is certainly not a case, more specifically, in which "nothing changes its position on or within the earth" (561). For the falling stone obviously changes its position within the earth, and observable deviations from pure rectilinear fall – consisting of continuous *phoronomic* changes of position from west to east – are precisely what then serve to verify the circular motion in question.

Moreover, the way in which Kant goes on to extend this example in the immediately following passage makes its divergence from the point it is supposed to exemplify even more apparent:

> Or, if I also remove the stone further [out] from the surface of the earth, and it does not remain over the same point of the surface, but moves away from it from east to west, then I will infer to the very same previously mentioned axial rotation of the earth, and both kinds of observation will be sufficient to prove the actuality of this motion. The change of relation to the external space (the starry heavens) does not suffice for this, since it is mere appearance [*Erscheinung*], which may proceed from two in fact opposing grounds, and is not a cognition derived from the explanatory ground of all appearances [*Erscheinungen*] of this change, that is, experience. (561)

Thus, if I throw the stone perpendicularly upwards (rather than drop it perpendicularly downwards), it will, in addition, move from east to west (rather than from west to east) relative to the earth's surface, since the (eastward) linear tangential velocity of rotation at the surface is now insufficient to maintain it above the same point of the surface as it rises. So not only do we here have, once again, an obvious phoronomic change of position relative to the surface of the earth, but Kant also suggests that he is not imagining the earth to be rotating in empty space after all. Rather, we begin from an *appearance* of (merely phoronomic) relative motion between the earth and the starry heavens, and the experimental verification in question is then supposed to inform us of the true "explanatory ground" of this appearance found only in what Kant calls *experience*.[117]

[117] This second verification also appears to be rather fanciful, since it was not possible to observe the deviation from east to west in question by means available in Kant's time. It is for this reason, in fact, that Alois Höfler, in his explanatory note in the Akademie edition (4, 651), insists that Kant likely intended to write "from west to east" here and, accordingly, that Kant had in mind a case of dropping the stone off a high tower (which would assimilate this case to the first case and constituted a possible – although still quite difficult – verification at the time) rather than throwing it upwards from the earth's surface. Nevertheless, I agree with Konstantin Pollok's lucid discussion of this issue in the notes to his edition of Kant's text (1997a, pp. 148–49) that Kant's original "from east to west" should be maintained here.

It is by no means surprising, therefore, that Kant turns to the example of the shape of the earth at precisely this point. What he now seems to be suggesting is that the truth or actuality of the rotational motion of the earth to which he has so far appealed (in the first part of the discussion) is in fact to be grounded in a different – *and entirely non-phoronomical* – representation:

> But that this [rotational] motion, even though it is no change of relation to the empirical space, is nevertheless not absolute motion, but rather a continuous change in the relations of matters to one another, which, although represented in absolute space, is thus actually only relative, and, for just this reason alone, it is true motion – this rests on the representation of the mutual and continuous *withdrawal* of any part of the earth (outside the axis) from any other part lying diametrically opposite to it at the same distance from the center. (561–62)

The withdrawal in question, as explained, results in a centrifugal bulging of the earth perpendicular to the axis of rotation, whereby "the reduction of the distance in question, which gravity by itself would induce in the body [i.e., the return to a perfect spherical shape that would occur if there were no rotation – MF] is thereby continuously made up, and, in fact, without any dynamical repulsive cause" (562; see note 95 above, together with the paragraph to which it is appended). So we now have a representation that illustrates the point at issue exactly. There is no phoronomic change of place either on or within (or above) the earth but rather a constant deviation of the earth from its otherwise spherical shape (in a state of oblate spheroidal rather than spherical equilibrium) that manifests "a continuous ***dynamical*** change, demonstrable through experience, in the relations of matter *within its space*" – namely, "a continual diminution of attraction in virtue of a striving to escape, as an action or effect of the circular motion" (561, bold emphasis added).

What Kant appears to be suggesting, therefore, is that he has now found a (dynamical) explanation or "ground" for the two different appearances of merely relative (phoronomical) motion considered in the first part of his discussion (involving the earth, the surrounding starry heavens, and a moving stone near the earth's surface). The dynamical ground in question is the representation of a bulging earth rotating in empty space, where a "mutual and continuous *withdrawal* of any part of the earth (outside the axis) from any other part lying diametrically opposite to it at the same distance from the center" (561–62) gives rise to an oblate spheroidal shape deviating from the perfectly spheroidal shape that would have resulted by the action of gravitational attraction alone. The crucial question, however, involves the precise nature and character of this "mutual and continuous *withdrawal*." How, in particular, is it supposed to illustrate Kant's central commitment to an "absolute space" nonetheless determined through merely "relative" motion?[118]

I shall approach these questions by asking how we know that the bulging of the earth does *not* proceed from "any dynamical repulsive cause" but rather from an "actual motion, which relates, however, to the space inside the moved matter (namely its center), and not to that outside it" (562; see again the paragraph to which note 95 above is appended). Kant takes this example, as explained in section 33 above, to be precisely analogous to Newton's example of the two balls connected by a cord, where the observed tension in the cord is similarly explained by the centrifugal effect of their mutual rotation rather than a repulsive force. So how do we know, in that case, that the mutual rotation is in fact the cause? Newton writes (P414; see the paragraph to which note 102 above is appended): "[I]f the cord was examined and its tension discovered to be the very one which the motion of the balls required, it would be valid to conclude that the motion belonged to the balls and that the bodies [playing the role of the fixed stars – MF] were at rest." And I have argued, in the remainder of section 33, that the analogue of this determination in the case of bodies connected by the invisible force of universal gravitation rather than a visible cord is, in effect, the entire argument of Book 3 of the *Principia*. Here Newton begins with the purely relative (phoronomical) motions of the bodies within the solar system against the background of the fixed stars, and he then arrives at a fundamentally dynamical determination of the true or actual motions – independently of the fixed stars – within his evolving theory of universal gravitation. So to say, in Kant's example, that the observed bulging of the earth is due to true rotational motion (instead of a repulsive force), is to say that the centrifugal endeavor corresponding to the observed *relative* rotational motion between the earth and the fixed stars leads to precisely the observed bulging in the context of the theory of universal gravitation.[119] We have now reached the point, therefore,

[118] This commitment is explicitly (if not entirely perspicuously) stated when Kant first introduces absolute space as an idea of reason (559–60; see again the paragraph to which note 69 above is appended): "an absolute space to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively mutual, but none as absolute motion or rest (where, while one is said to be moved, the other, in relation to which it is moved, is nonetheless represented as absolutely [*schlechtin*] at rest." The same idea recurs immediately before Kant introduces the "mutual and continuous *withdrawal*" in the passage quoted in the preceding paragraph (561–62).

[119] See, in particular, notes 108 and 109 above, together with the paragraphs to which they are appended.

where it is necessary to consider both Newton's treatment of this case in the argument of Book 3 and Kant's understanding of Newton's treatment in more detail.

It is of central importance, as I also suggested in section 33, that the problem of the shape of the earth represented the first great test case for truly *universal* gravitation in the eighteenth century, and that Kant understood this problem rather well (compare again note 99 above, together with the paragraph to which it is appended). In particular, Newton and Huygens had derived two different predictions for the precise amount of the earth's equatorial bulging, where both began from an initial hydrostatic equilibrium in which a perfectly spherical *non-rotating* earth maintains its shape by its internal gravity. Huygens, however, rejected universal gravitation between each part of the earth and every other part, and he instead assumed gravitation within the earth to be directed towards the center in accordance with uniform Galilean gravity. Newton, by contrast, maintained universal inverse-square gravitation between each part and every other part, and, on this basis, he derived the result that gravity within the earth is directed towards the center and decreases linearly with the distance as one approaches it.[120] But centrifugal force also decreases linearly with the distance as one moves towards the center of the earth. For Newton, therefore, gravitation exerts less counterbalancing influence against the centrifugal bulging than it does for Huygens, and, accordingly, there is more than twice as much centrifugal bulging at the equator than there is for Huygens. Whereas, for Huygens, the diameter at the equator

[120] This argument depends, as Newton makes clear in his discussion of the shape of the earth in Proposition 19 of Book 3, on Propositions 72 and 73 of Book 1. Proposition 73 reads (P593): "If toward each of the separate points of any given sphere there tend equal centripetal forces decreasing in the squared ratio of the distance from those points, I say that a corpuscle placed inside the sphere is attracted by a force proportional to the distance from the center of the sphere." This is then followed by Proposition 74, which states the corresponding result for a body outside the attracting sphere (P593): "With the same things being supposed as in prop. 73, I say that a corpuscle placed outside a sphere is attracted by a force inversely proportional to the square of the distance from the center of the sphere." It is essentially this proposition (together with the following Proposition 75) on which Newton had already relied in the previous Proposition 8 of Book 3 (see note 113 of my chapter on the Dynamics). Note that these two Propositions concern perfect spheres and assume that the distribution of matter within them is sufficiently uniform. In Corollary 3 of Proposition 91 of Book 1 Newton is indeed able to extend his result on attraction within a sphere to more general spheroids (see note 124 below), but he is never in a position to arrive at any conclusive results concerning the actual distribution of matter within the earth – a point to which I shall return below.

exceeds the diameter at the poles by 1/578 of the earth's axis, for Newton this excess is given rather by 1/230.[121]

That Kant was familiar with this fundamentally important difference between Huygens's treatment (based on uniform Galilean gravity) and Newton's (based on universal gravitation) is, as suggested, clear from the fifth chapter of the *Theory of the Heavens*. The subject of this chapter is not the shape of the earth but the origin of Saturn's rings and the rotation of Saturn around its axis. In the course of his discussion, however, Kant has occasion to be concerned with the amount of bulging of Saturn at the equator, and, in this connection, he comments on the problem of the shape of the earth. I here present the whole of Kant's comments (together with my own explanatory footnotes in brackets):

According to the Huygensian hypothesis, which assumes that the gravity in the interior of a planet is everywhere equal, the difference between [the polar and equatorial] diameters has a ratio to the diameter at the equator that is twice as small as that of centrifugal force [at the equator] to gravity under the poles. For example, since, in the case of the earth, the centrifugal force at the equator is 1/289 of gravity under the poles, so, according to the Huygensian hypothesis, the diameter of the equatorial surface must be 1/578 greater than the axis of the earth. The reason is this. Since gravity, on this assumption, is always as great in the interior of the earth's mass, at all distances from the center, as it is at the surface, but the centrifugal force decreases when approaching the center, the latter is not everywhere 1/289 of gravity; rather, the entire diminution of the weight of a fluid column in the equatorial surface, for this reason, does not amount to 1/289 but to one-half of this, i.e., 1/578.[[122]] By contrast, in the hypothesis of Newton, the centrifugal force, which excites the axial rotation,[[123]] has an equal ratio to

[121] See the classic discussion in Todhunter (1873), which concentrates on the mathematical development of the subject from Newton to Laplace. Huygens had taken his result (first obtained in 1690) as furnishing a potentially decisive empirical refutation of Newtonian universal gravitation – and, indeed, Huygens thought that voyages investigating the variation of surface gravity at different latitudes had already confirmed his view. A number of such voyages throughout the first half of the eighteenth century continued to investigate the problem, and, although no truly decisive evidence in favor of Newtonian universal gravitation was found (because the density below the surface is not in fact sufficiently uniform), they did eventually rule out Huygens's theory (which is independent of the distribution of density). I am especially indebted to George E. Smith for inspiring my interest in this subject and for helping me better to understand it.

[122] Newton had first introduced the device of two fluid columns joined at right angles at the center of the earth, along a pole and the equator respectively, in Proposition 19 of Book 3. Huygens then adopted this device to derive his opposing calculation in 1690. See Todhunter (1873, chapters I and II).

[123] Kant here says that the centrifugal force "excites" the axial rotation (rather than the other way around) because, according to the *Theory of the Heavens*, large massive bodies are originally formed by the action of gravitational attraction in an initially undifferentiated chaos of matter, and, as matter falls towards such an emerging center of attraction, it is deflected away from its rectilinear approach to the center by repulsive forces and thereby acquires a counterbalancing

gravity at every place in the whole spherical surface up to the center – because this force decreases in the interior of the planet (if it is assumed to be everywhere of uniform density) with the distance from the center in the same proportion as the centrifugal force; therefore, the latter is always 1/289 of the former. This produces a lightening of the fluid column in the equatorial surface and also its raising by an amount of 1/289; [but this] difference, in the [Newtonian] system in question, is further increased by the circumstance that the shortening of the axis [of rotation] entails an approach of the parts to the center, and therefore an increase in gravity, while the lengthening of the equatorial diameter entails a withdrawal of parts from precisely the same center, and therefore a diminution of gravity, and, for this reason, the flattening of the Newtonian spheroid is so increased that the difference of the diameters is raised from 1/289 to 1/230.[[124]] (1, 295–96)

Thus, there appears to be no doubt that Kant understood the issue between Huygens and Newton rather well.[125]

Now Kant unreservedly accepted Newtonian universal gravitation from the very beginning of his career. In the *Metaphysical Foundations* Kant argues that what he calls the "original" or "fundamental" force of attraction "extends in the universe from each part of matter to every other

centrifugal tendency due to rotation (compare note 59 of my chapter on the Dynamics). For Kant, therefore, the centrifugal tendency of rotation possessed by any heavenly body (together with the resulting oblate shape) is an essential feature of its cosmogenetic formation.

[124] The point Kant is making here involves Newton's extension of his results for (fluid) spheres to more general spheroids – as in §13 of Book 1, including Proposition 91 and its corollaries, to which Newton also appeals in Proposition 19 of Book 3. In the case of an ellipsoid, in particular, the gravitational attraction perpendicular to the axis of rotation is further diminished by the elongation of such a diameter independently of the centrifugal force, and combining these two effects together then results in a difference between the diameter at the equator and the axis of rotation that is significantly greater than would be produced by centrifugal force alone. For a discussion of Newton's reasoning see Todhunter (1873, §§19–28).

[125] In the notes to his translation of the *Theory of the Heavens* (Kant 1981, pp. 273–74n. 26) Stanley Jaki quotes a passage from Maupertuis (1744, pp. 22–24), in the first part of *Éléments de géographie*, and suggests that Kant is closely following this passage here. In truth, Kant's discussion at this point is considerably fuller than Maupertuis's. At the end, however, in connection with the issue discussed in note 124 above, the correspondence is rather close. Compare (using Jaki's translation) Maupertuis (1744, p. 24):

> By attributing gravity to the mutual attraction of all parts of the matter which forms the earth, according to the inverse square of their distances, Newton no longer viewed gravity as having to be the same everywhere. If the figure of the earth depended on gravity, gravity itself depended on the figure that the earth had; and the earth once being flattened by the centrifugal force, this figure alone made the gravity smaller at the equator than at the pole, independently of the centrifugal force. Newton did his calculation according to this subtle theory and found that the diameter of the equator had to exceed the axis of the earth by 1/230 part of its length.

It also appears, as Jaki further suggests, that Kant is depending on the second part of Maupertuis (1744), *Discours sur les differentes figures de corps celestes*, in his discussion of the oblate spheroidal shapes of other planets like Saturn.

part to infinity" (516) – and he bases this claim, in particular, on the fundamental force of attraction being both essential to all matter and acting immediately at a distance (512): "The *attraction essential to all matter* is an immediate action of it on others through empty space."[126] Kant also holds, finally, that the fundamental attraction is necessarily "a penetrating force, and for this reason alone it is always proportional to the quantity of matter" (516) – and that this is true, in turn, because "through true attraction *all parts* of the [one] matter act immediately *on all parts* of the other" (524, emphasis in the original).[127] In this sense, Kant's commitment to universal gravitation is considerably stronger than Newton's, and, accordingly, Kant shows relatively little interest in the empirical questions surrounding the issue between Huygens and Newton.[128] Nevertheless, the circumstance that he was quite familiar with this issue makes it especially clear, on my reading, that Kant is using the example of the shape of the earth to underscore the importance of truly universal (Newtonian) gravitation in the general remark to phenomenology.

I argued earlier in section 33 for the claim that, in invoking Newton's example of the two balls connected by a cord so prominently in his discussion of the rotation of the earth in the general remark, Kant also intends to invoke the main steps in Newton's argument for universal gravitation. Kant appears to have clearly in mind, in particular, not only Newton's discussion of the shape of the earth in Propositions 18, 19, and 20 of Book 3 but also the earlier argument for universal gravitation in Propositions 1–14, where this argument culminates, as explained, in an "a priori" determination, on the basis of precisely the theory of universal gravitation, of the principal states of true *orbital* rotation in the solar system independently of the fixed stars.[129] Newton's discussion of the true *axial* rotation of

[126] See note 163 of my chapter on the Dynamics, together with the paragraph to which it is appended.

[127] As explained in section 28 above, the most important reason for the strength of Kant's commitment to universal gravitation – as already built in to the (empirical) concept of matter he articulates – is that he takes this to be an essential part of a constructive explanation of the Newtonian mathematization of the (universal) concept of mass or quantity of matter. I shall return to this point below.

[128] Compare again note 121 above. Such lack of interest in the empirical question is evident even in 1755. Throughout his discussion in the *Theory of the Heavens*, for example, Kant simply assumes that the Newtonian – as opposed to the Huygensian – theory is correct. Indeed, far from using the occasion to discuss the empirical basis for the Newtonian theory, Kant's main point is that we can here *presuppose* the Newtonian theory and then argue, in cases where it does not appear to hold, that the densities of the planets in question (here Saturn and Jupiter) are not sufficiently uniform. I shall return to the place of this empirical question in Newton's argument below.

[129] For my detailed discussion of both Newton's and Kant's perspective on such determination in section 33 see notes 101–9 above, together with the paragraphs to which they are appended.

the earth in Propositions 18, 19, and 20 represents a continuation of this argument, where the earth's true rotation is determined independently of the fixed stars in the same way.[130] Kant's discussion of this rotation in the general remark explicitly refers to Newton's discussion of the rotating balls in both the main text and the corresponding footnote (561–62), where Kant also explicitly draws a parallel between orbital and axial rotation (562; see note 95 above, together with the paragraph to which it is appended). He explicitly draws a parallel, as well, between the tension of the cord in Newton's example and gravity in the immediately following discussion of the third proposition (562; see note 100 above, together with the paragraph to which it is appended). That, in the *Theory of the Heavens*, Kant describes the crucial differences between Huygensian (terrestrial) and Newtonian (universal) gravitation rather precisely, serves to highlight the clear indications in the text of the general remark to phenomenology in the *Metaphysical Foundations* and, on my reading, to make the central importance of (Newtonian) *universal* gravitation there unmistakable.

If this is correct, however, then what is the role of the example of determining the earth's true rotation via Coriolis forces that Kant discusses at the beginning of his discussion of the earth's rotation? This example, as observed, does not strictly conform to the general conditions Kant sets out for determining the rotation of the earth purely dynamically – independently of *any* (merely) phoronomical change (see the paragraph to which note 117 above is appended, together with the preceding paragraph). But it is now clear, in addition, that it involves only the rectilinear acceleration of fall due to terrestrial gravity and does not yet invoke truly universal (Newtonian) gravitation. All that matters, in particular, is that the falling stone acquires a tangential component of (uniform) rectilinear motion perpendicular to the direction of fall in accordance with the law of inertia, which then combines with the (accelerated) rectilinear motion of fall

[130] As also emphasized in section 33, the logic of Newton's determination of the states of true orbital rotation in the solar system has an essentially open-ended character and rests, in the end, on nothing more nor less than the confidant hope that all perturbations will continue successfully to be accommodated within his evolving theory (see note 107 above, together with the paragraph to which it is appended and the preceding paragraph). The same holds for the determination of the true axial rotation of the earth in so far as the actual distribution of mass within the earth remains unknown (see notes 120, 121, and 128 above, together with the paragraphs to which they are appended). Indeed, the problem of the exact shape of the rotating earth turns out to be far more difficult than that of the planetary perturbations. While the latter were largely successfully accommodated within Newton's theory by the end of the nineteenth century (with the exception of extremely small effects requiring correction by general relativity), the exact distribution of mass density within the earth remains unknown today. I am here indebted, once again, to George E. Smith.

to produce a deviation from the expected straight trajectory towards the center of the earth. In this context, moreover, we have no reason to take account of the (very small) acceleration of the earth towards the stone as well. In the context of truly universal gravitation, however, we do need to take account of precisely this mutually counterbalancing acceleration. So Kant, in the first part of his discussion, is entertaining a preliminary stage in the consideration of a rotating earth before the full theory of universal gravitation is invoked in the discussion of its bulging shape.

The point of the first part of Kant's discussion, I suggest, is to serve as a counterpart for the transition between terrestrial gravity and universal gravitation in the argument of Book 3 of the *Principia*. That Kant begins his procedure for reducing all motion and rest to absolute space from our initial perspective on an (apparently) stationary earth and then determines the earth's rotation, in the first part of his discussion, by means of terrestrial gravity alone, suggests that Kant here has in mind primarily the moon test of Propositions 3 and 4 of Book 3. For in Proposition 3 Newton infers the existence of an inverse-square force responsible for the moon's orbit directed towards the center of the earth.[131] And in Proposition 4 he identifies this force with terrestrial gravity by considering the inverse-square proportion "brought down" to the surface of the earth. It is precisely at this point, therefore, that Newton first establishes a link between terrestrial and celestial gravity – which then leads, by the argument of Propositions 5, 6, and 7, to truly universal (and everywhere mutual) gravitation between each piece of matter and every other.

The suggestion that Kant here has in mind the moon test allows us to begin to make sense of the example of a stone thrown vertically upward in this part of his discussion. Whereas the example appears rather fanciful as an experimental verification of the earth's true rotation (compare note 117 above), it makes more sense as an allusion to the moon test – and, in particular, to what I called the "inverse moon test" Newton describes in his *System of the World*.[132] Newton there imagines a stone that is projected

[131] Newton infers a centripetal force directed towards the center of the earth from Phenomenon 6 (801): "The moon, by a radius drawn to the center of the earth, describes areas proportional to times." So Newton is here invoking Proposition 2 and 3 of Book 1, where a centripetal force is derived from Kepler's area law in accordance with the law of inertia. The inverse-square proportion, however, is derived from the very slow motion of the moon's apogee in accordance with Proposition 45 of Book 1: see note 107 above, and recall that this motion of the moon's apogee was not yet firmly established at the time and Newton does not list it among his Phenomena. At the end of his derivation of the inverse-square proportion in Proposition 3 of Book 3, Newton adds (803): "[T]his will be even more fully established by comparing this force with the force of gravity as is done in prop. 4 below."

[132] See note 124 of my chapter on the Dynamics, together with the paragraph to which it is appended.

from the top of a high mountain with progressively greater horizontal velocities until it finally orbits the earth as a heavenly body (the nearest of which, of course, is the moon). Kant here imagines a stone that is projected vertically upward, which, however, acquires a horizontal motion from west to east in virtue of precisely the earth's rotation. And, since the orbital (as opposed to diurnal) motion of the moon relative to the earth (as for all other heavenly bodies) is also from west to east, the example appears to be also well suited (from Kant's point of view) to suggest the inverse moon test here.[133]

Admittedly, if this is all there were to Kant's example, the suggestion of an allusion to the moon test might easily appear to be forced.[134] As explained, however, Kant had earlier appealed to the example of a stone in the earth's gravitational field while alluding to the inverse moon test in the remark to the fifth proposition of the Dynamics.[135] The context of that remark was Proposition 8 of Book 3, where Newton shows that the attraction of any planet on a body external to it is precisely the same (if the planet is perfectly spherical and its density is sufficiently uniform) as if all the mass of the planet were concentrated at its central point. We thereby provide important additional support for the identification of celestial and terrestrial gravity via the moon test. For we can now – and only now – conclude that *both* terrestrial and celestial gravity are, at the surface of the earth, necessarily directed precisely towards its center.[136] But Proposition 8 of Book 3 depends on Propositions 74 and 75 of Book 1, just as Proposition 19 of Book 3 (which is central to Newton's treatment of the centrifugal bulging of the earth) depends on Propositions 72 and 73 of Book 1 (see note 120 above). The former says that the attraction of a (sufficiently uniform) sphere on an *external* body is directed towards the center in inverse proportion to the square of the distance from that center, the latter that the attraction of a (sufficiently uniform) sphere on an *internal*

[133] To be sure, the stone acquires a contrary motion from east to west due to Coriolis force as well. Yet this westward motion is quite negligible in comparison with its eastward motion, and, for precisely this reason, it is better to take this (second) example as a consequence of an already established rotating earth rather than a verification of it.

[134] For one thing, Newton's description of his inverse moon test essentially involves a horizontal projection from the top of a mountain rather than a vertical projection directly upwards. And a mere upward projection is in fact insufficient for attaining an orbit, no matter how fast the (vertical) velocity. Nevertheless, as explained in note 140 below, Kant himself still has good reasons for closely associating the rotational motion of any planet (including the earth) with the orbital motion of its satellites.

[135] See again note 124 of my chapter on the Dynamics, together with the paragraph to which it is appended and the remainder of section 15.

[136] Compare again note 113 of my chapter on the Dynamics, together with the paragraph to which it is appended.

body is directed towards the center in direct proportion to the distance from the same center.[137] So it is an essential property of Newtonian universal gravitation that it acts quite differently within the earth and above the earth's surface. It is directly proportional to the distance from the center up to the surface but in the inverse-square proportion to this distance from every point on the surface all the way out to the most distant celestial bodies.[138]

In light of everything that we have seen about the wider context of Kant's treatment of the shape of the earth, therefore, it now appears reasonable to assume that he has this essential property of Newtonian universal gravitation in mind throughout his discussion of the true rotation of the earth in the general remark. And, on this assumption, the point of considering both a stone dropped vertically downward into the earth and a stone projected vertically upward from the earth's surface in the first part of his discussion then comes clearly into focus. The stone dropped vertically downward can be thought of as probing the earth's internal

[137] Newton's argument for this last proportionality is especially straightforward. Proposition 70 has already shown that an internal particle is not attracted at all by the matter outside the spherical surface in which it lies; hence, the attraction on this particle is compounded out of the inverse-square attractions of all the particles *within* the sphere bounded by the surface in question. The attraction (according to Proposition 72) is therefore directed towards the center in a ratio inversely proportional to the square of the distance and directly proportional to the sum of the particles themselves – which, in turn, is directly proportional (as a volume) to the cube of the distance. In a footnote to the fifth chapter of Part Two of the *Theory of the Heavens* Kant exhibits a good understanding of the basis for this argument (1, 296):

> For, according to the Newtonian law of attraction, a body that is found in the interior of a sphere is only attracted by that part of it comprised within a sphere described at the distance of the body from the center. The concentric part found beyond this distance, because of the equilibrium of its attractions, which annul one another, does nothing to move the body either towards or away from the central point.

It is also worth noting, that, while Newton does extend his result on *internal* attraction to more general spheroids (in accordance with Corollary 3 to Proposition 91: see notes 120 and 124 above), the inverse-square proportion from the center (by Corollary 2 to this same proposition) cannot be extended in the same way in the case of *external* attraction above the surface.

[138] If, on the Huygensian approach, we assume uniform (Galilean) gravity within the earth, then we also have a difference between this case and the celestial case – for, in light of the moon test and the calculations of the *Principia*, Huygens was willing to accept the inverse-square law governing all cases of celestial gravity (including all gravitational effects from the earth's surface up to the orbit of the moon). What is essential to Huygens's approach is only that truly universal gravitation (especially between each part of matter and every other part) be definitively rejected. Accordingly, Huygens also considers the case where one assumes inverse-square gravitation (in proportion to the distance *from the center*) both outside and inside the surface of the earth. In this case, however, one obtains almost exactly the same result as on the assumption of uniform (Galilean) gravity below the surface: namely, that the excess of the equatorial over the polar diameter is given by 1/579 (rather than 1/578): see Todhunter (1873, §§56, 57). I am once again indebted to George E. Smith for this point.

gravitational field, while the stone projected vertically upward probes the earth's external gravitational field.[139] Moreover, since the stone projected vertically upward – by the moon test and Proposition 8 of Book 3 – can be thought of as probing the earth's external (inverse-square) gravitational field from its surface all the way up to the moon, this provides significant confirmation, on my reading, for the suggestion that Kant is alluding to both the moon test and Proposition 8 precisely here.[140]

We now have all that we need to confirm the suggestion that Kant is alluding to the other key steps in Newton's argument for universal gravitation as well (compare note 128 above, together with the paragraph to which it is appended and the following paragraph). For the argument of the moon test in Propositions 3 and 4 of Book 3 immediately leads, in Propositions 5 and 6, to the result that all bodies gravitate towards every planet and thereby have weights towards each planet that are directly proportional (at a given distance) to their quantities of matter.[141] Proposition 7 then uses this result, together with the Third Law of Motion, to show that the accelerations thereby produced in such surrounding bodies are also directly proportional (at a given distance) to the quantities of matter of the planets themselves.[142] The Corollaries to Proposition 8 put all of these earlier results together (including Propositions 1 and 2, which derive inverse-square centripetal forces around Jupiter, Saturn, and the sun) in developing a procedure for determining the relative quantities of matter

[139] By contrast, if all that Kant were interested in is an empirical verification of the earth's rotation via Coriolis forces, it would appear to be entirely mysterious why he envisions dropping a stone into a deep hole descending to the center of the earth – rather than, say, dropping such a stone off a high tower above the surface. For, as Höfler points out in his notes to the Akademie edition (note 117 above), only a downward motion from a high tower could yield a practicable verification at the time – which experiments of 1792, 1802, and 1831 then appeared to confirmed.

[140] The fourth chapter of Part Two of the *Theory of the Heavens* considers the origin of moons and the motion of planets around their axes. The key point is given in its first sentence (1, 283): "The striving of a planet to form itself from the [surrounding] sphere of elementary matter is, as the same time, the cause of its axial rotation[, and it also] generates the moons that may orbit around it." The idea is that elementary matter falling towards a center of attraction acquires a rotational motion as it falls, which is thereby communicated to both the resulting planet and any moons that may form around it (compare note 123 above). There is thus a systematic relationship, for Kant, between the (axial) velocity of rotation of a planet and the orbital velocities of its surrounding moons.

[141] Compare again note 181 of my chapter on the Dynamics, together with the paragraph to which it is appended. As explained, this result means (in the terminology derived from Stein) that every planet is surrounded by an inverse-square acceleration field acting on all the bodies gravitating towards it.

[142] Compare again note 192 of my chapter on the Dynamics, together with the paragraph to which it is appended. As explained, whereas the earlier result (Proposition 6) shows, in modern terms, that inertial mass is equal to passive gravitational mass, the present result (Proposition 7) shows that inertial mass is equal to active gravitational mass.

of the earth, the sun, Jupiter, and Saturn from the accelerations of their respective satellites. This procedure culminates with the determination of the center of gravity of the solar system – which, according to Proposition 12, is always very close to the center of the sun.[143] Thus, contrary to first appearances, both the moon test and the determination of the center of gravity of the solar system do indeed seem to be (implicitly) present in Kant's discussion of his procedure for reducing all motion and rest to absolute space in the general remark to phenomenology.

The relationship between the two parts of Kant's discussion of determining the true rotation of the earth can then be understood more precisely as follows. The first part, focussed on the deviations from rectilinear fall produced by Coriolis forces, corresponds to Propositions 3 and 4 of Book 3, where truly universal gravitation is not yet in place. But the second part, focussed on deviations from perfect sphericity produced by centrifugal forces, corresponds to Propositions 18, 19, and 20, where truly universal gravitation is already in place. Indeed, we are here involved with universal gravitation in its strongest form, acting between each *part* of any body and every *part* of any other – precisely the aspect of Newton's theory that Huygens was most concerned to reject (see note 138 above). From the perspective on the earth's rotation achieved in the second part of Kant's discussion, moreover, the first part of his discussion now needs to be corrected in an essential way. For we can no longer ignore the clear implication of truly universal gravitation that the rectilinear accelerations considered in the first part need to be supplemented by contrary accelerations of the earth. Just as the earth attracts the stone, inducing a downward acceleration of terrestrial gravity, the stone must similarly attract the earth, inducing a corresponding (but much smaller) upward acceleration in it. Similarly, when the stone is projected vertically upward, a corresponding (but much smaller) opposite acceleration of the earth is also produced, and, as the earth and stone slow down and then accelerate towards one another once again (due to their mutual gravitational attraction), they finally return to an equilibrium position around their common center of gravity.

In the description of the stone in the first part of Kant's discussion, by contrast, neither its falling motion nor its rising motion (both under the

[143] Compare again note 119 of my chapter on the Dynamics, together with the paragraph to which it is appended. As explained, it is in precisely this way that Kant applies his Copernican conception of space and motion to all the motions in the solar system – and this then represents the first step, for Kant, in an extension of this conception far beyond the solar system following the cosmological conception of the *Theory of the Heavens*.

influence of *terrestrial* gravity) is counterbalanced by any other motion – nor, for that matter, are the eastward and westward tangential motions manifesting Coriolis accelerations. In particular, although a stone falling into or rising above a rotating earth certainly satisfies the law of inertia (for its horizontal component is conserved in accordance with precisely this law while its vertical component represents the acceleration of terrestrial gravity), it does not yet (as described so far) satisfy the equality of action and reaction. For in order to satisfy this law we need also to take into account the counterbalancing (but much smaller) accelerations of the earth itself – and thus move, in effect, to the center of mass frame of the earth–stone system. Hence, although the motions in the first part of the discussion can indeed indicate the true rotation of the earth in accordance with Kant's second proposition, they do not yet provide us with an "absolute space to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively-mutual" (559), in accordance with Kant's third proposition.[144] It is for precisely this reason, therefore, that we do not yet have an "explanatory ground" (561) of these motions – which "unites all appearances" (560).

It is no wonder, then, that Kant immediately turns to the equality of action and reaction, and thus to his third proposition, in the following paragraph of the general remark. He suggests that we begin with the determination of states of true rotation in accordance with his second proposition, and thus with "active dynamical influences, given through experience, [as] are required in the second case (gravity or a tensed cord)" (562; compare the paragraph to which note 100 above is appended). Given any such "active dynamical influences" (attractive or repulsive), we can then proceed to infer corresponding equal and opposite (rectilinear) motions of the two bodies (or parts of bodies) thereby interacting with one another.[145] For example, if one body is rotating relative to another

[144] See notes 113 and 114 above (including the references there to earlier discussions), together with the paragraphs to which they are appended.

[145] It is important to bear in mind, in this context, that a community of *motions*, for Kant, presupposes a community of the original (dynamical) *forces* that produce these motions – a point that is re-emphasized at the beginning of his proof of the third proposition (558): "According to the Third Law of Mechanics (Proposition 4), the communication of motion of bodies is possible only by the community of their original moving forces, and the latter only by mutually opposite and equal motion." See the full text of Kant's proof quoted in note 110 above (together with the paragraph to which this note is appended). The statement of Kant's Third Law reads (558): "In every motion of a body, whereby it is moving relative to another, an opposite and equal motion of the latter is *necessary*." The qualification in the second clause – "whereby it is *moving* relative

under the influence of gravitational forces (e.g., the moon relative to the earth or the earth relative to the sun), it follows that the second body must be rotating relative to the first body as well (the earth relative to the moon and the sun relative to the earth). Any two such bodies, more generally, must be mutually rotating around their common center of gravity and thus mutually gravitating towards one another. Similarly, in the case of the rotating earth assuming an oblate spheroidal shape under the influence of both its centrifugal endeavor of rotation and the universal (Newtonian) gravitational attraction among all of its parts, each pair of parts lying opposite to one another along a diameter mutually rotate around the center of the earth and mutually gravitate towards this same center.[146]

I am now in a position to explain how Kant's procedure for reducing all motion and rest to absolute space "unites all appearances" (560) in the solar system. The appearances with which we begin are those apparent motions observable from a provisionally stable earth, but, as suggested, we could just as well begin by starting with any primary body in the solar system that, like the earth, has satellites of its own. We can therefore begin, in particular, with the Phenomena initiating Book 3 of the *Principia* describing the relative motions of these satellites with respect to the sun, Jupiter, Saturn, and the earth (compare note 73 above, together with the

to another" (emphasis added) – indicates that the (first) body in question has a *causal* relation to the other body: it *imparts* a motion to this (second) body by an original (dynamical) moving force.

[146] Assuming a uniform distribution of matter within the earth, this description, using the observed (relative) rotation between the earth and the fixed stars to compute the centrifugal endeavor, then results in Newton's calculations of the magnitude of centrifugal bulging – which, I observed, Kant appears to take entirely for granted (see note 128 above). It is instructive to return, in the present context, to the question raised in note 96 above concerning the two different frames of reference – inertial and rotating – in which this centrifugal endeavor may be described. Which, in particular, is Kant considering in the passage concerning the "mutual and continuous *withdrawal*" (561) of diametrically opposed parts of the earth already quoted in the paragraph to which note 96 above is appended? It appears that he may have both in mind. He is clearly considering what we now call an inertial frame in his second proposition on determining true rotation – which speaks of a "striving ... to proceed in the straight line tangent to the circle" (557; see the paragraph to which note 86 above is appended). However, the language of "mutual and continuous *withdrawal*" appears to be more in keeping with the perspective of a rotating frame – in which there are outwardly directed centrifugal accelerations (perpendicular to the tangential "strivings" in the inertial frame) between the diametrically opposed parts of the earth. Yet Kant's language is also quite consistent with our modern understanding, according to which these accelerations are the effect of what we now call a "pseudo force." For he insists that the outward accelerations in question are the effect of "actual motion" rather than "any dynamical repulsive cause" (562), and so he would certainly not apply the equality of action and reaction to them (see note 145 above). I shall return to the precise relationship between Kant's procedure for reducing all motion and rest to absolute space and our modern conception of an inertial reference frame in the technical note below.

paragraph to which it is appended). However, there are two essentially different types of appearances generated by taking any of these primary bodies to be provisionally at rest: those connected with their axial rotations and those connected with their orbital rotations. The former turn out to be exemplary of mere *semblance*, while the latter are associated with provisionally acceptable alternative judgements among which we then have a free choice.[147] In particular, we have a free choice whether we then continue our procedure from the point of view of the sun, Jupiter, Saturn, or the earth.

Taking the fixed stars to be at rest (as Newton does in Phenomena 1, 2, and 4), and taking the remaining primary planets (aside from the earth) to be orbiting the sun rather than the earth (as Newton does in Phenomena 3 and 5), we can now run the argument of Book 3 on each of the sun, Jupiter, Saturn, and the earth. In particular, a version of the moon test applies to each of these bodies, showing that gravity all the way up from their surfaces to their satellites generates an inverse-square "acceleration field" throughout the region in question (compare again note 141 above, together with the paragraph to which it is appended).[148] Then, just

[147] Compare the discussion from the paragraph to which note 73 above is appended through the remainder of section 32. The two cases of mere semblance considered there are the daily rotation of the heavens around the earth (as opposed to the contrary daily rotation of the earth itself) and the retrograde motions of the planets around the earth (as opposed to their non-retrograde motions around the sun). In deciding that these two cases correspond to mere semblance we arrive at disjunctive judgements that positively exclude their contraries in accordance with the law of inertia. By contrast, in the case of the remaining appearances associated with alternative judgements (the earth revolves around the sun or vice versa, Jupiter revolves around the sun or vice versa, and so on), it eventually turns out, in accordance with the equality of action and reaction, that the two bodies in question mutually revolve around a common center of gravity. In these cases, therefore, no alternative judgement with which we may (arbitrarily) begin can positively exclude its contrary. We thus obtain distributive rather than disjunctive judgements in these cases. We should bear in mind, however, that any axial rotation of a body (like the rotating earth) necessarily involves orbital rotations of corresponding *parts* of the same body (see again note 91 above, together with the paragraph to which it is appended), and the equality of action and reaction therefore applies to these parts as well – resulting in associated distributive judgements concerning *them*.

[148] In his brief Introduction to the *Theory of the Heavens* Kant takes it to be evident that the main result of the moon test applies to all of the planets (as well as the sun). After explaining Kepler's third law (and the derivation of the inverse-square law from it) in the case of the planets with respect to the sun, Kant continues (1, 244–45):

> Precisely the same law that holds among the planets, in so far as they orbit around the sun, is also found in the case of the smaller systems, namely, those consisting of the moons moved around their primary planets. Their periodic times are proportioned to the distance in precisely the same way [in accordance with Kepler's third law – MF] and establish precisely the same relation of the falling force with respect to the planets as that which governs the force of the planets with respect to the sun [namely the inverse-square law – MF]. All of this is established, in every case without contradiction, by means of infallible geometry from uncontested

as we do starting from the earth, we can run the rest of the argument of Book 3 up to the determination of the center of mass of the solar system in Proposition 12. A crucial step, in each case, is the application of the equality of action and reaction throughout the system – and, in particular, between any body with which we may happen to begin and the sun. For it is in precisely this way that we move from any given alternative judgement serving as our (arbitrary) starting point, in accordance with Kant's first proposition, to a fully distributive (or alternatively-mutual) judgement serving as our (unique and determinate) end point, in accordance with Kant's third proposition. It is in precisely this way, accordingly, that we finally arrive at (a good approximation to) what Kant calls "absolute space."

To unite all appearances, in this sense, is thus to unify all these different alternative judgements by means of a unique distributive judgement, which embeds all the correspondingly different appearances of motion within a single and unique description relative to the common center of gravity of the entire system. Kant's procedure for reducing all motion and rest to absolute space in the Phenomenology thereby provides an important illustration of the way in which, in the first *Critique*, he takes "the unity of the cosmos [*Weltganzen*]" to be primarily effected by the third analogy:

> The unity of the cosmos, in which all appearances are to be connected, is obviously a mere consequence of the principle of the assumed community of all substances that are simultaneous; for, were they isolated, they would not, as parts, constitute a whole, and, were their connection (interaction of the manifold) not already necessary for the sake of their simultaneity, one could not infer from the latter, as a mere ideal relation, to the former, as a real [relation]. We have shown, however, that community is properly the ground for an empirical cognition of coexistence, and, properly speaking, one only infers back from the latter to the former, as its condition. (A218/B265n.)

Thus, what we might call the spatial aspect of the unity of time – the unity of all *simultaneous* appearances in time – is a product of the third analogy of experience, and this ideal relation of (spatio-)temporal simultaneity is a product of real relations of interaction between the coexistent

observations. We then have the idea, in addition, that this falling force is just the same impetus that is called gravity on the surface of the planet, and which decreases gradually [outward] from there in accordance with the law in question. One perceives this from the comparison of the quantity of gravity on the surface of the earth with the force driving the moon towards the center of its orbit, which relate to one another precisely as attraction in the entire cosmic system [*Weltgebäude*] – namely, in inverse proportion to the square of the distance. This is the reason one also calls this often-appearing central force gravity.

(phenomenal) substances in question.[149] In the *Metaphysical Foundations* Kant then instantiates this conception concretely via the equality of action and reaction and the force of universal gravitation.[150]

At the end of his discussion of absolute space and absolute motion in the Phenomenology Kant extends his conception far beyond the solar system to embrace the cosmos as a whole. In this striking passage, which I have already quoted in full in section 32 above (see the paragraph to which note 51 above is appended), Kant asserts that only the rectilinear motion of the cosmos [*Weltganzen*] would be a truly absolute motion. As a result, "any proof of a law of motion, which amounts to showing that its opposite would have to result in a rectilinear motion of the entire cosmic system [*Weltgebäude*], is an apodictic proof of its truth" (562–63). Moreover, "the law of *antagonism* in all community of matter through motion" is just such a law (563): "For any deviation from it would shift the common center of gravity of all matter, and thus the entire cosmic system, from its place – which would not happen, by contrast, if one wanted to imagine

[149] Kant emphasizes the spatial aspects of community and simultaneity more strongly in the second edition, where the principle of the third analogy is reformulated as follows (B256): "All substances, in so far as they can be perceived as simultaneous in space, are in thoroughgoing interaction." (In the first edition we have [A211]: "All substances, in so far as they are simultaneous, stand in thoroughgoing community.") Similarly, in the general remark to the system of principles added to the second edition, after explaining that substances thought according to the pure understanding alone (in the manner of Leibniz) can only be thought in community by the mediation of God, Kant continues (B293): "But we can make the possibility of community (of substances as appearances) conceivable very well, if we represent them to ourselves in space, and therefore in outer intuition. For the latter already contains within itself a priori formal outer relations as conditions of the possibility of real [relations] (in action and reaction, and thus community)." For the full text of the relevant passage see the paragraph to which note 33 of the Introduction is appended.

[150] It is no accident, therefore, that, in the new first paragraph of the proof of the principle of the third analogy added to the second edition of the *Critique*, Kant uses the example of precisely the earth-moon system to illustrate his point (B257–58):

> Thus, I can direct my perception first to the earth and then to the moon, and, because the perceptions of these objects can mutually [*wechselseitig*] follow one another, I say that they exist simultaneously. Now simultaneity is the existence of the manifold in the same time. But one cannot perceive time itself in order to conclude that the things are posited in the same time … Therefore, a concept of the understanding of the mutual succession of determinations of these things simultaneously existing externally to one another is required, in order thereby to represent the simultaneity objectively. But now the relation of substances, in which the one contains determinations whose ground is in the determinations of the other, is the relation of influence, and, if the former mutually contains the ground of the determinations of the other, the relation of community or interaction [*Weschselwirkung*]. Therefore, the simultaneity of substances in space can be cognized in no other way in experience, except under the presupposition of an interaction of [these substances] among one another; thus this is also the condition of the possibility of the things themselves as objects of experience.

Compare the paragraph to which note 118 of my chapter on the Mechanics is appended, together with the references to earlier discussion in the note itself.

this system as rotating on its axis." Thus here, in particular, Kant finally makes clear that his procedure for reducing all motion and rest to absolute space depends on the Copernican conception of space and motion deriving from his *Theory of the Heavens* (compare note 112 above, together with the paragraph to which it is appended).

Kant's remarks on the "law of antagonism" are particularly interesting. This, as we know, is the Third Law of Mechanics, which Kant has already demonstrated in the fourth proposition of the Mechanics (compare the paragraph to which note 55 above is appended). Here, however, Kant proposes an entirely new demonstration based on the idea that any deviation from this law "would shift the common center of gravity of all matter, and thus the entire cosmic system, from its place" (563). And, since Kant's earlier demonstration in the Mechanics focusses on impact or collisions, his two different demonstrations (in the Mechanics and Phenomenology) thereby correspond to Newton's two demonstrations of the equality of action and reaction in the Scholium to the Laws of Motion in the *Principia*. For, while Newton's first demonstration (P424–27) concerns collisions, the second concerns attractions:

> I demonstrate the third law of motion for attractions briefly as follows. Suppose that between any two bodies A and B that attract each other any obstacle is interposed so as to impede their coming together. If one body A is more attracted toward the other body B than that other body B is attracted toward the first body A, then the obstacle will be more strongly pressed by body A than by body B and accordingly will not remain in equilibrium. The stronger pressure will prevail and will make the system of the two bodies and the obstacle move straight forward in the direction from A toward B and, in empty space, go on indefinitely with a motion that is always accelerated, which is absurd and contrary to the first law of motion. For according to the first law, the system will have to persevere in its state of resting or of moving uniformly straight forward, and accordingly the bodies will urge the obstacle equally and on that account will be equally attracted to each other. (P427–28)

In the second edition, moreover, Newton extends this argument to show that "gravity is mutual between the earth and its parts" (P428): if we divide the earth into any two parts, their "weights" (gravitational forces) towards one another must be equal, for, "if these weights were not equal, the whole earth, floating in an aether free of resistance, would yield to the greater weight and in receding from it would go off indefinitely."

That Kant, in the present passage, is now proposing a demonstration modeled on Newton's second case further underscores a point made at the beginning of this section. Whereas in the Mechanics Kant focusses primarily on the communication of motion "by means of repulsive forces, and thus by pressure … or impact" (537), here in the Phenomenology the focus is squarely on attraction and the force of universal gravitation (see again the paragraph to which note 112 above is appended). There is an important difference, however, between the demonstration that Kant suggests here and Newton's: there is no reference in Kant to an obstacle equally pressed on both sides. Newton, in effect, is assuming the equality of action and reaction in cases of static equilibrium, and, on this basis, he is extending this same principle to not necessarily contiguous dynamical attractions as well.[151] Kant, by contrast, is proposing that the (non-rotating) common center of mass frame of a system of interacting bodies is a suitable representative of absolute space for the purpose of describing any interactions among them. The equality of action and reaction, in this frame, necessarily holds by construction, and, from Kant's point of view, it can therefore experience no rectilinear motion at all – and certainly no *accelerated* motion – except on the basis of one or another interaction with matter outside the system.

In the case of the solar system, for example, all gravitational interactions among its constituent bodies necessarily satisfy the equality of action and reaction relative to its common center of gravity, and these bodies are all maintained in their states of (true) rotational motion around this center – and thus in a kind of equilibrium – by the way in which gravitational forces are thereby balanced by centrifugal endeavors. So the only way in which the solar system can itself move (rectilinearly), from Kant's point of view, is if it is attracted, in turn, by matter outside it. Similarly, if the earth were rotating around its axis in an otherwise empty space, it would still maintain itself in a state of oblate spheroidal rather than perfectly spheroidal equilibrium (between gravitational attraction and expansive pressure). For we now have a new expansive pressure (combined with both its original expansive pressure and its internal gravitation) due to precisely its centrifugal rotational endeavor.[152] Absolutely no other motion of the earth – and certainly no *accelerated* motion – would then be possible.

[151] I owe this way of putting the matter to George E. Smith, who has suggested that Newton is here making an inductive extrapolation from the already well-known case of contiguous static equilibrium to the at the time much more controversial case of dynamical equilibrium (conservation of momentum) in non-contiguous attractions at a distance.

[152] The original (hydrostatic) pressure of the fluid earth is exerted equally and symmetrically in all directions proceeding (diametrically) from the center towards the surface, and this pressure is balanced by the equal and symmetrical centripetal accelerations produced by universal gravitation. But the new expansive pressure due to the earth's rotation is not equal and symmetrical in this sense but rather picks out a preferred axis (of rotation). It thereby results in a bulging at the equator and a flattening at the poles.

But the earth is not rotating in an otherwise empty space, and neither, for that matter, is the solar system. Indeed, it is essential to the cosmological conception of the *Theory of the Heavens* (and, accordingly, to Kant's Copernican conception space and motion as well) that the solar system is only the first system in an indefinitely extended sequence of concentric galactic structures. The primary bodies in the solar system rotate around their common center of gravity; the solar system rotates along with the other inhabitants of the Milky Way galaxy around the latter's common center of gravity; this galaxy rotates around the common center of gravity of a larger system of such galaxies; and so on *ad infinitum*. This is why Kant speaks here of the cosmos [*Weltganzen*] or cosmic system [*Weltgebäude*], and, accordingly, of a proof of the law of antagonism based on the impossibility of shifting this entire cosmic system (rectilinearly) from its place. That Kant seems clearly to be echoing Newton's demonstration of the equality of action and reaction for attractions supplies further confirmation, on my reading, for the suggestion that he is also echoing Newton's argument for universal gravitation throughout the general remark. For Kant here appears to have in mind a transition from a rotating earth, to a rotating solar system, to a rotating (galactic) system of such systems, and so on – all the way out (in the limit) to the "common center of gravity of all matter" (563).

A final point of interest raised by Kant's discussion concerns the sharp contrast he draws between rectilinear and circular motion of the entire cosmic system: the former (by the argument just reviewed) is "absolutely [*schlechterdings*] impossible" (563), whereas the latter, in some sense, remains possible. This point was already considered in section 32 above, where I argued that it represents a decisive reason to reject the view that Kant understands the relativity of rectilinear motion along the lines of what we now call Galilean relativity (compare the paragraph to which note 52 above is appended). Kant does not limit the relativity in question to uniform rectilinear motion, and, indeed, it appears that accelerated rectilinear motion – in accordance with the equality of action and reaction – is paradigmatic of what Kant has in mind.[153] Circular motion of the entire cosmic system, by contrast, remains always possible – or at least

[153] The present discussion further confirms this point. For, not only are alternatively-mutual *accelerated* rectilinear motions central to reducing all motion and rest to absolute space in the context of gravitational interactions, but, in the passage from Newton's Scholium to the Laws of Motion where he demonstrates the equality of action and reaction for attractions, Newton explicitly says that, were this principle to be violated, a system of gravitationally interacting bodies would "move straight forward" and "go on indefinitely with a motion that is always *accelerated*" (P428, emphasis added).

it remains always possible "to think such a motion" (563). For such a rotation, even in infinite empty space, could perfectly well take place without violating the equality of action and reaction in the slightest.

In what precise sense, however, is the circular motion of the entire cosmic system always possible, and what exactly does Kant mean by suggesting, nonetheless, that "to assume it, so far as one can foresee, would be entirely without any conceivable use" (563)? In Kant's Copernican conception of space and motion the universe consists of a never-ending sequence of concentric rotating systems – where every such system is in fact *actually* rotating, no matter how far out one goes. Indeed, the actuality of such circular motion, in each case, is shown by the precise counterbalancing of gravitational and centrifugal endeavors, in conformity with Kant's second proposition of the Phenomenology. But, if every member of the sequence is actually rotating, then it would seem (since the sequence is cumulative) that the entire sequence should be actually rotating as well. The sequence should converge, in some sense, on the common center of gravity of all matter, and, relative to this center, "the system of all matter" (562) should, in fact, be rotating.

It is precisely here, however, that the argument of the antinomies comes into play and, in particular, the argument of the first antinomy concerning the extent of the (material) world in space. As explained in section 14 above, there is a close connection between this argument and Kant's Copernican conception of space and motion, in so far as the latter is predicated upon a never-ending sequence of ever wider concentric spatial regions corresponding to ever more comprehensive (material) relative spaces or reference frames.[154] And it follows from Kant's argument in the first antinomy that we can neither consider the sequence of (material) relative spaces as a completed infinite totality of cumulative rotating structures nor as a finite such totality terminating in empty space. According to the first point, there actually exists no limit of the potentially infinite sequence in question – there is no existing object in the phenomenal world that could correspond to this limit.[155] According to the second point, moreover, we cannot attain a finite most comprehensive

[154] See the paragraph to which note 85 of my chapter on the Dynamics is appended, together with the two following paragraphs.

[155] The point is not merely that there can exist no such limit outside the sequence. Since, for Kant, the sequence can (at most) be *potentially* as opposed to *actually* infinite, we can also not consider the sequence itself as our representative of the desired limit – in the manner of Dedekind, Cauchy, and Weierstrass. There is no possible object in Kant's phenomenal world corresponding to this actually infinite set.

such structure either – which would then be actually rotating in infinite (empty) space.

As I also explained in section 14, according to Kant's empirical use of the regulative principle of reason with regard to this particular antinomy, we do not even know, strictly speaking, that the sequence of ever more comprehensive galactic structures is *potentially* infinite. The empirical regress, in Kant's terminology, proceeds *in indefinitum* rather than *in infinitum*, and thus we do not know, strictly speaking, that there is always a larger (but still finite) such structure beyond any given finite structure. To say that absolute space, in Kant's sense, is a regulative idea of reason is to say only that reason demands that we continually seek for ever larger such structures without providing any guarantee at all that we will actually find one.[156] Instead of either of the two meanings that the concept of absolute space could have in this context – as a largest (finite) rotating galactic structure bounded by infinite (empty) space or the potentially infinite sequence of all such structures bounded by no external (empty) space at all – we have only a never to be completed quest demanded by the nature of reason itself. Therefore, once again, there is no possible object of experience that could correspond to the termination of this quest.

We have also seen, finally, that Kant briefly touches on the relativity of space and motion in a footnote to the antithesis of the first antinomy, where, in particular, he makes the following intriguing remark (A429/B457; see the passage to which note 88 of my chapter on the Dynamics is appended): "[M]otion or rest of the world in infinite empty space, a determination of the relation of the two [empirical intuition and empty space – MF] that can never be observed, is, as a predicate, a mere entity of thought [*Gedankendinges*]." But how is this consistent, in the case of *circular* motion, with Kant's claim in the Phenomenology that such motion is always empirically determinable, even in infinite empty space? The answer, in Kant's terms, is that the actual motion thereby determined consists in a relation of the rotating matter to the space inside it, not to any external space (see the passage to which note 95 above is appended). So such a motion, for Kant, is not – and cannot be – a determination (cognition) of the relation between this matter and infinite empty space. Nevertheless, we can always *think* that such a system is, after all, rotating in infinite empty space, and, in this sense, we can always *think* that the system of all matter is indeed rotating around its axis. Since, however, there can (by the argument of the first antinomy) never be an actual

[156] See note 86 of my chapter on the Dynamics.

empirical intuition corresponding to this state of affairs, the concept of such a system remains a mere "entity of *thought*" (emphasis added).[157]

Technical note: inertial frames and Kantian absolute space

I have suggested at a number of points that Kant's conception of reducing all motion and rest to absolute space can be illuminated by comparison with the modern conception of an inertial frame of reference.[158] And I have observed that one central difference between Kant's conception and our modern one is that the latter is committed to a class of inertial frames, all of which are moving uniformly and rectilinearly relative to one another, while Kant aims to construct something like a single privileged such frame uniquely fixed at the center of gravity of all matter. It is on account of precisely this uniqueness or singularity, I have suggested, that Kant's notion of what he calls "absolute space" is more like Newtonian absolute space than the modern conception of inertial frame. Yet Kantian absolute space differs from Newton's in being what Kant considers a limiting idea of reason rather than the concept of an actual object – a forever unreachable regulative ideal that we can only successively approximate in experience but never actually attain. In this note I shall first clarify the relationship between Kant's procedure for reducing all motion and rest to absolute space and the modern conception of an inertial frame. I shall then consider the limiting process by which this procedure (from a modern point of view) converges on a unique such frame.

The essence of the modern conception is to define an inertial frame of reference as one in which the Newtonian Laws of Motion are

[157] In these terms, and in light of Kant's distinctions among different types of empty concepts (and objects) in the amphiboly (A290–92/B346–49), a *circular* motion of the system of all matter would therefore be an "*empty concept without an object* (*ens rationis*)" (A292/B398; see again note 88 of my chapter on the Dynamics). By contrast, a *rectilinear* motion of this same system would be an "*empty object without a concept* (*nihil negativum*) – where "[t]he object of a concept that contradicts itself is nothing, because the concept is nothing, the impossible" (A291–92/B348). The concept of such a rectilinear motion, as we have seen, is "absolutely impossible [*schlechterdings unmöglich*]" (563), since such a motion (with no counterbalancing equal and opposite motion) would violate the equality of action and reaction – which "like everything sufficiently demonstrable from mere concepts, is the law of an absolutely necessary [*schlechterdings notwendigen*] contrary motion" (562). In the corresponding footnote to the antithesis of the first antinomy, however, Kant may not be considering either the distinction between circular and rectilinear motion or the second proposition of the not yet written Phenomenology. Yet it also seems perfectly possible that Kant does have this more subtle idea in mind, especially when he returns to the question here in the Phenomenology itself.

[158] See, for example, note 28 of the Introduction, note 52 of my chapter on the Phoronomy, and notes 52 and 109 above – all together with the paragraphs to which they are appended.

satisfied.[159] The Third Law of Motion is particularly important here because the requirement that any acceleration of one body be counterbalanced by a contrary acceleration of another, in accordance with the equality of action and reaction, implements the idea that every *true* acceleration is the product of a genuine dynamical force. True (versus merely apparent) accelerations, on this conception, therefore come in oppositely directed pairs, where each of the two bodies in question counts as a source or seat of the dynamically interacting forces mutually responsible for the two (true) accelerations. This requirement, as I have suggested, suffices to eliminate what we now call "pseudo forces" – such as centrifugal and Coriolis forces – from consideration and thus distinguishes an inertial frame of reference, in particular, from a rotating frame (see again note 109 above, together with the paragraph to which it is appended). More precisely, for any isolated system of interacting bodies, we can choose the center of mass of this system as the origin of our frame, and then apply the equality of action and reaction to eliminate all rotational pseudo forces. By Corollary 4 to Newton's Laws of Motion, the center of gravity of our frame will move uniformly and rectilinearly, and, by Corollary 5, the frame will be non-rotating as well. We have thereby constructed a privileged inertial frame for describing all the true accelerations arising from the interactions in question.

Kant's procedure for reducing all motion and rest to absolute space, on my reading, is very close to this modern construction. His overriding emphasis on the equality of action and reaction, the explicit connection he draws between what he calls "absolute space" and the center of mass of a system of interacting bodies, and his insistence on the *mutual* character of all true motions together enable him to determine a relevant center of mass and then to eliminate all rotational pseudo forces in precisely the manner just sketched. Moreover, the way in which Kant implements such a construction concretely by closely following the argument of Newton's Book 3 does in fact lead, in effect, to a non-rotating frame of reference fixed at the center of gravity of the solar system.[160] Nevertheless, since Kant does not have the modern conception of an inertial frame, he does not say that the true motions are just those defined *relative to such a frame*. Nor does he say, in his own language, that the true motions are

[159] For sophisticated discussions of this conception and its late nineteenth-century background see Torretti (1983, §1.5) and DiSalle (1988, 1991, and 2002).

[160] See the paragraphs to which notes 52, 56, and 62 above are appended, as well as the paragraph to which note 151 above is appended and the following paragraph.

just those defined relative to absolute space. He instead speaks, as we have seen, of "an absolute space to which all relative motions can be referred, in which everything empirical is movable, precisely so that in it all motion of material things may count as merely relative with respect to one another, as alternatively-mutual, but none as absolute motion or rest" (559; see note 118 above, together with the paragraph to which it is appended).

Kant insists, in other words, that all true or actual motions be defined in such a way that they are *relative, mutually, to the bodies engaged in the interaction in question*. Yet, from a modern point of view, while this is indeed straightforward in the case of true rectilinear accelerations, it can easily lead to confusion when applied to true rotations. In Kant's pivotal discussion of the centrifugal bulging of the earth, in particular, he says that the truth or actuality of the rotation in question "rests on the representation of the mutual and continuous *withdrawal* of any part of the earth (outside the axis) from any other part lying opposite to it on a diameter at the same distance from the center" and concludes that this "actual motion" relates "to the space inside the moved matter (namely its center), and not to that outside it" (561–62; see again the paragraph to which note 95 above is appended). As I have suggested, therefore, he here seems to be considering a rotating frame of reference where the "mutual and continuous *withdrawal*" appears as a pair of outwardly directed (centrifugal) accelerations along a line through the earth's center – so that it is in precisely this way that they may then "count as merely relative with respect to one another" (559). But these accelerations – unlike the counterbalancing centripetal accelerations due to gravity – do not, from a modern point of view, count as true accelerations at all. In an inertial frame (as opposed to a rotating frame) there are no such accelerations (corresponding to what we take to be a pseudo force); there is, instead, a true angular velocity of the earth generating a centrifugal endeavor directed parallel (rather than perpendicular) to the rotation in question. (See again note 96 above, together with the paragraph to which it is appended.)

So is Kant himself confused here? I think not. For, in the first place, he is clearly considering what we now take to be an inertial frame in the second proposition of the Phenomenology – which explicitly speaks of a "striving … to proceed in the straight line tangent to the circle" (557; see the paragraph to which note 86 above is appended). And he appears to be recurring to this same description at the beginning of the long paragraph on the earth's true rotation in the general remark, where he speaks of a "continual diminution of attraction in virtue of a striving to escape" (561; see the paragraph preceding the one to which note 116 above is appended).

Further, and in the second place, Kant also says in the passage at issue that the relevant "mutual and continuous *withdrawal*" is the effect of "actual motion" rather than "any dynamical repulsive cause" (562). And, since he applies the equality of reaction only to genuine dynamical forces (see note 145 above), he treats this withdrawal, in effect, as precisely the manifestation of what we now take to be a pseudo force. In other words, the true or actual motion in question involves the mutual (orbital) *rotation* around a common center of two diametrically opposed parts of the earth rather than their mutual (rectilinear) acceleration away from this center. So the frame of reference in which Kant determines this motion (as true) – what he calls "absolute space" – is, after all, what we now take to be an inertial frame. (See note 146 above, together with the paragraph to which it is appended.)

Kantian absolute space functions as a regulative idea of reason in so far as he conceives it as a kind of limit of the never to be completed procedure for reducing all motion and rest to absolute space. As explained, however, Kant does not have in mind the concept of limit in the modern (mathematical) sense. For, by the argument of the first antinomy, he cannot consider the sequence of ever more comprehensive (material) relative spaces generated by this procedure as an actually infinite completed totality (see note 155 above, together with the paragraph to which it is appended). As a result, there is no object at all corresponding to what Kant calls "absolute space" but only the regulative demand of reason that we should always pursue the sequence one step further while never accepting any given finite stage as terminal. The question remains, however, of exactly what kind of *potentially* infinite sequence we are thereby seeking and of what it means, in particular, for such a sequence to be *converging* (even if it does not converge to any object as limit). One reason that this question is especially pressing, moreover, is that it does not appear, in moving from the earth, to the center of gravity of the solar system, to the center of gravity of the Milky Way galaxy, and so on, that this sequence of centers of gravity of ever more comprehensive rotating systems is itself convergent. It does not appear, for example, that the distances between these centers is becoming smaller and smaller – quite the contrary.

So what kind of convergence is involved here? The most natural suggestion, from a modern point of view, is that we are thereby obtaining better and better approximations to an inertial frame of reference – a frame in which every true acceleration is counterbalanced by a corresponding oppositely directed such acceleration in accordance with the equality of action and reaction. Thus, for example, if we confine ourselves to a frame centered on the earth, this principle is not exactly satisfied. The accelerations of falling bodies (due to terrestrial gravity) are not counterbalanced by any corresponding upward acceleration, and the attractions of the earth for celestial bodies such as the moon (and even the sun) are similarly not counterbalanced by any corresponding attractions of these celestial bodies for the earth. It is only by moving to the common center of gravity of all the bodies in the solar system that we then attain a frame of reference in which the equality of action and reaction is exactly satisfied by all of their gravitational attractions. Moreover, in Kant's Copernican conception of space and motion we cannot rest at the center of gravity of the solar system. This system, too, is experiencing gravitational attractions by bodies external to it in the Milky Way galaxy, and, therefore, the equality of action and reaction demands that both these bodies and the center of gravity of the solar system are experiencing true accelerations directed towards one another – and so on *ad infinitum*. Thus, it will not be the case that *all* true accelerations satisfy the equality of action and reaction at any stage prior to the center of gravity of all matter, and, as we move from one stage of the sequence to another, *ever more* true accelerations are successively accommodated. Finally, just as the earth can legitimately be considered as at rest for the purpose of describing all motions in the sublunary region (for its accelerations, in this context, are extremely small), the sun can legitimately be considered as at rest – at least to a first approximation – for the purpose of describing the motions in the solar system. The center of gravity of the solar system then represents a better approximation; and so on.

However, there is a significant complication to this picture arising from Newton's Corollary 6 to the Laws of Motion, according to which accelerated motions also satisfy a certain kind of principle of relativity. In particular, if a number of moving bodies in a given system are all accelerated together equally and in the same direction, then their motions relative to one another are completely undisturbed.[161] Moreover, the force of universal gravitation, as Newton knew well,

[161] The statement of Corollary 6 reads (P423): "If bodies are moving in any way whatsoever with respect to one another and are urged by equal accelerative forces along parallel lines, they will all continue to move with respect to one another in the same way as they would if they were not acted on by these forces." This Corollary, as Newton presents it, is thus a direct supplement to Corollary 5 stating the relativity of uniform inertial motion. Corollary 6 extends such relativity to accelerated motion as well, provided that all the bodies in the system are equally accelerated along parallel lines, and, in this sense, it can be viewed as a kind of forerunner of Einstein's principle of equivalence in the general theory of relativity. For a sophisticated recent discussion see DiSalle (2006).

generates motions of precisely this kind. All bodies in a gravitational field experience equal accelerations at equal distances, and, in many circumstances, they move along (almost) parallel lines as well. Thus, for example, all falling bodies in a limited region near the surface of the earth accelerate downwards with (very nearly) the same constant acceleration, and, in addition, they all fall (very nearly) parallel to one another. Similarly, Jupiter and all its moons are accelerating towards the sun, and, since the sun is extremely distant in comparison, they are also accelerating (very nearly) in parallel lines. Thus, all the motions within the system of Jupiter and its moons take place (very nearly) as if this system were not, after all, accelerating towards the sun. The same holds for the motions in the sublunary region of the earth in the context of its acceleration towards the sun; for all the motions within the solar system in the context of its acceleration towards the center of gravity of the Milky Way galaxy; and so on. In other words, each of these systems can already be considered (very nearly) as defining an inertial frame of reference (for all the motions of the bodies *within* the system), and, in this sense, the process of finding ever more accurate approximations to a true inertial frame of reference does not fully capture the force of the convergence we are after.

It is true that none of these less comprehensive systems defines an exactly inertial frame, for the accelerations in question do not in fact take place along exactly parallel lines. So we do make (very small) corrections as we move from a less comprehensive to a more comprehensive such system. It is also true, accordingly, that only the most comprehensive system – defined by the center of gravity of all matter – is exactly a true inertial frame. In light of precisely Corollary 6 to the Laws of Motion, however, there is an additional important feature of this procedure as well. If we take different such less comprehensive systems to be inertial frames (even approximately), we are then faced with the anomalous situation that there can be two such (approximate) inertial frames that are not, in the context of a more comprehensive system, moving inertially (even approximately) relative to one another. For example, both the system consisting of Jupiter and its moons and the system consisting of Saturn and its moons are, in this sense, (approximate) inertial frames. Nevertheless, they are in no sense moving inertially relative to one another (even approximately) in the context of the solar system.[162] In this sense, therefore, the

[162] This point is central to the interpretation of the principle of equivalence in DiSalle (2006, §§4.4–4.5), which I am closely following here.

point of moving from less comprehensive to more comprehensive systems is not simply to make the (very small) corrections mentioned above due to the failure of (exact) local inertiality. Rather, as we move from less comprehensive to more comprehensive systems, we also overcome the just-mentioned anomaly with respect to successively more inclusive rotating systems. In the limit, when we finally attain the center of gravity of all matter, we will have constructed a global inertial frame in which there is no remaining anomaly at all.[163]

35 PHENOMENOLOGY AND THE DYNAMICAL THEORY OF MATTER

What is the relationship between the argument of the Phenomenology and the dynamical theory of matter that Kant articulates in the Dynamics and then applies to the concept of quantity of matter in the Mechanics? The most general and fundamental relationship between the two involves Kant's attempt to explain the possibility of Newton's mathematization of the concept of quantity of matter in the Mechanics discussed in section 28 above. As explained, the balancing argument of the Dynamics depicts matter as an infinite and continuously distributed aggregate in a given space, whose determinate density results from a balancing of the two fundamental forces. The traditional concept of quantity of matter – bulk or amount of matter as the product of density and volume – is thereby connected with both the mechanical concept of mass or inertia and the universal penetrating force of attraction. In the context of Newton's theory of universal gravitation, in particular, we are thereby able to construct a threefold system of interconnections between the static concept of weight and both the mechanical concept of mass and the concept of quantity of matter as the product of density and volume. The main point of Kant's balancing argument, from this point of view, is not to propose another

[163] Again, I owe this way of putting the matter to DiSalle (2006, §3.4), which makes precisely this point when discussing Kant's conception of absolute space. From this point of view, in particular, Kant thereby anticipates some of the force of Einstein's principle of equivalence, and we can then make further sense of Kant's repeated failure to embrace what we now call the principle of Galilean relativity governing all inertial frames of reference (see again note 52 above, together with the paragraph to which it is appended). For, from this point of view, Kant's conception is sensitive to the fact that nothing like a true global inertial frame of reference can be constructed (even approximately) until we move all the way out through the sequence of ever more comprehensive rotating systems so as to construct a *unique* such frame suitable for playing the role of absolute space.

contribution to speculative matter theory but rather to articulate a distinctive concept of matter (Kant's own dynamical concept) possessing just those structural features that are necessary to explain Newton's successful mathematization.

The crux of Kant's argument, on my reading, is the idea that only the theory of universal gravitation has successfully managed the transition from the terrestrial (static) concept of weight to the universal (mechanical) measure of mass for all matter in the universe and, at the same time, has given a precise (and similarly universal) mathematical meaning to the concept of quantity of matter explained in terms of density and volume (see again the paragraph to which note 152 of my chapter on the Mechanics is appended, together with the following paragraph). Moreover, Newton's argument for universal gravitation, as recently emphasized once again (see notes 141 and 142 above, together with the paragraph to which they are appended), thereby leads to the determination of the center of mass (center of gravity) of the solar system. I have suggested, accordingly, that the strength of Kant's commitment to truly universal gravitation rests on precisely the metaphysical foundation that he has constructed, within his dynamical theory of matter, for the Newtonian mathematization of the concept of mass and thus quantity of matter (see note 127 above, together with the paragraph to which it is appended).

It is for precisely this reason, as I have also suggested (note 128 above), that Kant takes Newton's solution to the problem of the shape of the earth completely for granted and does not consider Huygens's alternative solution – based on the rejection of universal gravitation – to be a serious competitor. So Kant's commitment to universal gravitation does not rest solely on the circumstance that this theory, for the first time, has shown us how empirically to decide the contentious question of Copernicanism. It also rests, perhaps even more fundamentally, on Kant's understanding of the mathematical structure of a truly universal concept of mass or quantity of matter. The main point, in this connection, is that Huygens's rejection of universal gravitation – and of the Third Law of Motion applied between spatially separated celestial bodies – leaves him unable to account for this structure. For it leaves him unable successfully to manage the transition from the terrestrial concept of weight to the universal mathematical measure of mass. There is thus a significant relationship indeed between Kant's procedure in the Phenomenology for reducing all motion and rest to absolute space and the dynamical theory of matter that he had earlier developed in the Dynamics and Mechanics.[164]

The penultimate paragraph of the Phenomenology consists of a lengthy discussion (comprising some two and a half pages) of various concepts of empty space. Kant distinguishes three different concepts of empty space – empty space in the *phoronomical*, *dynamical*, and *mechanical* senses – corresponding to the three different characterizations of motion found in the three propositions of the Phenomenology. As observed, the first proposition characterizes rectilinear motion as merely possible and thereby determines the modality of motion with respect to *phoronomy*; the second characterizes circular motion as actual and thereby determines the modality of motion with respect to *dynamics*; the third characterizes equal and opposite motion as necessary and thereby determines the modality of motion with respect to *mechanics*.[165] Kant begins the paragraph in question by suggesting just this correspondence (563): "To the various concepts of motion and of moving forces the various concepts of *empty space* also have their relation."

By far the greatest part of Kant's discussion (comprising one and a half pages) concerns empty space in the dynamical sense. The central question left open by Kant's dynamical theory of matter – the question of how exactly the original expansive force of matter is counterbalanced by the original attraction – plays a dominant role throughout this discussion. Kant introduces the question and treats it at length when discussing

[164] To see what is at issue here it is important to appreciate that it is possible empirically to decide the issue of Copernicanism without yet invoking the concept of mass or the Third Law of Motion. For, in the argument sketched at the end of section 18 above (see again note 192 of my chapter on the Dynamics, together with the paragraph to which it is appended), one can stop with the determination of the constant k for each primary body in the solar system that characterizes the absolute strength of its surrounding gravitational acceleration field independently of the distance from the center of that body. One can empirically determine k for each such body from the accelerations produced in its satellites (provided it has satellites), and one can thereby empirically determine a "center of motion," "center of gravity," or "center of acceleration" without yet taking it to be a center of *mass*. The Third Law is applied only after this determination – and, in effect, results in the identification of what we now call "active gravitational mass" with inertial mass. Indeed, as Smith (1999) explains, Newton himself formulated an argument for Copernicanism based on precisely this "center of gravity" at a relatively early stage of his composition of the *Principia*, before he had articulated either the fundamental relationship between weight and (inertial) mass or the Third Law of Motion or (of course) the law of universal gravitation. What the Newtonian concept of mass and Third Law of Motion add to this argument, from Kant's point of view, is a unified treatment of the true motions arising from any interactions whatsoever – not only gravitational but also, for example, involving impact or pressure.

[165] For the modality of motion with respect to phoronomy see the paragraph to which note 30 above is appended, for dynamics note 109 above, and for mechanics note 110 above.

empty space in the dynamical sense, in terms of "the possibility of the composition [*Zusammensetzung*] of a matter in general, if only this were better understood" (563). Moreover, he returns to it when discussing empty space in the mechanical sense, in terms of "the mystery of nature, difficult to unravel, as to how matter sets limits to its own expansive force" (564).[166] Accordingly, Kant takes up the question of the aether (and its incomparably small density) from the Dynamics when discussing empty space in the dynamical sense (564) and returns to it when discussing empty space in the mechanical sense (564).[167]

We can already see, therefore, that Kant's discussion is by no means focussed solely on the preceding Dynamics. On the contrary, he is concerned with empty space in both the mechanical and phoronomical senses as well, and, in this way, he attempts to show more explicitly how the discussion of this concept in the Dynamics is related to both the Mechanics and the Phoronomy. Thus, for example, whereas Kant first introduces the question of the aether (and its incomparably small density) at the end of the general remark to dynamics (see note 167), he does not return to this question explicitly in the following Mechanics. In the general remark to phenomenology, however, Kant first introduces the question of the aether in his discussion of empty space in the dynamical sense as "accumulated empty space (*vacuum coacervatum*, which separates bodies, e.g.,

[166] At the end of section 16 above (see notes 148 and 149 of my chapter on the Dynamics, together with the paragraph to which they are appended), I emphasized that Kant deliberately leaves open exactly how the counterbalancing compression due to the original attraction is effected – whether solely by the attractive forces of the matter under consideration or also involving other attractive forces due to matter external to it. I called attention to the second note to the eighth proposition of the Dynamics (518): "[I]t may now be that the [original attraction in conflict with the original repulsion] derives from the individual attraction of the parts of the compressed matter among one another, or from the unity of this with the attraction of all cosmic matter [*aller Weltmaterie*]." And I pointed out that the cosmic matter in question appears to involve an aether distributed throughout the cosmos exerting external compression on the bodies (while being brought to this pressure by the attractive forces of the bodies themselves). Section 17 above then linked this last conception to the specific physical realization of the balancing argument that is sketched in the *Danziger Physik*, and also to the present discussion in the Phenomenology. In general, whereas the balancing argument that Kant develops in the fourth and fifth propositions of the Dynamics is an integral part of his metaphysical dynamical theory of matter, the particular mechanism that realizes this balancing is then left open (from a metaphysical point of view) as a (mere) physical hypothesis.

[167] Kant first introduces the aether, at the end of the general remark to dynamics, as a way of avoiding a necessary commitment to empty space (534):

> In this way, one would not find it impossible to think a matter (as one represents the aether, say) that fills its space completely without any emptiness, and yet with an incomparably smaller quantity of matter at the same volume than all bodies which we can subject to our experiments. The repulsive force in the aether must, in relation to its own force of attraction, be thought as incomparably greater than in all other matter known to us.

the heavenly bodies, from one another) … in order to derive therefrom the possibility of a motion in the universe [*Weltraum*] free of all external resistance" (563). He then explains that empty space in the mechanical sense concerns precisely "the accumulated emptiness within the cosmos [*Weltganzen*] in order to provide the heavenly bodies with free motion" (564) – and argues, in accordance with the general remark to dynamics, that this can in fact be secured by a space-filling aether "because the resistance, even in the case of completely filled spaces, can still be thought as small as one wishes" (564). Kant's discussion here, therefore, is intended explicitly to connect the discussion of an (incomparably rare) space-filling aether in the Dynamics with a problem about "free motion" arising against the background of the Mechanics.

Another indication of the way in which Kant attempts to depict such interconnections among the earlier chapters of the *Metaphysical Foundations* is provided by the first sentence of the paragraph under discussion (563, quoted above), where Kant says that the various concepts of empty space are related to both the various concepts of motion and the various concepts of moving forces. It seems clear that the various concepts of motion in question are just those discussed in the three propositions of the Phenomenology (compare note 165 above). But what are the various concepts of *moving forces* at issue here? Kant goes on to mention both repulsive and attractive forces in the following discussion, and both kinds of forces figure in the preceding discussion as well – especially the fundamental force of attraction. Nevertheless, since he appears to be centrally concerned in the following discussion with connecting empty space in the dynamical and mechanical senses, and since this kind of connection would better correspond with his threefold classification of motions (phoronomical, dynamical, and mechanical), it seems that the distinction between *dynamical* and *mechanical* moving forces is more salient here.[168]

Kant begins his discussion with empty space in the phoronomical sense:

> Empty space in the *phoronomical* sense, which is also called absolute space, should not properly be called an empty space; for it is only the idea of a space, in which I abstract from all particular matter that makes it an object of experience, in order to think therein the material space, or any other empirical space, as movable, and thereby to think motion, not merely in a one-sided fashion

[168] There is no such thing as *phoronomical* moving force. Nevertheless, merely possible motions, whose modality is determined with respect to phoronomy, are, for Kant, rectilinear but not necessarily uniform (section 32 above), so that moving forces (both dynamical and mechanical) are still thereby implicated in these motions as well. I shall return to this point below.

[*einseitig*] as absolute, but always mutually [*wechselseitig*], as a merely relative predicate. It is therefore nothing at all that belongs to the existence of things, but rather [belongs] solely to the determination of concepts, and to this extent there *exists* no empty space. (563)

So empty space in the phoronomical sense is just the absolute space to which all motion and rest are to be reduced in accordance with the basic argument of the Phenomenology.[169] In particular, although the notion of pure or absolute space, considered as a mere idea of reason, is first introduced in the Phoronomy, the procedure for constructing it is not fully articulated until the Phenomenology.

In the Phoronomy, as observed in section 1 above, Kant characterizes "absolute space" in the first explication (480): "That space in which all *motion* must finally be thought (and which is therefore itself absolutely immovable) is called pure, or also *absolute space*." He goes on to say that "absolute space is *in itself* nothing and no object at all" (481) and explains that it "signifies only any other relative space, which I can always think beyond the given space, so as to include it and suppose it to be moved" (481). He concludes (482): "To make this [absolute space] into an actual thing is to transform the *logical universality* of any space with which I can compare any empirical space, as included therein, into a *physical universality* of actual extent, and to misunderstand reason in its idea." Kant thus introduces his Copernican conception of an ever more inclusive indefinitely extended sequence of relative spaces governed by a forever unattainable limiting idea of reason.

But it is only in the Phenomenology that Kant finally explains how this Copernican conception is to be concretely and determinately executed, by moving from our parochial perspective here on earth to the center of gravity of the solar system, from there to the center of gravity of the Milky Way galaxy, and so on *ad infinitum*.[170] For it is only the "idea of a[n absolute] space" articulated in the Phenomenology that now allows us "thereby to think motion, not merely in a one-sided fashion [*einseitig*] as absolute, but always mutually [*wechselseitig*], as a merely relative predicate" (563). It is only in this way, in particular, that "[a]bsolute space is

[169] Compare note 72 above, together with the paragraph to which it is appended. I was there engaged in a reading of the crucial phrase "alternatively-mutual [*alternativ-wechselseitig*]" that Kant introduces at the beginning of his discussion of reducing all motion and rest to absolute space in the general remark.

[170] For the lack of stability and determinacy in the conception of absolute space in the Phoronomy, as seen from the perspective of the Phenomenology, see the paragraph to which note 42 above is appended.

therefore necessary, not as a concept of an actual object, but rather as an idea, which is to serve as a rule for considering all motion therein merely as relative; and all motion and rest must be reduced to absolute space, if the appearance thereof is to be transformed into a determinate concept of experience [*Erfahrungsbegriff*] (which unites all appearances)" (560).[171] Kant thus extends the Newtonian procedure for determining the center of gravity of the solar system to the cosmos as a whole by invoking "the law of *antagonism* in all community of matter through motion" in reference to "the common center of gravity of all matter" (563; see the paragraph following the one to which note 150 above is appended).

When Kant says that absolute space is empty space in the *phoronomical* sense, therefore, he is not simply repeating the argument of the Phoronomy. He is reconsidering this argument in the fundamentally transformed context of the Phenomenology – where, in particular, we are now also able to take the following Dynamics and Mechanics into account. Moreover, Kant is not relying only on the first proposition of the Phenomenology (which determines the modality of motion with respect to phoronomy). In order fully to implement his Copernican conception, he must essentially appeal to the second and third propositions as well (which determine the modality of motion with respect to both dynamics and mechanics).[172] Empty space in the phoronomical sense immediately involves us with the argument of the Phenomenology as a whole, and, accordingly, it also thereby involves us with a reconsideration of all three previous chapters of the *Metaphysical Foundations* in light of this argument.[173]

Kant next considers empty space in the dynamical sense (563): "Empty space in the *dynamical* sense is that [space] which is not filled – i.e.,

[171] Again, this passage immediately follows the crucial passage introducing the notion of an "alternatively-mutual" determination of motion (559–60) that is central to my reading of the argument of the Phenomenology.

[172] In notes 72 and 73 above, together with the paragraph to which they are appended, I suggested that an alternatively-mutual judgement has one foot, as it were, in the first proposition (as "alternative") and another in the third (as "mutual"). It encapsulates the way in which the first proposition *prospectively* envisions a later application of the third (compare also the paragraph to which note 148 above is appended, together with the following paragraph).

[173] In particular, the concept of empty (or absolute) space in the phoronomical sense also essentially involves us with the dynamical theory of matter developed in the Dynamics and applied in the Mechanics – but only, as I discussed at the beginning of this section, with the most general or "metaphysical" features of this theory aimed at explaining the possibility of Newton's mathematization of the concept of quantity of matter. As discussed below, the central "physical" question left open by this theory (see note 166 above, together with the paragraph to which it is appended) is then what is primarily at issue in Kant's discussion of empty space in the dynamical and mechanical senses.

wherein nothing else movable resists the penetration of the movable, and therefore no repulsive force acts." This characterization recalls the first explication of the Dynamics (496): "To fill a space is to resist every movable that strives through its motion to penetrate into a certain space. A space that is not filled is an *empty space*." In the following first proposition of the Dynamics Kant argues that this kind of resistance must be due to the action of a repulsive force (497; see section 10 above), and in his remark to the first explication he makes an important distinction between *filling* and *occupying* a space. The latter designates merely the *extension* of a thing (e.g., a geometrical figure) in space in virtue of its "immediate presence in all points of this space," where it is so far entirely undetermined "what action arises from this presence, or even whether there is any action at all" (497; see section 9 above). He then returns to the distinction between filling and merely occupying a space in the final paragraph of the Dynamics in a discussion of "the well-known question as to the admissibility of empty spaces within the world" (534). An empty space, once again, is precisely one in which no specifically *repulsive* forces act (535): "[A]ttractive force is attributed to matter in so far as it *occupies* a space around itself, through attraction, without at the same time *filling* this space – which space can therefore be thought as empty even where matter is active, because matter is not active there through repulsive forces and hence does not fill this space."[174]

This characterization of empty space in the dynamical sense is followed by a rather elaborate classification of different kinds of such empty spaces, according to where exactly the space in question occurs:

> It can either be *within* the world (*vacuum mundanum*), or, if the latter is represented as limited, empty space *outside* the world (*vacuum extramundanum*); the former, too, can be represented either as dispersed (*vacuum disseminatum*, which constitutes only a part of the volume of matter), or as accumulated empty space (*vacuum coacervatum*, which separates bodies, e.g., the heavenly bodies, from one another). (563)

This latter distinction, Kant continues, "is certainly not essential, since it rests only on a difference in the locations assigned to empty space within the world, but it is still employed for different purposes – the first, in

[174] See note 133 of my chapter on the Dynamics. Kant makes essentially the same point in the note to the seventh explication of the Dynamics (516): "By contrast [to repulsive force], no intervening matter sets limits to the action of an attractive force, whereby matter occupies a space *without filling it*, so that it thereby acts on other distant matter *through empty space*." Compare also note 155 of my chapter on the Dynamics, together with the paragraph to which it is appended and the following paragraph.

order to derive therefrom specific differences in density, the second, in order to derive the possibility of a motion in the universe free of all external resistance" (563). What is most interesting, in the present context, is precisely the distinction between these two purposes.

In the general remark to dynamics Kant is exclusively concerned with the first purpose – with the fundamental idea of what he calls the "mechanical" (or "mathematical-mechanical") natural philosophy for explaining specific differences in density "by combination of the absolutely full with the absolutely empty" (532). On this approach, in particular, we represent the density of matter in a given space by the ratio between the volume occupied by a single uniform type of absolutely impenetrable matter within this space and the total volume – which, accordingly, may also contain interspersed interstices of absolutely empty space. Kant opposes this conception with his own view of *relative* impenetrability, based on the continuously increasing resistance to compression of expansive force, and he advocates a "dynamical" (or "metaphysical-dynamical") natural philosophy that explains specific differences in density by specifically different degrees of repulsive force in specifically different materials.[175] In the latter approach, moreover, one can then easily arrive at the idea of an incomparably rare cosmic aether, which, nonetheless, continuously fills the space it occupies through and through. And it is in precisely this way that Kant then introduces such an aether at the end of the general remark to dynamics (note 167 above).[176]

By contrast, the second purpose for postulating an empty space within the world (as accumulated empty space or the *vacuum coacervatum*) is not mentioned in the Dynamics at all. Indeed, it makes its very first appearance in the *Metaphysical Foundations* here in the penultimate paragraph of the Phenomenology. The point of postulating this kind of empty

[175] Compare sections 11 and 20 above. On the "mechanical" conception of absolute impenetrability there is actually no repulsive force in Kant's sense at all, and, as explained in section 10 above, the argument for repulsive force in the first proposition of the Dynamics essentially *assumes* relative (continuously acting) impenetrability at the very beginning.

[176] In the discussion of "empty space within the world" (534) at the conclusion of the general remark to dynamics, Kant first explains that the *possibility* of such a space cannot be denied – since space is required for all forces of matter, especially attractive force. Nevertheless (535): "[N]o experience, or inference from experience, or necessary hypothesis for its explanation, can justify us in assuming empty spaces as *actual*." The reason is that "all experience gives us only comparatively empty spaces for our cognition, which can be completely explained from the property of matter of filling its space with greater or ever smaller expansive force to infinity, in accordance with all desired degrees, without requiring empty spaces" (535). The density of the space-filling aether is incomparably small (but not zero), because its repulsive force is incomparably great in comparison with its attractive force. It is therefore maximally dispersed, and its expansive force, for precisely this reason, is then incomparably small.

space, once again, is "to derive the possibility of a motion in the universe free of all external resistance" (563). Or, as Kant puts it on the following page (564), we required an "accumulated emptiness within the cosmos [*Weltganzen*] in order to provide the heavenly bodies with free motion." He also states, as we have seen, that this end can actually be secured by the space-filling aether, "because the resistance, even in the case of completely filled spaces, can still be thought as small as one wishes" (564; see the paragraph following the one to which note 167 above is appended). Thus, the incomparably rare cosmic aether Kant had introduced in the Dynamics in connection with his "metaphysical-dynamical" approach to specific differences in density is here being employed for a quite different purpose – to provide a motion "free of resistance" for the heavenly bodies in the cosmos without assuming an *absolutely* empty space separating and surrounding these bodies.

I have suggested that Kant's discussion of this kind of empty space (*vacuum coacervatum*) indicates a significant relationship between the Dynamics and the Mechanics. In particular, Kant emphasizes a fundamental distinction between dynamical and mechanical resistance at the very beginning of both chapters. The Dynamics is concerned with a resistance to *penetration* rather than a resistance to motion (496–97; see the paragraph to which note 14 of my chapter on the Dynamics is appended). The Mechanics is concerned with precisely a resistance to *motion*, and it arises in all cases of the communication of motion. When one body communicates some of its own motion to another, the loss of motion experienced by the first body is due to the mechanical resistance of the second, and mechanical resistance to motion (unlike dynamical resistance to compression) is not limited to the communication of motion by means of repulsive forces. Such mechanical resistance, on the contrary, arises equally in the communication of motion by universal gravitation, where an attracting body imparts a change of momentum to an attracted body and (by the equality of action and reaction) undergoes a corresponding change of momentum of its own.[177]

The incomparably rare aether introduced in the general remark to dynamics exerts an incomparably small dynamical resistance to compression. Repulsive force in the aether is incomparably great in comparison with attractive force, and the aether is thereby maximally expanded or dispersed throughout the entire cosmos. Therefore, Kant says, this

[177] See especially the first two paragraphs of my chapter on the Mechanics – and, more generally, compare all of section 22 above.

incomparably rare aether must also have "an incomparably smaller quantity of matter at the same volume than all bodies which we can subject to our experiments" (534; see again note 167 above). It follows, then, that a heavenly body moving through the cosmos and displacing a corresponding volume of aether must experience an incomparably small exchange of momentum with this volume. The aether thereby exerts an incomparably small mechanical resistance to the motion of the heavenly bodies – which, as far as the aether is concerned, thus move "freely" through cosmic space.

It is in precisely this way, for Kant, that the *vacuum disseminatum* discussed in the general remark to dynamics is connected with the *vacuum coacervatum* intended to secure a motion free of mechanical resistance for the heavenly bodies. So it is precisely this connection that Kant is attempting to depict in the present discussion of empty space in the dynamical sense. It is clear, however, that the *vacuum coacervatum* is concerned with mechanical resistance, and it therefore appears, more appropriately, in the following discussion of empty space in the mechanical sense:

> As for empty space in the third, or *mechanical sense*, it is the emptiness accumulated within the cosmos in order to provide the heavenly bodies with free motion. It is easy to see that the possibility or impossibility of this does not rest on metaphysical grounds, but on the mystery of nature, difficult to unravel, as to how matter sets limits to its own expansive force. Nevertheless, if one grants what was said in the general remark to dynamics concerning the possibility of an ever increasing expansion of specifically different materials, at the same quantity of matter (in accordance with their weight), it may well be unnecessary to suppose an empty space for the free and enduring motion of the heavenly bodies; because the resistance, even in the case of completely filled spaces, can still be thought as small as one wishes. (564)

Kant thus makes the connection between the general remark to dynamics and the mechanical resistance exerted by the space-filling aether fully explicit.[178]

What exactly does Kant have in mind by the "free and enduring [*freien und dauernden*]" motion of the heavenly bodies? They do not move free of all external forces in accordance with the law of inertia, of course, because their motions are entirely subject to the force of universal

[178] Kant thereby gives a centrally important illustration of the connection between dynamical and mechanical moving forces as well, by depicting how the dynamical expansive force of the aether (due to its incomparably great fundamental force of repulsion) is then connected with the possibility of an incomparably small mechanical exchange of momentum (i.e., mechanical moving force) between the aether and the heavenly bodies.

gravitation according to Book 3 of the *Principia*. However, if we now consult Book 3 on this question, it turns out that, immediately before the climactic Propositions 11 and 12 determining the center of mass of the solar system as "at rest" for the purpose of describing this system, we find Proposition 10 (P815): "The motions of the planets can endure for an extremely long time [*diutissime conservari posse*]." The crux of Newton's argument for this proposition is the claim that, "since the resistance of mediums decreases in the ratio of their weights and density … it follows that up in the heavens, where the weight of the medium in which the planets move is diminished beyond measure, the resistance will nearly cease" (P816): "And therefore in the heavens, which are devoid of air and exhalations, the planets and comets, encountering no sensible resistance, will move through those spaces for an extremely long time."

Newton takes up this argument again in the General Scholium added to the second edition, which begins with the famous statement (P939): "The hypothesis of vortices is beset with many difficulties." For example, it would be difficult to coordinate the solar vortex with the smaller vortices surrounding planets encircled by satellites. Moreover: "Comets go with very eccentric motions into all parts of the heavens, which cannot happen unless vortices are eliminated" (P939). Newton then summarizes the argument of Proposition 10:

> The only resistance which projectiles encounter in our air is from the air. With the air removed, as it is in Boyle's vacuum, resistance ceases, since a tenuous feather and solid gold fall with equal velocity in such a vacuum. And the case is the same for the celestial spaces, which are above the atmosphere of the earth. All bodies must move very freely in these spaces, and therefore planets and comets must revolve continually [*perpetuo*] in orbits given in kind and in position, according to the laws set forth above. They will indeed persevere in their orbits by the laws of gravity, but they certainly could not originally have acquired the regular position of the orbits by these laws. (P940)[179]

[179] Compare the argument at the end of Newton's discussion of Proposition 10 (P816):

> In the spaces nearest to the earth, of course, nothing is found that creates resistance except air, exhalations, and vapors. If these are exhausted with great care from a hollow cylindrical glass vessel, heavy bodies fall within the glass vessel very freely and without any sensible resistance; gold itself and the lightest feather, dropped simultaneously, fall with equal velocity and, in falling through a distance of four or six or eight feet, reach the bottom at the same time, as has been found by experiment. And therefore in the heavens …

As observed in note 40 of my chapter on the Mechanics, Kant alludes to Newton's discussion of using the air pump to confirm the equality of accelerations in fall in connection with his discussion of using weight to determine quantity of matter. Newton's remark at the very end of the above passage leads to his well-known objections to a fully mechanical explanation of the regular constitution of the solar system (all planetary orbits are in the same direction and in

It appears quite likely, therefore, that Kant's remarks on the motions of the heavenly bodies through the aether in the Phenomenology proceed against the background of Newton's discussion of the same topic in the *Principia*. So it appears quite likely, in particular, that Kant means by "free and enduring" exactly what Newton means: the motions in question are affected only by gravity, and the heavenly bodies continually maintain their orbits under the influence of this force alone.[180]

If this is correct, however, then what is the significance of Kant's point? Why is it important to insist that the space-filling aether introduced at the end of the general remark to dynamics exerts an incomparably small mechanical resistance to the motions of the heavenly bodies? What Kant says here is related to – but by no means the same as – the point of his argument in the second remark to the seventh proposition of the Dynamics (see section 18 above). He there (515) considers the circumstance that Newton leaves it open that the original force of attraction (i.e., universal gravitation) might be merely apparent, so that it could be explained, in turn, by the pressure exerted by an external aether. Kant rejects this possibility himself, for the reason that we would then have difficulty in applying the equality of action and reaction directly to the gravitational attractions between distant bodies (such as Jupiter and Saturn), and we would thereby have difficulty in carrying out Newton's argument in Proposition 7 of Book 3 – according to which the attractive force exerted by the bodies in question is directly proportional (at a given distance) to their masses. Newton needs to assume, in particular, that conservation of momentum can be applied directly to these attractions, so that the resulting interactions between the bodies involve only negligible exchanges of momentum with the surrounding aether.[181]

almost the same plane, and so on), with the conclusion that "[t]his most elegant system of the sun, planets, and comets could not have arisen without the design and dominion of an intelligent and powerful being" (P940). One of the main goals of Kant's *Theory of the Heavens* is to combat this conclusion (and to defend Leibniz on this question against Clarke) with his own conception of how the regular constitution of the solar system could have indeed arisen purely "mechanically" in accordance with the nebular hypothesis.

[180] The second chapter of Part Two of the *Theory of the Heavens* not only exhibits considerable familiarity with Newton's calculations of planetary masses and densities in the corollaries to Proposition 8 of Book 3, but also with some of the calculations that Newton makes in the argument of the following Proposition 10 – for example, with Newton's estimate that the weight of the earth is approximately four to five times greater than it would be if it consisted only of water (see 1, 276).

[181] Compare again the paragraph to which note 192 of my chapter on the Dynamics is appended, together with the two following paragraphs. As explained in that note, Newton is here demonstrating the equality of inertial mass with what we now call active gravitational mass (note 142 above).

Nevertheless, whereas the argument of the seventh proposition of the Dynamics is thereby related to the corresponding argument at the end of the general remark to phenomenology, the two arguments (as suggested) are by no means the same. For the former is not intended to show that the cosmic aether has no significant effect at all on the motions of the heavenly bodies but only that it is not significantly involved in the specifically *gravitational* interactions between them. The original attraction (unlike the original repulsion) "pass[es] straight through" the intervening aetherial medium that actually separates them (516; see the passage to which note 156 of my chapter on the Dynamics is appended). Therefore, it is entirely compatible with the seventh proposition of the Dynamics that the cosmic aether could play no significant role in the momentum exchanges produced by gravitational interactions while, at the same time, exerting a significant mechanical resistance to the motions of the heavenly bodies. In other words, the specifically gravitational accelerations between these bodies could still satisfy the equality of action and reaction while there were also other accelerations of these same bodies (paired with corresponding exchanges of momentum) produced by the resistance of the aether.

The argument we are considering from the general remark to phenomenology is addressed to this last possibility. It aims to show that – or, more precisely, how – the aether can completely fill all cosmic space without also producing any appreciable (mechanical) resistance to the motions of the heavenly bodies. And why should we believe that the heavenly bodies do in fact move freely in this sense? Newton's argument in Proposition 10 of Book 3 is that any appreciable resistance would have an appreciable effect on the motions of the heavenly bodies – and that *this kind of effect is simply not observed*. For example (P816): "If Jupiter is a little denser than water, then in the space of thirty days (during which this planet describes a length of 459 semidiameters) it would, in a medium of the same density as our air, lose almost a tenth of its motions." But since (by the argument of the Scholium to Proposition 22 of Book 2) the density of the atmosphere at a distance of two hundred miles above the earth's surface is some 75 trillion times less than at the surface, "the planet Jupiter, revolving in a medium with the same density as that upper air, would not, in the time of a million years, lose a millionth of its motion as a result of the resistance of the medium" (P816).[182] In other words, if there were any appreciable

[182] The connection between this calculation and the calculation that the earth is about four to five times denser than water mentioned in note 180 above is that Newton has also argued, in Corollary 3 to Proposition 8, that the earth is four times denser than Jupiter – and so he can

resistance on the motion of Jupiter, its orbit would not have the character that it in fact does – which character, as Newton emphasizes in the General Scholium, is *precisely that which follows from his theory of universal gravitation*.[183]

Newton's theory describes the orbits of the heavenly bodies using no forces other than universal gravitation, and this theory, in turn, is derived from the Phenomena with which Book 3 begins. Hence, if any other forces did have an appreciable influence, they would eventually show up in the observable phenomena. To be sure, the theory of universal gravitation allows us to correct small errors in our descriptions of the initial Phenomena of Book 3. Taking account of the planetary perturbations, for example, reveals that Kepler's laws actually hold only approximately. The crucial point, however, is that this kind of correction involves only further (but more sophisticated) applications of the theory of universal gravitation itself, with no need, at least so far, to invoke additional forces from outside this theory. To precisely the extent to which we can then continue to succeed in this evolving program, we are justified in our (ever increasing) confidence that no other forces are appreciably involved – including no appreciable mechanical resistance deriving from a space-filling aether.[184]

Consider, by contrast, what our situation would be like if there were any appreciable resistance exerted by the aether – or, more generally, if forces other than gravity were appreciably implicated in the orbits of the heavenly bodies. Even if the theory of universal gravitation were completely correct, in so far as specifically gravitational accelerations all took place in accordance with this theory, it would be extremely difficult to extricate this theory from the now much more complicated observable motions. And, in any case, it would be quite impossible to derive (and

now conclude that Jupiter is a little denser than water in turn. He then appeals to the density of air being some 860 times less than that of water in his calculations of the motions of Jupiter through the air.

[183] See again the passage to which note 179 above is appended (P940, emphasis added): "[B]odies must move very freely in these spaces, and therefore planets and comets must revolve continually in orbits given in kind and in position, *according to the laws set forth above*."

[184] See notes 105, 106, and 107 above, together with the paragraphs to which they are appended and the intervening paragraph. According to Proposition 13, in particular, we first discuss the motions of the heavenly bodies "from the phenomena," and, after the theory of universal gravitation has been found, we then "deduce the celestial motions from these principles a priori" (P817). This procedure is not circular, however, because we encounter small deviations along the way, which, so we hope, the theory of gravitation can then continue fully to accommodate. As observed in note 130 above, the Newtonian program did indeed continue successfully to accommodate the planetary perturbations through the end of the nineteenth century – with the exception of extremely small effects that were eventually explained by general relativity at the beginning of the twentieth century.

then correct) the theory from phenomena in the manner of Book 3 of the *Principia*. Kant, I suggest, is here locating this fundamentally Newtonian point within the context of his own account of absolute space in the Phenomenology. In particular, we can only make the motion of matter into an object of experience, for Kant, by his procedure for reducing all motion and rest to absolute space – which, as explained, extends the argument of Book 3 to the cosmos as a whole (compare again the paragraph to which note 171 above is appended). The success of the argument of Book 3, for Kant, is thus a necessary condition for our being able to make motion into an object of experience in the first place. And in this way, I suggest, Kant is thereby introducing a transcendental dimension into Newton's argument in Proposition 10 of Book 3 that no appreciable resistance exerted by the aether is, as a matter of fact, ever observed.

In order to substantiate this suggestion, and also better to appreciate how the point Kant is making here fits into the overall argument of the *Metaphysical Foundations*, let us take up the text of the penultimate paragraph of the Phenomenology where we left off. In discussing empty space in the dynamical sense Kant has distinguished two different purposes that could be served by postulating either a *vacuum disseminatum* or a *vacuum coacervatum* – the former to explain specific differences in density, the latter to secure the free motion of the heavenly bodies. The culmination of the discussion of the second purpose, as observed, occurs in the following discussion of empty space in the *mechanical* sense. But the remainder of the present discussion of empty space in the dynamical sense is concerned almost exclusively with the first purpose and thus with the *vacuum disseminatum*:

> That it is not *necessary* to assume empty space for the *first purpose* has already been shown in the general remark to dynamics; but that it is *impossible* can in no way be proved from its concept alone, in accordance with the principle of noncontradiction. Nevertheless, even if no merely logical reason for rejecting this kind of empty space were to be found here, there could still be a more general physical reason for expelling it from the doctrine of nature – that of the possibility of the composition [*Zusammensetzung*] of a matter in general, if only this were better understood. (563)

So Kant is here referring to the proposed account of differences in specific densities in accordance with his metaphysical-dynamical approach expounded in the general remark to dynamics. He is also returning to the question deliberately left open in the general balancing argument of the fifth and sixth propositions of the Dynamics as to how exactly the fundamental repulsive force of matter (and thus its original expansive force) is counterbalanced by original attraction (compare note 166 above, together with the paragraph to which it is appended).

Kant continues, at this point, as follows:

> For, if the *attraction* that one assumes for the explanation of the cohesion [*Zusammenhang*] of matter should be merely apparent, not true attraction, but rather, say, merely the effect of a *compression* [*Zusammendrückung*] by an external matter distributed throughout the universe (the aether), which is itself brought to this pressure only by a universal and original attraction, namely, gravitation (which view has many reasons in its favor), then empty space within matters would be, although not logically, still dynamically and thus physically impossible – for this matter would itself expand into the empty spaces that one assumed within it (since nothing would resist its repulsive force here) and would always conserve them as filled. (563–64)

So Kant is here referring to his more specific (and hypothetical) realization of the balancing argument, discussed at the end of section 16 and the whole of section 17 above, according to which the internal expansive force of matter is typically counterbalanced by the external pressure exerted by an all pervasive, originally elastic medium: i.e., the aether.[185]

Kant emphasizes that his "more general physical reason for expelling [the *vacuum disseminatum*] from the doctrine of nature" (563) is indeed merely hypothetical (564): "It should not surprise anyone, however, that this refutation of empty space proceeds entirely hypothetically, for the assertion of empty space fares no better." This is precisely why the proposed explanation is one that would make empty space, in this sense, "not logically, [but] still dynamically and thus *physically* impossible" (564, emphasis added). It is also entirely in keeping with the circumstance that the general balancing argument articulated in the fifth and sixth propositions of the Dynamics belongs to Kant's a priori metaphysical treatment of the concept of matter, while the more specific realization of this

[185] Compare again notes 148 and 149 of my chapter on the Dynamics, together with the paragraph to which they are appended. The entire passage from which this one is extracted was already considered in section 17 (see the two paragraphs following the one to which note 161 of my chapter on the Dynamics is appended). As I explained there, cohesion [*Zusammenhang*], properly speaking, is a property only of liquid and solid matter (compare note 162 of my chapter on the Dynamics, together with the paragraph to which it is appended). Kant's full view of how the "composition [*Zusammensetzung*] of a matter *in general*" (563, emphasis added) is (hypothetically) effected thus appears to have the following form. Cohesive matters (liquids and solids) maintain this state by the external pressure exerted by the cosmic aether, whereas permanently elastic matters, by contrast, have limits set to their expansive force primarily by the external gravitational attraction of solid or liquid matters immersed within them – as atmospheric air experiences a compression effected by the earth, for example, or the aether itself experiences a compression effected by the heavenly bodies as a whole (and, in turn, maintains these same bodies in their state of cohesion by the resulting external pressure).

argument now at issue has the status of a (mere) physical hypothesis (see notes 166 and 173 above). Yet the *vacuum disseminatum* or empty space in the dynamical sense is essentially connected with the *vacuum coacervatum* or empty space in the mechanical sense: "the emptiness accumulated within the cosmos in order to provide the heavenly bodies with free motion" (564; see the paragraph to which note 178 above is appended, together with the two preceding paragraphs). So how is the idea that Kant's discussion, at this point in the Phenomenology, is concerned with a (mere) physical hypothesis consistent with my suggestion that Kant thereby intends to introduce a transcendental dimension into Newton's argument in Proposition 10 of Book 3 that "[t]he motions of the planets can endure for an extremely long time" (P815)?

The problem can be articulated more precisely as follows. Kant's treatment of the *vacuum coacervatum* introduces a transcendental dimension into Proposition 10 in so far as the overall success of Newton's argument from phenomena in Book 3 *presupposes* the free and enduring motions in question. But precisely this empirical success appears to be also the basis for such central and essential features of Kant's more general metaphysical treatment of the concept of matter as the immediacy and universality of the fundamental force of attraction (demonstrated in the seventh and eight propositions of the Dynamics) and the law of the equality of action and reaction (demonstrated in the fourth proposition of the Mechanics). For these features, on my reading, are also presupposed by the success of Newton's argument. They are therefore presupposed by Kant's procedure, developed throughout the *Metaphysical Foundations*, for reducing all motion and rest to absolute space – which, I have argued, is modeled on precisely the argument of Book 3. So how can we count these features as belonging to Kant's properly metaphysical treatment of the concept of matter while, at the same time, relegating his discussion of the *vacuum disseminatum* and the *vacuum coacervatum* to a merely physical treatment?

Moreover, as explained in sections 9 and 10 above, the dynamical concept of the filling of space that Kant introduces at the very beginning of the Dynamics depends on the empirical concept of dynamical as opposed to mathematical impenetrability.[186] And what is the empirical basis for Kant's specifically dynamical concept of the filling of space? As observed

[186] For the distinction between mathematical and dynamical impenetrability see the paragraph to which note 17 of my chapter on the Dynamics is appended, together with the following paragraph. For the sense in which the concept of impenetrability, in particular, is essential to what Kant calls the *empirical* concept of matter (in contrast to the *pure* concept of the real in space) see the paragraph to which note 7 of that same chapter is appended.

in section 29 above, Kant returns to the question of absolute or mathematical impenetrability – as opposed to his own favored concept of relative or dynamical impenetrability – in the general remark to mechanics. He develops a conception of continuously acting causality, modeled on Galileo's treatment of the acceleration of fall and Newton's generalization of this treatment, and then appeals to this same conception in finally decisively rejecting the competing model of instantaneous discontinuous action traditionally associated with the concept of absolute hardness. The argument depends on the previously articulated dynamical theory of matter, and so, as observed, it is ultimately circular. Nevertheless, it has the great virtue of exhibiting the crucial importance of the model of continuously acting causality in the argument for universal gravitation in Book 3 of the *Principia* and suggesting, accordingly, that the very same model should now be extended to impact as well.[187] Hence, it is only the continuing empirical success of Newton's argument for universal gravitation that ultimately gives Kant a decisive empirical basis for preferring his own dynamical concept of matter to the opposing mathematical concept.

So far, therefore, Kant's discussions of the *vacuum coacervatum* and such fundamental propositions as the immediacy and universality of the fundamental force of attraction and the law of the equality of action and reaction appear to be completely on a par. We begin, in both cases, with the impressive continuing empirical success of Newton's argument and thereby presuppose, in effect, that the bodies in the solar system present us with an approximately closed system in which no other forces than universal gravitation exert an appreciable influence. In both cases, in particular, we presuppose that there are no appreciable exchanges of momentum between the aether and the bodies in the solar system. No such exchanges appear to occur when considering only the specifically gravitational interactions among these bodies, and, more generally, no such exchanges appear to occur at all.[188] Moreover, this empirical presupposition acquires a transcendental dimension, in both cases, simply because without it neither Newton's determination of the center of mass of the solar system nor (*a fortiori*) Kant's procedure for reducing all

[187] For the resulting circularity arising from Kant's appeal to the dynamical theory of matter see note 171 of my chapter on the Mechanics, together with the paragraph to which is appended. For the connection between Kant's use of the model of continuously acting causality and the argument of the *Principia* see the remainder of section 29 above.

[188] See again note 181 above, together with the paragraph to which it is appended and the following paragraph. As explained, the difference between the two cases is that the theory of universal gravitation could still be true in the second case but not (of course) in the first.

motion and rest to absolute space could succeed. Yet the application of the equality of action and reaction to the motions of the bodies in the solar system (such as Jupiter and Saturn) – which are assumed to interact with one another immediately and at a distance – is secured in the seventh proposition of the Dynamics as an essential part of Kant's general dynamical theory of matter. Kant's discussion of the *vacuum coacervatum*, by contrast, is confined to the general remark to phenomenology in the context of a merely hypothetical particular realization of this general theory.

So what, if anything, privileges the general dynamical theory of matter articulated in the Dynamics and then applied in the Mechanics? Why does it belong to what Kant himself calls a metaphysical as opposed to a merely physical consideration of the concept of matter? An illuminating response to this question emerges from my discussion at the beginning of the present section. Kant's general dynamical theory of matter is not intended to provide a hypothetical account of the empirically given properties of matter in competition with other such accounts. Nor is it intended to establish these properties a priori, entirely independently of their actual empirical basis. It is intended, rather, to provide a constructive analysis of the conditions for successfully applying mathematics to our empirical concept of matter – and, in particular, to the concept of mass or quantity of matter that Newton deploys in the *Principia*. Thus, in particular, the reason that such fundamental propositions as the immediacy and universality of the original attraction and the law of the equality of action and reaction belong to what Kant calls metaphysics is that these propositions, in his view, articulate just those features of the concept of matter that explain the possibility of Newton's successful mathematization of its quantity.[189]

Kant's discussion of the *vacuum coacervatum* in the general remark to phenomenology, by contrast, does not explain the application of mathematics to any additional constituent concept belonging to the empirical concept of matter. Indeed, since the Phenomenology, as observed, introduces no new determinations of matter at all but only explains how matter (with all of the determinations that have already been explicated in the previous chapters) is then related to our cognitive faculties, there is no

[189] See the paragraph to which note 164 above is appended, together with the two preceding paragraphs. The crucial point is that only Newton's theory of universal gravitation has shown us how to extend the terrestrial measure of weight to a completely universal measure of mass for all bodies in the universe in connection with the traditional – and equally universal – concept of quantity, bulk, or amount of matter.

room for this kind of explanation in any case.[190] Thus, for example, while velocity becomes a mathematical magnitude in the Phoronomy, and both quantity of matter and quantity of motion become mathematical magnitudes in the Mechanics, no new physical quantities become mathematical magnitudes in the Phenomenology. The role of the Phenomenology, from this point of view, is rather to exhibit the significance of the applications of mathematics that have already been explained in the previous chapters by delineating their precise roles and functions within the Newtonian argument for universal gravitation. In particular, while the discussion of the *vacuum coacervatum* in the general remark indeed has a transcendental dimension, in so far as it involves a (further) necessary condition for the success of Newton's argument, it does not explain the possibility of any new applications of mathematics. So it does not, by Kant's lights, belong to what he understands as metaphysics.

I argued in the Introduction that what Kant calls a special metaphysical natural science – as opposed to the general metaphysics of nature articulated in the first *Critique* – is only possible on the basis of *both* metaphysics *and* mathematics. In particular, the special metaphysics of corporeal nature has the task of explaining the possibility of mathematically constructing the constituent concepts belonging to the empirical concept of matter. The application of mathematics whose possibility is explained in the Mechanics – the mathematization of the concept of mass or quantity of matter – is especially important, because we here see how the content of specifically empirical (as opposed to pure) intuition becomes increasingly important in Kant's metaphysical foundations for natural science. Kant thereby offers an explanation of the possibility of the spectacular mathematization of the *empirical* world achieved by modern natural science culminating in Newton. Moreover, because of the central role of the analogies of experience in this explanation, it also contributes, at the same time, to the explanation of the possibility of what Kant calls experience.

The central role of the analogies of experience – corresponding to the categories of substance, causality, and community – explains why Kant calls this enterprise metaphysics. For these are precisely the fundamental concepts of the Leibnizean metaphysical tradition he inherited. I

[190] See the two paragraphs following the one to which notes 1 and 2 above are appended, where I relate this peculiarity of the Phenomenology to the corresponding postulates of empirical thought in the first *Critique*. Just as the modal categories do not further characterize a given empirical object as either a magnitude, a reality, or a substance, for example, the Phenomenology adds no further constituent concepts to those already definitive of Kant's concept of matter: motion, the filling of space, and moving force.

discussed the role of the analogies in considerable detail in my chapter on the Mechanics, and the most important result of that chapter concerns the metaphysical concept of substance. In section 25 above, in particular, I discuss how the principle of conservation of the total quantity of matter – a principle closely related, for Kant, to conservation of the total quantity of motion or momentum – is implicated in the principle of conservation of the total quantity of *substance* that is first formulated explicitly in the second edition of the *Critique*. And the centrality of the concept of substance in Kant's argument again bears witness to the importance of the Leibnizean metaphysical tradition. But the circumstance that Kant then finds it necessary, in the course of his argument, explicitly to reject the Leibnizean commitment to substantial simplicity indicates that he has now found it necessary radically to revise this tradition in light of precisely the novel mathematization of the concept of quantity of matter accomplished by Newton.

Finally, the importance of Newton's mathematization of the concept of quantity of matter, as already suggested, also explains Kant's attitude towards the issue between Huygens and Newton concerning the shape of the earth (see again note 164 above, together with the paragraph to which it is appended). Kant is well aware that the differences between them concern precisely Newton's commitment to truly universal gravitation between each part of matter and every other part. He is also well aware that their different approaches result in differing empirical predictions (at least in principle) for the exact amount of the earth's centrifugal bulging. In sharp contrast with other natural philosophers of the time, however, Kant appears to be quite unconcerned with possible empirical tests of this issue – both in the *Theory of the Heavens* and, more importantly, in the general remark to phenomenology in the *Metaphysical Foundations*.[191]

Yet Kant's at first sight peculiar attitude towards this question is not based on either ignorance of or disregard for the properly empirical basis for Newton's mathematization. On the contrary, Kant is explicit that it depends on both the true universality and immediacy of gravitation between each part of matter and every other and the equality of all gravitational accelerations at equal distances first discovered by Galileo.[192]

[191] See notes 128, 129, and 130 above, together with the paragraphs to which they are appended. I argue there, and throughout section 34 above, that the fundamental difference between Huygens and Newton on this issue is centrally important (albeit implicitly) throughout Kant's discussion of determining the earth's true rotation in the Phenomenology.

[192] The former is responsible for gravitation being a penetrating force and thereby underlies the equality of inertial mass with what we now call active gravitational mass. The latter is the generalized form of Galileo's discovery that all bodies fall the same in a gravitational field and

He is also well aware that these fundamental features of Newtonian universal gravitation presuppose corresponding empirical phenomena: the (approximate) validity of Galileo's law of fall for terrestrial gravity, Newton's generalization and extension of this law to both terrestrial and celestial gravity in accordance with the moon test of Propositions 3 and 4 of Book 3, and Newton's further generalization and extension of his argument to truly universal gravitation in Propositions 5–20 of Book 3 in accordance with all of the initial Phenomena presented at the beginning. Indeed, I have provided what I take to be compelling reasons, throughout my reading, to consider precisely this Newtonian argument "from phenomena" as Kant's model for how *appearances* are transformed into *experience* in the Phenomenology. The upshot, once again, is that the general dynamical theory of matter on which Kant builds a foundation for Newton's argument is by no means intended to turn clearly empirical properties of the force of universal gravitation into a priori demonstrative truths. It is intended, rather, to elucidate the way in which the Newtonian mathematization of the concepts of mass, force, and interaction can be considered as a specific realization or instantiation of the categories of substance, causality, and community in Kant's constructive analysis. And it is for precisely this reason, for Kant, that his general dynamical theory then counts as metaphysical as opposed to (merely) physical.

36 REASON, THE EMPIRICAL CONCEPT OF MATTER, AND THE CATEGORIES OF MODALITY

The final paragraph of the Phenomenology (and thus of the *Metaphysical Foundations* as a whole) consists of a single long sentence reflecting on the faculty of reason, which emphasizes, in particular, the necessary striving of this faculty to comprehend the totality of all conditions for a given conditioned:

> And so ends the metaphysical doctrine of body with the *empty* and, for precisely this reason, [the] inconceivable, wherein it shares the same fate as all other attempts of reason when it strives after the first grounds of things in a retreat to principles – where, since its very nature entails that it can never conceive

thereby underlies the equality of inertial mass with what we now call passive gravitational mass (see again note 142 above, together with the paragraph to which it is appended). As explained in section 24 above, Kant gives priority in the first proposition of the Mechanics to determining quantity of matter via weight – against the background of precisely the empirical property of the equality of fall discovered by Galileo (see notes 37–41 of my chapter on the Mechanics, together with the paragraphs to which they are appended).

anything except in so far as it is determined under given conditions, and since it can therefore neither come to rest at the conditioned nor make the unconditioned comprehensible, nothing is left to it, when thirst for knowledge summons it to comprehend the absolute totality of all conditions, but to turn away from the objects back to itself, in order to investigate and to determine, not the ultimate limits of things, but instead the ultimate limits of the powers [*Vermögens*] it reserves to itself. (564–65)

I shall argue in the present section that this paragraph is no mere rhetorical flourish, which could therefore be safely ignored. On the contrary, when properly read against the background of his preceding discussion of the various concepts of empty space, it sheds much light on Kant's conception of the interconnections between the two intellectual faculties of reason and understanding, on the relationship between the argument of the Phenomenology and the postulates of empirical thought in the first *Critique*, and on how, exactly, this last chapter of the *Metaphysical Foundations* serves to sum up and retrospectively comment upon the argument of the three previous chapters.

To begin with, Kant's suggestion of how reason should proceed here – "to turn away from the objects back to itself [*von den Gegenständen auf sich selbst zurückzukehren*]" (565) – appears to be closely connected with the famous Copernican analogy for the revolution in metaphysics proposed in the Preface to the second edition of the *Critique*.[193] For the basic idea there, as discussed in section 30 above, is that, just as Copernicus proposed to make better progress in astronomy by locating the source of (some of) the observed motions in the heavens in the observer rather than the heavenly bodies observed (the starry heavens), Kant is now proposing to make better progress in metaphysics by turning our attention towards the cognitive faculties [*Vermögens*] of the subject (sensibility, understanding, and reason) and away from the objects as they may exist in themselves.[194] Moreover, as I also explained in section 30, these well-known

[193] See note 6 above, together with the paragraph to which it is appended. I there quote Kant's initial statement of the Copernican analogy (Bxvi):

It is precisely the same here as with the first thoughts of Copernicus, who, after he was not able to progress well in the explanation of the motions of the heavens when he assumed that the entire starry host rotates around the observer [*Zuschauer*], attempted [to see] whether it might not succeed better if he allowed the observer to rotate and, on the contrary, the stars [to remain] at rest.

[194] See notes 8 and 9 above, together with the paragraph to which they are appended – where, in particular, we see that both sensibility and understanding are implicated in Kant's proposed revolution. In his introductory remarks (Bxiv–xv), beginning with the passage quoted in note 6 above (Bxiv), Kant makes it clear that the faculty of *reason* is primarily at issue when it comes to the possibility of metaphysics. As quoted in note 7 above, Kant's description of his proposed revolution in metaphysics reads (Bxvi):

remarks from the second edition Preface are themselves closely connected with Kant's description of the task of the Phenomenology in his remark to the first explication: namely, to explain how the mere (sensible) appearance of motion is transformed into an object of experience through a (conceptual) determination by the faculty of understanding. So the suggestion that now, at the very end of the Phenomenology, Kant intends thereby retrospectively to comment upon the entire argument of this chapter appears to be by no means far-fetched.

This suggestion is further supported, moreover, by the circumstance that the second edition Preface goes on to explain how the antinomies of pure reason provide secure confirmation of Kant's proposed Copernican hypothesis:

> For that which necessarily drives us to go beyond the limits of experience and all appearances is the *unconditioned*, which reason necessarily and rightly demands in the things in themselves for every conditioned, and thereby demands that the series of conditioned be completed. But if we find, when we assume that our empirical cognition conforms to the objects as things in themselves [*richte sich nach den Gegenständen als Dingen an sich selbst*], that the unconditioned can *in no way be thought without contradiction*; and, on the contrary, that the *contradiction disappears* when we assume that our representation of things, as they are given to us, does not conform to them as things in themselves, but these objects, as appearances, rather conform to our mode of representation [*sondern diese Gegenstände vielmehr, als Erscheinungen, richten sich nach unserer Vorstellungsart*]; and, therefore, that the unconditioned must not be found in things in so far as we are acquainted with them (as they are given to us), but rather in things in so far as we are not acquainted with them (as things in themselves) – then [all] this would show that what we assumed initially only as an experiment is well grounded. (Bxx–xxi)

It appears quite likely, therefore, that Kant's reflections on reason and the unconditioned at the end of the Phenomenology are indeed closely connected with the discussion of his Copernican hypothesis (one year later) in the second edition Preface.[195]

> Until now one assumed that all of our cognition must conform to the objects [*müsse sich nach den Gegenständen richten*]; but all attempts to decide something about them a priori through concepts, whereby our cognition would be expanded, came (under this presupposition) to nothing. One should therefore once attempt [to see] whether we might make better progress in metaphysics by assuming that the objects must conform to our cognition [*müssen sich nach unserem Erkenntnis richten*].

[195] We already know that in an important footnote to the second edition Preface (Bxxii) Kant compares his own "proof" in the body of the *Critique* of his initially proposed Copernican analogy with Newton's proof of the original Copernican hypothesis: see note 2 above, its references back to the Phoronomy chapter, and the paragraph to which it is appended. I can now add that the footnote in question (quoted in note 10 of my chapter on the Phoronomy) is attached to precisely the paragraph presently under consideration (Bxviii–xxii), in which the antinomies figure

I observed in section 14 above that Kant's characteristic conception of absolute space as an idea of reason is intimately connected with the argument of the first antinomy.[196] It is by no means surprising, then, that Kant's discussion of how matter is determined with regard to the predicate of motion at the beginning of the Phenomenology provides us with a striking illustration of what he says at the very end. The understanding can determine an object of experience with respect to motion or rest only relative to a given "condition [*Bedingung*]" – an empirically given relative space constituting a *correlate* of the motion or rest of the object in question. But for any such given condition or correlate we can always ask whether it is moving in turn, and this necessarily requires us to specify a further condition (an expanded empirically given relative space) as correlate.[197] Moreover, as Kant says, we can neither stop such a regress at any given finite stage nor consider the totality of stages all at once. So our only choice is to relinquish, once and for all, the conception of absolute space as a genuine *object* (of experience) in favor of an indefinitely extended empirical progress of our own understanding and reason. In this empirical progress, in particular, we apply the three analogies of experience, as instantiated by Kant's three Laws of Mechanics, successively to approximate (but never actually to reach) the purely regulative idea Kant calls "absolute space." And the result, as observed in section 35 above, is precisely what Kant calls empty space in the phoronomical sense in the general remark to phenomenology.[198]

prominently. The preceding footnote to this same paragraph draws a further analogy between Kant's proposed revolution in metaphysics and chemistry (Bxxi):

> This experiment of pure reason has much in common with that of the *chemists*, which they sometimes call the experiment of *reduction* but in general the *synthetic procedure*. The *analysis* of the *metaphysician* divided pure a priori cognition into two heterogeneous elements, namely, that of things as appearances and then of things in themselves. The *dialectic* connects the two again into *unanimity* with the necessary idea of reason of the *unconditioned*, and [it] finds that this unanimity will never arise except by the latter distinction – which is therefore the correct one.

[196] See note 86 of my chapter on the Dynamics, together with the paragraph to which it is appended and the two following paragraphs. In these terms, Kant's conception of absolute space as a forever unreachable regulative idea of reason is intimately connected, in particular, with what Kant calls "the regulative principle of pure reason in regard to the cosmological ideas" (§8 of the antinomy chapter).

[197] For the notion of "correlate [*Correlat*]" see the passage (554; from the remark to the first explication) to which note 43 above is appended. At the end of this passage Kant speaks of the necessity "to indicate the conditions [*Bedingungen*] under which the object (matter) must be determined in one way or another by the predicate of motion" (554).

[198] See notes 169–73 above, together with the paragraphs to which they are appended. I emphasize there that this concept of empty space in the phoronomical sense involves a fundamental reconsideration of the discussion of absolute space in the Phoronomy in light of the argument of the Phenomenology.

I also observed, at the very beginning of section 14, that Kant's most explicit appeal to the antinomies of pure reason in the *Metaphysical Foundations* appears in his pivotal second remark to the fourth proposition of the Dynamics. This remark initiates a transition from the fundamental force of repulsion to the fundamental force of attraction by discussing the relationship between the infinite divisibility of matter and the critical doctrine of transcendental idealism. Just as (in accordance with the first antinomy) Kant decisively rejects absolute space considered as an actual object in favor of a never to be completed empirical regress into more and more expanded relative spaces, he also (in accordance with the second antinomy) decisively rejects the Leibnizean–Wolffian conception of monads as ultimately simple "physical points" in favor of a never to be completed empirical regress into smaller and smaller parts of continuously distributed matter.[199] In addition, just as Kant's conception of a maximally comprehensive absolute space reconceives it as a regulative idea of reason, his conception of the infinite divisibility of material substance invokes a merely infinitesimal "idea of a space" as the limit of a corresponding empirical regress in this case.[200]

Finally, as explained in section 20 above, Kant makes a parallel move in the general remark to dynamics while rejecting the conception of the ultimate constituents of matter characteristic of the mechanical natural philosophy: that is, absolutely impenetrable elementary corpuscles and the void. Whereas his critical conception of continuously distributed matter begins with the observable large-scale properties of matter and then attempts (experimentally) to investigate its more fine-grained internal structure "from the outside in," the mechanical natural philosophy takes the absolutely impenetrable and the absolutely empty as

[199] See notes 90–98 of my chapter on the Dynamics, together with the paragraphs to which they are appended. I there emphasize the fundamental difference between Leibniz and his Leibnizean–Wolffian followers on this issue (since Leibniz's ultimately simple monads are certainly not in space). I also emphasize that Kant is clear about this difference both in the second antinomy and here in the *Metaphysical Foundations*. Indeed, as discussed in notes 99–102 of this same chapter, together with the paragraphs to which they are appended, Kant even attempts (in the second remark to the fourth proposition) to appropriate Leibniz himself on behalf of transcendental idealism.

[200] See notes 103 and 104 of my chapter on the Dynamics, together with the paragraph to which they are appended. I explain that Kant introduces the merely infinitesimal idea of a space in connection with the original expansive force of matter and the Boyle–Mariotte law in an attempt to appropriate Newton's treatment of this law that is partially analogous to his attempted appropriation of Leibniz. I argue that the point of this twin attempt at appropriation, in the end, is to prepare the way for Kant's radical transformation of the Leibnizean conception of the simplicity of substance on behalf of a parallel reinterpretation of the Newtonian concept of quantity of matter.

primitive or self-explanatory and then attempts (hypothetically) to arrive at the observable large-scale properties of matter "from the inside out." In this case, from Kant's point of view, we erect an ultimate barrier to the further progress of scientific inquiry beyond which (or below which) it is simply impossible to penetrate. We thereby violate his conception of the regulative use of reason – according to which we can by no means "*anticipate* what is given *in the object* prior to all regress in itself" (A509/B537) and thus "cannot say [prior to the regress] *what the object is* but only *how the empirical regress is to be undertaken*" (A510/B538).[201]

In the terms of Kant's discussion of the various concepts of empty space in the general remark to phenomenology, therefore, we have now arrived at the question of empty space in the dynamical sense. We are considering, more specifically, the proposed explanation of specific differences in density based on interspersed pores of empty space – the *vacuum disseminatum* – in an otherwise absolutely dense (absolutely incompressible) material. Kant had earlier considered and rejected this explanation in the general remark to dynamics, on the grounds that "no experience, or inference from experience, or necessary hypothesis for its explanation, can justify us in assuming empty spaces as *actual*" (535; see note 176 above): "For all experience gives us only comparatively empty spaces for our cognition, which can be completely explained from the property of matter of filling its space with greater or ever smaller expansive force to infinity, in accordance with all desired degrees, without requiring empty spaces." So the empty, in this sense, must also be "inconceivable" (564) precisely because reason "can never conceive anything except in so far as it is determined under given conditions" (564–65) – and we are entertaining a supposed determining condition (the void) that can never be given in experience.[202]

[201] See notes 243–47 of my chapter on the Dynamics, together with the paragraphs to which they are appended. As I explain, although there are indeed fundamental differences between the Newtonian and the mechanical natural philosophies, they share essentially the same conception of the ultimate constituents of matter (compare notes 220 and 249 of this same chapter, together with the paragraphs to which they are appended).

[202] Compare the discussion from the first antinomy quoted in note 86 of my chapter on the Dynamics (A517/B545):

> Here, just as much as in the other cosmological questions, the ground of the regulative principle of reason is the proposition that in the empirical regress *no experience of an absolute limit*, and thus no condition which, as such, is *empirically absolutely unconditioned*, can be met with. And the reason is that such an experience would have to contain a limitation of the appearances by means of nothing, or the empty, which the continued regress could encounter by means of perception – which is impossible.

Kant's invocation of the faculty of reason and its necessary striving to comprehend the absolute totality of empirical conditions in the final paragraph of the Phenomenology is therefore inextricably connected with the argument of the *Metaphysical Foundations* as a whole. It is also connected, at the same time, with Kant's consideration of empty space in both the phoronomical and dynamical senses in the general remark to phenomenology. But how is it connected with the main argument of the Phenomenology itself, which has the very specific goal of characterizing different attributions of motion to matter (rectilinear, circular, equal and opposite) in terms of the three modal categories of possibility, actuality, and necessity? Of course this goal, as we already know, involves Kant's characteristic conception of absolute space – and thus, once again, empty space in the *phoronomical* sense. And so our further task, at this point, is to describe in more detail how Kant's procedure for reducing all motion and rest to absolute space is also essentially connected with both the modal categories and the necessary strivings of the faculty of reason.

I have just observed that reason makes an appearance at the very beginning of the Phenomenology in the passage concerning the two "correlates [*Correlata*]" that are always involved in any determination of motion (554; compare note 197 above, together with the paragraph to which it is appended). Indeed, this passage explicitly invokes the faculty of reason in describing a threefold progression through the modal categories, which, on Kant's view, is necessarily involved in the empirical determination of matter with respect to the predicate of motion:

> But now motion is change of relation in space. There are thus always two correlates here, such that either, *first*, the change can be attributed in the appearance to one just as well as to the other, and either the *one* or the *other* can be said to be moved, because the two cases are equivalent; or, *second*, one must be thought in experience as moved to the exclusion of the other; or, *third*, both must be necessarily represented ***through reason*** as equally moved. (554, bold emphasis added)[203]

Kant is here discussing the first antinomy, not the second. Nevertheless, as he indicates, essentially the same considerations apply in both cases. Indeed, as we have seen, Kant considers both empty space in the phoronomical sense (corresponding to absolute space) and empty space in the dynamical sense (corresponding to the *vacuum disseminatum*) in the general remark to phenomenology. Invoking either one would involve something "empirically absolutely unconditioned," i.e., "a limitation of the appearances by means of nothing, or the empty."

[203] This passage comes from the remark to the initial explication of the Phenomenology: see again the fuller passage to which note 43 above is appended.

I also observed earlier that the threefold progression described in this passage reflects Kant's more general conception of the modal categories in the postulates of empirical thought. There Kant remarks on the "peculiarity" of these categories (A219/B266), "that, as determination of the object, they do not in the least augment the concept to which they are ascribed as predicate, but only express the relation [of this concept] to the faculty of cognition [*Erkenntnisvermögen*]." He also describes an analogous progression from "the understanding and its empirical employment, to the empirical power of judgement, and to reason (in its application to experience)" (A219/B266). The passage from the beginning of the Phenomenology (554) just mentioned makes a closely analogous point with respect to the modalities of motion.[204]

This "peculiarity" of the modal categories is suggested much earlier in the *Critique*, where Kant makes a corresponding point about the modal forms of judgement (A74–75/B99–100):

The modality of judgements is a completely peculiar function of them, which has the distinctive character [by contrast with the other logical forms] that it contributes nothing to the content of the judgement (for, aside from quantity, quality, and relation, there is nothing more that constitutes the content of a judgement), but concerns only the value of the copula in relation to thought in general. *Problematic* judgements are such that one takes the affirmation or negation as merely *possible* (arbitrary); *assertoric* where it is considered as actual (true); *apodictic* in which one views it as *necessary*.*

And, in the footnote, Kant also suggests a correspondence with our three characteristically intellectual cognitive faculties (A75/B100): "It is just as if, in the first case, thought were a function of the *understanding*, in the second of the *power of judgement*, in the third of *reason*."

This threefold procedure is then illustrated by the example of the hypothetical syllogism:

The problematic proposition is therefore one that expresses only logical possibility (which is not objective), i.e., the free choice to allow such a proposition to hold, a merely arbitrary reception of it in the understanding. The assertoric [proposition] speaks of logical actuality or truth – as, for example, in a hypothetical syllogism the antecedent is problematic in the major premise and appears assertorically in the minor premise – and it indicates that it is already bound

[204] See again note 45 above, together with the paragraph to which it is appended. As explained, in order to make the two passages line up completely we would need to equate the reference to "experience" in the Phenomenology with "the empirical power of judgement" and, accordingly, to subsume the reference to "the appearance" in the Phenomenology under "the understanding and its empirical employment." And, once again, I shall return to this question below.

up with the understanding and its laws. The apodictic proposition thinks the assertoric [proposition] as itself determined by these laws of the understanding, and therefore as asserted a priori – and, in this way, it expresses logical necessity. Now, since everything is here incorporated into the understanding step by step, in so far as one first judges something problematically, then assumes it assertorically as true, and finally asserts it as inseparably bound up with the understanding, i.e., as necessary and apodictic, one can call these three functions of modality just so many moments of thought in general. (A75–76/B100–1)

In a hypothetical syllogism the antecedent is merely problematic in the major premise, because it appears only as a not yet asserted condition of a compound conditional statement. In the minor premise, by contrast, it is definitely asserted in its own right and therefore accepted as true. But what does it mean for this very same proposition (the *antecedent* of the major premise) to be then "determined by [the] laws of the understanding" in such a way that it finally counts as apodictally (a priori) true and necessary? The answer to this question, in turn, only becomes clear in Kant's later discussion of the modal categories in the principles chapter.

Just as Kant calls the modal functions of judgement "moments of thought in general [*Momente des Denkens überhaupt*]" (A76/B101), his later discussion of the modal categories is organized around "postulates of empirical thought in general [*Postulate des empirischen Denkens überhaupt*]" (A218/B265). The modal categories, however, are not mere (analytic) logical forms but rather, as Kant says, express the forms of an empirical synthesis in general.[205] The three postulates of empirical thought governing such synthesis are as follows:

1. That which agrees with the formal conditions of experience (according to intuition and concepts), is *possible*.
2. That which coheres [*zusammenhängt*] with the material conditions of experience (sensation), is *actual*.

[205] Kant strongly emphasizes this point immediately following the passage from the postulates just quoted above (A219/B266–67):

For this very reason the principles of modality are also nothing more than explications of the concepts of possibility, actuality, and necessity in their empirical use, and thus, at the same time, restrictions of all the categories to their merely empirical use, without allowing or permitting their transcendental use. For, if these are not to have a merely logical meaning and to express the form of *thought* analytically, but are to concern *things* and their possibility, actuality, or necessity, they must then extend to possible experience and its synthetic unity, in which alone objects of cognition are given.

By contrast, Kant's earlier discussion of the modal forms of judgement appears to involve merely analytic judgements – such as "if there is perfect justice then obstinate evil is punished" and "the world exists either through blind chance or through inner necessity or through an external cause" (A73–74/B98–99).

3. That whose coherence [*Zusammenhang*] with the actual is determined in accordance with the universal conditions of experience, is (exists as) *necessary.* (A218/B265–66)

The formal conditions of experience mentioned in the first postulate include all pure concepts of the understanding (as applied to our pure forms of intuition).[206] The material conditions mentioned in the second postulate include given sensations or empirical perceptions. So Kant is suggesting in the third postulate that empirical judgements conforming to the second postulate can now be further determined by the universal formal conditions of experience mentioned in the first postulate and that the laws of the understanding through which a necessary or apodictic judgement can arise are precisely the *synthetic a priori* categorical principles.[207]

In order to illustrate this procedure concretely, and also appreciate its relevance for the main argument of the Phenomenology, let us now return to my discussion of determining the state of true (or actual) rotation of the earth in section 34 above. Here, as explained, there is a crucial transition between two different stages in the empirical determination that the earth (as opposed to the surrounding starry heavens) is truly rotating. Kant begins by considering how this rotation can be verified by Coriolis accelerations resulting in observed deviations from rectilinear fall towards the center of the earth in accordance with (Galilean) terrestrial gravity. For example, a falling body dropped into a rotating earth experiences a horizontal velocity equal to the (linear) velocity of rotation at the surface, which is then conserved in its quantity (in accordance with the law of inertia) as the body falls further towards the center.[208] Kant concludes, however, by considering how this same rotation can be verified by the centrifugal bulging of the earth at

[206] See note 33 above (where the first postulate is quoted and discussed), together with the paragraph to which it is appended. So what is at issue here are the *schematized* categories and thus the *synthetic* principles of pure understanding.

[207] By contrast, in the examples discussed in connection with the modal logical forms of judgement (note 205 above) Kant appears to be envisioning *analytic* proofs (in traditional rational theology) of the initially merely problematic propositions. We analytically demonstrate (by a traditional rational argument for the existence of God, for example) that there is, in fact, perfect justice, or we analytically demonstrate (by a similar argument) that the world *cannot* exist by blind chance. Such a derivation would show that the antecedent of the major premise is *necessarily* true, and thus that its consequent is as well

[208] For verifying the earth's true rotation by means of Coriolis accelerations see note 116 above, together with the paragraph to which it is appended. In the two paragraphs following the latter I point to infelicities in Kant's discussion of this stage that motivate a transition to the second stage.

the equator. Here the effect of mutual gravitational attraction between each part of the earth and every other part (which, acting alone, would result in a perfectly spherical shape) is systematically diminished by the centrifugal endeavor of rotation (resulting in an oblate spheroidal shape). And it is only at this second stage, Kant suggests, that we have finally "reduced all motion and rest to absolute space" (560) by finding an "explanatory ground" (561) of the motions in question that "unites all appearances" (560).[209]

Since it involves an application of the law of inertia in accordance with the second proposition of the Phenomenology, the first stage corresponds to an attribution of the modality of *actuality* to the earth's rotation. The second stage, however, involves an application of the equality of action and reaction in accordance with the third proposition of the Phenomenology, and it therefore corresponds to an attribution of the modality of *necessity* to this same rotation.[210] The first stage results in an empirical judgement based on our perceptions of the falling body under the influence of Galilean terrestrial gravity, but this judgement does not yet count as fully grounded or determined, for Kant, until it is properly united with all other relevant appearances "in accordance with the *universal* conditions of experience" (A212/B266, emphasis added). Therefore in the present context, it must be properly united with all other relevant appearances (i.e., all other relevant apparent motions) in accordance with all of Kant's three Laws of Mechanics. Galilean *terrestrial* gravity must then be subsumed under the Newtonian theory of *universal* gravitation; for it is only within this theory that the earth both attracts and is attracted by all falling bodies, that every heavenly body in the solar system both attracts and is attracted by all other such bodies, that every part of the earth both attracts and is attracted by every other part, and so on. And it is only at this stage, finally, that we have thereby found a proper relative space or reference frame (determined by the center of mass of the solar system)

[209] I begin the transition from the first to the second stage in the paragraph to which note 116 above is appended, together with the two following paragraphs. I arrive at the "explanatory ground" that "unites all appearances" in the paragraphs to which notes 144 and 146 above are appended.

[210] That the first and second stages are related in this way begins to emerge in the paragraph to which note 119 above is appended, which initiates an explanation of how Kant's discussion here is to be embedded within the Newtonian argument for universal gravitation. I then explicitly arrive at the fundamental importance of the equality of action and reaction in this connection in the paragraph to which note 144 above is appended.

for considering simultaneously all the relevant (apparent) motions together.[211]

I have said that Kant's discussion of true circular motion involves a transition from attributing the modality of actuality to the rotation of the earth to attributing the modality of necessity to this same rotation. Indeed, it is only in this way that Kant's discussion in the Phenomenology can correspond to his treatment of the modal categories in the postulates of empirical thought – where, as explained, the third postulate arrives at the category of necessity by *further* determining that which has already been judged to be actual (see the paragraph to which note 207 above is appended). This may appear puzzling, however, because *circular* motion is supposed to fall under the modality of actuality (in accordance with the second proposition), whereas *equal and opposite* motion then falls under the modality of necessity (in accordance with the third proposition). In cases of circular motion, moreover, only one of the two correlates involved is represented as moved (to the exclusion of the other), whereas both of the two correlates involved are represented as (equally) moved in cases of equal and opposite motion. If the earth is represented as (truly) rotating then the surrounding starry heavens have no share in this motion at all, but if two bodies are represented as mutually moving in accordance with the third proposition then *both* must necessarily share in the motion (equally and oppositely).[212] So how can the earth's true rotation possibly be subsumed under the modality of necessity?

The answer to this question is implicit in my detailed discussion of true circular motion in section 33 above. The crucial point is that Kant is primarily concerned with determining states of *axial* rotation within a system of at least two bodies (or parts of bodies), and it is only on this basis that we can then determine (true) states of *orbital* rotation of any single body.[213] In particular, we have a state of (true) axial rotation in a system of two bodies if and only if both bodies in the system experience mutual orbital rotations (around a common center of gravity) relative to

[211] It is therefore significant, in particular, that Newton's discussion of the shape of the earth in Propositions 18, 19, and 20 of Book 3 (which, as I have argued, serves as a model for Kant's discussion in the Phenomenology) takes place after he has determined the center of mass of the solar system in Proposition 12. Here from a modern point of view, once again, the "proper" frame of reference determined in this case is a *non-rotating* and thus (approximately) inertial frame of reference fixed at the center of mass of the solar system: see note 146 above, together with notes 96 and 109 above. (Compare also the technical note at the end of section 34.)

[212] Compare the paragraph to which note 203 above is appended – and, for more details, see the paragraph to which note 57 above is appended (including the note itself).

[213] Compare note 91 above, together with the paragraph to which it is appended.

one another. In the case of Newton's two balls mutually rotating around a common center of gravity, for example, we determine the true state of axial rotation of this system by the observable tension in the connecting cord, which counterbalances the resulting centrifugal endeavor by its attractive elasticity. Similarly, in the case of the earth, we determine its state of true axial rotation via its observable centrifugal bulging, as here counterbalanced by the mutual gravitational attraction between each part of the earth and every other. And in the context of universal gravitation, more generally, if one body (or part of a body) is rotating relative to another under the influence of gravitational forces, the latter must be rotating relative to the former as well. Any two such bodies, therefore, must be mutually rotating around their common center of gravity and thus mutually gravitating towards one another.[214] Hence, for any state of true rotation, whether axial or orbital, we must eventually appeal to the equality of action and reaction applied to mutual gravitational accelerations within a system of interacting bodies (or parts of bodies) – which accelerations, for this very reason, now count as *necessary*.[215]

For Kant, it is because all states of true rotation, whether axial or orbital, thereby involve a system of several bodies (or parts of bodies) that we here have a systematic interconnection (which unites all relevant appearances) among a number of empirical judgements. And it is for precisely the same reason that any given such empirical judgement has now been further determined in accordance with the faculty of reason:

> [The principles of modality] add to the concept of a thing (something real), of which they otherwise say nothing, the cognitive faculty from which it [the concept] springs and where it has its seat – so that, if it is merely in the understanding in connection [*Verknüpfung*] with the formal conditions of experience, it is called possible; if it is interconnected [*im Zusammenhang*] with perception (sensation, as matter of the senses), and is determined from it [perception] by means of the understanding, then the object is actual; if it is determined through the interconnection [*Zusammenhang*] of perceptions in accordance with concepts, then the object is called necessary. The principles of modality therefore

[214] In the case of the rotating earth, in particular, each pair of *parts* lying opposite to one another along a diameter mutually rotate around the center of the earth and mutually gravitate towards this same center: see the paragraph to which note 146 above is appended.

[215] In what sense, however, are the corresponding true rotational motions (whether axial or orbital) themselves necessary? The point is that the equality of action and reaction is crucially involved here as well, in so far as this law, from a modern point of view, characterizes the privileged inertial frames of reference in which the (true) rotational motions are defined. For Kant's understanding of the role of action and reaction in this context (as part of his understanding of absolute space) see the paragraphs to which notes 158–60 above are appended, together with the two following paragraphs.

assert nothing else about a concept except the action of the cognitive faculty through which it is generated. (A234/B286–87)

Kant here rephrases what he had earlier said in his initial presentation and explanation of the modal principles (A218–19/B265–66; compare the paragraphs to which notes 204 and 207 above are appended). He now makes it completely clear, on the one hand, that the understanding is also essentially involved in the second stage – for how else, in fact, could any product of "the empirical power of judgement" (A219/B266) then arise? But he also makes it clear, on the other hand, that the third stage – corresponding to "reason (in its application to experience)" (A219/B266) – differs from the first in operating on the *interconnection* of perceptions with one another ("in accordance with concepts") rather than on the (individual) perceptions themselves.

Thus, in the example from the general remark to phenomenology, we begin by determining the earth's rotation by means of Coriolis accelerations experienced by falling bodies dropped into a vertical hole. We here have a single empirical judgement based on a single given perception – and, as in all empirical judgements, the categories of the understanding must therefore come into play. More precisely, since we here apply the law of inertia in accordance with the second proposition of the Phenomenology, the category of causality (which is realized or instantiated by the law of inertia) must be applied as well. When we determine this same state of true rotation by the earth's centrifugal bulging, however, we are now considering it within the full Newtonian theory of universal gravitation, and, accordingly, we are now considering the motion of the earth (and all of its parts) in the context of all of the other motions in the solar system.[216] All empirical judgements about any (putatively) true motion in the solar system have now been interconnected with one another in a coherent *system* of such judgements – and this means, for Kant, that the faculty of reason ("in its application to experience") must also come into play.

In the appendix to the transcendental dialectic on the regulative use of reason Kant explains why the idea of the systematic unity of nature as a whole yields a genuinely transcendental (and not merely logical) principle:

[216] The crucial difference between Kant's second and third propositions of the Phenomenology, from this point of view, is that in the latter we are now applying the category of *community* as well – so that we are now considering the rotation of the earth in community with all other motions in the solar system: see note 149 above, together with the paragraph to which it is appended.

For the law of reason to seek [such unity] is necessary, because without this we would have no reason at all – without this, however, we would have no interconnected [*zusammenhangenden*] use of the understanding, and, lacking this, no sufficient mark of empirical truth; thus in regard to the latter we must presuppose the systematic unity of nature as objectively valid and necessary. (A651/B679)

The idea of the systematic unity of nature as a whole is merely regulative, because we can never succeed in embedding the *totality* of all empirical judgements into a completely interconnected system. Nevertheless, as we progress further and further under the guidance of this idea, we succeed in embedding ever larger complexes of empirical judgements into an interconnected system in conformity with the laws of the understanding. And, when we do so, all the empirical judgements in question are thereby further determined by the understanding in precisely the context of such a (partially completed) system. At this point – and only at this point – all of these judgements now count as necessary according to the postulates of empirical thought. Before this point, however, although we may have attained true judgements in fact (in accordance with the empirical power of judgement), we do not yet have a "*sufficient* mark of empirical truth" (emphasis added).

Kant's procedure for reducing all motion and rest to absolute space described in the general remark to phenomenology provides a concrete realization of this process. We begin from the appearances presented to our parochial perspective here on earth, and we then progress through a sequence of ever larger rotating systems towards the never to be fully realized regulative idea of an absolute space determined by the center of gravity of all matter. We thereby obtain a sequence of *partial* (but cumulative) realizations of absolute space under the guidance of this idea. We move from the earth to the center of gravity of the solar system, from there to the center of gravity of the Milky Way galaxy, from there to the center of gravity of a rotating system of such galaxies, and so on *ad infinitum*. At each finite stage we have a completely interconnected system governing all (gravitational) states of true rotation of the bodies that we have considered so far. From the point of view of the center of gravity of the solar system, for example, all such states of true rotation (of the earth and all of its parts, of all satellites relative to their primary bodies, and so on) have in fact been completely determined. At each finite stage, therefore, we have a sufficient mark of empirical truth for all the rotational motions in question – which, according to Kant's three propositions determining the modalities of motion, must now count as necessary.[217] What we do not yet have, and cannot ever have, is a corresponding mark of empirical truth for all the rotational motions in the cosmos considered as a totality.

It may still be questioned, however, whether Kant's discussion of the systematic unity of nature in the appendix to the dialectic can be relevant to the use of reason ("in its application to experience") described in the postulates of empirical thought. For the former discussion explicitly concerns a merely *regulative* use of reason, whereas the latter, on the contrary, concerns *constitutive* principles of the understanding. We saw in section 20 above, for example, that Kant discusses examples in the appendix to the dialectic derived from contemporary (Stahlian) chemistry. He also asserts in the Preface to the *Metaphysical Foundations* that chemistry is not yet a proper science but rather a mere "systematic art or experimental doctrine" (471) that is not yet constitutively grounded in pure natural science.[218] Indeed, this is precisely why chemistry is Kant's main example of the (merely) regulative use of reason, where we begin with the observable large-scale properties of matter and then attempt to determine its internal structure "from the outside in" (compare note 201 above, together with the paragraph to which it is appended). But Kant's determination of motion with respect to the categories of modality, by contrast, is clearly a constitutive procedure, and, accordingly, it should not be assimilated to

[217] In the technical note at the end of section 34 above I explained that the sequence of rotating systems corresponds, from a modern point of view, to a sequence of ever better approximations to a true inertial frame of reference. This does not mean, however, that the states of true rotation (both axial and orbital) determined at any finite stage are themselves only approximate. Rather, what is left out, at any finite stage, are additional accelerations, directed towards bodies *outside* the system in question, going beyond the accelerations involved in the rotational states of bodies within the system. Compare the paragraph to which note 161 above is appended, together with the preceding and following paragraphs.

[218] For chemistry as a mere systematic art or experimental doctrine see note 224 of my chapter on the Dynamics, together with the paragraph to which it is appended. In the passage quoted in the note Kant asserts that the "mere laws of experience" found in chemistry "carry with them no consciousness of their *necessity*" (468), and he enlarges on this in what follows (469):

> [I]n accordance with demands of reason, every doctrine of nature must finally lead to natural science and conclude there, because [the] necessity of laws is inseparably attached to the concept of nature, and therefore makes claim to be thoroughly comprehended. Hence, the most complete explanation of given appearances from chemical principles still always leaves behind a certain dissatisfaction, because one can adduce no a priori grounds for such principles, which, as contingent laws, have been learned merely from experience.

Thus chemistry, according to Kant has not yet received the constitutive grounding that reason demands for all genuine laws of nature, whereas the Newtonian theory of universal gravitation, by contrast, does have such a constitutive grounding. I shall return to this point below.

the discussion of the (merely) regulative use of reason in the appendix to the dialectic.[219]

The first point to notice is that the postulates of empirical thought do have an important regulative dimension. In Kant's preliminary discussion of the analogies of experience as a whole he sharply distinguishes between *mathematical* and *dynamical* principles of pure understanding. The former comprise the axioms of intuition and anticipations of perception, the latter the analogies of experience and postulates of empirical thought. The former, Kant says, are constitutive, while the latter are (merely) regulative:

> An analogy of experience will therefore be only a rule, in accordance with which unity of experience (not, like perception itself, as empirical intuition in general) is to arise from perceptions, and, as [a] principle, it will hold of the objects (the appearances) merely *regulatively*, not *constitutively*. Precisely the same will also hold for the postulates of empirical thought in general, which together concern the synthesis of mere intuition (the form of appearance), of perception (the matter of appearance), and of experience (the relation of these perceptions): namely, that they are only regulative principles, and are distinguished from the mathematical [principles], which are constitutive – not, to be sure, in their certainty (which is established a priori in both), but still in the manner of their evidence, i.e., in their intuitiveness (and thus also in their [manner of] demonstration). (A180/B222–23)

So it follows from Kant's characterization of the postulates of empirical thought, in particular, that the third postulate (concerning necessity) is regulative in a twofold sense. For it corresponds to the analogies of experience, which are themselves (merely) regulative according to the previous sentence.[220]

[219] Kant clearly asserts at the beginning of the appendix to the dialectic that transcendental ideas of reason – in explicit contrast to concepts of the understanding – "are never of constitutive use" (A644/B673; see the paragraph to which note 262 of my chapter on the Dynamics is appended). Immediately before this passage Kant concludes (A644/B673; see the same paragraph):

> Therefore, reason properly has only the understanding and its purposive operation as object, and, as the latter unites the manifold in the object through concepts, the former, for its part, unites the manifold of concepts through ideas, in that it posits a certain collective unity as the goal of the activities of the understanding, which are otherwise occupied only with distributive unity.

[220] Kant's association of the three postulates of empirical thought with "intuition," "perception," and "experience" respectively suggests a parallel correspondence with the axioms of intuition, anticipations of perception, and analogies of experience – just as the three propositions of the Phenomenology correspond to the Phoronomy, Dynamics, and Mechanics. I shall return to this point below.

Yet it is puzzling in the extreme to claim that the dynamical principles of pure understanding (both analogies and postulates) are merely regulative, for this would appear to assimilate them entirely to the purely regulative principles of reason – contrary to Kant's own explicit intentions. Fortunately, Kant clarifies the situation substantially at the end of his discussion of the regulative use of reason in the appendix to the dialectic:

> In the transcendental analytic we have distinguished, among the principles of the understanding, the *dynamical*, as merely regulative principles of *intuition*, from the *mathematical*, which are constitutive with respect to the latter. Regardless of this, however, the dynamical laws in question are certainly constitutive with respect to *experience*, in that they make the *concepts*, without which no experience takes place, possible a priori. Principles of pure reason, by contrast, are not even constitutive with respect to empirical *concepts*, for no schema of sensibility corresponding to them can be given, and they can therefore have no object *in concreto*. (A664/B692)

The crucial distinction, then, is between being constitutive with respect to intuition and being constitutive with respect to experience.

The axioms of intuition and anticipations of perception are constitutive of perception or empirical intuition in so far as all empirical intuitions whatsoever are given in the forms of our pure sensibility – and they are therefore necessarily subject, in turn, to an a priori conceptual synthesis in space and time governed by the categories of quantity and quality.[221] The categories of quantity and quality, we might say, are thereby necessary constituents of all perception of appearances. The dynamical principles, by contrast, are constitutive of experience rather than perception or empirical intuition, in so far as experience involves a relationship of *interconnection* [*Zusammenhang*] between given perceptions that is in no way immediately

[221] The case of the axioms of intuition (corresponding to the categories of quantity) is relatively straightforward, for the relevant principle implies that empirical intuition is necessarily subject to what pure mathematics has already established with respect to pure intuition (A165–66/B206): "The synthesis of spaces and times, as the essential form of all intuition, is that which, at the same time, makes possible the apprehension of appearances, and thus every outer experience, and therefore all cognition of objects of [outer experience]; and what mathematics proves of the former in its pure use also holds necessarily of the latter." The case of the anticipations of perception (corresponding to the categories of quality) is a bit more complicated, because we here need to add a new a priori element – the concept of a "degree of influence on the sense" (B208; compare note 9 of my chapter on the Dynamics) – which is specifically characteristic of *empirical* (rather than pure) intuition. Nevertheless, Kant holds, the specific degree (but not the specific quality) of such an influence can still be anticipated a priori, that is, constructed (A178–79/B221): "Thus, for example, I will be able to compose the degree of illumination out of approximately 200,000 illuminations of the moon and give it as a priori determined, that is, construct it." Compare note 24 of my chapter on the Dynamics, together with the paragraph to which it is appended.

given in the (individual) perceptions themselves. Rather, what the dynamical categories give us a priori, Kant says, is only "a rule for seeking [such a relationship – MF] in experience, and a mark by which it can be discovered" (A180/B222), while purely regulative principles of reason, by contrast, can never have a corresponding object of experience at all. In the case of the principle of causality, for example, a cause for a given empirical event cannot be anticipated or constructed a priori, but we do have a "schema of sensibility" in accordance with which it can be discovered: namely, when we find a previous event on which the given event necessarily follows in accordance with a rule.[222] The regulative principle of the (complete) systematic unity of nature as a whole, however, can never be fully instantiated in experience at all, but only successively approximated in the never completed progress of our empirical cognition.

In these terms, therefore, Kant's procedure of determining true states of rotation in ever larger cosmic systems provides, once again, a concrete realization of what we now see to be a rather subtle relationship between regulative and constitutive uses of reason. The procedure as a whole aims at the forever unreachable center of gravity of all matter, which, accordingly, can never be fully instantiated in experience but only successively approximated. Nevertheless, at any finite stage we do succeed in finding a system of necessary (gravitational) interactions between every body considered so far and every other, and these interactions are indeed sufficient to determine all the true rotations in question as necessary in the sense of the third postulate.[223] Newton's law of universal gravitation has now

[222] The schema of the categories of relation as a whole "[contains and makes representable] the relation of perceptions to one another at every time (i.e., in accordance with a rule of time determination)" (A145/B184). What Kant adds concerning the categories of modality is then especially interesting: "[F]inally, the schema of modality and its categories contains and makes representable time itself, as the correlate of the determination of an object whether and how it belongs to time" (A145/B184). Kant continues: "The schema of necessity is the existence of an object at every time" (A145/B184). This makes sense, for, according to Kant's discussion of the third postulate of empirical thought (A226–27/B279–80), "there is no existence that could be cognized as necessary under the condition of other given appearances except the existence of effects from given causes in accordance with laws of causality. Thus, it is not the existence of things (substances), but only that of their state, about which we can cognize their necessity – and, indeed, from other states that are given in perception, in accordance with empirical laws of causality." What exists "at every time," therefore are empirical causal laws (empirical "rules of time determination"). And, more generally, what the dynamical categories require is precisely the existence of such laws governing the relations of (causal) interconnection among given perceptions.

[223] What is necessary in this realization is thus the existence of certain *states* (of true orbital rotation) characterizing the motions in any such (finite) system of bodies. That the (gravitational) interactions in question are *sufficient* fully to determine these states follows from note 217 above. In the terms of note 219 above, therefore, the (systematic) unity of each (finite) system in the sequence is *distributive*, while that of the entire sequence (as a totality) can only be *collective*.

been established as a necessary (but still empirical) law, and it is precisely in virtue of this law that all states of true rotation in any such (finite) system of bodies are also determined as necessary (see again note 215 above, together with the paragraph to which it is appended).[224]

However, I have not yet completely addressed the difficulties raised above. For I have not yet explained the crucial difference between the empirical regress into ever larger rotating systems, in accordance with the first antinomy, and the complementary such regress into ever smaller internal parts of matter, in accordance with the second. Both of these regresses are fundamental to the overall argument of the *Metaphysical Foundations*, and both centrally illustrate the purely regulative use of reason. Yet the latter regress, as just emphasized, is integral to Kant's conception of chemistry as a "systematic art or experimental doctrine" (471), and chemistry, for Kant, has not yet attained the status of a proper natural science. For Kant, therefore, there is no constitutive grounding for chemical phenomena at *any* stage of the regress into ever smaller internal parts of matter corresponding to the constitutive grounding that exists at *every* (finite) stage of the complementary regress into ever larger rotating systems. Why, exactly, do the postulates of empirical thought (and therefore the categories of modality) faithfully apply (as just argued) in the latter case while not applying at all in the former?

I shall approach this question by considering more carefully the way in which the three postulates of empirical thought are supposed to correspond to Kant's three propositions on the modality of motion in the Phenomenology. I have argued so far that the second and third propositions of the Phenomenology correspond to the second and third postulates of empirical thought. We determine a (putative) state of true rotation as *actual* in accordance with the "empirical power of judgement" and determine this same state as *necessary* in accordance with "reason (in its application to experience)" (A219/B266; see the paragraph to which note 204 above is appended, together with the preceding paragraph). The former judgement is "determined from [perception] by means of

[224] For a detailed discussion of the sense in which the law of universal gravitation, for Kant, counts as a *necessary* (but still empirical) law in the sense of the third postulate see Friedman (1992a). I point out there that Kant illustrates the transition from mere empirical rules to true laws of nature in a striking unpublished *Reflexion* (written between the late 1770s and mid 1780s) using precisely the transition from Kepler to Newton (R 5414; 18, 176): "Empirically one can certainly discover rules, but not laws – as Kepler in comparison with Newton – for to the latter belongs necessity, and hence that they are cognized a priori." I discuss this aspect of the law of universal gravitation further, in the context of the reading of the Phenomenology I am developing here, in Friedman (2012).

the understanding," whereas the latter is "determined through the *inter-connection* of perceptions in accordance with concepts" (A234/B286–87, emphasis added; compare the paragraph to which note 216 above is appended, together with the preceding paragraph).

Yet the parallel correspondence in the case of the first proposition appears to be quite problematic. For empirical intuition and "the *material* conditions of experience" enter in only with the second postulate, whereas the first involves solely "the *formal* conditions of experience (according to intuition and concepts)" (A218/B265–66, emphasis added; compare note 206 above, together with the paragraph to which it is appended). The first postulate determines of a given concept whether "it is merely in the understanding in connection with the formal conditions of experience," whereas only the second can determine whether "it is interconnected with perception (sensation, as matter of the senses)" as well (A234/B286–87). Determinations in accordance with the first postulate, in other words, involve pure understanding and pure intuition alone, whereas empirical intuition, properly speaking, first enters the progression of determinations with the second.[225] But this does not correspond to Kant's first proposition on the modality of (rectilinear) motion. For here, as in all such determinations of motion, we begin from two *empirically given* correlates (a putatively moving body and a given relative space), and the problem is then to determine which of the two is truly in motion. The result is an alternative judgement in which either description (in the case of rectilinear motion) can be adopted arbitrarily, and we are certainly not proceeding, with such a judgement, in pure intuition.[226]

[225] As suggested in note 220 above (in the context of the paragraph to which it is appended), the three postulates of empirical thought correspond, respectively, to the axioms of intuition, anticipations of perception, and analogies of experience respectively. Perception or the matter of appearance enters explicitly only in the anticipations, and, in particular, Kant explains (A180/B223) that the first postulate concerns a "synthesis of mere intuition (the form of appearance)," while only the second concerns a "[synthesis] of perception (the matter of appearance)."

[226] The Phoronomy – which corresponds in the *Metaphysical Foundations* to the axioms of intuition – is primarily concerned with a construction (of the composition of motions) in pure intuition. More precisely, it is concerned with a transition from pure to empirical intuition, in so far as this construction (of motion as a magnitude) is then to be applied to empirically given objects (compare the paragraph to which note 37 above is appended). This is consistent with a correspondence between the Phoronomy and the axioms of intuition, because the point of the latter is to apply whatever pure mathematics establishes in pure intuition to empirical intuition as well (see note 221 above). The first proposition of the Phenomenology, by contrast, is concerned with a transition from mere *appearance* or empirical intuition to *experience*, and it is for precisely this reason that the standpoint of this proposition is in fact quite different from that of the Phoronomy (compare the paragraph to which note 40 above is appended).

So how can it be that perception or specifically empirical intuition is – and must be – explicitly considered in the first proposition of the Phenomenology? The answer, on my reading, is that the conceptual background at this stage of Kant's argument includes not only the pure concepts of the understanding but also the *empirical concept of matter* whose explication is the task of the *Metaphysical Foundations* as a whole. By the time we arrive at the Phenomenology, moreover, this concept has already been explicated in the three preceding chapters as the movable in space, the movable in so far as it fills a space, and the movable in so far as it (as such a thing) has a moving force. The task of the Phenomenology is not to add any further conceptual determination but rather to explain how the motion of this movable (as already determined *conceptually* in the preceding chapters) can now be determined as an object of experience in accordance with the three categories of modality. It is only in this way, in particular, that the movable, as an object of experience, can then be related (sequentially) to "the understanding and its empirical employment, to the empirical power of judgement, and to reason (in its application to experience)" (A219/B266).[227]

Thus, when Kant considers applications of the category of *possibility* to the (rectilinear) motion of matter in the first proposition of the Phenomenology, he has already incorporated the conceptual determinations of matter from the three previous chapters into "the understanding (in its empirical employment)."[228] To incorporate a given concept into the understanding in accordance with the first postulate, however, is to establish that this concept is really (and not merely logically) possible. And, as Kant explains in his discussion of the first postulate (A222/B269–70), specifically *empirical* concepts "cannot acquire the character of their possibility a priori, like the categories, as conditions on which all experience depends, but only a posteriori, as such that are given through experience itself – and their possibility must either be cognized a posteriori and empirically, or it cannot be cognized at all." In the case of empirical concepts, more generally, their "possibility can only be derived from [their

[227] See note 33 above, together with the paragraph to which it is appended. Just as the pure concepts considered in the first postulate include all pure concepts of the understanding, the *empirical* concepts at play in the first proposition of the Phenomenology comprise all conceptual determinations introduced in the previous chapters. I shall return to this point below.

[228] In the terms of note 204 above (including its reference back to the earlier note 45), we have already seen how "experience" in the Phenomenology lines up with "the empirical power of judgement" – and, of course, how "reason" in the Phenomenology lines up with "reason (in its application to experience)." What remains, accordingly, is to subsume the reference to "appearance" in the Phenomenology under "the understanding and its empirical employment."

actuality] in experience" (A223/B270). It follows that the empirical concept of matter can be really possible only if it is actually instantiated as well, and so we cannot abstract completely from perception or the material conditions of experience when considering this concept – even when considering it under the first postulate.[229]

In the corresponding first proposition of the Phenomenology, therefore, the content of the empirical concept of matter must already be in place. In particular, Kant is presupposing that empirical intuitions or perceptions of the putative object of experience in question – putative attributions of rectilinear motion to one of two given material correlates – belong to "the material offered to us by perception" (A222/B269) from which the empirical concept of matter is "borrowed" (A220/B267; note 229). So when I now make an appropriate alternative judgement, I am presupposing that both of the two bodies (material correlates) fall under the concept of the movable as that which fills a space through the two fundamental forces and, in turn, exerts moving force through its motion on (and thus exchanges momentum with) other bodies. It is for precisely this reason, in the end, that we can (provisionally) begin the procedure of reducing all motion and rest to absolute space from the point of view of either the earth or the sun, while leaving it entirely open (prospectively) that an equal and opposite (rectilinear) motion will be later attributed to the other body as well. It is for precisely this reason, on my reading, that, when Kant says that such (rectilinear) motions are merely possible, he means that both of the two attributions, from the point of view of experience, count as equally good starting points for the three-stage progression by which all states of true motion are then (successively) empirically determined.[230] It

[229] The whole of Kant's discussion of (real) possibility in the first postulate (A220–23/B267–70) is well worth considering carefully here, and I quoted the essential passages in the Introduction (see notes 42 and 43 of the Introduction, together with the paragraphs to which they are appended). The most important of these, in the present context, begins as follows (A222/B269):

> However, if one wanted to make entirely new concepts of substances, of forces, and of interactions out of the material offered to us by perception, without borrowing [*entlehnen*] the example of its connection from experience itself, then one would fall into mere phantoms of the brain, whose possibility would have no indications at all – since one does not accept experience as instructress, and yet these concepts are borrowed [*entlehnt*] from it. These feigned concepts …

There follow the passages (A222–23/B269–70) just quoted in the main text. In the case of the empirical concept of matter articulated in the *Metaphysical Foundations* we are considering specifically material (and therefore movable) substances, which act on one another by the two fundamental forces of attraction and repulsion, and all interactions involving these forces (all cases of the communication of motion) are governed by the four propositions of the Mechanics.

[230] See again the discussion that begins in the paragraph to which note 59 above is appended, proceeds with the example of (provisionally) choosing either the earth or the sun as the center of the motions in the solar system (in accordance with Newton's Phenomenon 4 of Book 3), and

is in precisely this sense, finally, that the initial *appearance* of mere relative motion between two given correlates has now been subsumed under "the understanding and its empirical employment" (A219/B266).[231]

To say that the empirical concept of matter is really possible, therefore, is to presuppose that Kant's entire procedure for reducing all motion and rest to absolute space is already (prospectively) in view. Moreover, since this procedure is modeled on the argument of Book 3 of the *Principia*, all the empirical phenomena presupposed by Newton's argument (the initial Keplerian "Phenomena" of Book 3, for example) are essentially implicated in Kant's empirical concept as well. Indeed, since Kant (unlike Newton himself) is envisioning an extension of the Newtonian argument through an indefinitely extended sequence of nested rotating systems, Kant is presupposing a rather intricate and elaborate de facto structure for the entire cosmos as well. Nevertheless, despite this quite substantial empirical content, the concept of matter has a unique status and role within any system of classification for empirical natural science. As the very highest empirical concept in such a system it sits directly under the pure concepts of the understanding, which are then successively realized in all lower-level empirical concepts by an appropriate further specification of the concept of matter into species, sub-species, and so on. This particular empirical concept thereby mediates the application of the categories to all other empirical concepts and thus mediates the application of the categories to experience such.[232]

concludes with a consideration of *alternatively-mutual* attributions of motion in the paragraph to which note 73 above is appended.

[231] The issue last broached in note 228 above has thus been finally resolved. When we trace this issue back, moreover, we ultimately arrive at note 45 above, together with the paragraph to which it is appended. The crucial point is that, when Kant says (in the case of his first proposition) that "the change can be attributed in the appearance to one just as well as the other, and either the *one* or the *other* can be said to be moved, because ***the two cases are completely equivalent*** [*gleichgültig*]" (554, bold emphasis added), the corresponding alternative judgement can only be made from the point of view of "experience (a cognition that determines the object validly for all appearances" (555). The faculty of understanding must therefore already be in play, and, in addition, we are now considering the empirical concept of matter as already fully incorporated within this faculty.

[232] See notes 263–69 of my chapter on the Dynamics, together with the paragraphs to which they are appended. This discussion considers the problem of specific variety treated in the general remark to dynamics – the problem, that is, of explaining how the universal (empirical) concept of matter in general is further specified into species, sub-species, and so on. And here, as explained, Kant has the empirical science of chemistry primarily in view. In the midst of his discussion of examples from contemporary (Stahlian) chemistry, moreover, Kant makes an explicit link – in reference to the (regulative) principle of always seeking higher genera – between empirical concepts and the possibility of experience (A654/B682, emphasis added): "According to [this principle] we necessarily presuppose homogeneity in the manifold of a possible experience (although we cannot determine its degree a priori), because without this no empirical concepts, *and thus no experience*, would be possible."

According to the Preface to the *Metaphysical Foundations*, however, the empirical concept of matter is still the basis for an a priori natural science.[233] It provides such a basis through a combination of both metaphysics and mathematical construction (469–70), and the real possibility of this concept, in particular, depends on precisely mathematical construction.[234] Yet it does not follow, as I argue in the Introduction, that mathematical construction is sufficient for its real possibility – for otherwise the concept of matter would be a pure mathematical concept. Instead, Kant's metaphysical foundation for pure natural science explains a priori how the *application* of mathematics to the component concepts of the empirical concept of matter is possible. It thereby explains, in particular, how certain central concepts in the (Newtonian) mathematical theory of motion – such as the concept of motion and the concept of quantity of matter – first acquire the status of mathematical magnitudes. These concepts remain empirical (rather than purely mathematical) concepts, but they thereby become empirical concepts with definite mathematical structures.[235]

Kant further explains in the Preface that metaphysics is required in addition to mathematics, because natural science is concerned with *existent* rather than merely possible things and existence is a concept that cannot be constructed in pure intuition.[236] Moreover, the fundamental principles governing the existence of things are precisely the analogies of experience, which are realized or instantiated by Kant's three Laws of Mechanics in

[233] Compare especially the important passage from the Preface where Kant asserts that

> a complete analysis of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions in space and time, and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*. (472; see the paragraph to which note 45 of the Introduction is appended)

[234] This is the famous (and very difficult) paragraph in the Preface where Kant asserts that "a [special] doctrine of nature will contain only as much proper science as there is mathematics capable of application there" (470; see the passage to which note 38 of the Introduction is appended).

[235] The passage (partially) quoted in note 233 above begins by asserting that "in order to make possible the application of mathematics to the doctrine of body, which only through this can become a natural science, principles for the *construction* of the concepts that belong to the possibility of matter in general must be introduced first" (472). As explained in note 46 of the Introduction, such *principles* for construction belong precisely to metaphysics as opposed to mathematics: see also the paragraph to which this note from the Introduction is appended, together with the remainder of my discussion there.

[236] Kant writes (469; see the paragraph to which note 35 of the Introduction is appended): "*Properly* so-called natural science presupposes, in the first instance, metaphysics of nature. For laws, that is, principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be presented a priori in any intuition."

the *Metaphysical Foundations*. So the central concepts of the metaphysical tradition Kant inherited – the concepts of substance, causality, and interaction – are thus instantiated in pure natural science by specifically material (and therefore movable) substances that act on one another by original dynamical forces (of attraction and repulsion) and are such that their interactions occur via the communication of motion (compare note 229 above). And, once again, the central empirical concept whose mathematical structure is thereby explained is precisely the Newtonian concept of quantity of matter. For, as we have seen, Kant's metaphysical articulation of the most general features of his empirical concept of matter aims to capture just those features of the argument of Book 3 of the *Principia* that make the Newtonian mathematization of this quantity first possible.

It is in precisely this way, for Kant, that Newton's deduction from the phenomena of the law of universal gravitation in Book 3 receives an a priori grounding in *both* metaphysics *and* mathematics, and so it is in precisely this way, in particular, that the law of universal gravitation counts as both constitutively grounded and necessary. By contrast, as Kant also explains, we do not yet have anything like a Newtonian-style dynamical force law governing the microscopic interactions ultimately responsible for chemical phenomena. Hence, until we do have such a law (an eventuality that Kant takes to be very unlikely), no putative law of chemistry will have become constitutively grounded and necessary, and mathematics will not yet have achieved a proper application there.[237]

Once again, however, more needs to be said in order fully to appreciate the crucial difference between the empirical regress into ever larger rotating systems that Kant envisions in the context of the Newtonian theory of universal gravitation and the complementary such regress into ever smaller internal parts of matter that he envisions in the context of contemporary (Stahlian) chemistry. For this chemistry, as explained in section 20 above, does not proceed by looking for microscopic Newtonian-style force laws in any case, but rather by an open-ended purely experimental program guided, in Kant's words, "in accordance with the idea of a mechanism" (A646/B674) – where he here has in mind, more specifically, continuum (as opposed to atomistic) models of matter in the tradition of Euler's hydrostatics.

[237] See again the pivotal passage from the Preface (470–71), which concludes with the assertion that "[chemical] principles are merely empirical, and allow of no a priori presentation in intuition; consequently, they do not in the least make the principles of chemical phenomena conceivable with respect to their possibility, for they are not receptive to the application of mathematics" (471; fully quoted in the paragraph to which note 224 of my chapter on the Dynamics is appended).

But such continuum models, as we know, are central to Kant's dynamical theory of matter, and, accordingly, he takes original elasticity and the fundamental force of repulsion to be just as much essential features of the concept of matter in general as gravity, weight, and the fundamental force of attraction.[238] So there is still room to ask why, exactly, the *Metaphysical Foundations* provides a constitutive grounding for the empirical regress into ever larger rotating systems in the context of Newtonian universal gravitation while providing no such grounding – via precisely original elasticity and the fundamental force of repulsion – for the complementary regress into ever smaller internal parts of matter in the context of contemporary (Stahlian) chemistry. Although we already know, in general terms, that there is indeed a crucial asymmetry between the two cases, it will still prove to be illuminating, as we shall see, to consider the nature of this asymmetry in more detail.

Let us consider, in particular, the hydrostatic concept of (expansive) pressure and its relationship to the fundamental force of repulsion. This concept, as explained in section 12 above, underlies Kant's conception of the original (expansive) elasticity characteristic of all matter as such, and it has also achieved a successful mathematization at the hands of Euler. Yet, as explained in section 19 above, the relationship of this original elasticity (expansive pressure) to the fundamental force of repulsion remains obscure, and, in any case, it does not relate to the fundamental force of repulsion in the way that the Newtonian concept of mass or quantity of matter relates to the fundamental force of attraction. Indeed, since the former force acts infinitesimally, in inverse proportion to the cube of the *infinitely small* distance, it involves only what Kant calls "the idea of a space, which serves to make intuitive the expansion of a matter as a continuous quantity" (522). So it appears that the most reasonable reading of the relationship between (expansive) pressure and the fundamental force of repulsion is that we start with the empirical Boyle–Mariotte law relating (expansive) pressure to *finite* volumes and then arrive at the fundamental force of repulsion simply as what emerges in the limit as the volumes in question become infinitely small.[239] In the end, therefore, the

[238] Compare the important passage from the second note to the eighth proposition of the Dynamics, where Kant asserts that "*original elasticity* [as grounded in the universal repulsion – MF] … and weight [as grounded in universal attraction – MF] constitute the sole universal characteristics of matter that are comprehensible a priori, the former internally, and the latter in external relations; for the possibility of matter itself rests on these two [properties]" (518; see note 265 of my chapter on the Dynamics, together with the paragraph to which it is appended).

[239] See note 211 of my chapter on the Dynamics, together with the paragraph to which it is appended and the four preceding paragraphs.

fundamental force of repulsion becomes a limiting idea of reason, corresponding to what Kant calls a mere "*idea* of a[n infinitely small] space" (emphasis added). This force is ultimately grounded, accordingly, in the very empirical regress into ever smaller parts of (continuous) matter that we are now in the process of considering.

The problem, however, is not that the fundamental force of repulsion has a necessary empirical basis in this way. For the fundamental force of attraction has an analogous empirical basis in the complementary regress into ever larger rotating systems resulting from Kant's extension of the argument of Book 3 of the *Principia* to the cosmos as a whole (compare the paragraph to which note 232 above is appended). Indeed, Kant strongly emphasizes precisely this aspect of the argument for universal gravitation in an important discussion towards the end of the appendix to the dialectic in the first *Critique* that immediately follows his final discussion of examples from contemporary (Stahlian) chemistry.[240] Kant is discussing three principles of reason governing the (never to be completed) articulation of a classificatory system of empirical concepts. The first is a principle of *unity* (or homogeneity) requiring ever higher genera above any given species, the second a principle of *variety* (or specification) requiring ever lower (and more diverse) sub-species below any given species, and the third a principle of *affinity* (or continuity) requiring a potential infinity of intermediate species between any higher and lower pair. These principles are merely regulative, however, because a complete system in accordance with them can never in fact be instantiated in experience; yet reason demands that we pursue such completeness nonetheless by seeking ever new empirical concepts under their regulative guidance (A662/B690): "Reason presupposes the cognitions of the understanding that are first applied to experience, and [it] seeks their unity through ideas, which extend much further than experience can reach."

An astronomical example then follows:

> The affinity of the manifold, without detriment to its variety, under a principle of unity, concerns not merely the things, but, much more, the properties and powers [*Kräfte*] of things. Therefore, e.g., when the orbits of the planets are given to us as circular in a (not completely rectified) experience, and we find variations, we conjecture that they [consist] in that which can alter the circle, in accordance with a constant law through all infinite intermediate degrees, into one of these deviating orbits; i.e., [we conjecture that] the motions of the planets, which are

[240] This final example concerns the differentiation of "absorbent earths" into "calx and muriatic earths" (A657/B685; see note 267 of my chapter on the Dynamics, together with the paragraph to which it is appended).

not circular, will approximately approach this property more or less, and we come upon the ellipse. Comets show an even greater variety in their paths, since they (as far as observation reaches) never return in a circle; yet we guess at a parabolic orbit, which is still akin to the ellipse – and, if the major axis of the latter is extended very far, it cannot be distinguished from [the former] in any of our observations. Thus, under the guidance of these principles, we arrive at unity of the genera of these paths according to their form; and we thereby further arrive, however, at unity of the cause of all the laws of their motion (i.e., gravitation). From there we afterwards extend our conquests further, seeking also to explain all variations and apparent deviations from these rules from the same principle. Finally, we even add more to this than experience can ever confirm – namely, in accordance with the rule of affinity, we even imagine hyperbolic cometary orbits, in which these bodies leave our solar system entirely, and, proceeding from sun to sun, unify in their orbits the most distant parts of a for us unlimited cosmic system, which is interconnected [*zusammenhängt*] through one and the same moving force. (A662–63/B690–91)

Kant is here reconstructing the route by which Kepler first abandons the traditional idea of circular orbits in favor of elliptical orbits, Newton extends this idea to parabolic orbits for comets, Newton then arrives at the inverse-square law of universal gravitation governing all the orbital motions in the solar system, and so on.[241] So Kant is here reconstructing the route by which Newton first argues from the Keplerian "Phenomena" that initiate Book 3 to universal gravitation and then Kant himself (together with Lambert) extends this Newtonian argument far beyond the bounds of the solar system to the cosmos as a whole.[242]

[241] Newton's discussion of comets occupies the final three propositions of Book 3 (Propositions 40–42). The heart of this discussion (Proposition 41) is a method for determining a parabolic cometary trajectory from three given observations, which is then applied in detail to the cases of several recent comets. Newton begins, in Proposition 40, by showing that comets move in orbits centered on the sun in accordance to the inverse-square law. These orbits (by Corollary 1) can be taken as ellipses with very elongated major axes, so that (according to Corollary 2) "[they] will be so close to parabolas that parabolas can be substituted for them without sensible errors" (P895). Section 3 of Book 1 (including Propositions 11–13) shows, in effect, that the inverse-square law is equivalent to motion in an ellipse, hyperbola, or parabola (i.e., a conic section) directed towards one focus of the figure in question. When we extend the inverse-square law to truly universal gravitation, however, it then follows (by Propositions 13 and 14 of Book 3) that precisely these (possible) orbits are only approximate, in so far as perturbations within the solar system due to the attractions of the planets on one another result in a precession of their axes. Kant's remark here about explaining "all variations and apparent deviations from these rules [orbits in conic sections] from the same principle [universal gravitation]" (A663/B691) appears to be a clear allusion to this situation: compare note 106 above, together with the paragraph to which it is appended and the two preceding paragraphs.

[242] The picture of comets ranging freely throughout the entire cosmos in hyperbolic orbits and thereby unifying the different rotating galactic structures (including our solar system) with one another is a prominent (speculative) device in both Kant's and Lambert's cosmologies: compare note 112 above, together with the paragraph to which it is appended and the two preceding

The crucial moment in this passage, from our present point of view, comes in the fourth sentence – where, after arriving at the more general concepts of elliptical and parabolic orbits beginning with the at first sight circular orbits of the planets in accordance with his (merely) regulative principles, Kant then arrives at the law of universal gravitation (A663/B691, emphasis added): "Thus, under the guidance of these principles, we arrive at unity of the genera of these paths according to their form; and we thereby further arrive, however, at unity of *the cause of all the laws of their motion* (i.e., gravitation)." The crucial moment, in other words, comes when we move from a purely inductive assent from lower species to higher genera via regulative principles to the knowledge of a genuine causal law capable of explaining, from a still higher point of view, everything that we have observed so far. In the Newtonian argument of Book 3, in particular, we arrive at the full law of universal gravitation in Proposition 7, which then enables us to determine the center of mass of the solar system in Propositions 11 and 12.[243]

Hence, in undertaking the empirical regress into ever larger rotating systems in accordance with the Newtonian argument we rather quickly reach a point, in a relatively small number of steps, where we have established a necessary and universal causal law that now constitutively grounds all the remaining steps. The necessity of this law, moreover, is just that demanded by the category of necessity in the postulates of empirical thought. So it is in precisely this sense that the empirical regress in question is also governed by a *constitutive* use of the faculty of reason (in its application to experience) in accordance with the modal categories. In undertaking the complementary empirical regress into ever smaller parts of matter in accordance with Kant's conception of contemporary (Stahlian) chemistry as "a systematic art or experimental doctrine" (471), by contrast, we never arrive, in any finite number of steps, at a necessary and universal causal law capable of constitutively grounding

paragraphs. As I emphasize in this earlier discussion (which eventually occupies the whole of section 34 above), such unity of the cosmos as a whole – effected, in the end, by gravitational force – is inextricably connected with Kant's reinterpretation of Newton's argument for universal gravitation. It is thereby inextricably connected, as well, with Kant's conception of what he calls "absolute space" as a forever unreachable regulative idea of reason.

[243] Kant's discussion in this passage appears to contain a clear allusion to precisely this stage in Newton's argument, for, as indicated in note 241 above, the next sentence appears to suggest the transition from Proposition 12 to the following Proposition 13 – where, after inductively ascending to the inverse-square law of universal gravitation from the Phenomena, Newton then proposes to reverse this procedure (P817; see the paragraph to which note 105 above is appended): "Now that the principles of motions have been found, we deduce the celestial motions from these principles a priori."

the remaining steps. Indeed, as explained towards the end of section 20 above, Kant's conception of contemporary (Stahlian) chemistry as an "experimental doctrine" under the guidance of a purely *regulative* use of the faculty of reason does not involve any explanatory theory of the inner structure of matter at all. Instead, following the example of the Galilean–Newtonian mathematical description of (the acceleration of) gravity, the point is completely to abstract from all questions concerning the true physical causes of such phenomenological quantities as hydrostatic pressure in favor of a rigorous mathematical description of their effects.[244]

What is of overriding importance here is that Kant does not follow such Galilean–Newtonian mathematical agnosticism in the case of gravity. He has no qualms, in particular, about immediate gravitational action at a distance, and he does not attempt to mitigate such qualms by insisting that the law of universal gravitation describes only the effects of gravity while leaving its true cause to be found out later.[245] On the contrary, one material substance exerts a genuine dynamical (causal) action on another, for Kant, just in case there is a corresponding balance in the quantities of motion (momenta) thereby exchanged.[246] And since such momentum exchanges must occur between distant pieces of matter in Newton's argument, there is no question at all, for Kant, that we here have a genuine (and immediate) causal action at a distance in the metaphysical sense as well.[247] Thus, just as Kant has reinterpreted the traditional metaphysical concept of substance so that material or phenomenal substance can in no way be metaphysically simple, he has here reinterpreted the traditional metaphysical concept of causality so that gravitational action at distance is now paradigmatic of such causality.

I have thus returned to the point that the specifically metaphysical resources for grounding the application of mathematics to the central

[244] See, in particular, note 244 of my chapter on the Dynamics, together with the paragraph to which it is appended.

[245] For the contrast between Newton's mathematical agnosticism and Kant's conception of universal attraction (acting immediately at a distance) as a true dynamical cause see notes 174 and 175 of my chapter on the Mechanics, together with the paragraph to which they are appended.

[246] Kant makes this clear in his note to the first proposition of the Mechanics (539; see again the paragraph to which note 28 of my chapter on the Mechanics is appended): "As the quantity of motion of a body relates to that of another, so also does the magnitude of their action [*Wirkung*], where this is to be understood as the *entire* action." My discussion of this passage in section 23 above (from which the above quotation is taken) proceeds against the background of the discussion of dynamical and mechanical moving forces in section 22.

[247] For Kant's perspective on the direct exchanges of momentum at a distance required by Newton's argument see notes 194–96 of my chapter on the Dynamics, together with the paragraphs to which they are appended.

empirical concepts of a proper natural science comprise, first and foremost, the categories of substance, causality, and community articulated in the analogies of experience. For these are the fundamental principles governing the *existence* of things, and existence is a concept that cannot be constructed (mathematically) in pure intuition (see again note 236 above, together with the paragraph to which it is appended). They are also the fundamental principles for explaining the application in experience of the category of *necessity* – or, what amounts to the same thing, for establishing the necessary connections between empirically given perceptions in virtue of which they can then count as experience.[248] This, in the end, is why exhibiting the universality and necessity of the law of universal gravitation in accordance with the argument of Book 3 of the *Principia* is also inextricably connected with Kant's reinterpretation of the concept of substance in matter so as properly to account for its status as a mathematical magnitude.[249] Kant's metaphysical reinterpretations of substance, causality, and community can only be accomplished on the basis of the three analogies of experience, in so far as they are realized in the *Metaphysical Foundations* by his three Laws of Mechanics.[250] And we thus have a final confirmation for one of the central ideas that have guided my reading from the beginning: Kant's strenuous efforts to transform the essential concepts of Leibnizean metaphysics are primarily aimed, in this context, at providing a radically new kind of metaphysical foundation for specifically Newtonian mathematical physics.

[248] For the relationship between the analogies of experience and the third postulate of empirical thought see note 220 above, together with the paragraph to which it is appended. For the analogies as those principles in virtue of which perceptions acquire necessary connections and thereby become experience see note 27 above, together with the paragraph to which it is appended.

[249] This point depends on the relations between quantity of motion and quantity of matter discussed in sections 23, 24, and 25 above. Recall also that the conservation of quantity of motion essentially involves the category of community as well, because of the inextricable connection between this conservation law and the equality of action and reaction: see, for example, the discussion in my chapter on the Dynamics referred to in note 247 above.

[250] This essential dependence on the three analogies of experience is precisely what is missing in the case of the concepts of expansive pressure and original elasticity figuring in Kant's complementary regress into ever smaller parts of matter. To be sure, these concepts do play a role in the demonstration of the infinite divisibility of material substance in the fourth proposition of the Dynamics (section 13 above). Yet, as explained, Kant here appeals only to the *unschematized* category of substance, whereas the schematized category (and thus the first analogy of experience) only comes into play when Kant then establishes the essential connection between quantity of substance and quantity of matter in the Mechanics: see note 61 of my chapter on the Mechanics, together with the paragraph to which it is appended and the two preceding paragraphs.

Conclusion: The complementary perspectives of the Metaphysical Foundations *and the first* Critique

I now return to the question, first raised in the Introduction, of the place of the *Metaphysical Foundations* in the critical system and, more specifically, its relationship to the *Critique of Pure Reason*. This is a very large question, however, which would require another substantial book fully to address. So I shall confine my attention here to the issues surrounding the relationship between the two works that are explicitly raised in the Preface to the *Metaphysical Foundations*, as now further illuminated by what Kant adds in the body of this text.[1] Especially relevant, in this connection, is the Mechanics chapter, where Kant considers the relationship of his three Laws of Mechanics to the three analogies of experience in considerable detail, and, in particular, he considers the former as realizations or instantiations of the latter in the objects of outer sense. The question naturally arises, therefore, of the role played by such specifically spatial or corporeal instantiations of the principles of pure understanding in the main argument of the *Critique*. Are they merely illustrative of or incidental to this argument, in such a way that it could perfectly well proceed without them? Or, on the contrary, are they rather absolutely essential – as necessary, for example, to complete the transcendental deduction of the categories that had been begun, but not successfully completed, in the first *Critique*?[2] I shall here chart a middle course between these two extremes. The realization or instantiation of the transcendental principles of the understanding in the special metaphysics of corporeal nature is indeed indispensable and therefore privileged, for it is the one and only

[1] Compare note 1 of the Introduction.

[2] The first alternative corresponds to Gerd Buchdahl's "looseness of fit" interpretation, which is most explicitly developed in connection with the *Metaphysical Foundations* in Buchdahl (1969, chapter VIII, §9). The second alternative is represented in Förster (1987), which argues that the *Metaphysical Foundations* was written to complete the demonstration of the objective reality of the categories. My own earlier discussions of the *Metaphysical Foundations* came very close to this view, especially in Friedman (1986).

such instantiation that fully realizes the transcendental principles in the phenomenal world. It does not follow, however, that the main argument of the *Critique* requires completion in the *Metaphysical Foundations*. Rather, the two works have different yet complementary perspectives on this same phenomenal world, about which they establish different yet complementary conclusions.

GENERAL AND SPECIAL METAPHYSICS

As I observed in the Introduction, the Preface to the *Metaphysical Foundations* begins by asserting that (467) nature (in its *material* meaning) "has two principal parts, in accordance with the principle division of our senses, where the one contains the objects of the *outer* senses, the other the object of *inner* sense." At least in principle, therefore, two different species of the metaphysics of nature are possible – two different kinds (470) of "*special* metaphysical natural science (physics or psychology), in which the above transcendental principles [of the first *Critique*] are applied to the two species of objects of our senses." What Kant calls *general* metaphysics, by contrast, is "the *transcendental* part of the metaphysics of nature," which concerns "the laws that make possible the concept of a nature in general, even without relation to any determinate object of experience, and thus undetermined with respect to the nature of this or that thing in the sensible world" (469). So general metaphysics or transcendental philosophy is expounded in the analytic of principles of the first *Critique*, and it constitutes the super-ordinate genus to which the two possible species of the metaphysics of nature are subordinate. The special metaphysics of specifically corporeal nature is expounded in the *Metaphysical Foundations* itself – where, in particular, the three Laws of Mechanics then instantiate the three analogies of experience.[3]

The transcendental philosophy of the first *Critique* is therefore distinguished from the *Metaphysical Foundations* in the first instance by its greater generality. The former concerns nature or the sum total of the objects of our senses, whether outer or inner, whereas the latter is restricted

[3] See notes 7 and 37 of the Introduction, together with the paragraphs to which they are appended. As I there observe in the first of these paragraphs (and further explain in what follows), it turns out that only the special metaphysics of corporeal nature can serve to ground a genuine science: no proper science of the object of inner sense (the soul) is possible. I shall return to this situation below.

to the objects of specifically outer sense. Indeed, Kant had already made this point, by way of anticipation, in §15 of the *Prolegomena*:

Now we are nonetheless actually in possession of a pure natural science, which propounds laws a priori – and with all the necessity required for apodictic propositions – under which nature stands. I need here only call upon that propaedeutic to the doctrine of nature which, under the name of universal natural science, precedes all physics (which is based on empirical principles). There we find mathematics applied to appearances, and also merely discursive principles (from concepts), which constitute the philosophical part of the pure cognition of nature. But there is still much in it that is not entirely pure and independent of empirical sources: such as the concept of *motion*, of *impenetrability* (on which the empirical concept of matter rests), of *inertia*, and others, which prevents it from being able to be called an entirely pure natural science; moreover, it extends only to the objects of the outer senses, and thus yields no example of a universal science of nature in the strict sense, for [the latter] must bring nature in general under universal laws, whether it concerns the object of the outer senses or that of inner sense (the object of physics as well as psychology). But among the principles of this universal physics a few are found that actually have the universality we require, such as the proposition *that substance remains* and endures, that *everything that happens* always previously *is determined by a cause* in accordance with constant laws, etc. These are actually universal laws of nature that obtain fully a priori. (4, 295)[4]

Thus Kant here illustrates the greater (maximal) generality of the transcendental principles of the understanding by the first two analogies of experience.[5]

So far, therefore, there are two important features distinguishing the general metaphysics of the first *Critique* from the special metaphysics of the *Metaphysical Foundations*. For, on the one hand, the latter is not

[4] Compare note 47 of the Introduction.

[5] As I indicated in my chapter on the Mechanics, Kant makes the same point after the publication of the *Metaphysical Foundations* in an important passage from the *Critique of the Power of Judgement*. Kant there distinguishes between *transcendental* and *metaphysical* principles, and he again illustrates the distinction by reference to the first two analogies (5, 181):

Thus, the principle of the cognition of bodies as substances and as changeable substances is transcendental, if it is thereby asserted that their changes must have a cause; it is metaphysical, however, if it is thereby asserted that their changes must have an *external* cause. This is because in the first case bodies may be thought only by ontological predicates (pure concepts of the understanding), e.g., as substance, in order to cognize the proposition a priori; but in the second case the empirical concept of a body (as a movable thing in space) must be taken as the basis of the proposition.

Kant here has in mind his formulation of the law of inertia (his Second Law of Mechanics) in the *Metaphysical Foundations*: see note 92 of my chapter on the Mechanics, together with the paragraph to which it is appended.

"entirely pure and independent of empirical sources" (4, 295), since it explicitly depends on the *empirical* concept of matter, and, on the other hand, it extends only to the objects of outer sense (the objects of physics), while the latter concerns all objects of the senses in general – "the object of physics as well as psychology" (4, 295). The connection between these two features, in the words of the Preface to the *Metaphysical Foundations*, is (467) that "in accordance with the principle division of our senses [into the objects of outer and inner sense]," "a twofold doctrine of nature is possible, the *doctrine of body* and the *doctrine of the soul*, where the first considers *extended* nature, the second *thinking* nature." Moreover, any special metaphysical natural science thereby involves an empirical concept:

It concerns itself with a particular nature of this or that kind of thing, for which an empirical concept is given, but still in such a manner that, outside of what lies in this concept, no other empirical principle is used for its cognition (for example, it takes the empirical concept of matter or of a thinking being as its basis, and it seeks that sphere of cognition of which reason is capable a priori concerning these objects), and here such a science must still always be called a metaphysics of nature, namely, of corporeal or of thinking nature. (470)[6]

And, according to the following paragraph of the Preface, a third important distinguishing feature now follows.

Special metaphysics centrally depends on the application of mathematics – and, in particular, on the mathematical construction of concepts – in a way that the general metaphysics of the first *Critique* does not:

[T]he possibility of determinate natural things cannot be cognized from their mere concepts; for from these the possibility of the thought (that it does not contradict itself) can certainly be cognized, but not the possibility of the object, as a natural thing that can be given outside the thought (as existing). Hence, in order to cognize the possibility of determinate natural things, and thus to cognize them a priori, it is still required that the *intuition* corresponding to the concept be given a priori, that is, that the concept be constructed. Now rational cognition through the construction of concepts is mathematical. Hence, although a pure philosophy of nature in general, that is, that which investigates only what constitutes the concept of a nature in general, may indeed be possible even without mathematics, a pure doctrine of nature concerning *determinate* natural things (doctrine of body or doctrine of soul) is only possible by means of mathematics. (470)[7]

[6] See again note 3 above, including its references back to the Introduction.
[7] See note 38 of the Introduction, which is appended to the full quotation of the paragraph in question (470).

I considered this central yet very difficult paragraph in detail in the Introduction. I argued that Kant does not mean that the empirical concept of matter is itself supposed to be mathematically constructed in pure intuition. For this would directly contradict a number of his explicit statements, in both the *Metaphysical Foundations* and the first *Critique*, to the effect that neither his characteristically dynamical concept of matter nor any empirical concept in general can have its real possibility or objective reality presented a priori in pure intuition.

What Kant is saying, rather, is that the general metaphysics of the first *Critique* provides a priori principles governing the real possibility of objects of a nature in general: i.e., objects falling under the categories or pure concepts of the understanding, which are thus characterized solely by pure "ontological predicates" (5, 181; see note 5 above). A priori insight into the real possibility of the more determinate objects of a special metaphysical natural science, by contrast, requires additional a priori principles going beyond the transcendental principles of the pure understanding (which are not themselves mathematical), and the only available such a priori principles are mathematical. Hence a special metaphysics of any more determinate species of objects in nature must explain the possibility of applying mathematics to the specific empirical concepts involved in a proper natural science restricted to this domain. It must explain how these particular concepts acquire their precise mathematical structure and, in this sense, how their mathematical construction is possible.[8]

In the case of the special metaphysics of corporeal nature the relevant empirical concepts, which make "the concept of [the mathematical physicists'] proper object, namely, matter, a priori suitable for application to outer experience," are "the concept of motion, the filling of space, inertia, and so on" (472).[9] The concept of motion acquires the structure of a mathematical magnitude (in terms of speed and direction) in the sole proposition of the Phoronomy, which comes closer to a construction in pure

[8] See note 51 of the Introduction, together with the paragraph to which it is appended. According to §15 of the *Prolegomena*, the a priori universal natural science constituting the propaedeutic to empirical natural science contains "mathematics applied to appearances, and also merely discursive principles (from concepts), which constitute the philosophical part of the pure cognition of nature" (4, 295; see the passage to which note 4 above is appended). On my reading of the paragraph on possibility and the application of mathematics from the Preface to the *Metaphysical Foundations* (470), therefore, the point of the a priori principles of pure natural science articulated in that work is to explain the application of mathematics to the specific empirical concepts considered there.

[9] See again note 47 of the Introduction, together with the paragraph to which it is appended. It emerges from the note itself that this list of concepts is essentially the same as that given in §15 of the *Prolegomena*.

intuition than any other proposition in the *Metaphysical Foundations*.[10] When we arrive at the concept of (relative) impenetrability or the filling of space in the Dynamics, however, we have definitely left pure intuition behind. Moreover, Kant also introduces the concept of quantity of matter in the Dynamics in terms of a "measure of the intensive filling of space [i.e., density]" (521). Yet, according to the first proposition of the Mechanics, this concept can only be quantitatively estimated in general by "the quantity of motion at a given velocity" (537). For we can *dynamically* compare densities only among matters of the same kind (where one can arise from the other by mere compression), and this would only be possible in general in the system of absolute impenetrability that Kant rejects. In the end, therefore, we must resort to what Kant calls a *mechanical* comparison of quantities of matter by an equilibrium of momenta (quantities of motion) at a given velocity – as paradigmatically exemplified in the procedure of comparing the weights of different bodies in a balance.[11]

In general, two bodies have an equal quantity of matter when they descend equally in the two sides of a balance, and the quantity of one body is the (numerical) sum of that of two other bodies when the first body is equal in quantity to the (mereological) sum of the other two. The remaining problem is to extend this quantitative structure to all bodies in the universe regardless of their relationship to the surface of the earth, and precisely such an extension, Kant sees, is one of the central achievements of the Newtonian theory of universal gravitation. For this theory embeds the parochial comparison of quantities of matter in terms of weights relative to the earth's surface within a more general system of comparisons based on (more general) equilibria between changes of momenta involving gravitational accelerations: of falling bodies towards the earth and vice versa, of the moon towards the earth and vice versa, of the planets towards the sun and vice versa, and so on.[12] So it is the principle of the equality of action and reaction (Kant's Third Law of Mechanics), applied

[10] As explained especially in section 6 above, however, even this proposition is not solely a construction in pure intuition. For it depends on the preceding relativity principle in the Phoronomy, which, in turn, essentially involves the empirical concept of motion of an object in space (relative to one or another empirical frame of reference). Nevertheless, it can still consider the motion in question as that of a mere mathematical point, and it is this that distinguishes it from the concept of motion and its quantity developed later in the Mechanics. So what happens in the Phoronomy, more precisely, is a *transition* from pure to empirical intuition.

[11] See note 38 of my chapter on the Mechanics, together with the paragraph to which it is appended and the two preceding paragraphs.

[12] See the discussion in section 24 above that begins with the paragraph to which note 42 of my chapter on the Mechanics is appended and concludes with the paragraph to which note 51 of this same chapter is appended.

to interactions in accordance with universal gravitation, which, for the first time, allows us to extend the static measure of weight to a universal measure of quantity of matter valid for all bodies in general on the basis of the new (Newtonian) mechanical quantity of mass.[13] We presuppose the universal applicability of the mechanical laws of motion, the equality of inertial and gravitational mass, and the universally penetrating character of gravitational force in this procedure. And it is for precisely this reason that Kant builds all three into the characteristically dynamical concept of matter that he articulates in the *Metaphysical Foundations*.

It is for this reason, too, that the mechanical laws of motion and the above properties of gravitational force (the fundamental force of attraction) count as a priori for Kant. This emphatically does not mean, however, that he attempts to demonstrate by pure reason, independently of experience, what Newton has discovered by observation and experiment. The point is rather, on my reading, that Kant attempts to isolate just those features of the concept of matter in virtue of which Newton has successfully mathematized its quantity, and he analyzes or explicates this concept (in terms of motion, the filling of space, inertia, and so on) precisely to reflect these features. Kant explicitly recognizes, in particular, that this analysis is, in an important sense, contingent, in so far as there is an alternative mechanical concept in accordance with the system of absolute (as opposed to relative) impenetrability.[14] And his choice of this preferred (dynamical) concept over the alternative (mechanical) concept rests, in the end, on nothing more nor less than the empirical success of Newton's theory in comparison with the opposing mechanical philosophy. But Kant also sees (perhaps even more clearly than Newton) that the success of this theory has a crucially important conceptual dimension as well, for it also makes possible, for the first time, a precise implementation of what I have called the Copernican conception of space and motion. It shows how rigorously to extend the modern concept of motion (based on the law of inertia) from the terrestrial to the celestial realm and, at the same time, how rigorously to transform the traditional static magnitude of weight into a universally applicable mathematical measure of mass and quantity of matter.[15]

[13] For these three measures see again note 54 of my chapter on the Mechanics, the paragraph to which it is appended, and the surrounding discussion in section 24.

[14] This point is central to my analysis in section 10 above of Kant's introduction of the fundamental force of repulsion in the first proposition of the Dynamics.

[15] These points are discussed most recently and explicitly in section 35 in my chapter on the Phenomenology.

If Kant's preference for his characteristically dynamical concept of matter rests, in the end, on nothing more nor less than the empirical success of Newton's theory, then the real possibility or objective reality of this concept can certainly not be demonstrated by a merely mathematical construction in pure intuition. Indeed, it is for precisely this reason, according to the Preface to the *Metaphysical Foundations*, that the special metaphysics of corporeal nature counts as metaphysics as opposed to mathematics (469): "*Properly* so-called natural science presupposes, in the first place, metaphysics of nature. For laws, that is principles of the necessity of that which belongs to the *existence* of a thing, are concerned with a concept that cannot be constructed, since existence cannot be presented a priori in any intuition." But, as we know, these metaphysical laws concerned with the *existence* of things in nature (as opposed to merely mathematical objects constructed in pure intuition) are, first and foremost, the analogies of experience. Since, on the one hand, the analogies of experience are the principles of pure understanding corresponding to the categories of substance, causality, and community (interaction), what Kant calls "metaphysics" is thereby closely tied to the Leibnizean metaphysical tradition he inherited. And since, on the other hand, the specific realizations of these principles in the *Metaphysical Foundations* are the corresponding mechanical laws of motion (Kant's three Laws of Mechanics), Kant's own version of metaphysics thereby involves a radical reconfiguration of Leibnizean metaphysics in light of Newtonian physics. This reconfiguration culminates, in an important sense, in the account of Newton's mathematization of the concept of quantity of matter in the Mechanics, where the mechanical laws of motion (the second, third, and fourth propositions) both follow upon and illuminate the initial explanation of how quantity of matter can alone be quantitatively estimated (in the second explication and first proposition).[16]

I am now in a position to illuminate the sense in which the specific realization of the principles of pure understanding articulated in the *Metaphysical Foundations* provides an "*indispensable* service" for general metaphysics or transcendental philosophy (476, emphasis added). I begin by considering what the first *Critique* itself establishes concerning the application of mathematics, and I then turn to considering what the *Metaphysical Foundations* adds to this. In his discussion of the (general) principle of the (three) analogies of experience Kant says that the application of mathematics to appearances in general is justified by the

[16] These points are most recently discussed in section 36 above, and, in particular, in the concluding four paragraphs of my chapter on the Phenomenology.

two previous principles, i.e., the axioms of intuition and anticipations of perception:

> The preceding two principles, which I called the mathematical [principles], in consideration of the circumstance that they justified the application of mathematics to appearances, extended to appearances in accordance with their mere possibility, and taught how, with respect to both their intuition and the real in their perception, they could be generated according to the rules of a mathematical synthesis – and therefore [how], in both cases, numerical magnitudes [*Zahlgrößen*] can be used, and with them the determination of appearance as magnitude [*Größe*]. (A178–79/B221)

In particular, according to the (second edition version of the) principle of the axioms (B202): "*All **intuitions** are extensive magnitudes*."[17] According to the (second edition version of the) principle of the anticipations (B207): "*In all appearances the real, **which is an object of sensation**, has an intensive magnitude*, i.e., a degree."[18]

Extensive magnitudes, according to the first principle, are those consisting of (antecedently given) spatio-temporal parts and are thus "already intuited as aggregates (multitudes [*Menge*] of antecedently given parts)" (B204). Intensive magnitudes, by contrast, do not consist of spatio-temporal parts: they represent quantitative degrees of some quality or reality (such as heat, illumination, color, and so on) that may continuously vary at a given (unextended) spatio-temporal point. But this does not mean, as explained in section 3 above, that intensive magnitudes (in contrast to extensive magnitudes) fail to be additive or to possess an operation of composition. All magnitudes (in accordance with the traditional concept of magnitude) must be additive, for Kant, but the addition or composition of specifically *intensive* magnitudes is not immediately given by a composition of their spatio-temporal parts. For example, whereas the addition of lengths, areas, and volumes is immediately given (geometrically) in their spatial intuitions, the addition of degrees of heat or illumination "filling" any spatial region is not. So such an operation still needs to be exhibited or constructed in each specific case of any given quality or reality.[19]

[17] In the first edition (A161): "All *appearances* are, *with respect to their intuition*, **extensive** magnitudes."

[18] In the first edition (A166): "In all appearances *the sensation, and **the real**, which corresponds to it in the object (realitas phaenomenon)* has *an **intensive magnitude***, i.e., a degree."

[19] For my earlier discussion of these points in section 3 see note 40 of my chapter on the Phoronomy, together with the paragraph to which it is appended. In the example of "constructing" the degree of illumination of the sun out of some 200,000 degrees of illumination of the moon quoted in the note (A179/B221), it appears that the relevant operation of composition is derived from the geometrical (extensive) relations between the earth, sun, and moon together with the (inverse-square) photometric law (compare note 203 of my chapter on the Dynamics).

The crucial point, however, is that the anticipations of perception by themselves do not provide us with an addition or composition operation for any particular given quality. All that we know in general is that any such quality, as object of sensation, must have "a degree of influence on the sense" (B208; see note 9 of my chapter on the Dynamics). And all that we can infer from the pure concept of a reality in general is that these degrees, in the case of any given quality, can be ordered continuously.[20] In order to exhibit the relevant addition or composition operation we must know something more about the specific reality in question, and this requires that we explicitly bring a particular *empirical* concept into consideration. In the case of the empirical concept of motion (of an object in space) developed in the *Metaphysical Foundations*, for example, we can exhibit or construct the relevant operation of composition of velocities (where velocity, for Kant, is an intensive magnitude) by the consideration of different relative spaces or frames of reference in accordance with the relativity principle of the Phoronomy (section 4; compare note 10 above). In the case of Kant's preferred empirical concept of the quantity of matter, originally introduced in terms of a "measure of the *intensive* filling of space" (521, emphasis added), the relevant operation (as just emphasized) can only be explained by extending the traditional static concept of weight to the universal (Newtonian) concept of mass in accordance with Kant's critical version of the dynamical theory of matter.[21] The special metaphysics of corporeal nature can thus provide additional a priori insight into the possibility of these particular intensive magnitudes by

[20] See the concluding paragraph of the anticipations (A175–76/B217–18):

> [T]he real, which corresponds to sensations in general, in opposition to negation = 0, only represents something whose concept in itself contains a being, and signifies nothing but the synthesis in an empirical consciousness in general. Namely, in inner sense the empirical consciousness can be raised from 0 up to any greater degree, so that precisely the same extensive magnitude of intuition (e.g., illuminated surfaces) excites as great a sensation as an aggregate of many other (lesser illuminations) together. One can thus completely abstract from the extensive magnitude of the appearances and still represent in the mere sensation at a moment a synthesis of uniform increase from 0 up to the given empirical consciousness. All sensations, therefore, as such, are in fact only given a posteriori, but their property of having a degree can be cognized a priori. It is remarkable that we can only cognize a single *quality* [*Qualität*] in magnitudes in general, namely continuity, and [we can cognize] nothing further a priori in any quality (in the real of appearances) than their intensive *quantity* [*Quantität*], but all the rest remains left to experience.

[21] Compare note 13 above, together with the paragraph to which it is appended. The important point, as explained in section 23 above, is that quantity of matter – defined, following Newton, as the product of volume and density – has both an extensive and an intensive aspect.

interweaving mathematical constructions with the transcendental a priori principles of general metaphysics.[22]

It is no wonder, therefore, that Kant emphasizes the problem of specifically intensive magnitudes when he calls attention to the "indispensable" service provided to general metaphysics by the special metaphysics of corporeal nature in the Preface:

> It is also indeed very remarkable (but cannot be expounded in detail here) that general metaphysics, in all instances where it requires examples (intuitions) in order to provide meaning for its pure concepts of the understanding, must always take them from the general doctrine of body, and thus from the form and the principles of outer intuition; and, if these are not exhibited completely, it gropes uncertainly and unsteadily among mere meaningless concepts. This is the source of the well-known disputes, or at least obscurity, in the questions concerning the possibility of a conflict of realities, of intensive magnitudes, and so on, in which the understanding is taught only by examples from corporeal nature what the conditions are under which such concepts can alone have objective reality, that is, meaning and truth. And so a separated metaphysics of corporeal nature does excellent and indispensable service for *general* metaphysics, in that the former furnishes examples (instances *in concreto*) in which to realize the concepts and propositions of the latter (properly speaking, transcendental philosophy), that is, to give a mere form of thought sense and meaning. (478)[23]

The special metaphysics of corporeal nature provides an indispensable service to transcendental philosophy, then, by furnishing examples or instances *in concreto* where its categories and principles of the pure understanding are first realized. And in the case of intensive magnitudes, in particular, there is no way to attain complete a priori insight into their

[22] As explained in section 25 above, both intensive and extensive aspects of the concept of quantity of matter figure essentially in Kant's argument for the conservation of the total quantity of matter in his First Law of Mechanics, which thereby depends on the categories of both reality and substance (together with their corresponding transcendental principles). What is special about the empirical concept of matter, in this context, is that, as the very highest empirical concept of natural science, it is located directly below the pure concepts of the understanding – in such a way that it can then be directly "brought under" the four headings of the table of categories (474–76; see the paragraph to which note 24 of the Introduction is appended). For the way in which the empirical concept of matter is located directly below the pure concepts of the understanding, in such a way that it thereby mediates their application to all lower empirical concepts, see the paragraph to which note 232 of my chapter on the Phenomenology is appended. It is not clear whether the mathematical structure of other intensive magnitudes, such as degree of illumination (note 19 above), can be a priori grounded in this way.

[23] I quoted most of this passage in the paragraph to which note 10 of the Introduction is appended – but omitted the part about a "conflict of realities" and "intensive magnitudes" in an ellipsis. Both involve special problems concerning the categories of quality, where the definite empirical content provided by sensation or empirical intuition is first explicitly considered.

possibility – with respect to both their (order) structure as *continuous* magnitudes and their (additive) structure as *magnitudes* in general – without the concrete examples (velocity and quantity of matter) that are articulated in detail in the *Metaphysical Foundations*.[24]

It does not follow, however, that the transcendental deduction of the categories requires the exhibition of a particular system of objects of experience that instantiate the categories (and principles) of the understanding. In Kant's discussion of the real (as opposed to logical) possibility appropriate to various kinds of concepts in the postulates of empirical thought he carefully distinguishes between *empirical* and *pure* concepts:

> A concept that comprises a synthesis is to be taken as empty, and related to no object, if this synthesis does not belong to experience – either as borrowed [*erborgt*] from it, in which case it is an *empirical concept*; or as one that rests, as an a priori condition, on experience in general (the form of experience), in which case it is a *pure concept*, which nevertheless belongs to experience, because its object can only be found there. For from what may one derive the character of the possibility of an object that has been thought through a synthetic concept a priori, if it does not come about from the synthesis that constitutes the form of empirical cognition in general? (A220/B267)

Moreover, among the pure concepts, it is clear, one must also distinguish between pure *sensible* concepts (mathematical concepts) and pure *intellectual* concepts (categories). Whereas the real possibility or objective reality of such pure concepts, in both cases, depends on the synthetic a priori conditions of the possibility of experience in general, in the former (mathematical) case we can still "give it [the concept] an object completely a priori, i.e., construct it" (A223/B271) by an "image-forming [*bildende*] synthesis" (A224/B271).[25]

According to Kant's discussion of the essential differences between mathematics and transcendental philosophy in the discipline of pure reason, however, precisely this can never be done in the case of pure intellectual concepts or categories:

[24] I am here especially indebted to illuminating conversations with Daniel Warren concerning the relationship between general and special metaphysics in the Preface. Intensive magnitudes and the category of reality are central to his studies of Kant's dynamics: see Warren (2001a and 2001b). I was also stimulated by a conversation with Alexei Angelides concerning the different ways in which the application of mathematics is treated in the first *Critique* and the *Metaphysical Foundations*.

[25] Compare note 43 of the Introduction, together with the paragraph to which it is appended. Here, and in the remainder of this section, I am drawing upon Friedman (2001). In the latter I also engage with some of the leading alternative views concerning the objective reality of the empirical concept of matter in relation to both mathematical construction and the categories – including, aside from Förster (1987), Plaass (1965), Gloy (1976), and Cramer (1985).

> Now an a priori concept (a non-empirical concept) either already contains a pure intuition in itself, and then it can be constructed; or [it contains] nothing but the synthesis of possible intuitions that are not given a priori, and then we can indeed judge by means of it synthetically and a priori, but only discursively in accordance with concepts, and never intuitively by the construction of the concept. (A719–20/B747–48)

In the case of the pure concept of the understanding, in particular, we are involved with the "concept of a *thing* in general" (A720/B748), as an object of perceptual experience in general:

> Synthetic propositions that extend to *things* in general, whose intuition can by no means be given a priori, are transcendental. Hence transcendental propositions can never be given by the construction of concepts but only in accordance with a priori concepts. They contain merely the rule, in accordance with which a certain synthetic unity of that which cannot be intuitively represented a priori (of perceptions) is to be empirically sought. Yet they cannot present any one of their concepts a priori in any case at all, but do this only a posteriori, by means of experience, which first becomes possible in accordance with these synthetic principles. (A720–21/B748–49)

Objects instantiating the pure concepts of the understanding can only be given a posteriori by means of sensible and empirical intuitions (perceptions). Such objects, in principle, must be *empirical* objects (objects of experience).[26]

Yet the objective reality or real possibility of the pure concepts of the understanding does not – and cannot – consist in the actual existence of such objects (objects of experience), for otherwise the condition for the (real) possibility of these concepts would be indistinguishable from that of empirical concepts. Accordingly, Kant explicitly distinguishes the categories from empirical concepts in this respect in the postulates of

[26] This is why Kant insists that the mathematical construction of objects in pure intuition results, properly speaking, in "only the *form* of an object" (A223/B271, emphasis added), whose matter or content still needs to be given a posteriori. Essentially the same point is made, even more clearly and explicitly, in §22 of the second edition deduction (B147):

> By determination of [pure intuition] we can acquire a priori cognition of objects (in mathematics), but only in accordance with their form, as appearances; whether there can be things that must be intuited in this form remains thereby still undecided. Therefore, all mathematical concepts in themselves are not cognitions, except in so far as one presupposes that there are things that can be presented to us only in accordance with the form of this pure sensible intuition. *Things in space and time*, however, are only given in so far as they are perceptions (representations accompanied by sensation) and thus by means of empirical representation. Therefore, the pure concepts of the understanding provide cognition – even when they are applied to a priori intuitions (as in mathematics) – only in so far as the latter, and thus the pure concepts of the understanding by means of them, can be applied to empirical intuitions.

empirical thought. The latter, as "borrowed [*entlehnt*] from [experience]" (A222/B269), "cannot acquire the character of their possibility a priori, like the categories, as conditions on which all experience depends, but only a posteriori, as such that are given through experience itself – and their possibility must either be cognized a posteriori and empirically, or it cannot be cognized at all" (A222/B269–70; compare the paragraph to which note 44 of the Introduction is appended). The categories do acquire their real possibility a priori, however, and in a completely different fashion:

> Only in the circumstance, therefore, that these [pure concepts of the understanding] express a priori the relations of perceptions in every experience, does one cognize their objective reality, i.e., their transcendental truth; and [one does this] certainly independently of experience, but not independently of all reference to the form of an experience in general, and [to] the synthetic unity in which alone objects can be empirically cognized. (A221–22/B269)

Thus the pure concepts of the understanding contain only the form in accordance with which the empirical objects of perception must be experienced if objective experience is to be possible at all. They provide only the rule by which we must *search out* the appearances in order thereby to find, where possible, the objects of experience that instantiate them.[27]

Such a rule is precisely what Kant calls the *schema* of a pure concept of the understanding, and he carefully distinguishes this kind of schema, in particular, from that of a pure sensible (mathematical) concept. In the case of the latter, in particular, he contrasts the schema of the concept (the rule for constructing instances of the concept) from what he calls its corresponding "images [*Bilder*]" (particular instances thereby constructed).

[27] Kant illustrates this with respect to the concept of cause in a footnote in the discipline of pure reason (A722/B750):

> By means of the concept of cause I actually proceed from the empirical concept of an event (where something happens) – not, however, to the intuition that exhibits the concept of cause *in concreto*, but rather to the conditions of time in general, which may be found in experience in accordance with the concept of cause. I therefore proceed merely in accordance with concepts, and cannot proceed through the construction of concepts, because the concept is a rule of synthesis of perceptions, which are not pure intuitions, and thus cannot be *given* a priori.

Compare the discussion of the sense in which an analogy of experience is merely regulative in the postulates of empirical thought (A180/B222–23):

> [An analogy of experience provides] a rule for seeking it [e.g., a cause] in experience and a mark for discovering it there. An analogy of experience will thus only be a rule in accordance with which from perceptions unity of experience may arise (not, like perception itself, as empirical intuition in general), and it is valid as [a] principle of the objects (the appearances) not *constitutively* but merely *regulatively*.

He then introduces the notion of a schema for the pure concepts of the understanding by way of contrast:

> [T]he *image* is a product of the empirical capacity of the productive imagination; the *schema* of sensible concepts (such as figures in space) is a product, and as it were a monogram of the pure a priori imagination, by which and in accordance with which the images first become possible, which must be connected with the concept only by means of the schema, which they designate, and are not in themselves congruent to the concept. By contrast, the schema of a pure concept of the understanding is something that cannot be brought into an image at all, but is rather only the pure synthesis, in accordance with a rule of unity according to concepts in general, which the category expresses a priori; and [it] is a transcendental product of the imagination, which concerns the determination of inner sense in general, in accordance with conditions of its form (time), with respect to all representations, in so far as they are to cohere a priori in a concept according to the unity of apperception. (A141–42/B181).

Thus, for example, images instantiating the pure sensible concept of a triangle are particular individual triangles constructed in accordance with the corresponding schema, and they are "congruent" or homogeneous with the concept (which is essentially general) only in virtue of this schema. But the concept of a cause, by contrast, possesses no schema at all in this sense – no general rule for constructing particular instances a priori. Its schema, as explained, is rather a rule for *finding* instances in empirically given perceptions wherever possible, in order thereby to transform these given perceptions into objective experience.[28]

The pure concepts of the understanding acquire their objective reality or real possibility, therefore, not from a system of concrete empirical objects that instantiate them but rather from their schemata – from the a priori rules that we can and must provide for finding such instances in our perceptions so as to make objective experience possible. In the special metaphysics of corporeal nature, by contrast, we actually do find such a system of concrete empirical objects in our perceptions: namely, the

[28] To return to the example from the previous note, the schema of the concept of cause is "the real such that, if it is arbitrarily posited, something else always follows[; the schema] therefore consists in the succession of the manifold, in so far as it is subject to a rule" (A144/B183). Although we cannot, of course, construct a particular cause for a given event a priori, we know that we have *found* such a cause when there is a corresponding event in perception that is related to the given event by an empirical universal law of succession. Only then does the empirically given sequence of perceptions count as an objective experience, and it *becomes* an experience, in this case, precisely in virtue of the schema of the concept of cause. Compare Kant's discussion of the peculiar character of the (transcendental) proof of the second analogy in the discipline of pure reason in terms of the distinction between *principles* and *theorems* (A737/B765; see note 83 of my chapter on the Mechanics).

system of massive bodies near the surface of the earth and beyond that interact with one another in accordance with Newtonian gravitational physics and astronomy. This system instantiates Kant's characteristically dynamical empirical concept of matter, and it then instantiates the categories and principles of the pure understanding in virtue of the way in which this empirical concept is directly "brought under" these categories and principles (474–76; note 22 above). Yet there is an essential difference. Whereas the objective reality of the empirical concept of matter consists in the actual existence of this particular concrete instance, the objective reality of the categories does not. For the latter are pure rather than empirical concepts, and, accordingly, they acquire their objective reality from a priori rules of synthesis or schemata rather than empirically given concrete instances. Moreover, such schemata (unlike those of mathematical concepts) do not result in a priori constructed instances (images) but rather in a priori rules for attaining objective experience from empirically and sensibly given perceptions of objects found in nature. Hence, while the articulation of such rules for the individual categories in the following schematism and principles chapters is needed to complete the transcendental deduction of the categories, the special metaphysics of corporeal nature is not.

This last point becomes especially clear when we observe that there is an important sense in which the scope of possible instantiations of the categories is vastly greater than that of Kant's special metaphysics of corporeal nature. For the latter provides an a priori grounding for only the most general properties of all matter as such (original elasticity and mass, for example), while leaving all more particular properties of matter (even cohesion) for a physical rather than metaphysical treatment.[29] In this way, in particular, the natural phenomena at issue in this metaphysics are correspondingly limited to those accounted for by the Newtonian theory of universal gravitation. Extending our scientific knowledge beyond this theory – to thermal phenomena, electric and magnetic phenomena, chemical phenomena, biological phenomena, and so on – is left to the further progress of empirical science under the guidance of the regulative use of reason.[30] But the role of the pure concepts and principles of the

[29] See again note 265 of my chapter on the Dynamics, together with the paragraph to which it is appended. Accordingly, the problem of what Kant calls the "specific variety" of matter is relegated to the general remark to dynamics.

[30] The main example treated in the *Metaphysical Foundations* is chemistry. My discussion of Kant's perspective on contemporary chemistry occupies the bulk of section 20 above, beginning with the paragraph to which note 224 of my chapter on the Dynamics is appended and continuing through the remainder of this section. I shall briefly touch on the case of biological phenomena below.

understanding, by contrast, is to explain the general a priori conditions that make possible *any and all* objects of experience, including all those not yet accounted for at any given stage in the development of empirical science. In the case of the categories, as explained, the general conditions in question are just their schemata, and these remain the same whether the categories are applied in the Newtonian theory of universal gravitation or any other domain of theoretical cognition.[31] Thus, once again, it is precisely this a priori schematization of the categories – and not any particular empirically given concrete instantiation – which alone provides them with their objective reality.

So why is the special metaphysics of corporeal nature nonetheless indispensable for giving "sense and meaning" to the concepts and principles of general metaphysics (478)? This, on my reading, is because the particular concrete instantiation considered in the *Metaphysical Foundations* is the very first application of the categories and principles of the *Critique* to an actual empirically perceptual domain. In general, the categories can apply to such a domain only through the mediation of some empirical concepts instantiated in this domain, and the empirical concept of matter articulated in the *Metaphysical Foundations*, as the very highest empirical concept of natural science, thereby mediates the application of the categories to all other empirical concepts (see again note 22 above). Hence understanding exactly how the categories (and their schemata) apply to this domain does indeed provide an "excellent and indispensable service" for general metaphysics by teaching us "what the conditions are under which [the categories] can alone have objective reality, that is, meaning and truth" (478). In particular, we now learn – as emphasized both here and in the corresponding general remark to the system of principles added to the second edition of the *Critique* – that any particular instances (intuitions) falling under the categories must be taken "from the form and the principles of *outer* intuition" (478, emphasis added). Such instances, in other words, must be spatial, and they cannot be limited to inner sense alone.[32] This condition, as explained in the Introduction, establishes

[31] For example, the schema of the concept of causality – succession in accordance with a rule (note 28 above) – remains the same whether we are discussing deviations from the state of inertial motion due to "moving forces" (A206–7/B252), the liquid state of water on freezing being succeeded by the solid state (B162–63), the position of a drifting ship higher up in a stream being succeeded by its position lower down (A192–93/B237–38), or the cool air surrounding a hot stove becoming warm (A202/B247–48).

[32] In discussing this passage above I placed special emphasis on the problem of intensive magnitudes – on the conditions for providing a perceptually given quality or reality with both the (order) structure of a *continuous* magnitude and the (additive) structure of a *magnitude* in general

an important connection between the argument of the *Metaphysical Foundations* and the central revisions made in the second edition of the *Critique*, and so I shall now turn to a closer consideration of precisely that connection.

INNER AND OUTER SENSE

The Preface to the *Metaphysical Foundations* contains a striking argument according to which proper natural science, strictly speaking, consists only of the doctrine of body, so that Kant's metaphysical foundations of natural science, strictly speaking, consist only of the special metaphysics of *corporeal* nature (471; compare notes 3 and 6 above). As discussed in the Introduction, moreover, the first part of this argument hinges on the circumstance that "mathematics is not applicable to the phenomena of inner sense and their laws" (471). More precisely, only a very limited amount of mathematics would be applicable, "which, however, would be an extension of cognition standing to that which mathematics provides for the doctrine of body approximately as the doctrine of the properties of the straight line stands to the whole of geometry" (471).[33] I argued in the Introduction for a close connection between this argument and that of the refutation of idealism added to the second edition of the first *Critique*, which establishes the priority of outer sense over inner sense in the constitution of even inner experience.[34] I suggested, in addition, that a deeper ground for

(see note 24 above, together with the paragraph to which it is appended). We now see, in particular, that these conditions essentially involve the requirement that the realities in question must be given in perception in *both* space and time. Indeed, this is already suggested by the example of comparing the degrees of illumination of the sun and the moon in the discussion of the (general) principles of the analogies (A179/B221; see note 19 above) – which, of course, is also an example taken "from corporeal nature" (478). In the general remark to the system of principles Kant focusses almost exclusively on the categories of relation and does not discuss the category of reality. But he does discuss the concept (or category) of *magnitude* and says this towards the very end (B293): "In precisely the same way it can easily be shown that the possibility of things as *magnitudes* [*Größen*], and thus the objective reality of the category of magnitude [*Größe*], can also be exhibited only in outer intuition, and by means of it alone can it then be applied also to inner sense." Kant, on my reading, is not here restricting himself to the consideration of specifically *extensive* magnitudes.

[33] The passage is quoted more extensively in the paragraph to which note 8 of the Introduction is appended. Kant there argues that "the only option one would have would be to take the *law of continuity* in the flux of inner changes into account" (471): "For the pure intuition in which the appearances of the soul are supposed to be constructed is *time*, which has only one dimension."

[34] Kant's introductory remarks (B274–75) to the refutation of idealism distinguish the *problematic* idealism of Descartes from the *dogmatic* idealism of Berkeley. And, since the latter has been adequately refuted in the transcendental aesthetic, the proof Kant offers here is addressed only to the former (B275): "The required proof must therefore show that we have *experience* and not merely *imagination* of outer things; which will not be able to happen except if one can prove that

this priority depends on the importance of the (Newtonian) mathematical theory of motion in the argument of the *Metaphysical Foundations*.[35] It is now time, however, to qualify this suggestion, and to focus, in particular, on the equally important differences between the argument of the refutation of idealism in the first *Critique* and the related argument concerning the priority of outer over inner sense in the *Metaphysical Foundations*. This will further enhance our understanding of the fundamentally different perspectives represented in these works as well.[36]

To begin with, when Kant explicitly discusses motion in the first *Critique*, he almost always discusses what I have called purely mathematical motion in pure intuition: namely, the motion of a *point* in space. This is true, in particular, of all the texts that I cited in the Introduction in order to build a bridge between the argument of the Preface to the *Metaphysical Foundations* and the refutation of idealism: the transcendental exposition of time added to the transcendental aesthetic as §5, the general remark to the system of principles, and §24 of the second edition transcendental deduction. Motion, in all of these texts, signifies the *change of position* of a mathematical point in space. By contrast, the *Metaphysical Foundations* is primarily concerned with the motion of an empirical object or *body* in space, and, as Kant asserts in the second explication of the Phoronomy, the motion of such a thing is better characterized as "the *change of its external relations* to a given space" (482). For a (three-dimensional) body (such as the earth), as opposed to a mere (zero-dimensional) mathematical point, can rotate relative to a given empirical space (such as that determined by the fixed starts) without changing its position.[37] Thus the application of the mathematical theory of motion that is most central to the *Metaphysical Foundations* – the Newtonian determination of the true motions in the solar system starting from our parochial perspective here on earth – hardly makes an appearance in the relevant texts from the first *Critique*.

Yet there is one significant exception: the second remark to the proof of the refutation of idealism. Here, as we have seen, Kant states (B277) that "[a]ll empirical employment of our cognitive faculties in the determination

even our *inner* experience, which Descartes did not doubt, is only possible under the presupposition of *outer* experience."

[35] This discussion begins with the paragraph to which note 11 of the Introduction is appended and concludes with the paragraph to which note 14 of the Introduction is appended.

[36] As I already indicated in note 10 of the Introduction, I am especially indebted to conversations with Daniel Warren for a better appreciation of these differences.

[37] Compare note 11 of my chapter on the Phoronomy. The central distinction between mathematical and empirical motion is extensively discussed in section 6 in that chapter.

of time fully agrees with [the refutation of idealism]," and he illustrates this claim by the circumstance that "we can undertake all time determination only by the change of external relations (motion) in relation to the permanent in space (e.g., motion of the sun with respect to objects on the earth)" (B277–78). There is no doubt here that Kant does have in mind the motion of an empirical object or body (as opposed to a mere mathematical point) and, moreover, that the argument of the refutation of idealism is thereby closely connected with the Newtonian determination of the true motions in the solar system.[38] Nevertheless, this important passage is only an illustration or application of the argument of the refutation of idealism. The argument itself appeals to much more abstract considerations, which, in particular, do not explicitly appeal to motion at all. Similarly, the later discussion in the general remark to the system of principles, which does explicitly appeal to motion (albeit purely mathematical motion of a point), is put forward as what Kant calls a *confirmation* of the refutation of idealism (B293).

So what is the argument of the refutation itself? The proposition to be proved reads (B275): "*The mere, but empirically determined, consciousness of my own existence proves the existence of objects in space outside me.*" The proof immediately follows:

> I am conscious of my existence as determined in time. All determination of time presupposes something *permanent* in perception. But this permanent cannot be something in me; because precisely my existence in time can only be determined in the first place by this permanent [thing]. Therefore, the perception of this permanent is only possible by means of *a thing* outside me and not by means of the mere *representation* of a thing outside me. Consequently, the determination of my existence in time is only possible by means of the existence of actual things that I perceive outside me. But consciousness in time is necessarily connected with consciousness of the possibility of this determination of time: Therefore, it is also necessarily connected with the existence of things outside me, as the condition of the determination of time; i.e., the consciousness of my own existence is, at the same time, an immediate consciousness of the existence of other things outside me. (B275–76)

This argument, notoriously, is extraordinarily brief and abstract. As I indicated, it makes no mention at all of the representation of motion in either of the two senses discussed above: motion as change of place (of a mere mathematical point in space) or as change of external relations (of

[38] See note 46 of my chapter on the Phoronomy, together with the paragraph to which it is appended and the remainder of section 3; compare notes 77 and 78 of this same chapter, together with the paragraphs to which they are appended and the remainder of section 6.

a body) to a given (empirical) space. Moreover, it is so brief and abstract as to be almost unintelligible as it stands, and, accordingly, it has been the subject of unusually intense discussion and controversy ever since it first appeared. I here intend to say just enough to illuminate my main theme – the different yet complementary perspectives of the *Metaphysical Foundations* and the first *Critique*.

Kant himself was dissatisfied with the argument as it stands, and, as is well known, he provided a replacement for the pivotal third sentence in a footnote to the (second edition) Preface:

Because I find some obscurity in the expression of the proof from the third to the sixth lines, I ask that this passage be revised as follows: "*But this permanent cannot be an intuition in me. For all grounds of determination of my existence that can be found within me are representations, and they themselves require, as such, a permanent different from them, in relation to which their change, and thus my existence in the time wherein they change, can be determined.*" (Bxxxix)

Kant also continued to reconsider the argument well after the publication of the second edition.[39] For my purposes, however, it is sufficient to confine our attention to the relevant remarks Kant makes in the two editions of the *Critique*, and I shall take my starting point, in particular, from his suggested revision in the second edition Preface.

What does Kant add by emphasizing that the "grounds of determination of my existence that can be found within me" are *representations*? As we have seen, there is an important passage in the transcendental aesthetic – to the effect that time as the form of inner sense is a more general a priori condition of appearances than space as the form of outer sense – which also emphasizes that the contents of inner sense consist exclusively of representations:

[B]ecause all representations, whether they have outer things as object or not, still belong in themselves, as determinations of the mind, to [its] inner state, and this inner state belongs under the formal conditions of inner intuition, and therefore to time, [it follows that] time is an a priori condition of all appearances in general – and, in fact, [it is] the immediate condition of inner [appearances] (of our souls) and precisely thereby also the mediate condition of the outer appearances. (A34/B50)[40]

All representations are states or determinations of the mind [*Gemüts*] and, as such, are inner appearances of our souls [*Seelen*]. Time as the form of

[39] See, for example, the very detailed discussion in Guyer (1983).
[40] The full passage is quoted in the paragraph following the one to which note 183 of my chapter on the Mechanics is appended. I shall return to the issues raised there below.

inner sense thus applies (mediately) to all objects of outer sense as well, in so far as some (but not all) representations have "outer things as their object" (A34/B50).

Now this priority of inner over outer sense – where the former is the *immediate* condition of inner appearances but only the *mediate* condition of outer appearances – may seem, at first sight, to give aid and comfort to Cartesian subjective idealism. For it may seem to suggest that the existence of outer objects would then need to be proved by some kind of mediate inference from our given representations of outer sense, which, like all representations, must belong, first and foremost, to inner sense. But this, on the contrary, is precisely what the refutation of idealism denies, as Kant forcefully emphasizes in his first remark:

[Cartesian idealism] assumed that the only immediate experience is inner [experience], and that one only *inferred* outer things from this, but, as in all cases where one infers from given effects to *determinate* causes, only unreliably, because the cause of the representations – which [cause] we perhaps falsely ascribe to outer things – can also lie in us ourselves. But here it is proved that outer experience is properly immediate,* [and] that only by means of it is the determination of our own existence in time (not, to be sure, the consciousness of our own existence [itself]) possible. (B276)[41]

The remark continues:

Certainly the representation: *I am*, which expresses the consciousness that can accompany all thinking, is that which immediately contains within itself the existence of a subject, but as yet no *cognition* of this subject, and thus also no empirical [cognition], i.e., experience. For there still belongs to this, besides the thought of something existing, also intuition, and in this case inner [intuition], in relation to which, i.e., to time, the subject must be determined, for which purpose outer objects are always required, so that, as a result, inner experience is itself only mediate and only possible by means of outer [experience]. (B276)

So three points have now become clearer.

There is, in the first place, a close connection between this argument and the paralogisms of pure reason. The argument depends on the idea that, although the representation 'I am' (as a consciousness that accompanies all thinking) does indeed contain a consciousness of the existence

[41] The footnote begins (B276): "The *immediate* consciousness of the existence of outer things is not assumed in the above proposition but proved, whether we may comprehend the possibility of this consciousness or not."

of a subject, it does not yet amount to a cognition or experience of this subject.[42] Moreover, in the second place, we require intuition as well as thought to have cognition or experience of such a subject.[43] And finally, in the third place, the required intuition needs to be *determined* in accordance with the categories in order to count as objective cognition or experience – in this case of the subject as determined in time. The experience in question is therefore made possible by the analogies of experience and, in particular, by the category of *substance*, which alone can determine an empirically given reality as permanent or persisting in time.[44]

So the question at the heart of the refutation of idealism appears to reduce to the following. Can I find, among my changing representations in inner sense, a permanent substantial subject "in relation to which their change, and thus my existence in the time wherein they change, can be determined" (Bxxxix)? Can we find such a substantial subject among the inner appearances "of our souls" (A34/B50)? Kant's answer to this question is an unequivocal 'no': an appropriately permanent empirically given reality can only be found among the objects of outer sense. For, as Kant explains in the second remark to the refutation of idealism, "we also have nothing at all permanent, which could underlie the concept of a substance, as intuition, except merely *matter*" (B278). He makes essentially the same point in the general remark to the system of principles (B291): "If, for example, we take the pure concepts of *relation*, we find, first, that in order to supply something *permanent* in intuition corresponding to the concept of *substance* (and thereby to verify the objective reality of this concept), we require an intuition *in space* (of matter), because space alone

[42] Kant equates the 'I think' and the 'I exist' in a footnote to the second edition paralogisms (B422):

> The 'I think' is, as already stated, an empirical proposition, and it contains the proposition 'I exist' within itself. But I cannot say that everything that thinks exists; for then the property of thinking would make all beings that possess it into necessary beings. Hence my existence can also not be viewed as inferred from the proposition 'I think', as Descartes held (since otherwise the major premise 'Everything that thinks exists' would have to be presupposed); rather, it is identical to this [proposition].

[43] This is a completely general point, which Kant has already taken pains to emphasize in §22 of the second edition deduction (B146): "To *think* an object and to *cognize* and object are therefore not one and the same. Two components, namely, belong to cognition: first, the concept, whereby an object is thought in general (the category), and second, the intuition, whereby it is given." It is to remind us of this point, it appears, that Kant (in the second edition Preface) changed "a thing" to "an intuition" in the first part of the pivotal third sentence of the refutation (Bxxxix).

[44] Recall that the *schema* of the category of substance is "the permanence of the real in time, i.e., the representation of [this real] as a substratum of empirical time determination in general, which therefore remains while everything else changes" (A144/B183; see notes 66 and 67 of my chapter on the Mechanics, together with the paragraph to which they are appended).

is determined as permanent, but time, and thus everything in inner sense, continually flows." In both places, in addition, Kant suggests clear links to the later argument of the paralogisms.[45] In general, then, although the representation 'I am' undoubtedly contains the representation of the existence of a subject (of all my representations), it in no way acquaints me with a corresponding substance. The soul, as putative object of inner sense (in which all my representations inhere), is no empirically cognizable substance at all.

The argument of the refutation of idealism therefore hinges on the schema of the category of substance – an appropriately permanent perceptible reality given in empirical intuition – and on the further claim that the permanence in question must be necessary or demonstrable: i.e., we must be able to establish it as a synthetic a priori truth on the basis of the analogies of experience.[46] Thus, in the Mechanics of the *Metaphysical Foundations*, as explained in section 25 above, Kant demonstrates the permanence (conservation) of the total quantity of matter by showing how what he calls the "measure of the intensive filling of space" (521) simultaneously meets the demands of both the category of reality and the category of substance.[47] The quantity of matter, in this way, thereby provides

[45] In the second remark to the refutation Kant adds (B278):

> The consciousness of my self in the representation *I* is no intuition at all, but rather a merely *intellectual* representation of the self-activity of a thinking subject. Therefore this *I* also has not the least predicate of intuition, which, as *permanent*, could serve as the correlate of the determination of time in inner sense: as, for example, *impenetrability*, as *empirical* intuition, is in matter.

In the general remark to the system of principles, as we know, Kant sums up the point of his discussion by asserting that it is not only important for confirming the earlier refutation of idealism, but also for anticipating the later argument of the paralogisms (B293–94; see again the paragraph to which note 9 of the Introduction is appended, together with the following paragraph).

[46] This requirement rules out examples of merely de facto persistent empirical intuitions in inner sense such as the "master-sound" discussed in Strawson (1959, chapter 2) in the context of an auditory "analogue" to a genuinely spatial ordering of re-identifiable particulars. In his later discussion of permanence, the refutation of idealism, and the first analogy Strawson (1966, chapter III, §3) does not reintroduce this example explicitly, but he does explicitly qualify the argument so as to require only "an analogue of space." So he still avoids the requirement that there be a particular empirically given reality (specified by a particular empirical concept) that is *demonstrably* permanent in the sense of the second analogy – especially in the form of a quantitative conservation law as in the second edition version. Indeed, Strawson (1966) takes Kant to be rather obviously confused in attempting to demonstrate such a conservation law within his metaphysics of experience.

[47] The discussion of this point in section 25 begins with the paragraph to which note 67 of my chapter on the Mechanics is appended and concludes with the paragraph to which note 72 of this same chapter is appended. The crucial point is that Kant's understanding of quantity of matter as the product of density and volume involves an essential interdependence between its intensive aspect (density) and extensive aspect (volume), and it is for precisely this reason that the reality in question must be distributed in *space* (as well as time).

us with a measure of the quantity of substance as well. What is essential to his demonstration, Kant says (541), is that substance "is possible only in space and in accordance with its conditions, and thus possible only as object of the *outer* senses." Moreover, he is now prepared to extend this conclusion to all instantiations of the category of substance whatsoever – which, quite generally, must also be possible only in space and among the objects of outer sense. This emphatically does not mean, however, that the (empirical) concept of quantity of matter developed in the Mechanics is the only possible realization of the category of substance. Rather, we have now learned something more concerning "what the conditions are under which [the category of substance] can alone have objective reality, that is, meaning and truth" (478) – namely, that *all* instantiations of this category are necessarily spatially extended. The category of substance, quite generally, can only be instantiated via the concept of "the *real* in space [*das Reale im Raume*]" (A173/B215).

To see that this conclusion is indeed more general than that of the first Law of Mechanics demonstrated in the *Metaphysical Foundations*, recall that Kant explicitly distinguishes the concept of the real in space from the empirical concept of matter while first introducing the former concept in the anticipations of perception (A173/B215, emphasis added): "I may here not call [the real in space] impenetrability or weight, for these are *empirical* concepts." The empirical concept of matter, however, includes the two fundamental forces of repulsion (responsible for impenetrability) and attraction (responsible for weight), and Kant is deliberately abstracting from all such (empirically given) forces here. The concept of the real in space, unlike the empirical concept of matter, is thus a *pure* concept (a combination of the concept of reality and the pure intuition of space).[48] So Kant is perfectly within his rights when, in a marginal note to the statement of the first analogy in the first edition of the *Critique*, he explains (23, 30): "Here the proof must be so developed that it applies only to substances as phenomena of outer sense, and therefore from space – which, together with its determinations, exists at all times."[49]

[48] See the paragraph to which note 8 of my chapter on the Dynamics is appended, together with the preceding paragraph.

[49] See note 62 of my chapter on the Mechanics, together with the paragraph to which it is appended. The following paragraph, including note 63 of this same chapter, explains that an *unextended* object (such as the soul) can exhibit nothing that one could call permanent in accordance with the first analogy and thus no "quantum of substance" (23, 31). So it is also no wonder that Kant modifies the statement of the first analogy into a quantitative conservation principle in the second edition (B224, emphasis added): "In all change of the appearances substance is permanent, and its *quantum* in nature is neither increased nor diminished." Moreover, although the new

Hence the first analogy as Kant understands it in the second edition of the *Critique* – in connection with both the refutation of idealism and his revised formulation of the principle itself – is more general than the First Law of Mechanics. For the latter depends on a specific realization of the "quantum of substance" in terms of Kant's critical version of the dynamical theory of matter while the former does not.[50] As emphasized at the beginning of this chapter, moreover, the greater generality of the second analogy in comparison with Kant's Second Law of Mechanics is even more evident. Whereas the former asserts only that all changes (in the states) of a substance must have a cause, the latter restricts the scope of this assertion to say that all changes *in matter* must have an *external* cause. And this restriction appears to be closely connected, in turn, with the circumstance that, although the general metaphysics of the first *Critique* applies to all objects of our senses as such (both outer and inner), the special metaphysics of corporeal nature is restricted to the objects of outer sense.[51] Yet we already know – on the basis of the first analogy,

"proof" added to the second edition does not explicitly mention space, it does suggest that the permanent substance in question must be spatial as well as temporal (B224–25): "All appearances are in time, in which, as substratum (as permanent form of inner intuition) *simultaneity* as well as *succession* can alone be represented. The time, therefore, in which alone all change of appearances is to be thought, remains and does not change; because it is that in which succession or simultaneity can be represented only as its determinations." By thus emphasizing simultaneity, in particular, Kant calls attention to the fact that the "time" under consideration has both spatial and temporal dimensions. It is, in contemporary terms, what we would now call *space*-time (compare note 194 of my chapter on the Mechanics, together with the paragraph to which it is appended).

[50] The more general notion of quantum of substance depends on the concept of the real in space, and it presupposes that this quantum involves *some* intensive measure of the degree of the reality in question. The construction of such a quantum thereby implies the existence of *some* further empirical concept – such as Kant's characteristically dynamical concept of the quantity of matter – wherein we can exhibit the required structure of a measurable mathematical magnitude. However, although the first analogy itself is independent of any particular such construction, the only one that we have so far is that presented in the *Metaphysical Foundations*. And it appears to be for precisely this reason that Kant invokes his specifically dynamical concept of matter – if only by way of illustration – in both the second remark to the refutation of idealism and the general remark to the system of principles (B291; see the paragraph to which note 45 above is appended). Kant's allusion to this particular (empirical) concept of matter is especially explicit in the second remark to the refutation, where he mentions *impenetrability* as the empirical correlate of permanence in matter (B278; see note 45 above).

[51] See note 5 above, together with the paragraph to which it is appended. It will be recalled that the statement of Kant's Second Law of Mechanics reads (543): "Every change in matter has an external cause." The following proof depends on the idea that (543) matter "has no other determinations except those of external relations in space, and [it] therefore undergoes no change except by motion." The argument concludes (543; see the first paragraph of section 26 above): "With respect to the latter, as change of one motion into another, or of a motion into rest, or conversely, a cause must be found (by the principle of metaphysics)." That the change in question (for which an external cause is needed) is "of one [state] of motion into another" echoes Kant's observation in the second analogy that a body changes its state of motion only when it moves non-uniformly

refutation of idealism, and paralogisms – that the putative object of *inner* sense, the soul, is in fact no object of experience at all. In particular, it is not an empirically cognizable substantial subject of its own changing states or determinations. So in what sense, exactly, is the restriction of Kant's Second Law of Mechanics to external causes related to the more fundamental restriction of the special metaphysics of corporeal nature to the objects of outer sense?

The answer to this question, as we have seen, emerges in the following remark to the Second Law of Mechanics, which explains that this law essentially depends on the "*lifelessness*" of matter, "as matter in itself" (544; see the paragraph to which note 91 of my chapter on the Mechanics is appended):

> *Life* is the capacity of a *substance* to determine itself to act from an *internal principle*, of a *finite substance* to change, and of a *material substance* [to determine itself] to motion or rest, as change of its state. Now we are acquainted with no other internal principle in a substance for changing its state except *desiring*, and no other internal activity in general except *thinking*, together with that which depends on it, the *feeling* of pleasure or displeasure, and *desire* or willing. But these actions and grounds of determination in no way belong to the representations of the outer senses, and so neither [do they belong] to the determinations of matter as matter. (544)

Thus matter as such – as the movable in space – is necessarily lifeless, and the changes of its states (changes in motion) must therefore result from external causes. Kant appears to allow, however, that there can be living material substances as well as non-living material substances. So, in particular, there appears to be room in the phenomenal world of sensible experience for spatially extended material substances, such as animals and human beings, possessing internal principles for changing their states of motion.[52]

(A207/B252; see again note 93 of my chapter on the Mechanics). This is a further respect in which Kant's Second Law of Mechanics involves an empirical concept – that of natural inertial motion – which is not constitutive of the concept of a change in general.

52 For the case of human beings, in particular, compare the refutation of Mendelssohn's proof of the permanence of the soul in the second edition paralogisms. After concluding that we have no proof of the permanence of "the supposed substance" of the soul, Kant adds (B415): "[T]he permanence of the soul, as mere object of inner sense, remains unproved, and indeed indemonstrable, although its permanence in life, where the thinking being (as human being) is equally an object of outer sense, is clear in itself." There is thus a significant difference between the restriction (to specifically spatial and in this sense material substances) imposed by Kant's second edition version of the first analogy plus refutation of idealism and the further restriction (to non-living material substances) imposed by his Second Law of Mechanics.

Kant's argument in the *Metaphysical Foundations* here comes into contact with his philosophy of biology, and it thereby comes into contact, once again, with the role of the regulative use of reason in extending Kant's discussion of empirical natural science beyond the Newtonian physics constitutively grounded by his special metaphysics of corporeal nature. In the *Critique of the Power of Judgement*, partly to address this situation more perspicuously, Kant introduces a new a priori principle specifically belonging to the faculty of judgement. This principle (of reflective judgement), like the regulative idea of systematicity discussed in the appendix to the dialectic of the first *Critique*, is concerned with the systematic unity of nature under empirical scientific laws. Yet Kant now articulates its essentially teleological character more clearly and explicitly characterizes it as a principle of *purposiveness* [*Zweckmäßigkeit*]. Indeed, in the second part of the third *Critique* (the critique of teleological judgement) we learn that not only is nature in general framed by synthetic a priori teleological principles (of the faculty of judgement) but there is also a sub-domain of (material) nature, that of living organisms, such that natural objects belonging to this domain must themselves be teleologically understood – by directly applying the concept of purposiveness to them.[53] Moreover, we ourselves are living organisms and, as such, belong to the teleological sub-domain of (material) nature. As distinctively human beings, in particular, who possess the intellectual faculties of understanding, judgement, and reason, we thereby constitute the final and highest purpose [*Endzweck*] of nature. We do not do so, however, for the sake of our happiness but rather for the sake of the moral law resulting from the autonomous self-legislation of our (pure) *practical* reason (5, 435–36): "[O]nly in human beings, but also only in them as subjects of morality, is there to be found unconditioned legislation with respect to purposes, which only thereby makes the human being capable of being a final purpose to which the whole of nature is teleologically subordinated."[54]

[53] Kant draws this conclusion in the dialectic of teleological judgement in the third *Critique*, as a result of the necessary incompleteness of purely "mechanical" explanations for subsuming the entire domain of (material) nature. Moreover, the discussion following the resulting antinomy of teleological judgement (§70) contains a clear allusion to Kant's remark to his Second Law of Mechanics (§73; 5, 394): "[T]he possibility of a living matter (whose concept contains a contradiction, because lifelessness, inertia, constitutes the essential character of matter) cannot even be thought." The solution to the antinomy then involves a "unification of the principle of the universal mechanism of matter with the teleological principle" (§78).

[54] This assertion occurs in §84 of the third *Critique*, "On the final purpose of the existence of a world, i.e., of creation itself." It is preceded by §80, "On the necessary subordination of the principle of mechanism under the teleological [principle] in the explanation of a thing as a natural purpose," and followed by §87, "On the moral proof of the existence of God." The entire

These considerations cast further light on the peculiar intermingling of mathematical and non-mathematical a priori principles in Kant's metaphysics of nature (both special and general) discussed in the previous section. In particular, the special metaphysics of corporeal nature can only proceed by means of essentially mathematical principles governing quantitatively articulated empirical concepts (such as a mathematically expressed conservation law for the total quantity of matter or the quantitative equality of action and reaction). But the (transcendental) metaphysics of nature in general does not involve the mathematical construction of any particular empirical concept (such as that of quantity of matter). The objective reality of the categories or pure concepts of the understanding, unlike that of the empirical concept of matter, does not require particular concrete instances given in experience but only general schemata for subsuming any and all such instances that may eventually be given. To be sure, any such possible instantiations must also be subsumed under particular empirical concepts, and these empirical concepts must also be capable of mathematical articulation in principle.[55] Nevertheless, it is one thing to require the existence of an appropriate such empirical concept, it is quite another to require that the mathematical construction of this concept be actually exhibited – and precisely here, according to Kant, lies the characteristic difference between general and special metaphysics.[56]

The categories and principles of the pure understanding, then, although they are not themselves mathematical, still need to be realized within the forms of our pure intuition, space and time. Indeed, they

argument then culminates in the concluding §91, "On the kind of taking as true [*Fürwahrhaltens*] by means of a practical faith [*Glauben*]," where Kant thereby establishes the priority of pure practical over theoretical reason.

[55] This follows from the priority of the categories of quantity in the process of articulating experience in accordance with the postulates of empirical thought – and, in particular, from the circumstance that the mathematical principles of pure understanding (axioms and anticipations) are applied prior to the dynamical principles (the analogies and postulates) in constituting experience. For my most recent discussion of these questions see note 27 of my chapter on the Phenomenology, together with the paragraph to which it is appended, along with note 220 of this same chapter, together with the paragraph to which it is appended.

[56] The best example of the relationship between general category and particular instantiation via a specific empirical concept remains the relationship between the pure concept of the quantum of substance (which must be conserved according to the revised version of the first analogy) and the empirical concept of quantity of matter (as actually mathematically constructed in the Mechanics of the *Metaphysical Foundations*). Recall that what Kant calls "mathematical" principles of pure understanding are not themselves principles of mathematics; rather, they are properly transcendental principles intended to explain the possibility of applying mathematics to appearances in general (compare again note 46 of the Introduction). As I have explained, however, the *Metaphysical Foundations*, unlike the first *Critique*, has the ambition of explaining the possibility of specific applications of mathematics to specific empirical concepts.

even need to be realized, first and foremost, within our pure intuition of space, for only so are the objects of empirical intuition subject to the very substantial body of mathematical knowledge encapsulated in the laws of geometry. And it is only in this way, in particular, that there is any possibility of the mathematical construction of empirical concepts that is required by (but not actually exhibited in) the categories and principles of the understanding.[57] Yet a full and complete realization of the categories and principles has only been achieved (so far) in the sub-domain of material nature that is described by mathematical Newtonian physics and grounded, from Kant's point of view, on the special metaphysics of corporeal nature articulated in the *Metaphysical Foundations*. In order to extend the constitutive grounding of our experience beyond this domain we need additional a priori principles belonging to the regulative use of the faculty of reason, and this faculty, unlike the understanding, cannot be schematized within the forms of pure intuition.[58] So the (regulative) a priori principles guiding our experience of thermal phenomena, electric and magnetic phenomena, chemical phenomena, biological phenomena, and so on are less mathematical, in this respect, than the categories and principles of the understanding. Nevertheless, to the extent that we have successfully applied mathematics to the empirical concepts of these domains, we have found at least the beginnings of a properly constitutive grounding for them as well.[59] We have a fortuitous convergence of categorical, mathematical, and regulative principles on the basis of

[57] This is how I read the arguments for the necessity of specifically outer intuitions in the Preface to the *Metaphysical Foundations* and general remark to the system of principles: see note 32 above, together with the paragraph to which it is appended.

[58] See Kant's discussion of this point at the end of the appendix to the dialectic, immediately following the explanation of why ideas of reason are not even "constitutive with respect to *experience*" (A664/B692; see the paragraph following the one to which note 220 of my chapter on the Phenomenology is appended). Kant continues (A665/B693): "[A]lthough no schema can be discovered in *intuition* for the complete systematic unity of all concepts of the understanding, an *analogon* of such a schema can and must be given, which is the idea of a *maximum* in the division and unification of the cognition of the understanding in a principle."

[59] The inverse-square photometric law governing degree of illumination is a good example of such a preliminary application of mathematics to an empirical concept (here to a particular empirically given reality): see notes 19 and 22 above, together with their references back to earlier chapters. And we might imagine that this law, in turn, can figure in a preliminary application of the category of causality to thermal phenomena as well – such as Kant's example of the heat produced by the illumination of a stone by the sun in §29 of the *Prolegomena*. For here the recent mathematization of the concept of heat by Joseph Black and the (optical) theory of radiant heat developed by Wilhelm Scheele appear to be also in play: see Friedman (1992a, n.38, together with the paragraph to which it is appended).

which alone, according to Kant, experience (i.e., empirical cognition) is possible.[60]

The case of biology, however, introduces an essentially new kind of non-mathematical a priori principle, the principle of purposiveness. This principle, in particular, applies to a sub-domain of natural phenomena – the structure and behavior of living organisms – which, according to Kant, can never be constitutively grounded "mechanically" (note 53 above). Yet, although we do know that such phenomena can never be mechanically grounded *completely*, we cannot and should not set a priori limits to the extent to which we continue to search for such explanations. Whereas we must always ultimately appeal to the purpose of the eye, for example, in attempting to explain its structure, we must also appeal to mathematical and physical considerations in describing the actual causal mechanisms responsible for its successful function.[61] The idea of a purpose must always guide our search for such causal mechanisms, but the search continues indefinitely nonetheless. Even in biology, therefore, a fortuitous convergence of categorical, mathematical, and regulative principles (now including the new principle of purposiveness) is required to make possible our experience or empirical cognition of this domain as well.

The case of biology, as we have seen, also introduces essentially *moral* a priori principles deriving from the autonomous self-legislation of pure practical

[60] Such a fortuitous convergence of categorical, mathematical, and regulative principles is actually required in all cases of the constitutive grounding of an empirical science, including that of the Newtonian theory of universal gravitation. Here, however, the regress guided by the regulative use of reason has already terminated in a fully constitutively grounded causal law: see the paragraph to which note 240 of my chapter on the Phenomenology is appended, together with the remainder of section 36 above.

[61] Kant makes this point in §IX of the (unpublished) *First Introduction* to the *Critique of the Power of Judgement* (20, 236):

> [I]t is clear that in such cases [of organic phenomena] the concept of an objective purposiveness of nature serves merely *on behalf of reflection* about the object, not for the *determination* of the object through the concept of a purpose, and the teleological judgement concerning the inner possibility of a natural product is merely a reflecting, not a determining judgement. For example, in saying that the crystal lens of the eye has the *purpose* of effecting, by a second refraction of the light rays, the focussing of the rays emerging from a point [on the lens] onto a point on the retina of the eye in turn, one says only that the representation of a purpose in the causality of nature in the production of the eye is to be thought so that such an idea serves as a principle to guide the investigation of the eye in this respect, as well as to help find the means that one could devise to promote this effect.

The idea that the eye was created for the purpose of effective sight thereby guides our investigation into its optical and mechanical properties – which serves the further aim, in turn, of our efforts to promote and improve effective sight. I am indebted to Paul Guyer for calling my attention to this passage from the *First Introduction* (together with the continuation of the discussion in §X quoted in the following note). For his own perspective on the relationship between constitutive, regulative, teleological, and moral principles in Kant see Guyer (2005).

reason (note 54 above). In the envisioned development of empirical science from the Newtonian physics constitutively grounded in the *Metaphysical Foundations* to embrace thermal phenomena, electric and magnetic phenomena, chemical phenomena, and finally biological phenomena, we therefore find a corresponding sequence of increasingly non-mathematical a priori principles. We move from categorical principles, to the regulative principle of systematicity, to the principle of reflective judgement governed by the idea of purposiveness, and finally to moral principles governed by the idea of the highest good. Here the increasing distance from mathematical principles corresponds to an increasing distance from the faculty of sensibility and the pure intuitions of space and time. Whereas categorical principles require spatio-temporal schematization in order to have determinate content, the regulative principle of systematicity is capable of no such schematization but only the idea of a maximum of unity in the spatio-temporal cognitions of the understanding (note 58 above). The principle of reflective judgement leads to the idea of essentially purposive natural objects (living organisms), whose existence in space and time is inconceivable on purely mechanical principles (note 53 above). And the principles of morality, finally, have determinate synthetic a priori content entirely independently of space and time. This, in fact, is why the self-legislation of pure practical reason is *autonomous* (and not merely spontaneous) and, in the end, why this faculty is prior to theoretical reason.[62]

[62] Kant's discussion of the role of teleology in our understanding of the structure and function of the eye continues in §x of the *First Introduction* (20, 240–41):

> [I]f I judge of the eye that it *ought* [*sollen*] to be suitable for seeing, and, although the figure, the constitution of all of its parts, and their composition are, judged by merely mechanical laws, entirely contingent for my power of judgement, I nevertheless think a necessity for being constructed in a certain way in its form and construction that precedes the formative causes [*bildenden Ursachen*] of this organ, without which the possibility of this natural product is conceivable for me by no mechanical laws of nature .... This ought [*sollen*] contains a necessity that is clearly distinguished from physical-mechanical [necessity], in accordance with which a thing is possible by mere laws of efficient causes [*wirkenden Ursachen*] (without a preceding idea of it).

What is here added to mechanical causation by the idea of a purpose is a new type of necessity governing the existence of the organ in question – and this, moreover, appears to be precisely a *moral* necessity, ultimately judged by reference to the final purpose [*Endzweck*] of creation: see again note 54 above, together with the paragraph to which it is appended. This same note suggests the way in which the priority of pure practical over theoretical reason is finally conclusively established in the third *Critique* by the moral argument for the existence of God. The point, briefly, is that the idea of a mere wise author of nature (as discussed in the appendix to the dialectic of the first *Critique*) establishes no determinate goal for our pursuit of systematic unity, in so far as it does not contain, in particular, the idea of the *moral goodness* of this creator. Thus only practical as opposed to mere theoretical reason can establish a definite final purpose guiding all of our endeavors, both practical and theoretical. The understanding, in contrast to *theoretical* reason, does have a determinate content of its own, but this is only in virtue of the necessarily spatio-temporal schematization of the categories.

In terms of the different yet complementary perspectives of the *Metaphysical Foundations* and the first *Critique*, therefore, it appears that the former focusses almost exclusively on the way in which the transcendental categories and principles of the understanding are realized or instantiated in the essentially mathematical concepts and principles of Newtonian physics. The much more general treatment of these categories and principles in the first *Critique*, by contrast, has also in view their ongoing application to phenomena going far beyond those covered in Newtonian physics – so as eventually to embrace, if only partially, even biological phenomena as well. And at this point, finally, we also forge a connection between theoretical and (pure) practical reason and thus with reason's autonomous self-legislation of the moral law. Thus, in particular, the first *Critique*, unlike the *Metaphysical Foundations*, has the task of showing how, in the words of the second edition Preface, we must "deny *knowledge* [*Wissen*] in order to make room for *faith* [*Glauben*]" (Bxxx). In arriving at this point, however, I have wandered rather far from the theme of inner and outer sense, and I need further to explore the relationship between the refutation of idealism and the paralogisms of pure reason in order to re-establish this theme. I also need further to explore the relationship of the special metaphysics of the *Metaphysical Foundations* to both the refutation and the paralogisms.

THE SELF AND NATURE

The one place where the argument of the *Metaphysical Foundations* makes clear contact with that of the paralogisms is in Kant's remark to the proof of his First Law of Mechanics (542–43). Here, as we have seen, Kant contrasts the proof that he has just given for the permanence of the total quantity of matter with a corresponding (spurious) proof of the permanence of the soul in a way that very clearly anticipates the refutation of Mendelssohn's proof of the permanence of the soul in the second edition paralogisms (B413–15).[63] Moreover, the connection with the argument of the (second edition) paralogisms becomes even clearer if we note that the passage from the remark in question already quoted above (section 25) continues as follows:

The *I*, as the general correlate of apperception, and itself merely a thought, designates, as a mere prefix, a thing of undetermined meaning – namely, the

[63] Compare note 52 above. My earlier detailed discussion of this "refutation" occurs in section 25 above, beginning with the paragraph to which note 63 of my chapter on the Mechanics is appended.

subject of all predicates – without any condition of all that would distinguish this representation of the subject from that of something in general: a substance, therefore, of which, by this expression, one has no concept of what it may be. (542–43)[64]

The gist of the point that Kant emphasizes here is that one cannot make an inference from the mere form of thought expressing the transcendental unity of apperception – the mere 'I think' as *subject* of all of my thoughts – to the existence of a corresponding substantial *object* of inner sense. But this is the gist of the main argument of the paralogisms as well, as Kant explains in the discussion preceding his refutation of Mendelssohn's proof.

In particular, the revisions added to the second edition paralogisms begin with the following sentence:

Since the proposition *I think* (taken problematically) contains the form of every judgement of the understanding in general, and accompanies all categories as their vehicle, it is clear that the inferences from it can contain a merely transcendental use of the understanding, which excludes all admixture of experience; and of [this use's] progress we can, after what we have shown above, already grasp that there is no very favorable prospect. (B406)[65]

The most general point, once again, is that thought can only generate determinate cognition when applied to given intuitions (note 43 above). The ensuing argument that the thinking self or subject is not itself a substantial object begins by restating this general point regarding all cognition of any object whatsoever and then applying it to cognition of my self:

[64] This continues the passage quoted in the paragraph from my chapter on the Mechanics cited in the previous note. There follows the passage contrasting this concept of a substantial subject corresponding to the *I* unfavorably with the concept of matter as the movable in space quoted in note 64 of this same chapter. Kant concludes by returning to the former (spurious) concept (543): "The thought *I*, by contrast, is *no concept at all*, but only inner perception, and so nothing at all can be inferred from it (except for the total distinctness of an object of inner sense from that which is thought merely as object of the outer senses) – including, in particular, the permanence of the soul as substance."

[65] The revisions actually begin with a final clause added to the previous paragraph (common to both editions), where Kant promises a brief and uninterrupted exposition of the entire argument (A348/B406). The sentence quoted above echoes an earlier passage from the text common to both editions (A343/B401):

I think is thus the sole text of rational psychology, from which it is supposed to develop its entire wisdom. One easily sees that this thought, if it is supposed to be referred to an object (my self), could not contain anything but transcendental predicates of this [object]; since the least empirical predicate would spoil the rational purity and independence of this science from all experience.

> It is not in virtue of the circumstance that I merely think that I cognize any object, but rather I can only cognize any object in so far as I determine a given intuition with regard to the unity of consciousness in which all thinking consists. Thus I do not cognize my self in virtue of the circumstance that I am conscious of my self as thinking, but rather when I am conscious to myself of the intuition of my self as determined in relation to the functions of thinking. (B406)

Kant proceeds to distinguish the "determining" from the "determinable" self, where the former is the thinking self in accordance with the unity of apperception and the latter the manifold of inner intuition waiting thereby to be determined. Only the latter, Kant says, can possibly become an *object* of cognition.[66]

The pivotal (first) argument of the revised exposition then follows:

> In all judgements I am always the *determining* subject of that relation that constitutes the judgement. That [the proposition that] the I that I think can always be considered in thinking as *subject*, and as something that not only attaches to thinking merely as predicate, must hold is an apodictic and even an *identical proposition*. But it does not mean that I, as object, am, for myself, a self *subsistent being* or *substance*. The latter goes very far [beyond the former – MF], and thus also requires data that are not at all to be found in thinking, and perhaps (in so far as I merely consider the thinking [thing] as such) more than I will ever encounter anywhere (in it). (B407)

The required data, of course, are data of intuition. And the point is that there is no guarantee at all, as far as the mere functions of thought are concerned, that the appropriate such data – involving a permanent reality given in intuition in accordance with the schema of the category of substance – are anywhere (in intuition) to be found. We cannot infer

[66] The text reads (B406–7):

> All modes of self-consciousness in thinking, in themselves, are therefore not yet concepts of the understanding of objects (categories) but merely logical functions, which give no object at all to cognition, and not even my self as object. It is not the consciousness of the *determining* self but only that of the *determinable* self, i.e., my inner intuition (in so far as its manifold can be combined in accordance with the universal condition of the unity of apperception in thinking) that is the *object*.

This contrast between the *determining* and the *determinable* self echoes the passage from §24 of the second edition deduction (B153–54) quoted in the paragraph to which note 200 of my chapter on the Mechanics is appended – where the former self is "the original capacity [of the understanding] to combine the manifold of intuition, i.e., to bring it under an apperception" and the latter is "inner sense [as] the mere *form* of intuition, but without combination of the manifold therein, and thus not yet any *determinate* intuition at all."

from the *I think* (taken problematically) to the existence of the self as substantial object.[67]

This latter (mistaken) inference constitutes the principal error, Kant argues, of the (supposed) discipline of rational psychology, which aims to obtain a priori knowledge of the existence and nature of the soul as substance from the mere concept of a thinking being.[68] Moreover, it seems clear that Cartesian rationalism, in particular, is paradigmatic of the kind of view that Kant here wants to reject.[69] Yet I have just called attention to the connection between the refutation of Mendelssohn's proof of the permanence of the soul added to the second edition of the *Critique* and the second remark to the second proposition of the Mechanics of the *Metaphysical Foundations* – and it thus becomes clear that Leibnizean rationalism constitutes another (and perhaps even more important) target of Kant's attack. For Mendelssohn's proof depends on the supposed *simplicity* of the soul as substance, and the main argument of the second proposition of the Mechanics is that the required substantial permanence can only be proved of material substance in Kant's sense. Such a substance must be continuously extended in space (consisting of an aggregate

[67] According to the refutation of idealism and general remark to the system of principles, it turns out that appropriate data cannot be found in *inner* intuition alone. Kant suggests such a conclusion here in the following (fourth) argument of the new exposition addressing this question (409):

> [That] I distinguish my own existence, as a thinking being, from other things outside me (to which my body also belongs) is just as much an analytic proposition; for *other* things are precisely those that I think as *different* from me. However, I do not thereby know at all whether this consciousness of my self without things outside me, through which representations are given to me, would even be possible, and [whether] I could therefore exist merely as thinking being (without being a human being).

The following refutation of Mendelssohn's proof also clearly suggests a negative answer to this question (note 52 above). Nevertheless, the argument of the (second edition) paralogisms is more general and requires only the result that we cannot infer from the mere concept of a thinking being to an existent substantial subject.

[68] Kant states this conclusion after presenting the (four) main arguments of his new exposition (B409): "Therefore, not the least is gained concerning the cognition of my self as object by the analysis of the consciousness of my self in thinking in general. The logical exposition of thinking in general is falsely taken for a metaphysical determination of the object."

[69] See note 42 above for the explicit reference to Descartes in the new exposition of the second edition (B422). There is also such a reference in the earlier portion of the text common to both editions (A346/B405): "The proposition: I think, is here taken only problematically; not in so far as it may contain a perception of an existence (the Cartesian *cogito, ergo sum*), but rather in accordance with its mere possibility, in order to see which properties may follow from this so simple proposition for its subject (whether or not such a thing may now exist)." There are also a number of explicit references to Descartes in the fourth paralogism in the first edition – which is replaced, in the second edition, by the new refutation of idealism inserted into the postulates of empirical thought.

of parts external to one another) and is thus in no way simple. So we cannot establish the permanence of either a purely mental substance or an ultimately simple physical monad.[70]

I observed in section 14 above that Kant is clear that Leibniz's own conception of monads is very far from the "physical points" embraced by some of his Leibnizean–Wolffian followers. For properly Leibnizean monads, according to Kant, are mind-like – *and therefore entirely non-spatial* – simple beings, which are given (at least to themselves) in immediate self-consciousness.[71] So when Kant in the paralogisms rejects the inference from the *I* of apperception to the simplicity of the substance supposedly corresponding to this *I*, he is naturally read as here taking Leibniz as his target.[72] And the salience of his fundamental divergence from Leibniz in this connection is both strengthened and deepened by my earlier discussion of the treatment of the law of continuity in the general remark to mechanics in the *Metaphysical Foundations* (section 29 above). Kant there sharply distinguishes the *metaphysical* law of continuity – belonging to general metaphysics or transcendental philosophy – from the *mechanical*

[70] See again the paragraph to which note 63 of my chapter on the Mechanics is appended, together with the preceding and following paragraphs. Kant's point, explored in detail in section 25 above, is that a purely *intensive* magnitude (such as the degree of consciousness or the degree of any dynamical force) can vary independently of spatial extension and thus continuously decrease to zero without annihilating any substantial subjects. For the case of physical monads, in particular, see note 60 of my chapter on the Mechanics, together with the paragraph to which it is appended.

[71] See the paragraph to which note 94 of my chapter on the Dynamics is appended. In the text from the second antinomy quoted there Kant puts it this way (A440–42/B468–70): "The proper meaning of the word *Monas* (according to Leibnizean usage) should only extend to that simple which is *immediately* given as simple substance (e.g., in self-consciousness), and not as element of the composite – which one could better call the atom."

[72] The second argument of the new exposition in the second edition concerns simplicity (B407–8):

> That the *I* of apperception, and thus in all thinking, is a *singular* [thing], which cannot be dissolved into a plurality of subjects, and thus designates a logically simple subject, already lies in the concept of thinking, and is therefore an analytic proposition; but that does not mean that the thinking *I* is a simple *substance*, which would be a synthetic proposition. The concept of substance always relates to intuitions, which in me cannot be anything other than sensible, and thus lie entirely outside the field of the understanding and its thinking – but only the latter is properly here in question when it is said that the *I* in thinking is simple.

This contrast between what lies in the understanding alone and what belongs to intuition resonates with Kant's criticism of Leibniz in the amphiboly. See especially the passage quoted in note 106 of my chapter on the Mechanics, together with the closely related passage to which this note is appended. In the latter passage Kant contrasts a *substantia phaenomenon* in space with a substance thought by the pure understanding alone. Because Leibniz considers only the pure understanding, Kant concludes, "after he had taken away everything that may signify an external relation, and therefore also *composition*, [Leibniz therefore] made of all substances, because he represented them as noumena, even the constituents of matter, simple substances with powers of representation – in a word, **monads**" (A266/B321–22).

law of continuity just demonstrated. He also insists, somewhat surprisingly, that this metaphysical law can "find no place" in the demonstration in question (553). Since the reason, Kant says, is that the latter law, unlike the former, "would have to be extended to all changes in general (inner as well as outer)" (553), this discussion, as we have seen, thereby illuminates the way in which general metaphysics, unlike special metaphysics, involves a centrally important priority of inner over outer sense.[73]

I first suggested that the *Metaphysical Foundations* represents a fundamentally different perspective from that of the first *Critique* in this same earlier discussion (section 29), and I arrived at the conclusion that Leibniz is very probably the ultimate source of what Kant calls the *metaphysical* law of continuity.[74] In particular, Kant likely has in mind Leibniz's treatment of the law of continuity in the Preface to the *New Essays on Human Understanding*, which appeared in 1765 and produced a strong impression on Kant and contemporary German philosophy more generally. In the relevant passage from this Preface Leibniz also envisions two parallel and complementary sub-parts of knowledge or science in general: "pneumatology" or the pure doctrine of the soul and "physics" or the pure doctrine of body. And, although Kant envisions just such a twofold division of what he calls "natural science" in the Preface to the *Metaphysical Foundations*, what is important, in this connection, is that he completely rejects the idea of a "pneumatology" or pure doctrine of the soul – just as he does in the paralogisms of pure reason in the first *Critique*.

The central idea of the Leibnizean monadology, according to Kant, is that pneumatology or the pure doctrine of the soul provides a metaphysical foundation for physics or the pure doctrine of body. The ultimate constituents of reality are monads, unextended mind-like simple substances modeled on our own inner experience of the soul, and the empirically given reality of bodies interacting mechanically in space is a

[73] The at first sight rather surprising disconnect between the metaphysical and mechanical laws of continuity is discussed in the paragraph to which note 179 of my chapter on the Mechanics is appended; my discussion of the resulting greater generality of the former in comparison with the latter extends to the paragraph to which note 183 of this same chapter is appended. The relevant priority of inner over outer sense turns out to be just that highlighted in the third "conclusion from these concepts" concerning time in the transcendental aesthetic, according to which "time is an a priori condition of *all* appearances in general – and, in fact, [it is] the immediate condition of inner [appearances] (of our souls) and precisely thereby also the mediate condition of the outer appearances" (A34/B50, emphasis added; see note 40 above, together with the paragraph to which it is appended).

[74] For the fundamentally different perspectives of the two works see the paragraph to which note 186 of my chapter on the Mechanics is appended; the discussion of Leibniz as the source of the metaphysical law begins in the following paragraph.

derivative well-founded phenomenon of this underlying monadic reality. It is absolutely central to (the critical) Kant, however, that this monadological metaphysics entirely misconstrues the route from self-reflection by means of the pure understanding to our empirical knowledge of nature. For Leibniz thereby appears to suggest (again according to Kant) that we have knowledge of our own mind by a kind of self-apperception as it is in itself, and we then project this knowledge onto the ultimate monadic constituents of matter.[75] Yet such knowledge of our own mind – as an object of cognition – is precisely what we do not have according to the argument of the paralogisms. So our knowledge of nature, for Kant, is explained in a fundamentally different fashion. I begin, to be sure, with the apperception of my self: with the representation *I think*. But this representation, for Kant, needs to be schematized in terms of sensibility and, in the first instance, in terms of the pure intuition of time. The understanding, as Kant conceives it, must then determine the manifold of inner sense (in accordance with its form), and the analogies of experience, as we know, provide the central a priori rules for this kind of determination of time. Thus it is only by an intricate and laborious route from self-apperception through the schematism to the principles, for Kant, that empirical cognition of nature – that is, experience – is possible at all.[76]

[75] See note 199 of my chapter on the Mechanics. I there quote a passage from §30 of the *Monadology* (which was well known to Kant), according to which, through knowledge of necessary truths, "we rise to *reflective acts*, which enable us to think of that which is called 'I' and enable us to consider that this or that is in us." It is also worth noting that the passage from the second edition paralogisms concerning simplicity quoted in note 72 above continues as follows (B408):

> It would also be miraculous if what otherwise requires so much care, in order to distinguish in what intuition exhibits what may be substance therein, and even more, whether this could also be simple (as in the case of the parts of matter), were to be immediately given to me in the most meager representation of all, as if by a revelation.

It is hard to resist the conclusion that Kant is specifically targeting Leibniz in this passage, especially if we read it in connection with the discussion in the amphibology contrasting "absolutely" with merely "comparatively" inner properties of matter in the investigation of nature (A277–78/B333–34; see the paragraph to which note 255 of my chapter on the Dynamics is appended).

[76] See again the paragraph to which note 200 of my chapter on the Mechanics is appended, together with the preceding and following paragraphs. At the end of this last paragraph I observe that the refutation of idealism represents a continuation of the argument, according to which our empirical self-knowledge of our own *internal* existence in time is entirely parasitic on the *external* time determination already established among spatially extended material substances existing in both space and time. But there are two important points to be noted here. On the one hand, the argument of the paralogisms does not require this continuation but depends only on the need to schematize the category of substance in time – so that, in particular, there is no asymmetry, in this respect, between my knowledge of outer objects in space (as mere appearances), on the one side, and my (putative) knowledge of my self as the (supposed) substantial object of inner sense, on the other. Thus Kant already has powerful considerations available to counter the "rational psychology" of Descartes and Leibniz even without the refutation of idealism (compare note 67

In my earlier discussion of this point I observed that the procedure of time determination described in the analogies terminates in the Mechanics of the *Metaphysical Foundations*, where Kant finally makes clear how his critical conception of a metaphysical foundation for physics must fundamentally diverge from the "pneumatological" foundation envisioned by Leibniz. He thereby makes clear, at the same time, how the foundation that he is now articulating must radically revise the monadological metaphysics of the Leibnizean tradition in light of the triumph of specifically Newtonian physics.[77] Yet, as I emphasized in this earlier discussion (and am now in the process of explaining in greater detail), the special metaphysics of corporeal nature involves a different and less general perspective on the phenomenal world in space and time than that adopted in the general metaphysics of the first *Critique*. In particular, inner sense is indeed prior to outer sense from the perspective of the latter work, in so far as the understanding in its pure apperception must be applied, in the first instance, precisely to time as the form of *inner* sense.

This emphatically does not mean, however, that a proper science of the soul – the putative *object* of inner sense – is possible. Indeed, according to the paralogisms, the soul cannot be an object of our experience and knowledge at all. Yet there is, nonetheless, a philosophy or metaphysics of the soul, as a fundamental part of general rather than special metaphysics. Rational psychology, a discipline that attempts to obtain a priori knowledge of the soul as object, is certainly impossible, but transcendental philosophy, by contrast, considers the soul as *subject*, as the spontaneity of the faculty of understanding that determines the manifold of inner intuition so as to make knowledge of the objects of experience first possible. Indeed, this theory of how the self as subject affects the manifold of inner intuition – by "an action of the understanding on sensibility and its first application to objects of an intuition possible for us (and at the same time the ground of all other applications)" (B152) – lies at the heart of the second edition transcendental deduction. Here the self (the

above). On the other hand, the argument of the refutation of idealism does not depend on the (Newtonian) mathematical theory of motion (which is given a metaphysical foundations within Kant's special metaphysics of corporeal nature) but only on the more general claim that the analogies of experience necessarily require a realization in *both* space and time – a point clearly suggested by Kant's revised formulations in the second edition (see note 49 above, together with the paragraph to which it is appended, and compare note 202 of my chapter on the Mechanics).

[77] See the paragraph following the one to which note 203 of my chapter on the Mechanics is appended, together with the two following paragraphs.

I think) is not the object of inner sense but rather that which, as "the source of all combination" (B154), "[determines] *inner sense* in accordance with its form" (B155).[78]

In general metaphysics – the transcendental philosophy of the first *Critique* – we therefore consider the principles of the understanding as derived from this spontaneity of the subject, as "principles of the objective determination of all representations, in so far as cognition can arise therefrom, which principles are all derived from the principle of the transcendental unity of apperception" (B142). In the special metaphysics of corporeal nature developed in the *Metaphysical Foundations*, by contrast, this theory of the determining spontaneity of the subject is entirely absent. Rather than considering the principles of the understanding as derived from the transcendental unity of apperception, these same principles are now taken simply as given, as premises for the further derivation of the principles of pure natural science. Moreover, the principles of the understanding, through this particular application to the objects of specifically outer sense, thereby acquire a mathematical content. But the derivation of the principles of the understanding themselves in the first *Critique* is prior to this procedure. These principles, as we have seen, are not mathematical, and the principle of the transcendental unity of apperception, in particular, lies even further from the application of mathematics to the objects of experience than they do. For the transcendental unity of apperception involves a purely intellectual rather than a sensible synthesis, of which the understanding "is conscious even without sensibility, but through which it is capable of determining sensibility inwardly with respect to the manifold, which may be given to it [the understanding – MF] in accordance with the form of its [sensibility's – MF] intuition" (B153).[79] The transcendental unity of apperception thereby has meaning – but not yet any determinate application to the objects of cognition – entirely independently of space and time. In considering the principles of the understanding as derived from this (purely intellectual) spontaneity of the subject

[78] These quotations are all taken from §24, "On the application of the categories to objects of the senses in general," which I have discussed at a number of points above. See, for example, the paragraph to which note 200 of my chapter on the Mechanics is appended, together with the preceding paragraph, and the paragraph to which note 73 of my chapter on the Phoronomy is appended.

[79] This passage also comes from the pivotal §24 of the second edition deduction – and, indeed, from the ellipsis in the quotation to which note 200 of my chapter on the Mechanics is appended. Here, and in the remainder of this section, I am drawing upon Friedman (2005).

we are thereby considering these principles, too, from a distinctively transcendental perspective.[80]

The crucial difference between the general metaphysics or transcendental philosophy of the first *Critique* and the special metaphysics of the *Metaphysical Foundations* therefore concerns the manner in which the categories and principles of the understanding are considered. Transcendental philosophy does not consider the soul – the putative object of inner sense – as a genuine object, for a philosophical consideration of the soul as object can only result in the pseudo-discipline of rational psychology. In considering the soul as subject, however, transcendental philosophy nonetheless establishes a characteristic perspective on the categories and principles of the understanding, viewed as products of the spontaneity of the subject with respect to the form of inner sense. Kant explains the point this way in the second edition paralogisms:

One sees from all of this that rational psychology has its origin in a mere misunderstanding. The unity of consciousness lying at the basis of the categories is here taken for [an] intuition of the subject as object, and the category of substance is applied to it. But it is only the unity in *thinking*, by which alone no object is given, and to which, therefore, the category of substance, which always presupposes [a] given *intuition*, cannot be applied, and thus this subject can in no way be cognized. The subject of the categories can therefore not acquire a concept of itself as an object of the categories by thinking this [unity – MF]. For, in order to think this, it must take as basis its pure self-consciousness, which is just what has to be explained. Similarly, the subject, in which the representation of time originally has its basis, cannot thereby determine its own existence in time; and if the second is impossible, then the first, as determination of its self (as thinking being in general), cannot take place by means of categories.* (B421–22)[81]

[80] At the end of the previous section I considered a somewhat different sequence of increasingly non-mathematical a priori principles, which begins with the categorical principles of pure understanding and ends with the principles of morality: see note 62 above, together with the paragraph to which it is appended. I am here considering the relationship between the (essentially mathematical) principles of pure natural science, the (non-mathematical) principles of pure understanding, and the (even less mathematical) principle of the transcendental unity of apperception. The latter principle, unlike the principles of the understanding, has a meaning independently of our faculty of sensibility. Unlike the principles of morality, however, it has no *determinate* meaning independently of sensibility. The point of emphasizing the principle of the transcendental unity of apperception at this point is to connect the distinctive generality of the principles of the understanding with the priority of inner over outer sense in general metaphysics or transcendental philosophy. I shall explore the connection between this principle and the principles of morality below.

[81] The beginning of the attached footnote is quoted in note 42 above, where Kant is considering the assertoric (rather than merely problematic) use of the 'I think', in which it includes the 'I exist' as well. This use of the 'I think', Kant says, results in an "empirical proposition" (B422). He continues (B422–23):

The spontaneity of the subject with respect to the form of inner sense does not result in the determination, by this subject, of itself as an object of inner sense. In "[determining] *inner sense* in accordance with its form" (B155), the subject rather makes possible the cognition of objects of experience – and these objects, as we know from the refutation of idealism and related texts, are, first and foremost, objects of *outer* sense.[82]

However, as Kant explains several pages later in the second edition paralogisms, although the spontaneity of the 'I think' can therefore not determine the subject as an existing thing, we also have a second form of spontaneity that can do precisely this:

> Suppose, however, that there were subsequently to be found occasion – not in experience, but in certain (not merely logical rules but) laws of the pure use of reason that stand fast a priori and concern our existence – to assume that we are *legislative* completely a priori with respect to our own *existence* and also self determining in this existence. We would thereby discover a spontaneity whereby our actuality would be determinable, without requiring for this purpose the conditions of empirical intuition; and we would become aware that there is something

> It expresses an indeterminate empirical intuition, i.e., [a] perception (and thus it proves that sensation, which therefore belongs to sensibility, already lies at the basis of this existential proposition), but it precedes the experience that is to determine the object of perception by the category [of existence – MF] in relation to time … An indeterminate perception here signifies only something real that has been given, and indeed only to thought in general, and therefore not as appearance, and also not as thing in itself (*Noumenon*), but rather as something that in fact exists and in the proposition 'I think' is designated as such. For it is to be noted that, if I have called the proposition 'I think' an empirical proposition, I do not thereby intend to say that the *I* in this proposition is an empirical representation; rather, it is a purely intellectual representation since it belongs to thinking in general. But without some or another empirical representation that provides the material for thinking, the act *I think* would not take place, and the empirical is only the condition of the application or use of the pure intellectual faculty.

Thus the 'I think' acquires an assertoric use, for Kant, when it is thought in relation to some or another given sensible manifold in the act of determining this manifold (as an object) in accordance with the categories. In the second footnote to §24 of the second edition deduction Kant illustrates this kind of self-consciousness by acts of *attention* – "in which the understanding always determines inner sense, in accordance with the combination that it thinks, into the inner intuition that corresponds to the manifold in the synthesis of the understanding" (B157). Compare also the following §25, which begins (B157): "[I]n the transcendental synthesis of the manifold of representations in general, and thus in the original synthetic unity of apperception, I am conscious of myself, not as I appear to myself, nor *as* I am in myself, but only *that* I am."

[82] As we have seen, this last idea, and thus a kind of preview of the refutation of idealism, occurs towards the end of §24 in a closely following passage (B156; see note 14 of the Introduction). Thus, whereas inner sense is prior to outer sense with respect to the determining spontaneity of the subject, outer sense is prior to inner sense with respect to the objects of experience thereby determined.

contained in the consciousness of our existence a priori that can serve to determine our existence – which is completely determinable only sensibly – in regard to a certain inner faculty in relation to an intelligible world (which is certainly only [a world of] thought). (B430–31)

The inner faculty in question is thus pure *practical* reason, and the determination of our existence that Kant has in mind results from our self-legislation of the moral law to ourselves. We thereby determine the actuality of our *will* in the phenomenal world by viewing ourselves, entirely independently of this world, as members of an ideal (legislative) realm of ends. The spontaneity of the faculty of (pure practical) reason, in sharp contrast with that of our pure apperception, can thereby determine our existence as subjects – including our existence as parts of the phenomenal world – with no need of mediation from either pure or empirical intuition.[83]

I have now reached, by a somewhat different route, the same point at which I arrived in the previous section. In order to have determinate *theoretical* content, the pure categories and principles of the understanding (including even the principle of pure apperception) require application to pure and empirical intuition. They therefore require, on my reading, an essentially mathematical application or realization in pure natural science.[84] The a priori *practical* principles generated by the faculty of reason, by contrast, are quite independent of such application. Nevertheless, both nature in general and our experience as a whole are teleologically subordinated to these same practical principles. Kant makes this especially clear, in the section of the second edition paralogisms I have been considering, in connection with his rejection of

[83] Pure practical reason determines the will though what Kant, in the *Groundwork of the Metaphysics of Morals*, calls the feeling of *respect* (*Achtung*), but he is careful to distinguish this feeling from all those involving inclination or sensation (4, 401):

But even if respect is a feeling, it is still not one *sensed* [*emfangenes*] through influence [*Einfluss*]; it is rather a feeling *self-effected* through a concept of reason, and is therefore specifically distinguished from all feelings of the first kind, which can be reduced to inclination or fear. What I immediately cognize as law for myself, I cognize with respect, which merely signifies the consciousness of the *subordination* of my will under the law without mediation of other influences on my sense.

Moreover, this consciousness – unlike the "indeterminate empirical intuition" implicated in the 'I think' (compare note 81 above) – determines my will with a definite content (the moral law) that is wholly independent of sensibility. This is the sense in which the self-legislation of pure practical reason is *autonomous* and not merely spontaneous: compare the paragraph to which note 62 above is appended.

[84] This is so even though the categories and principles themselves are essentially non-mathematical: see notes 56 and 57 above, together with the paragraphs to which they are appended.

the (spurious) dogmatic proofs of the immortality of the soul found in rational psychology:

> The proofs that are useful to the world thereby all remain with undiminished worth, and rather gain in clarity and natural conviction by setting aside these dogmatic pretensions, in that they transpose reason into its characteristic domain, namely, the order of ends [*Zwecke*] that is at the same time an order of nature. And reason then, as the practical faculty in itself, is also justified, without being limited to the conditions of the latter [the order of nature – MF], in extending the former [the order of ends – MF], and with it our own existence, beyond the limits of experience and of life. In accordance with the *analogy with nature* of living beings in this world, concerning which reason must necessarily assume as a principle that there is to be found no organ, no faculty, no impulse, nothing at all that is unnecessary or not proportioned to its use, and thus nonpurposive, but rather that all of its determination in life is to be judged as exactly suitable, [in accordance with this analogy – MF] the human being, who can alone contain within itself the ultimate final purpose [*letzten Endzweck*] of all this, would nonetheless have to be the only creature that would be excepted from it. (B424–25)

As explained in the *Critique of Practical Reason* (Book 2, chapter 2, §4), moreover, human beings are excepted from the order of nature – and, in particular, from the natural condition of mortality – precisely in so far as pure practical reason unconditionally commands us to seek the highest good, which can never be fully achieved at any finite stage of factual life.[85] Immortality therefore emerges as what Kant calls a postulate of pure practical reason, in virtue of which we can then have a well-grounded rational faith [*Vernunftglaube*], but absolutely no well-grounded theoretical knowledge, that the end unconditionally

[85] Thus the *idea* of the immortality of the soul – which is in this sense distinct from the human body – emerges precisely in connection with the natural teleology that Kant takes to be indispensable in the science of biology, but it can do so, for Kant, only in virtue of the entirely non-empirical (non-sensible) *practical* principles to which the whole of nature is ultimately subordinated. This helps to make sense of the circumstance that, in the antinomy of teleological judgement, Kant explicitly asserts that the concept of living matter contains a *contradiction* (note 53 above) – so that, strictly speaking, there can be no *living* material substances from the point of view of purely theoretical cognition. Compare note 52 above, together with the paragraph to which it is appended, and observe that the passage from the refutation of Mendelssohn quoted in the note appears to separate the "soul" from the "human being." Moreover, the passage from the remark to the proof of Kant's Second Law of Mechanics quoted in the paragraph to which note 91 of my chapter on the Mechanics is appended continues (544, emphasis added): "If we seek the cause of any change of matter in life, we will have to seek it forthwith in another substance, *different from matter*, yet combined with it." For Kant, it appears, our experience of life in our own person does give objective reality to the idea of a soul distinct from the body – but, like all ideas of reason, only from a purely *practical* point of view.

commanded by morality can be continually approximated without limit.[86]

The Preface to the *Metaphysical Foundations*, as we have seen, draws a fundamental distinction between general metaphysics or transcendental philosophy and the special metaphysics of corporeal nature – the doctrine of body restricted to objects of outer sense. In the course of emphasizing the importance of this distinction towards the end of the Preface, and as "an important reason for separating [the doctrine of body's – MF] detailed treatment from the general system of metaphysics, and presenting it systematically as a special whole" (477), Kant makes the following observation:

> [General] metaphysics has busied so many heads until now, and will continue to do so, not in order thereby to extend natural knowledge (which takes place much more easily and surely through observation, experiment, and the application of mathematics to outer appearances), but rather so as to attain cognition of that which lies wholly beyond all boundaries of experience, of God, Freedom, and Immortality. (477)

The general metaphysics or transcendental philosophy advanced in the first *Critique* (and then further extended in the second and third) does indeed portray nature in general and human experience as a whole as teleologically subordinated to essentially non-mathematical a priori principles extending far beyond the boundaries of all theoretical cognition of the phenomenal world. This world is thereby seen to be much more than a theatre for objective human experience and knowledge – which, from the point of view of the understanding, is necessarily constrained by the concepts and principles of mathematical natural science. It is also, and primarily, a vehicle for the realization of the moral law.

[86] The three postulates of pure practical reason are God, Freedom, and Immortality. When, in the Preface to the second edition of the first *Critique*, Kant says that he must deny knowledge in order to make room for faith (Bxxx; see the paragraph following to one to which note 62 above is appended), it is clear that precisely *rational faith* is at issue (Bxxix–xxx): "I can thus not even *assume* [*annehmen*] *God*, *Freedom* and *Immortality* on behalf of the necessary practical use of my reason, if I do not, at the same time, *deprive* [*benehme*] speculative reason of its pretension to extravagant insights." Thus, the distinctive perspective on the phenomenal world taken up in the first *Critique* not only involves embedding this world within the larger perspective of pure practical reason but also drawing a clear and sharp boundary around it beyond which our purely *theoretical* cognition cannot advance a single step.

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I KANT'S WRITINGS

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