Chapter 11

How Not to Trivialize the Identity of Indiscernibles

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The Principle of Identity of Indiscernibles (PII, hereafter) says that no two things differ *solo numero*. That means that when two things differ numerically there is also a further difference between them. This further difference I shall call *qualitative difference*.

This qualitative difference can be an internal difference between things or a difference as to how things are related to things. These differences can also be explained in terms of properties: in the former case qualitative difference consists in a difference with respect to intrinsic properties, in the latter case it consists in a difference with respect to relational properties.

Relational properties may depend on the identity of the *relatum* (or on the identity of *relata* of the *relatum*), or they may be purely qualitative. Properties that depend on the identity of a *relatum*, like *being two miles from the Eiffel Tower*, are often called *impure properties*.[1] Those that do not depend on the identity of a *relatum*, like *being two miles from a tall tower*, are called *pure* properties.

Since intrinsic properties do not depend on the identity of any *relatum*, they are also classified as pure. But given my understanding of 'qualitative difference', both pure and impure properties can make a qualitative difference.[2]

Given that the difference can be captured in terms of properties, PII is normally taken to assert either that there are no two things that share all their properties or, in its necessitated version, that there can be no two such things. Here is a formal statement of the non-necessitated version:

$$\text{PII } (x)(y)[(\text{F})(\text{F}x \equiv \text{F}y) \supset x = y)]$$ [3]

Both in the necessitated and the non-necessitated version the domain of properties over which one quantifies is crucial for the truth of PII. Indeed the more one restricts the domain of properties quantified over, the more likely for PII to come out false. And if one puts no restriction at all in the domain of properties quantified, then PII comes out true but, as it is often pointed out, *trivially* true.

The triviality of this version of PII depends on certain properties being included in the domain of property quantification. I shall call those properties that render PII trivial *trivializing properties*. Since PII, far from being a trivial principle, is one of the most substantive and controversial ideas in metaphysics, it is important to determine which properties must be excluded from the domain of quantification to get a metaphysically serious version of PII.

If one excludes from the domain of quantification all and only trivializing properties, then one ensures a non-trivial and to that extent a metaphysically serious version of PII. That version will be the weakest non-trivial version of PII. For every other non-trivial version of PII will entail the truth of the version that excludes all and only trivializing properties. But that it is the weakest does not make it unworthy of metaphysical discussion. That weakest version will make a non-trivial claim and establishing it may be as difficult as establishing other versions of PII.

I shall not discuss whether the weakest non-trivial version of PII is true. But it is significant that, as it will have become clear in Section 7, the weakest non-trivial version of PII will quantify over some impure properties, like *being the father of a* or *being one metre apart from a*. Thus I disagree with Peter Strawson, who said that the only version of PII which is worth discussing is one according to which there exists, for every thing, some description in purely universal or general terms such that only that thing answers to that description (Strawson, 1959, p. 120).[4] Indeed, normally the following three versions of the principle are distinguished (here I present only the non-necessitated variant of each version):

PII1: No two things share all their intrinsic properties
PII2: No two things share all their pure properties
PII3: No two things share all their properties

PII1 is the strongest version, PII3 the weakest. Philosophers typically claim or suggest that these are the only three versions of PII and that PII3 is a trivial version of PII (to cite only a few: Adams, 1979, p. 11; Forrest, 2002, §1; van Cleve, 2002, pp. 389–90). While I agree that PII3 is trivial, I disagree that the other two are the only non-trivial versions of the principle. Indeed, part of the significance of the discussion to follow is that shows the existence and importance of the following version of PII, weaker than PP2 but stronger than PII3:

PII2.5: No two things share all their non-trivializing properties

I take PII2.5 to be the weakest non-trivial version of PII. And since not all impure properties are trivializing properties, PII2.5 quantifies over some impure properties. The main aim of the chapter is then to specify the class of trivializing properties. But I want to produce a philosophically illuminating specification of such a class. That is, I want to be able to say what makes a certain property trivializing. This is why I shall specify the class of trivializing properties intensionally rather than merely extensionally.

In the next section I shall introduce the paradigmatic kind of trivializing properties. In Section 3 I shall discuss other trivializing properties and I shall argue against giving a characterization of trivializing properties in terms of the notion of entailment between properties. In Section 4 I shall discuss, and eventually find unsatisfactory, another way of characterizing trivializing properties, in terms of the notion of property containment. In Sections 5–6 I shall present and discuss my own characterization of trivializing properties. Section 7 is a brief conclusion.

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When one quantifies over all properties of things, what PII asserts is that no two things share all their properties. This version of PII is true but trivially true, as it has been widely recognized. To see why this version of PII is true, consider property (1) below, which is an instance of what I call *properties of identity*:[5]

(1) *being identical to a*.

If one quantifies over all properties, PII is true because if any things *a* and *b* share all their properties, including (1), then they are identical and so they are not two things. We can deploy the structure of the argument in the following way:

(i) *a* and *b* share all their properties.
(ii) *a* has the property of *being identical to a*.
(iii) *b* has the property of *being identical to a*.
(iv) Therefore, $a = b$.

Since (1) asserts the indiscernibility of *a* and *b*, the argument (i)–(iv) derives identity from indiscernibility by using a property of identity. (i) is the assumption of indiscernibility between *a* and *b* needed to get the argument started. (ii) follows, assuming properties of identity, from the general law that everything is self-identical, that is, $(x)(x = x)$. (i) and (ii) entail (iii), which states that *b* has the property of *being identical to a*, but if *b* has the property of *being identical to a*, it follows that *b* is identical to *a*, that is, (iv).[6] So, if *a* and *b* are indiscernible, they are identical; that is, PII is true.[7]

This argument for PII is simple and clear and it turns the denial of PII into a contradiction, since denying it would amount to saying that there are two non-identical particulars that share all their properties, including their properties of identity, and therefore they are identical. But note that this argument supports PII only because it is a case of a general argument that can be applied to every two particular things that are supposed to be indiscernible. Taken in itself the argument only proves that there is nothing indiscernible from *a*, not that there is no pair of indiscernibles. But, since everything is self-identical, this argument can be generalized. Other instances of the argument will use other properties of identity, such as *being identical to b*, *being identical to c*, or any others.

So properties of identity make PII true. But they make it *trivially* true. No doubt the proof of this version of PII has an undeniable air of triviality, but what matters here is not the triviality of the proof but the triviality of what is proved.[8] For it is trivial to claim that no numerically distinct things share all their properties, including their properties of identity. Properties of identity are trivializing properties, since they do not make a qualitative difference. They must be excluded from the domain of quantification to get a metaphysically serious version of PII.

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If properties of identity were the only trivializing properties, our *problem* would be trivial. But although they are paradigmatic trivializing properties, there are other trivializing properties. For if properties of identity trivialize PII, then so do conjunctive properties like (2):

(2) *being identical to a and being green*.

Suppose a certain thing *a* is green; one can then show that there is nothing indiscernible from it in the following way:

(v) *a* and *b* share all their properties.
(vi) *a* has the property of *being identical to a and being green*.
(vii) *b* has the property of *being identical to a and being green*.
(viii) Therefore, $a = b$.

This argument is generalizable in the relevant way, for even if not every thing is identical to *a* and green, everything has some conjunctive property one of whose conjuncts is a property of identity and the other is some other property. So conjunctive properties like (2) establish PII. But what they establish is trivial, since all the work in the proof is done by (1). The reason why in the argument above the real work is done by the property of *being identical to a* is that everything having the property of *being identical to a and being green* must have the property of *being identical to a*. So *a* and *b* cannot share (2) without being identical. In other words, property (2) entails property (1) in the sense that nothing having the former can lack the latter.[9] Since every property entails itself, if all and only trivializing properties entail properties of identity, then we have a solution to our problem:

D1 *F* is a trivializing property $=_{\text{def.}}$ *F* entails a property of identity.

But D1 is wrong, if only because there are trivializing properties that do not entail any properties of identity.[10] Consider property (3):

(3) *being numerically distinct from a*.

Property (3) is the complement of property (1), the property of *being identical to a*. I shall call any complement of a property of identity a *property of difference*. Property (3) does not entail property (1) and, in general, properties of difference do not entail properties of identity. Yet property (3) can be used to show there is nothing indiscernible from *a*. For since indiscernibles are those that share exactly the same properties, indiscernibles are those that lack exactly the same properties, and so we can run the following argument:

(ix) *a* and *b* lack exactly the same properties.
(x) *a* lacks the property of *being numerically distinct from a*.
(xi) *b* lacks the property of *being numerically distinct from a*.

(xii) Therefore, $a = b$.[11]

Again, although this argument refers to *a* and *b* only, it is generalizable to apply to any two things supposed indiscernible, and thus properties of difference establish PII. But the thesis established is trivial, for the work in such arguments is again done by properties of identity, since lacking a property of difference is equivalent to having a property of identity. So properties of difference must also be excluded from the domain of quantification of any metaphysically serious version of PII.

Properties of difference are not the only trivializing properties that do not entail properties of identity. Consider property (4):

(4) *being numerically distinct from a or not being green*.

Property (4) is the complement of property (2). Since no two things can lack property (4), it can be used to deploy an argument similar to (ix)–(xii). Since that argument is generalizable in the relevant way, properties like (4) prove PII. But it should be clear that the thesis thereby established is trivial since all the work in such arguments is done by the property of *being identical to a*. For lacking the property of *being numerically distinct from a or not being green* is equivalent to having the property of *being identical to a and being green*, and no two things can share this one because they would share the non-shareable property of *being identical to a*. This suggests that trivializing properties are those having or lacking which entails having a property of identity. Since lacking a property is having its complement, we may attempt to define trivializing properties as follows:

D2 *F* is a trivializing property $=_{\text{def.}}$ (1) *F* entails a property of identity or (2) the complement of *F* entails a property of identity.

But D2 is wrong, for not all trivializing properties satisfy it. Consider properties (5) and (6):

(5) *being identical to a or being green*
(6) *being identical to a or not being green*.

Neither (5) nor its complement entail a property of identity. The same is true of (6) and its complement. Many things could and do have them and many things could and do lack them. Nevertheless they make PII true. For nothing can have both of them. But they make it trivially true, because having both of them entails having property (1). It is only thanks to this entailment that together they make PII true.

The same is true of the complements of properties (5) and (6), properties (7) and (8) respectively:

(7) *being numerically distinct from a and not being green*
(8) *being numerically distinct from a and being green*.

For although neither having nor lacking either (7) or (8) entails having a property of identity, lacking both of them does entail having (1), the property of *being* *identical to a*. In general, properties like (7) and (8) are such that lacking both of them entails having a property of identity. So such properties make PII trivially true.

One might decide to exclude from the property domain of PII only one property from such pairs like (5) and (6). But this would be arbitrary. There is no reason to count either of them as trivializing that does not apply to the other. Thus we should count both of them as trivializing properties. The same is true for (7) and (8). This suggests we replace D2 by D3 below:

D3 *F* is a trivializing property $=_{\text{def.}}$ (a) *F* or its complement entails a property of identity, or (b) *F* or its complement is the conjunct of a conjunctive property which entails a property of identity and the other conjunct(s) neither individually nor jointly entail a property of identity.

D3 rightly makes (5) and (6) trivializing properties, since they are the conjuncts of (5)&(6), and so they satisfy condition (b) in D3. The same is true for (7) and (8). But D3 has a crucial defect: it counts as trivializing some properties that are not. Consider properties (9) and (10) below:

(9) *being green*.
(10) *being (identical to a or not green) and being green*.[12]

The defect of D3 is that it makes (9) a trivializing property, for (9) satisfies the second disjunct in its *definiens*. In effect, (9) is a conjunct of (10), which entails a property of identity, but the other conjunct of (10), namely (6), entails no property of identity. But (9) is the property of *being green*. And the property of *being green* is a paradigmatic non-trivializing property.

I do not see how to solve this difficulty in terms of the notion of entailment. But even if there is such a satisfactory solution, trying to define trivializing properties as those that somehow or other entail properties of identity is marred from the beginning. For any definition that counts properties that entail properties of identity as trivializing properties assumes that no pure properties entail properties of identity. But suppose things had pure individual essences. Imagine, for the sake of example, that *being the greatest philosopher* was the individual essence of Plato. In that case the property of *being the greatest philosopher* would entail a property of identity, namely the property of *being identical to Plato*. If all things have pure individual essences, then PII is true, and it is true thanks to these pure individual essences. But a property like *being the greatest philosopher* does not trivialize PII. If what one proves is that numerically different things must have different pure individual essences, then one has established that every numerical difference goes accompanied by a qualitative difference – and this is no triviality.[13]

The point can perhaps be better appreciated by considering Leibniz's position. For Leibniz all things (individual substances) have qualitative essences, expressed by their complete concepts (and therefore more complex than anything like the property of *being the greatest philosopher*). These essences entail properties of identity and so they guarantee PII. But no doubt Leibniz's was not a trivial version of PII.

Whether or not things have pure individual essences is not the question here. The point is simply that, whether or not things have such essences, D3 is inadequate as a characterization of trivializing properties. First, if things have such essences, D3 is extensionally wrong. Second, even if things do not have such essences, a *formulation* of a non-trivial version of PII should not presuppose that things do not have such essences. Third, even if things do not have such essences, and all trivializing properties satisfy D3, the mere conceptual possibility of things having pure individual essences shows that D3 does not tell us why trivializing properties trivialize PII.

4

We need a different *kind* of definition of trivializing properties. Since it seems clear that trivializing properties are those related in some special way to properties of identity, the question is: how are trivializing properties related to properties of identity, if not by entailment? The intuitive answer is that they *contain* properties of identity. And this marks the difference between a trivializing property like (6), which contains the property of identity (1), and a non-trivializing property like (9), which does not contain any property of identity.

This approach looks promising. Indeed Katz proposes a solution in terms of a notion of *containment*. But what Katz does is to specify the class of trivializing *predicates*, which is not the same as specifying the class of trivializing *properties* if only because presumably, as Katz acknowledges, there are properties which no predicate expresses.

What is Katz's definition of trivializing predicates? He first introduces what he calls *basic identity properties* (BIPs) as follows: F is a BIP if and only if (1) it is possible that $(\exists x)(\mathrm{F}x)$ and (2) it is necessary that $(x)(y)(\mathrm{F}x \;\&\; \mathrm{F}y \supset x = y)$. Let us call predicates expressing BIPs *BIP-predicates*. Katz says that a BIP-predicate is a trivializing predicate and that a predicate 'P' contains a BIP-predicate essentially provided 'P' contains a BIP-predicate but is not logically equivalent to a predicate that does not: '*x* is numerically distinct from *a*' contains a BIP-predicate essentially, but '*x* is green and (identical to *a* or numerically distinct from *a*)' does not. Then Katz says that a predicate 'P' expresses a trivializing property if and only if 'P' contains a BIP-predicate essentially or 'P' may be defined in terms of some predicate that does. This, of course, makes such properties like (1)-(8) above, and also (10) and others, trivializing properties. Needless to say, this does not make a property like (9) a trivializing property.[14]

What are we to say about Katz's definition of trivializing predicates? First, I would say that it wrongly makes trivializing properties those expressed by *superlative predicates*, that is, predicates like 'being the tallest man', 'being the widest river' and so on. Such superlative properties are BIPs. But superlative properties in general do not trivialize PII. Superlative properties, being BIPs, cannot be shared, for example no two things could be the tallest man. But they do not serve to prove PII, since not everything must have one of them. The most one can do with them is to assert that if something has a superlative property then that thing has no indiscernibles, but this, of course, is far from asserting that nothing has indiscernibles, which is what PII requires. If they do not make PII true, superlative properties cannot make it *trivially* true.

But does perhaps everything have a superlative property relative to a certain reference class? I am not sure. In a world like that imagined by Max Black (1952, p. 156), consisting of only two indiscernible iron spheres, there seems to be no superlative property that either sphere has relative to any reference class. In any case, even if everything has a superlative property relative to some reference class, the problem with Katz's proposal is that it makes those properties that are superlative relative to no class (or relative to the most inclusive class) trivializing, which they are not.

In any case the difficulty with superlative properties can be met by just letting BIPs be properties of identity – in that case superlative properties will not counts as trivializing properties for they will not be expressed by trivializing predicates.

Second, Katz does not explain what it is for a predicate to contain another. So it is not clear which predicates contain BIP-predicates and which do not, and therefore it is not clear which predicates express trivializing properties and which do not. For although it may be intuitively clear that the predicate 'is green' does not contain any BIP-predicate, intuition suggests that the predicate 'thinks about *a*' and 'is one metre apart from *a*' contain the BIP-predicate 'is identical to *a*'. But these predicates do not express trivializing properties, since properties like *thinking about a* and *being one metre apart from a* can be shared and they do not make PII true unless conjoined with properties like *being identical to a or not thinking about a* or *being identical to a or not being one metre apart from a*. But it is these latter properties that trivialize PII.

Third, it is possible to define trivializing *properties*, rather than predicates, since all we need is a precise notion of *property* containment. For instance, one may introduce a notion of property containment via some stipulations like the following:

Every property contains itself.

Every property that is a function of other properties contains those properties (i.e. a conjunctive property contains its conjuncts; a disjunctive property contains its disjuncts; a negative property contains its negated property).

The relation of property containment is transitive.

Then, following Katz, we say that a property *F* contains a property of identity essentially provided *F* contains a property of identity but is not logically equivalent to a property that does not. Then, to avoid the problem of superlative properties, we define trivializing properties in terms of their containment of *properties of identity*, rather than BIPs:

D4 *F* is a trivializing property $=_{\text{def.}}$ *F* contains a property of identity essentially.

D4 rightly counts properties (1)–(8) and (10) as trivializing properties. Furthermore, thanks to the precise specification of the containment relation, it rightly excludes properties like *thinking about a* and *being one metre apart from a* from the class of trivializing properties.

But D4 has several problems. First, it does not seem to count as trivializing the property of *being a member of {a}*. For *being a member of {a}* does not seem to be a property of identity, nor the complement of a property of identity, nor a conjunctive or disjunctive property having a property of identity as one of its conjuncts or disjuncts. But *being a member of {a}* is a trivializing property. For one could argue for PII thus: If *a* and *b* have all their properties in common, then since *a* has the property of *being a member of {a}*, *b* has this property as well; but since whatever is a member of $\{a\}$ is identical to *a*, $a = b$. But clearly what is doing the work here is the property of *being identical to a*, which must be had by whatever is a member of $\{a\}$.

Being a member of {a} seems to contain the property of *being identical to a* in the sense that it is a relational property whose *relatum* ($\{a\}$) is specified in terms that depend on the identity of *a* and so, in that sense, on the property of *being identical to a*. But if we redefine containment so as to make the property of *being a member of {a}* contain the property of *being identical to a*, then we should make sure we avoid making *thinking about a* or, even more to the point, *being the only lover of a*, contain a property of identity.

But even if this can be done, a definition of trivializing properties in terms of a notion of property containment will still be lacking, even if extensionally correct. The problem with such a definition is that it does not explain why trivializing properties are trivializing properties. Why should properties containing properties of identity trivialize PII? It is not evident why this should be the case. Furthermore, it is clear that merely containing a property of identity is not what makes a property trivializing, since there are properties, like *being green and (being identical to a or being numerically distinct from a)*, which contain properties of identity but do not trivialize PII. This is why D4 does not define trivializing properties purely in terms of containment, since the explanation of what it is for a property to contain a property of identity essentially makes reference not only to the properties it contains but also to the properties it is logically equivalent to. But the relation of equivalence is not a containment relation.[15]

It may be claimed that this lack of purity is not a symptom of explanatory deficiency. Why should it matter that the definition defines trivializing properties purely in terms of property containment? Even if it does not define them purely in terms of property containment, that does not show that D4 fails to provide an explanation of why trivializing properties are trivializing. Perhaps what explains why they are trivializing properties is that they contain a property of identity *and* are equivalent only to properties that do.

But why should such properties be trivializing properties? It is not clear why this should be the case. Furthermore, there are reasons to doubt that this is what explains why trivializing properties are trivializing. Consider property (5). It contains a property of identity and is not logically equivalent to a property that does not. But this does not show it is a trivializing property – after all, property (5) can be shared and so it does not suffice to establish PII. The same applies to (6).

Someone may say that even if (5) and (6) can be individually shared, they are such that in virtue of what they contain the pair of them cannot be shared (i.e. no two things can have both of them). But this does not explain why they are trivializing properties, since (6) and (9) are also such that in virtue of what they contain the pair of them cannot be shared. But (9) is not a trivializing property.

It may be claimed that the relevant difference between (5) and (9) is that (5) contains a property of identity while (9) does not. But we saw earlier that there are properties that contain properties of identity but do not trivialize PII. Furthermore, saying that it is in virtue of containing a property of identity that (5) is prevented from being shared with (6) does not work. For it is no less by virtue of containing the property of *being green* than containing a property of identity that (5) is prevented from being shared with (6). But it is by virtue of containing the property of *being green* that (9) is prevented from being shared together with (6).

5

This second objection to D4 applies only if we are interested in more than mere extensional correctness. But extensional correctness cannot be the goal of our enquiry. For extensional correctness *per se* does not provide an explanation of why trivializing properties trivialize PII. So even if we hit an extensionally correct definition of trivializing properties, we may still have serious difficulties in recognizing it as a correct definition, for there may be properties such that it is not intuitively clear whether they trivialize PII. Furthermore, even if we knew that a certain definition is extensionally correct, it may not be philosophically illuminating. For we might know that without having answered the question of why trivializing properties trivialize PII.

But knowing what features make trivializing properties trivialize PII puts us in a position to define trivializing properties: trivializing properties are those that have the features in question. So the question I shall answer in this section is *What makes trivializing properties trivialize PII?*

Trivializing properties are those that can be used to establish a trivial version of PII. So in order to find out what makes trivializing properties trivialize PII, we first need to understand why the trivial version of PII is trivial. Once we know this it should be easy to see what makes trivializing properties trivialize PII, namely that they have those features that enable them to be used to establish a trivial version of PII.

The trivial version of PII is the version established by arguments like those considered in Sections 2–3. I shall focus on argument (i)–(iv), which by using properties of identity is a paradigmatic trivializing argument. So why do arguments like (i)–(iv) establish a trivial thesis? PII is meant to be a thesis about the connection between qualitative identity and numerical identity, namely that qualitative identity entails numerical identity: there cannot be qualitative identity without numerical identity. Equivalently, there cannot be *solo numero* difference: things that differ numerically must also differ qualitatively. But (i)–(iv) establishes that if *a* and *b* share all their properties, and therefore are qualitatively identical, they are numerically identical because they share a property of identity. But sharing a property of identity is being numerically identical. So what the argument shows is that qualitatively identical things that are numerically identical are numerically identical. This is trivial.

In other words, the argument establishes only that any numerically different things differ in their properties of identity, without requiring that they differ in any other property. But difference with respect to a property of identity *is* numerical difference, not qualitative difference. So all the argument establishes is that numerically distinct things are numerically distinct. This is trivial.

So (i)–(iv) establishes a trivial version of PII because it only establishes the numerical difference of numerically distinct things. What the argument establishes is only the letter of the principle when formulated using unrestricted property quantifiers – *that no two things can share all their properties* – but it does not establish the spirit of the principle – that *there cannot be qualitative identity without numerical identity*.

What features of properties of identity account for this? That differing with respect to them is differing numerically. For since differing qualitatively is more than differing numerically, simply establishing a difference with respect to properties of identity establishes only a numerical difference, not a qualitative difference.

I am not saying that properties of identity are trivializing because differing with respect to them entails no more than a numerical difference. I am saying that they are trivializing because differing with respect to them *is* differing numerically. This is so even if differing with respect to properties of identity entails a qualitative difference. In that case one still cannot establish a qualitative difference by simply establishing a difference with respect to properties of identity: one needs to invoke the fact that a difference with respect to properties of identity entails a qualitative difference.

This also applies to properties of difference. They are the complements of properties of identity. So having a property of difference is lacking a property of identity and lacking a property of difference is having a property of identity. So differing with respect to properties of difference is differing with respect to a property of identity and so differing with respect to them is differing numerically.

Thus establishing a difference with respect to a property of difference only establishes a numerical difference, not a qualitative difference. So properties of difference are trivializing properties.

There are two things to distinguish. One is what the trivializing character of properties of identity and difference consists in; the other is what makes them have that character. The trivializing character of properties of identity and difference consists in that merely establishing a difference with respect to them only establishes a numerical difference between the things in question. What makes properties of identity and difference have that character is that being numerically different *is* differing with respect to those properties.

The trivializing character is common to all and only trivializing properties. Every property such that merely establishing a difference with respect to it only establishes that the things in question are numerically different is a trivializing property. Such a property can be used to establish a trivial version of PII and so it is a trivializing property. And every property such that merely establishing a difference with respect to it establishes more than a numerical difference is such that establishing a difference with respect to it establishes a qualitative difference. So, since establishing a difference with respect to it cannot be used to establish a trivial version of PII, such a property is not trivializing.

But although the trivializing character is common to all and only trivializing properties, only in the case of properties of identity and difference what accounts for their trivializing character is that differing with respect to them is differing numerically.[16]

So how can a property *F* be trivializing without being a property of identity or difference? Even if differing with respect to *F* is not differing numerically, differing with respect to *F may* consist in differing numerically. If differing with respect to F may consist in differing numerically, merely establishing a difference with respect to *F* only establishes a numerical difference. So if differing with respect to F may consist in differing numerically, *F* is a trivializing property.

Differing with respect to *F* may consist in differing numerically if and only if differing with respect to *F* may consist in differing with respect to a property of identity or property of difference. So properties such that differing with respect to them may consist in differing with respect to a property of identity or difference are trivializing properties.

How can a property be such that differing with respect to it may consist in differing with respect to a property of identity or difference? Consider conjunctive properties. A conjunctive property is such that having or lacking it is having or lacking other properties. So differing with respect to *F&G* is simply differing with respect to *F* or *G* or both. So differing with respect to *F&G* may consist in differing with respect to *F*.

So some conjunctive properties containing properties of identity are such that a difference with respect to them may simply consist in a difference with respect to a property of identity. Consider the property of *being identical to a and being green*. Differing with respect to it consists in differing with respect to either of its conjuncts. So differing with respect to it may consist in differing with respect to the property of *being identical to a*. And so differing with respect to the property of *being identical to a and being green* may simply be differing numerically. The same applies to conjunctive properties containing properties of difference, like the property of *being numerically distinct from a and being green*. The same is true of disjunctive properties like *being numerically distinct from a or not being green* and *being identical to a or being green.* This is why such conjunctive and disjunctive properties are trivializing properties.

It should be clear now *why* properties like *being green*, *being square* and *being hot* are not trivializing properties. Differing with respect to them must consist in more than simply differing numerically: it must consist in differing with respect to colour, shape and temperature. For similar reasons impure properties like *being father of a*, *loving b*, *being close to c* and *being in the same place as d* are not trivializing properties. Differing with respect to these properties must be more than differing with respect to a property of identity or difference: it must be differing with respect to fathering *a*, loving *b*, being close to *c*, and being in the same place as *d*. Let *e* be the father of *a*. Even if origin is essential, and so *e* cannot fail to be the father of *a* provided *a* exists, there is more to being the father of *a* than being such that *a* exists and being identical to *e*. The extra is all that is involved in fathering *a*. Similarly in the other cases.[17]

This also explains why superlative properties are not trivializing properties. Differing with respect to *being the tallest man* is more than differing with respect to a property of identity or difference: it is also differing with respect to height from other men. It also makes clear why there are some complex properties containing properties of identity that are *not* trivializing properties, namely those properties that contain properties of identity but are logically equivalent to properties that are not trivializing, like the conjunctive property of *being green and (being identical to a or being numerically distinct from a)*. Properties like these are such that differing with respect to them must consist in differing with respect to a non-trivializing property, in this case the property of *being green*. Thus establishing a difference with respect to those complex properties will be establishing more than a numerical difference. Therefore such complex properties are not trivializing properties, in spite of containing properties of identity.[18]

6

So far, so good. But how about the property of *being a member of {a}*? This is a trivializing property but it does not appear to satisfy our characterization of trivializing properties. For even if differing with respect to it will require differing with respect to *being identical to a*, it seems to require differing with respect to being a member of {*a*}, and so it seems that a difference with respect to it cannot simply consist in a difference with respect to the property of *being identical to a*.

Of course *a* is a member of {*a*} in virtue of being identical to *a* – that is, it has the property of *being a member of {a}* in virtue of having the property of *being identical to a*. But this cannot be what makes the property of *being a member of {a}* trivializing. For that one property is had in virtue of another only means that there is a particular relation between the two – the *in virtue of* relation.[19] It does not mean that differing with respect to one of those properties consists in differing with respect to the other.

But what if it were the case that all of the properties of a thing were had in virtue of being *that* thing? In that case *a* would have all of its properties simply in virtue of being *a*. Perhaps the world is like that. Perhaps things cannot share all their properties because they have their properties in virtue of being the things they are. Or perhaps things cannot share all their properties because every thing has a qualitative property that is necessarily peculiar to it in virtue of being the thing it is. In either case PII would be true but it would be non-trivially true. For even if things were qualitatively different in virtue of being numerically different, differing qualitatively would still be more than differing numerically.[20]

But that any thing numerically different from *a* must also differ from *a* with respect to being a member of {*a*} is a trivial fact. However mysterious the singleton membership relation is, it appears that differing with respect to being a member of a singleton is no more than differing numerically. How can this be?

This is because, if sets exist, the identity of the members fixes what sets they belong to. And this is, in turn, because given a set S with certain things as members, there is no more to being a member of S than being one of those things. So, given {*a*}, there is no more to being a member of {*a*} than being *a*, that is, being identical to *a*. Thus the property of *being a member of {a}* is the property of *being such that {a} exists and being identical to a*.

It is frequently asserted that *a* belongs to {*a*} in virtue of being *a* rather than being *a* in virtue of belonging to {*a*}. My proposed account of the property of *being* *a member of {a}* nicely explains why this is so: being a member of {*a*} consists in satisfying two conditions, one of which is being identical to *a*, but being identical to *a* does not consist in being a member of {*a*}. This also goes some way to dispel the mystery associated with the singleton membership relation. According to David Lewis, the relation of singleton to member holds in virtue of qualities or external relations of which we have no conception whatsoever. That is, we do not clearly understand what it is for a singleton to have a member (Lewis, 1991, p. 35). If my account of the property of *being a member of {a}* is right, then we have a conception of the relations in virtue of which a thing is a member of a singleton: these are existence and identity.

But there is a sense in which Lewis is right that the singleton membership relation is mysterious. Singletons are atoms and the connection with their members is primitive and thereby in some sense mysterious and opaque. So we do not know in virtue of what a certain singleton has a certain thing as its member. That is, we do not know in virtue of what the property of *being a member of {a}* is the property of *being such that {a} exists and being identical to a* rather than the property of *being such that {a} exists and being identical to b*. After all, if {*a*} exists, *b* has the property of *being such that {a} exists and being identical to b*, but this does not make it a member of {*a*}. But this is a mystery that we should expect. For there is nothing in virtue of what {*a*} is the singleton of *a* as opposed to the singleton of *b*: it just is. So there is nothing in virtue of what the property of *being a member of {a}* is the property of *being such that {a} exists and identical to a* rather than the property of *being such that {a} exists and identical to b*: it just is one rather than the other.

Nothing in my account of the property of *being a member of {a}* helps with this. All my account says is what the property of *being a member of {a}* consists in, and makes the property of *being identical to a* part of that property. So, given that being a member of {*a*} consists partly in being identical to *a*, my account makes clear why *a* is a member of {*a*} in virtue of being *a* rather than being *a* in virtue of being a member of {*a*}. But nothing in my account explains why being a member of {*a*} consists partly in being identical to *a* rather than being identical to *b*. To understand this we should, I think, know in virtue of what a singleton has its members. But there is nothing in virtue of which a singleton has its members, so that my account does not explain this should not be seen as a problem for it.[21]

It may be thought that a problem for my account is that it does not make clear why the singleton membership relation has the formal features it has, for example irreflexivity, asymmetry, intransitivity. But there is no reason why an account of what the property of *being a member of {a}* consists in should make clear why singleton membership has those formal features. This is not an account of singleton membership in general: it is an account of what it is for a thing to be a member of its singleton. What matters is simply that my account be compatible with those formal features of the singleton membership relation, and it is.

The property of *being such that {a} exists and being identical to a* is a conjunctive property one of whose conjuncts is the property of *being identical to a*, and so differing with respect to *being a member of {a}* may consist in differing with respect to the property of *being identical to a*. Thus differing with respect to *being a member of {a}* may be differing numerically. Even more, since whenever two things differ with respect to being a member of $\{a\}$ both of them are such that $\{a\}$ exists, differing with respect to *being a member of {a}* must, and therefore does, consist in differing numerically.

So there are properties such that differing with respect to them consists in differing numerically and there are properties such that differing with respect to them may consist in differing numerically. In both cases such properties can be used to establish a trivial version of PII, since establishing a difference with respect to them establishes no more than numerical difference. These are the trivializing properties.

But it will be impossible to establish a trivial version of PII by establishing a difference with respect to a property such that differing with respect to it must consist in more than differing with respect to a property of identity, and so it must consist in more than differing numerically. These are the non-trivializing properties.

7

We can now define trivializing properties as follows:

D5 *F* is a trivializing property $=_{\text{def.}}$ Differing with respect to *F* is or may be differing numerically.

D5 is an intensional definition of trivializing properties: it purports to tell us what being a trivializing property consists in rather than merely specifying the class of trivializing properties. Since it tells us what it is *to be* a trivializing property, D5 is philosophically illuminating in a way in which a mere specification of the class of trivializing properties is not.

By saying what trivializing properties are, D5 specifies a certain class of properties as the class of trivializing properties. D5 is right in this respect to the extent that the class it specifies includes the properties of identity and all the other trivializing properties we have considered. But is D5 extensionally correct? Do all and only trivializing properties satisfy D5? Yes. For, as I have argued, D5 is intensionally correct. So it is extensionally correct.

We now know what trivializing properties are. So we know what properties should not be quantified over in order not to trivialize PII. Since not all impure properties are trivializing properties, it should now be clear that one can quantify over some impure properties without trivializing it and so that there are at least three non-trivial versions of PII: PII1, PII2 and PII2.5.[22]

Notes

1 Often, but not always. In *Individuals* Strawson calls them *universals-cum-particulars* (1959, p. 137).

2 No doubt my understanding of the phrase 'qualitative difference' is idiosyncratic, since normally only pure properties would be taken to make a qualitative difference. But I have found no better phrase to express what I want to express, namely that difference

which is not merely numerical difference, that is, which is not a *solo numero* difference. As I said, what I mean by qualitative difference is any difference that is not merely a numerical difference. But differing with respect to some impure properties, for instance differing with respect to the impure property of *being two miles from the Eiffel Tower*, is more than differing merely numerically. In what follows, the reader should bear in mind that in this chapter qualitative difference is neither synonymous nor coextensive with difference with respect to pure properties.

3 If one takes second-order variables to range over sets, then the principle in the text is merely a set-theoretical analogue of PII, rather than PII itself. One can also express PII as a first-order principle, for example $(x)(y)(z)[(x \text{ has } z \equiv y \text{ has } z) \supset (x = y)]$, where '$x$ has z' is true if and only if z is a property of x. In this chapter I shall stick to the canonical second-order formulation in the text. But whether the principle must be formulated as a first- or second-order principle is not relevant for the purposes of the present chapter.

4 There is another dimension in which I would disagree with Strawson, since in that passage he also makes the necessitated version of PII the only one worth discussing (Strawson, 1959, p. 120). But I cannot discuss this issue here.

5 It is important to be clear what properties of identity are. Having recourse to the property abstraction λ-operator makes that clear. The λ-operator binds a variable from a first-order open sentence to designate the property expressed by that open sentence. Thus properties of identity are those that in their λ-expression the open sentence from which the λ-operator binds a variable consists only of an identity sign flanked by an individual variable and an individual constant. So properties like *being identical to a* and *being identical to b* are properties of identity because their λ-expressions are, respectively, '$(\lambda x)(x = a)$' and '$(\lambda x)(x = b)$'. But the properties of *being identical to something* or *being self-identical* are not properties of identity. Their λ-expressions, '$(\lambda x)(x = y)$' and '$(\lambda x)(x = x)$', do not satisfy our characterization.

6 That

then it is F. This is unexceptionable, and should not be confused with the more controversial principle that if a thing is F then it has property F.

7 Informal versions of (i)–(iv) appear in Brody (1980), p. 9, Katz (1983), p. 37, Legenhausen (1989), p. 626, and Whitehead and Russell (1925), p. 57.

8 The trivializing nature of properties of identity is recognized in, among others, Adams (1979), p. 11, Ayer (1954), p. 29, Black (1952), p. 155, Katz (1983), pp. 37–8, Legenhausen (1989), p. 626 and O'Connor (1954), pp. 103–4.

9 This is the sense in which I shall conceive of property entailment in this chapter and this is the usual way of conceiving property entailment. See, for instance, Carnap (1988), p. 17, Katz (1983), p. 44 and Lewis (1983), p. 199.

10 It may be thought that the problem with D1 is that, given that it is necessary that everything is self-identical and that if any thing is self-identical then that thing has a property of identity, all properties entail properties of identity and so, according to D1, all properties trivialize PII. This would show D1 to be wrong, since some properties, like *being green*, do not trivialize PII. But this is not a problem for D1. For even if it is necessary that everything is self-identical and that if any thing is self-identical then that thing has a property of identity, it does not follow that all properties entail properties of identity. For all that follows from this is that every property F is such that it is necessary that if any thing has F, then that thing has *some* property of identity. But it does not follow from it that for every property F there is a property of identity F^* such that it is necessary that if any thing has F, then that thing has F^*. And only in this latter case does every property F entail a property of identity.

11 I have found no version of (ix)–(xii) in the literature, but something similar to it is in

Katz (1983), p. 40. There are also arguments that derive discernibility from numerical difference, like the following two, which I shall deploy using the λ-operator:

(i_2)	$a \neq b$	(ix_3)	$a \neq b$
(ii_2)	$(\lambda x)(x = a)(a)$	(x_3)	$\neg(\lambda x)(x \neq a)(a)$
(iii_2)	$\neg(\lambda x)(x = a)(b)$	(xi_3)	$(\lambda x)(x \neq a)(b)$
(iv_2)	$(\exists F)(Fa \; \& \; \neg Fb)$	(xii_3)	$(\exists F)(\neg Fa \; \& \; Fb)$

These arguments are contrapositive versions of (i)–(iv) and (ix)–(xii). Informal versions of (i_2)–(iv_2) and (ix_2)–(xii_2), or of mixtures of them, appear in Adams (1979), p. 11, Ayer (1954), p. 29, Bergmann (1953), p. 77, Black (1952), p. XX, Broad (1933), pp. 172–3, Greenlee (1968), p. 760, McTaggart (1921), p. 96, O'Connor (1954), pp. 103–4, Odegard (1964), p. 204 and Russell (1959), p. 115. The arguments (i)–(iv), (i_2)–(iv_2), (ix)–(xii) and (ix_2)–(xii_2) are clearly related to each other, but they have never been clearly differentiated and, sometimes, they are thought of as a single argument (e.g. Adams, 1979, p. 11, footnote 11, seems to confound several of them).

12 Let us resort to λ-formulations to make clear what property (10) is. Where 'Gx' stands for '*x* is green', (10) is the following property: $(\lambda x)((x = a \vee \neg Gx) \; \& \; Gx)$.

13 To make the point of this paragraph I do not need to invoke *pure* individual essences. Invoking the possibility of impure individual essences that are not trivializing would have been good enough. But the point is more forcefully made by invoking the possibility of pure individual essences.

14 I have altered Katz's terminology. He calls BIP-predicates *identity-predicates* and he calls trivializing properties *identity-properties*. So what he actually says is that 'a predicate, P, expresses an identity-property if and only if P contains an identity predicate essentially or may be defined in terms of some predicate that does' (Katz, 1983, p. 41). I changed Katz's terminology because of its potential for confusion. 'Identity-predicate' suggests a predicate that expresses a property of identity, but as we shall see below not all BIPs are properties of identity. 'Identity-property' suggests a property of identity, but as we have seen not all trivializing properties are properties of identity. Katz was of course aware that not all trivializing properties are properties of identity, and although he did not realize that all BIPs are properties of identity, nothing here should be taken to imply that Katz used his terminology confusingly or confusedly. He used his terminology clearly and consistently, but nevertheless his terminology has potential for confusion.

15 Katz's definition of trivializing predicates has the same feature of not being purely in terms of containment.

16 Note that the trivializing character of properties of identity and difference consists in that merely establishing a difference with respect to them only establishes that the things in question are numerically different – not that the things in question differ only with respect to properties of identity and difference. The latter is not true. For instance, if *a* and *b* differ with respect to properties of identity and difference, then they differ with respect to conjunctive properties having their properties of identity and difference as conjuncts.

17 An interesting case is the property of *having all parts in common with a*. This is trivializing because among the parts of *a* is its improper part, namely *a* itself. So this property leaves open the possibility that *a* and *b* differ only with respect to their improper parts, in which case they differ only numerically. Some people think that no two things can share all their proper parts. If it is true that no two things can share all their parts, then there is a non-trivial version of PII that is true. But this does not make the property of *having all proper parts in common with a* trivializing. Differing with

respect to such a property is differing more than merely numerically. Furthermore, the insight that no two things can share all their parts, if indeed it is true, is not a trivial but a substantive metaphysical insight.

18 There are other properties containing properties of identity that are not trivializing, and in this case the explanation of why they are not trivializing must be different. Consider the property of *being identical to a or being numerically distinct from a*. This property is not trivializing because it must be shared by everything and so, since no two things can differ with respect to it, not even a trivial version of PII can be proved by its means. The property of *being identical to a and being numerically distinct from a* is also such that it cannot be shared, though this time because nothing can have it.

19 This relation between the properties in question is different from their being necessarily coextensive. For, assuming that necessarily *a* exists if and only if so does {*a*}, the necessary coextension of the properties is symmetrical: nothing can have one of the properties of *being identical to a* and *being a member of {a}* without having the other – but although *a* has the property of *being a member of {a}* in virtue of having the property of *being identical to a*, it does not have the latter in virtue of having the former.

20 It is important to emphasize that the non-triviality of PII in these situations would not be due to our ignorance that things have all their properties, or some properties necessarily peculiar to them, in virtue of their identity. Even if we discovered this, through metaphysical argument or any other means, it would be a discovery of a non-trivial fact.

21 Here I go beyond Lewis, who seems to think that there may be something in virtue of why singletons have their members.

22 I am grateful for comments to the following: Bill Brewer, David Charles, John Divers, Bernard Katz, Robin Le Poidevin, Hugh Mellor, Alex Oliver, Oliver Pooley, Ralph Wedgwood, Tim Williamson and, especially, Arnie Koslow.

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